Complementos de Matemática CMAT

Ficha 1 - Funções vetoriais

(1) $\lim_{t\to 1}\vec{f}(t) = \lim_{t\to 1}(t^2-2)\vec{i} + 2t\vec{j} + e^{t-1}\vec{k} = -\vec{i} + 2\vec{j} + \vec{k}$

(2) a) $\vec{f}'(t) = \left(\frac{1}{t}\right)'\vec{i} + \left(\frac{\ln(t)}{t}\right)'\vec{j} + (e^{-2t})'\vec{k} = -\frac{1}{t^2}\vec{i} + \frac{1-\ln(t)}{t^2}\vec{j} - 2e^{-2t}\vec{k}$

b) $\int_1^3 \vec{f}(t)\,dt = \vec{i}\int_1^3 \frac{1}{t}\,dt + \vec{j}\int_1^3 \frac{\ln(t)}{t}\,dt + \vec{k}\int_1^3 e^{-2t}\,dt =$ $\begin{cases} u = \ln(t) \\ du = \frac{1}{t}dt \end{cases}$

$= \vec{i}\left[\ln(t)\right]_1^3 + \vec{j}\int_1^3 u\,du + \vec{k}\left[\frac{e^{-2t}}{-2}\right]_1^3 = \vec{i}(\ln(3)-\ln(1)) + \vec{j}\left[\frac{\ln^2(t)}{2}\right]_1^3 + \vec{k}\left(\frac{e^{-6}}{-2} + \frac{e^{-2}}{2}\right) =$

$= \ln(3)\vec{i} + \frac{\ln^2(3)}{2}\vec{j} + \frac{e^{-2}-e^{-6}}{2}\vec{k}$

$\begin{cases} u = 1+e^t \\ du = e^t\,dt \end{cases}$

(3) a) $\int_0^1 \left(\frac{e^t}{1+e^t}\vec{i} + \frac{1}{1+e^t}\vec{j}\right)dt = \vec{i}\int_0^1 \frac{e^t}{1+e^t}\,dt + \vec{j}\int_0^1 \frac{1}{1+e^t}\,dt =$

$= \vec{i}\int_0^1 \frac{1}{u}\,du + \vec{j}\int_0^1 \frac{1+e^t-e^t}{1+e^t}\,dt = \left[\ln(u)\right]_0^1\vec{i} + \vec{j}\left[\int_0^1 \frac{1+e^t}{1+e^t}\,dt - \int_0^1 \frac{e^t}{1+e^t}\,dt\right] =$

$= \left[\ln(1+e^t)\right]_0^1\vec{i} + \vec{j}\left[\int_0^1 1\,dt - \int_0^1 \frac{1}{u}\,du\right] = \left[\ln(1+e)-\ln(2)\right]\vec{i} + \vec{j}\left(\left[t\right]_0^1 - \left[\ln(1+e^t)\right]_0^1\right) =$

$= \ln\left(\frac{1+e}{2}\right)\vec{i} + \vec{j}\left(1 - \left[\ln(1+e)-\ln(2)\right]\right) = \ln\left(\frac{1+e}{2}\right)\vec{i} + \left[1 - \ln\left(\frac{1+e}{2}\right)\right]\vec{j}$

b) $\int_0^1 \left(te^t\vec{i} + t^2e^t\vec{j} + te^{-t}\vec{k}\right)dt = \vec{i}\int_0^1 te^t\,dt + \vec{j}\int_0^1 t^2e^t\,dt + \vec{k}\int_0^1 te^{-t}\,dt =$

$\begin{cases} u = t \\ du = 1dt \end{cases}$ $\begin{cases} v = e^t \\ dv = e^t\,dt \end{cases}$ $\begin{cases} s = t^2 \\ ds = 2t\,dt \end{cases}$ $\begin{cases} w = e^t \\ dw = e^t\,dt \end{cases}$ $\begin{cases} x = t \\ dx = 1dt \end{cases}$ $\begin{cases} y = -e^{-t} \\ dy = e^{-t}\,dt \end{cases}$

$= \vec{i}\left[\left[te^t\right]_0^1 - \int_0^1 e^t\,dt\right] + \vec{j}\left[\left[t^2e^t\right]_0^1 - \int_0^1 2te^t\,dt\right] + \vec{k}\left[\left[-te^{-t}\right]_0^1 - \int_0^1 -e^{-t}\,dt\right] =$

$\begin{cases} g = 2t \\ dg = 2dt \end{cases}$ $\begin{cases} a = e^t \\ da = e^t\,dt \end{cases}$

$= \vec{i}\left[e - \left[e^t\right]_0^1\right] + \vec{j}\left[e - \left(\left[2te^t\right]_0^1 - 2\int_0^1 e^t\,dt\right)\right] + \vec{k}\left[-\frac{1}{e} + \left[-e^{-t}\right]_0^1\right] =$

$= \vec{i}\left(e-(e-1)\right) + \vec{j}\left[e - \left[2e - 2\left[e^t\right]_0^1\right]\right] + \vec{k}\left(-\frac{1}{e} + \left(-\frac{1}{e}+1\right)\right) = \vec{i} + (e-2)\vec{j} + \left(1-\frac{2}{e}\right)\vec{k}$

(4) Sabendo que $\begin{cases} \cosh(2t) = \frac{e^{2t}+e^{-2t}}{2} \\ \sinh(2t) = \frac{e^{2t}-e^{-2t}}{2} \end{cases}$; e $\vec{a}\cdot\vec{b} = a_xb_x + a_yb_y + a_zb_z$

$\begin{cases} u = t \\ du = 1dt \end{cases}$ $\begin{cases} v = \frac{e^{2t}}{2} \\ dv = e^{2t}\,dt \end{cases}$

$\vec{a}\cdot\vec{b} = 2\int_0^1 te^{2t}\,dt - 4\int_0^1 t\cosh(2t)\,dt + \int_0^1 2te^{-2t}\,dt =$

$= 2\int_0^1 te^{2t}\,dt - 4\int_0^1 \frac{te^{2t}}{2}\,dt - 4\int_0^1 \frac{te^{-2t}}{2}\,dt + 2\int_0^1 te^{-2t}\,dt =$

$= 2\int_0^1 te^{2t}\,dt - 2\int_0^1 te^{2t}\,dt - 2\int_0^1 te^{-2t}\,dt + 2\int_0^1 te^{-2t}\,dt = 0$

(5) Teorema: $\|\vec{a}\| = \text{constante} \Rightarrow \vec{a}\cdot\vec{a}' = 0$

$\|\vec{f}(t)\| = \sqrt{\left[\frac{2t}{1+t^2}\right]^2 + \left[\frac{1-t^2}{1+t^2}\right]^2} = \sqrt{\frac{4t^2}{(1+t^2)^2} + \frac{t^4-2t^2+1}{(1+t^2)^2}} = \sqrt{\frac{t^4+2t^2+1}{(1+t^2)^2}} = \sqrt{\frac{(1+t^2)^2}{(1+t^2)^2}} =$

$= \sqrt{1} = 1$ Como $\|\vec{f}(t)\|$ é constante, $\vec{f}(t)\cdot\vec{f}'(t) = 0$, pelo que estas funções vetoriais são ortogonais, logo é independente do parâmetro t.

6) $\vec{g}'(t) = \left(\vec{f}(t)\times\vec{f}'(t)\right)' = \cancel{\vec{f}'(t)\times\vec{f}'(t)}^{0} + \vec{f}(t)\times\vec{f}''(t) = \vec{f}(t)\times\vec{f}''(t)$

7) $\vec{g}'(t) = \left(\vec{f}(t)\cdot\vec{f}'(t)\times\vec{f}''(t)\right)' = \vec{f}'(t)\cdot\vec{f}'(t)\times\vec{f}''(t) + \vec{f}(t)\cdot\left(\vec{f}'(t)\times\vec{f}''(t)\right)' =$

$= \vec{f}'(t)\cdot\vec{f}'(t)\times\vec{f}''(t) + \vec{f}(t)\cdot\left[\cancel{\vec{f}''(t)\times\vec{f}''(t)}^{0} + \vec{f}'(t)\times\vec{f}'''(t)\right] \overset{(1)}{=}$

Como $\vec{f}'(t)\times\vec{f}''(t)$ resulta num vetor ortogonal a $\vec{f}'(t)$, então o produto escalar $\vec{f}'(t)\cdot\vec{f}'(t)\times\vec{f}''(t)$ é igual a 0.

$\overset{(1)}{=} \vec{f}(t)\cdot\vec{f}'(t)\times\vec{f}'''(t)$ c.q.p.

8) $t\vec{f}'(t) = \vec{f}(t) + t\vec{a} \Leftrightarrow \left(t\vec{f}'(t)\right)' = \left(\vec{f}(t) + t\vec{a}\right)' \Leftrightarrow \vec{f}'(t) + t\vec{f}''(t) = \vec{f}'(t) + \vec{a} \Leftrightarrow$

$\Leftrightarrow t\vec{f}''(t) = \vec{a} \Leftrightarrow \vec{f}''(t) = \vec{a}t^{-1}$ Logo, $\vec{f}''(1) = \vec{a}\times 1^{-1} = \vec{a}$

$\int \vec{f}''(t)\,dt = \vec{a}\int t^{-1}dt \Leftrightarrow \vec{f}'(t) = \vec{a}\left[\ln(t) + \vec{c}\right] = \vec{a}\ln(t) + \vec{c}$

$\vec{f}(t) = t\vec{f}'(t) - t\vec{a} \Leftrightarrow \vec{f}(t) = t\left[\vec{a}\ln(t) + \vec{c}\right] - t\vec{a}$ Como $\vec{f}(1) = 2\vec{a}$

$2\vec{a} = \vec{a}\cancel{\ln(1)}^{0} + \vec{c} - \vec{a} \Leftrightarrow \vec{c} = 3\vec{a}$ $\quad \vec{f}(t) = t(\ln(t) + 2)\vec{a}$

Logo, $\vec{f}(3) = 3(\ln(3) + 2)\vec{a}$.

9) $\vec{f}(t) = \sin(2t)\vec{a} + \cos(2t)\vec{b}$ $\quad \vec{f}'(t) = 2\cos(2t)\vec{a} - 2\sin(2t)\vec{b}$

$\vec{f}''(t) = -4\sin(2t)\vec{a} - 4\cos(2t)\vec{b} = -4\left(\sin(2t)\vec{a} + \cos(2t)\vec{b}\right) = -4\vec{f}(t)$

Como $\vec{f}''(t) = -4\vec{f}(t)$, conclui-se que os vetores $\vec{f}(t)$ e $\vec{f}''(t)$ são paralelos.

10) a) $\vec{r}(t) = (t, f(t)),\ t\in[a,b]$

b) $\vec{r}(\theta) = \left(f(\theta)\cos(\theta), f(\theta)\sin(\theta)\right),\ \theta\in[\alpha,\beta]$

11) $r: P + t\vec{PQ}$ $\quad P = (2,7,-1)$ $\quad \vec{PQ} = Q - P = (4,2,3) - (2,7,-1) = (2,-5,4)$

$r: (2,7,-1) + t(2,-5,4) \Leftrightarrow r: (2+2t, 7-5t, -1+4t),\ t\in\mathbb{R}$

$\vec{r}(t) = (2+2t, 7-5t, -1+4t),\ t\in[0,1]$

12) a) $\vec{r}(t) = (t, t^2),\ t\in[-1,3]$

b) $\vec{r}(t) = (-t, t^2),\ t\in[-3,1]$

c) $\vec{r}(\theta) = \left(2\cos(\theta), 2\sin(\theta)\right),\ \theta\in[0,2\pi]$

d) $x^2 + y^2 = 2x \Leftrightarrow x^2 - 2x + 1 - 1 + y^2 = 0 \Leftrightarrow (x-1)^2 + y^2 = 1$

$\vec{r}(\theta) = \left(\cos(\theta) + 1, \sin(\theta)\right),\ \theta\in[0,\pi]$

e) $\vec{r}(\theta) = \left(2\cos(\theta), -2\sin(\theta)\right),\ \theta\in[0,2\pi]$

13) a) $\begin{cases} x = 3t - 1 \\ y = 5 - 2t \end{cases} \Leftrightarrow \begin{cases} 2x = 6t - 2 \\ 3y = 15 - 6t \end{cases} \Leftrightarrow 3y + 2x = 13 \Leftrightarrow 2x + 3y = 13$

R: $2x + 3y = 13,\ x\in\mathbb{R}$

b) $\begin{cases} x = 2\cos(t) \\ y = 3\sin(t) \end{cases} \Leftrightarrow \begin{cases} x^2 = 4\cos^2(t) \\ y^2 = 9\sin^2(t) \end{cases} \Leftrightarrow \begin{cases} \frac{x^2}{4} = \cos^2(t) \\ \frac{y^2}{9} = \sin^2(t) \end{cases} \Leftrightarrow$

$\Leftrightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1 \, , \, x \in [-2, 2]$

c) $\begin{cases} x = \frac{1}{t} \\ y = \frac{1}{t^2} \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{1}{t^2} \\ y = \frac{1}{t^2} \end{cases} \Leftrightarrow x^2 - y = 0 \Leftrightarrow y = x^2 \, , \, x \in \left]\frac{1}{3}, +\infty\right[$

d) $\begin{cases} x = \tan(t) \\ y = \sec(t) \end{cases} \Leftrightarrow \begin{cases} x = \frac{\sin(t)}{\cos(t)} \\ y = \frac{1}{\cos(t)} \end{cases} \Leftrightarrow \begin{cases} x^2 = \frac{\sin^2(t)}{\cos^2(t)} \\ y^2 = \frac{1}{\cos^2(t)} \end{cases} \Leftrightarrow y^2 - x^2 = \frac{1 - \sin^2(t)}{\cos^2(t)} \Leftrightarrow$

$\Leftrightarrow y^2 - x^2 = \frac{\cos^2(t)}{\cos^2(t)} \Leftrightarrow y^2 = 1 + x^2 \Leftrightarrow y = \sqrt{1 + x^2} \, , \, x \in \mathbb{R}$

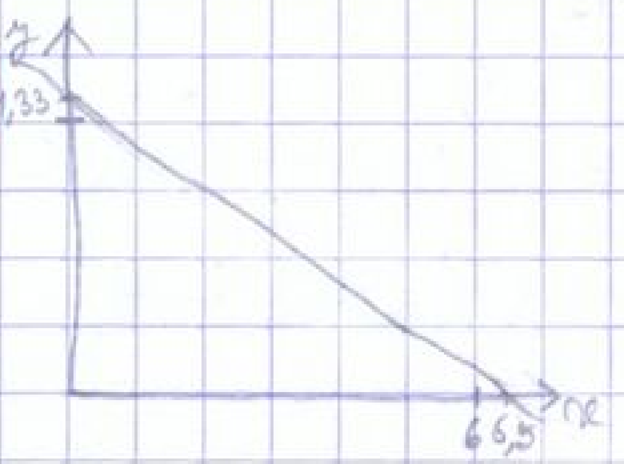
e) $\begin{cases} x = e^{2t} \\ y = e^{2t} - 1 \end{cases} \Leftrightarrow y - x = -1 \Leftrightarrow y = x - 1 \, , \, x \in \left]0, 1\right]$

f) $\begin{cases} x = 2\sin(t) \\ y = \cos(t) \end{cases} \Leftrightarrow \begin{cases} \left(\frac{x}{2}\right)^2 = \sin^2(t) \\ y^2 = \cos^2(t) \end{cases} \Leftrightarrow y^2 + \frac{x^2}{4} = 1 \Leftrightarrow y = \sqrt{1 - \frac{x^2}{4}} \Leftrightarrow$

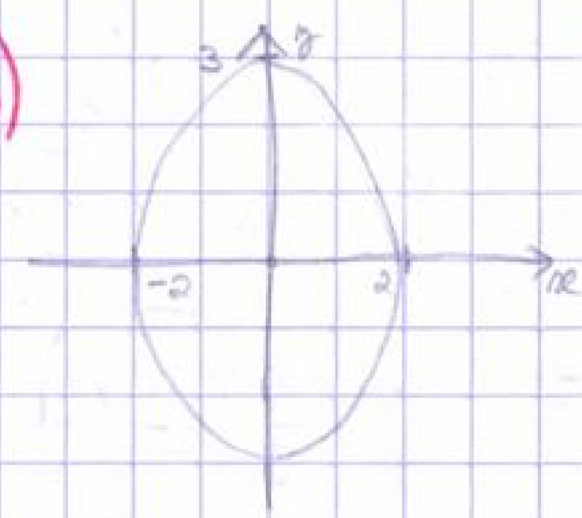
$\Leftrightarrow y = \frac{\sqrt{4 - x^2}}{2} \Leftrightarrow y = \frac{1}{2}\sqrt{4 - x^2} \, , \, x \in [0, 2]$

g) $\begin{cases} x = \sin(t) \\ y = 1 + \cos^2(t) \end{cases} \Leftrightarrow \begin{cases} x^2 = \sin^2(t) \\ y = 1 + \cos^2(t) \end{cases} \Leftrightarrow y + x^2 = 2 \Leftrightarrow y = 2 - x^2 \, ,$ $x \in [-1, 1]$

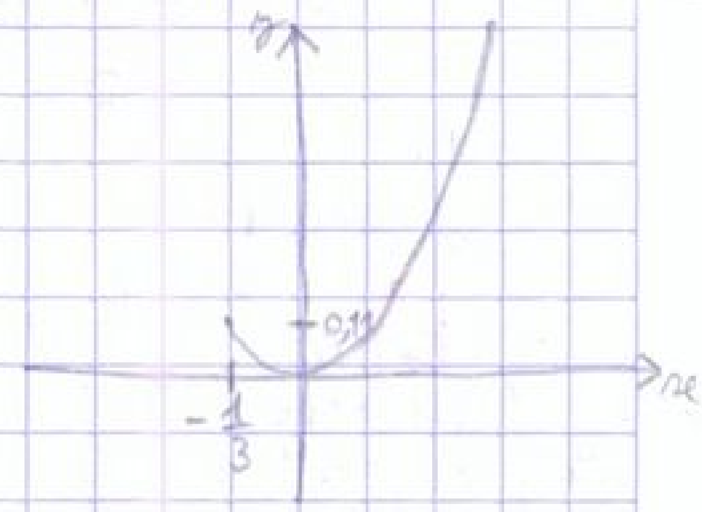
14 a)



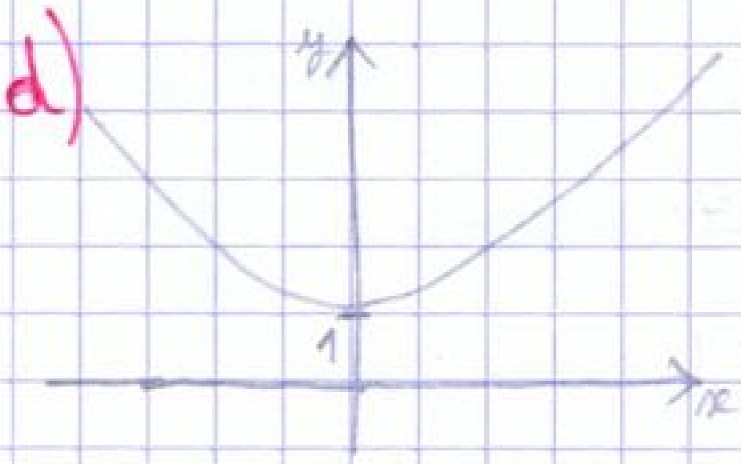
b)



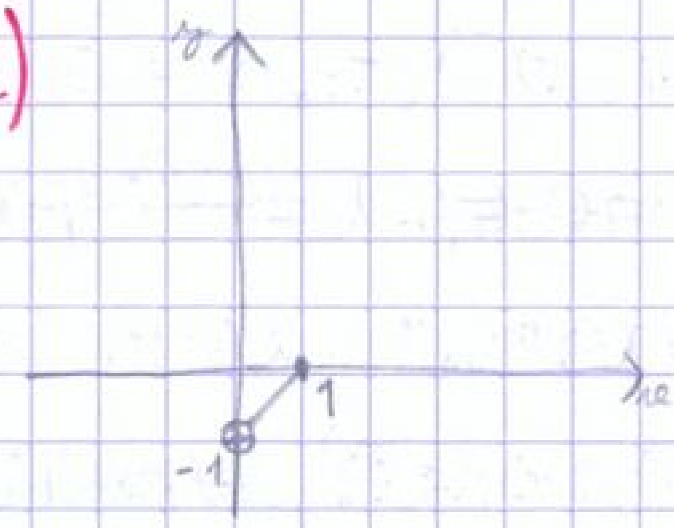
c)



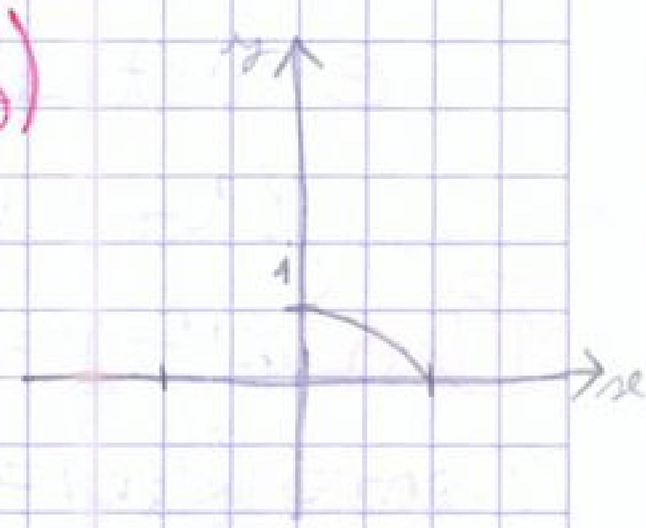
d)



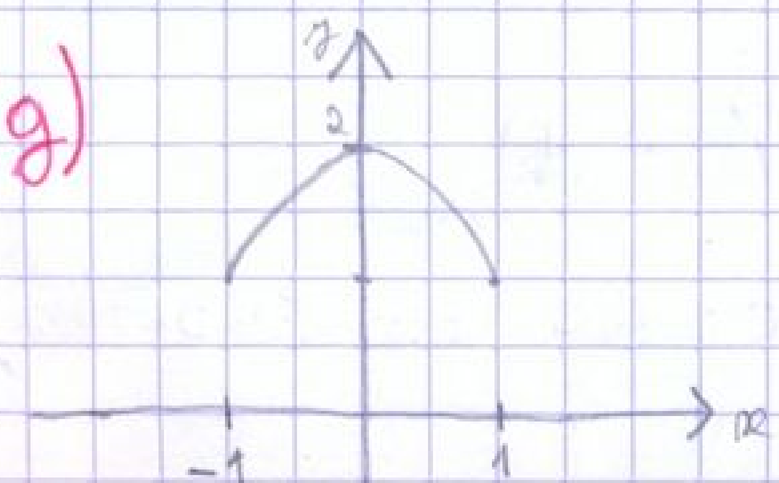
e)



f)



g)



15 a) $\vec{r}(\theta)=(\cos(\theta), \sin(\theta), 0), \theta\in[0,2\pi]$

b) $\vec{r}(t)=(t, t^2, t^3), t\in\mathbb{R}$

c) $\vec{r}(\theta)=(2\cos(\theta), 2\sin(\theta), e^{2\cos(\theta)}), \theta\in[0,2\pi]$ (≥ 0)?

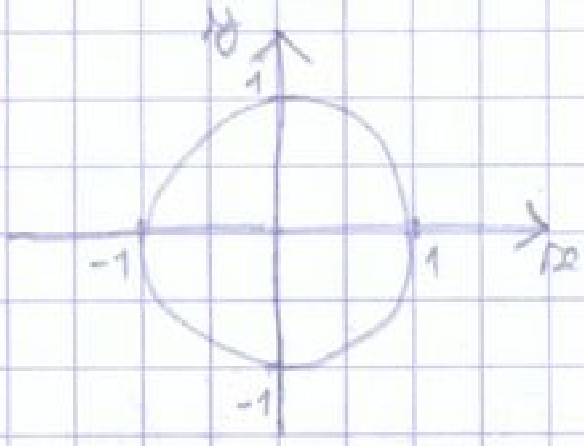
d) $4(x+1)^2+y^2=4 \Leftrightarrow (x+1)^2+\frac{y^2}{4}=1$

$\vec{r}(\theta)=(\cos(\theta)-1, 2\sin(\theta), 0), \theta\in[0,2\pi]$

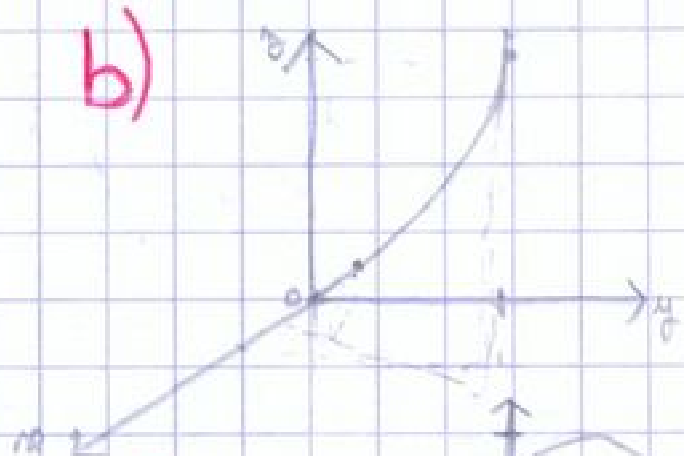
e) $x^2+y^2-2x-4y=-1 \Leftrightarrow x^2-2x+1-1+y^2-4y+4-4=-1 \Leftrightarrow$

$\Leftrightarrow (x-1)^2+(y-2)^2=4$ $\quad \vec{r}(\theta)=(2\cos(\theta)+1, 2\sin(\theta)+2, 0), \theta\in[0,2\pi]$

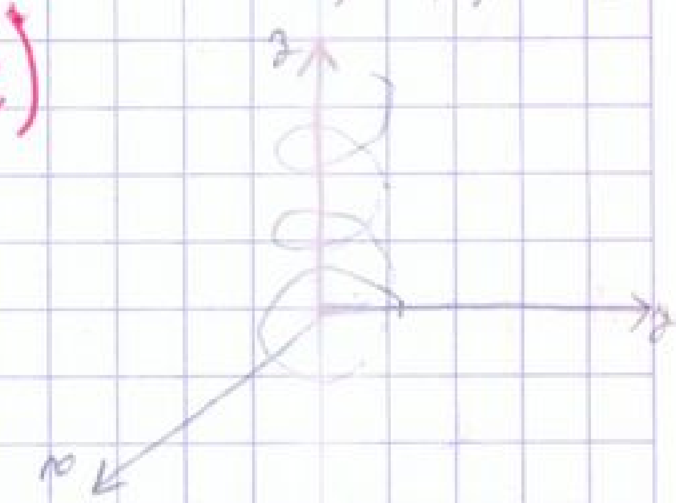
16 a)



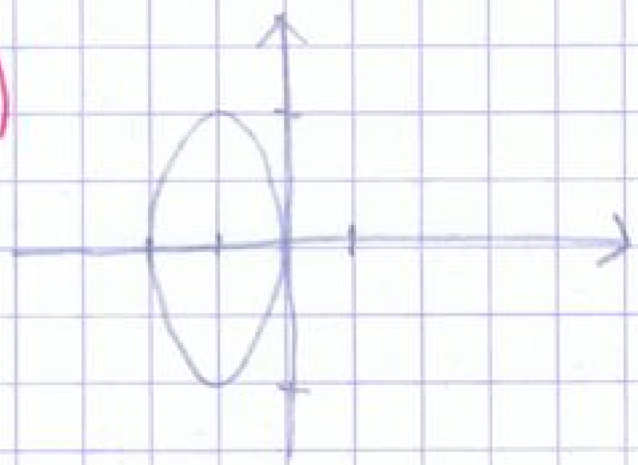
b)



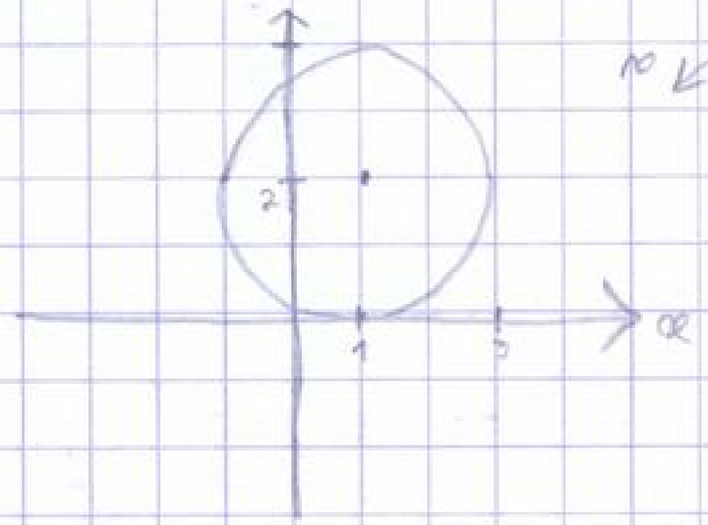
c)



d)



e)



17 $\vec{r}(t)=(e^{\alpha t}, 2e^{\alpha t}, 3e^{\alpha t}), t\in\mathbb{R}$

18 a) $\begin{cases} t=\frac{1}{2} \\ t^2=\frac{1}{4} \\ t^3=\frac{1}{8}\end{cases} \Leftrightarrow \begin{cases} t=\frac{1}{2} \\ t=\frac{1}{2} \\ t=\frac{1}{2}\end{cases}$ $\quad \vec{r}'(t)=(1, 2t, 3t^2), t\in[0,1]$

$\vec{r}'\left(\frac{1}{2}\right)=\left(1,1,\frac{3}{4}\right)$ $\quad r: \vec{r}(u)=\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)+u\left(1,1,\frac{3}{4}\right), u\in\mathbb{R}$.

b) $\vec{r}'(t)//\vec{a} \Rightarrow \vec{r}'(t)=\alpha\vec{a} \Leftrightarrow (1,2t,3t^2)=\alpha(4,4,3) \Leftrightarrow$

$\Leftrightarrow \begin{cases} 1=4\alpha \\ 2t=4\alpha \\ 3t^2=3\alpha\end{cases} \Leftrightarrow \begin{cases} \alpha=\frac{1}{4} \\ t=\frac{1}{2} \\ t=\frac{1}{2}\end{cases}$ $\quad \vec{r}\left(\frac{1}{2}\right)=\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$ R: O ponto $\left(\frac{1}{2},\frac{1}{4},\frac{1}{8}\right)$

c) $\vec{r}'(t)\perp\vec{a} \Rightarrow \vec{r}'(t)\cdot\vec{a}=0 \Leftrightarrow (1,2t,3t^2)\cdot(4,4,3)=0 \Leftrightarrow$

$\Leftrightarrow 4+8t+9t^2=0 \Leftrightarrow t=\frac{-8\pm\sqrt{64-144}}{18}$ ← impossível $\quad$ R: Não existem pontos nessas condições.

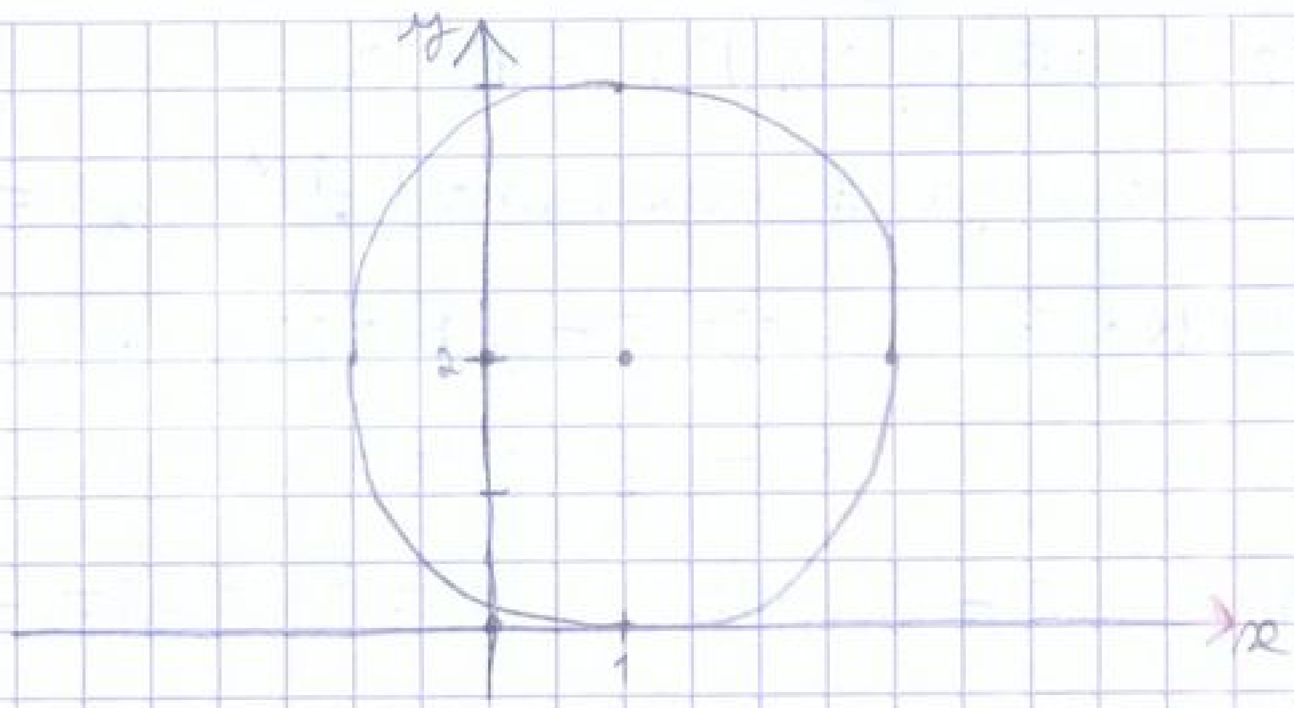
19 a) $\vec{f}'(t)=(3t^2, 5t^4), t\in\mathbb{R}$

Como $\vec{f}'(0)=(0,0,0)=\vec{0}$, conclui-se que a parametrização dada não é regular.

b) Considerando $u=t^3$, $\vec{g}(u)=(u, u^{\frac{5}{3}}), u\in\mathbb{R}$.

Como $\vec{g}'(u)=\left(1,\frac{5}{3}u^{\frac{2}{3}}\right)$ nunca se anula, em todos os pontos da curva, conclui-se que esta parametrização é regular.

20 a)



b) $2+2\sin(\theta)=0 \Leftrightarrow 2\sin(\theta)=-2 \Leftrightarrow \sin\theta=-1 \Leftrightarrow \theta=\frac{3\pi}{2}$

$\vec{f}\left(\frac{3\pi}{2}\right)=\left(1+2\cos\left(\frac{3\pi}{2}\right),0\right)=(1,0)$ $\quad \vec{f}'(\theta)=(-2\sin(\theta),2\cos(\theta)),\theta\in[0,2\pi]$

$\vec{f}'\left(\frac{3\pi}{2}\right)=(2,0)$ $\quad$ R: $r: \vec{r}(t)=(1,0)+\mu(2,0),\mu\in\mathbb{R}$

c) $1+2\cos(\theta)=0 \Leftrightarrow 2\cos(\theta)=-1 \Leftrightarrow \cos(\theta)=-\frac{1}{2} \Leftrightarrow \theta=\frac{2\pi}{3} \vee \theta=\frac{4\pi}{3}$

$\vec{f}\left(\frac{2\pi}{3}\right)=\left(0,2+2\sin\left(\frac{2\pi}{3}\right)\right)=(0,2+\sqrt{3})$ $\quad \vec{f}'\left(\frac{2\pi}{3}\right)=(-\sqrt{3},-1)$

$\vec{f}\left(\frac{4\pi}{3}\right)=\left(0,2+2\sin\left(\frac{4\pi}{3}\right)\right)=(0,2-\sqrt{3})$ $\quad \vec{f}'\left(\frac{4\pi}{3}\right)=(\sqrt{3},-1)$

São as retas: $\vec{r}(\mu)=(0,2+\sqrt{3})+\mu(-\sqrt{3},-1),\ \mu\in\mathbb{R}$

$\vec{r}(s)=(0,2-\sqrt{3})+s(\sqrt{3},-1),\ s\in\mathbb{R}$

21 a) $x^2+y^2+z^2=\sin^2(2t)+4\sin^4(t)+4\cos^2(t)=$

$=(2\sin(t)\cos(t))^2+4\sin^4(t)+4\cos^2(t)=4\sin^2(t)\cos^2(t)+4\sin^4(t)+4\cos^2(t)=$

$=4\left(\sin^2(t)\cos^2(t)+\sin^4(t)+\cos^2(t)\right)=4\left(\sin^2(t)\left(\cos^2(t)+\sin^2(t)\right)+\cos^2(t)\right)=$

$=4\left(\sin^2(t)+\cos^2(t)\right)=4\times1=4$ Logo, $x^2+y^2+z^2=4$, pelo que os pontos desta curva estão situados sobre uma superfície esférica centrada na origem, e de raio $\sqrt{4}=2$, c.q.p.

b) vetor tangente: $\vec{f}'(t)=\left(\sin(2t)',2\sin^2(t)',2\cos(t)'\right)=$

$=(2\cos(2t),4\sin(t)\cos(t),-2\sin(t))$ Projeção ortogonal sobre xO_y:

$\vec{r}=(2\cos(2t),4\sin(t)\cos(t))$ $\quad \|\vec{r}\|=\|(2\cos(2t),4\sin(t)\cos(t))\|=$

$=\sqrt{(2\cos(2t))^2+(4\sin(t)\cos(t))^2}=\sqrt{4\cos^2(2t)+(2\sin(2t))^2}=$

$=\sqrt{4\cos^2(2t)+4\sin^2(2t)}=\sqrt{4(\cos^2(2t)+\sin^2(2t))}=\sqrt{4}=2=$constante, c.q.p.

22 $x^2+y^2=4$ $\quad \vec{r}(\theta)=(2\cos(\theta),2\sin(\theta)),\theta\in[0,2\pi]$

linha tangente: $\vec{r}'(\theta)=(-2\sin(\theta),2\cos(\theta)),\theta\in[0,2\pi]$

É paralela à reta $\vec{r}(\mu)$ se tiver o mesmo declive, que é $\frac{1}{1}=1$

$-\frac{2\cos(\theta)}{2\sin(\theta)}=1 \Leftrightarrow -\cos\theta=\sin(\theta) \Leftrightarrow \tan\theta=-1 \Leftrightarrow$

$\Leftrightarrow \theta=\frac{3\pi}{4} \vee \theta=\frac{7\pi}{4}$ $\quad \vec{r}\left(\frac{3\pi}{4}\right)=(-\sqrt{2},\sqrt{2})$ $\quad \vec{r}\left(\frac{7\pi}{4}\right)=(\sqrt{2},-\sqrt{2})$

R: Os pontos $(-\sqrt{2},\sqrt{2})$ e $(\sqrt{2},-\sqrt{2})$

23 a) $y^2 = 4x \Leftrightarrow x = \frac{y^2}{4} \qquad \vec{r}(t) = \left(\frac{t^2}{4}, t\right),\ t \in \mathbb{R}$

b) $\vec{r}\,'(t) = \left(\frac{t}{2}, 1\right),\ t \in \mathbb{R}$

linhas tangentes que passam em P: $(x,y) = (-2,0) + k\left(\frac{t}{2}, 1\right),\ k \in \mathbb{R}$

$$\begin{cases} \frac{t^2}{4} = -2 + k \times \frac{t}{2} \\ t = k \end{cases} \Leftrightarrow \begin{cases} \frac{t^2}{4} = -2 + \frac{t^2}{2} \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} \frac{t^2}{4} = 2 \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} t^2 = 8 \\ \_\_\_\_ \end{cases} \Leftrightarrow$$

$$\Leftrightarrow \begin{cases} t = \pm\sqrt{8} \\ \_\_\_\_ \end{cases} \Leftrightarrow \begin{cases} t = \pm 2\sqrt{2} \\ k = t = \pm 2\sqrt{2} \end{cases}$$

$r_1: (x,y) = (-2,0) + k(\sqrt{2}, 1),\ k \in \mathbb{R}$

$r_2: (x,y) = (-2,0) + k(-\sqrt{2}, 1),\ k \in \mathbb{R}$.

24 a) $x^2 + 4y^2 = 8 \Leftrightarrow \frac{x^2}{8} + \frac{y^2}{2} = 1$

$\vec{r}(\theta) = (2\sqrt{2}\cos\theta, \sqrt{2}\sin\theta),\ \theta \in [0, 2\pi]$

b) $x + 2y = 7 \Leftrightarrow 2y = 7 - x \Leftrightarrow y = \frac{7-x}{2} \Leftrightarrow y = -\frac{1}{2}x + \frac{7}{2}$

Se é paralela a essa reta, a linha tangente tem o mesmo declive, $m = -\frac{1}{2}$

$\vec{r}\,'(\theta) = (-2\sqrt{2}\sin\theta, \sqrt{2}\cos\theta),\ \theta \in [0, 2\pi] \qquad \vec{v} = (-2, 1)$ ou $\vec{v} = (2, -1)$

$$\begin{cases} -2\sqrt{2}\sin\theta = -2 \\ \sqrt{2}\cos\theta = 1 \end{cases} \Leftrightarrow \begin{cases} \sin\theta = \frac{\sqrt{2}}{2} \\ \cos\theta = \frac{\sqrt{2}}{2} \end{cases} \Leftrightarrow \theta = \frac{\pi}{4} \quad (\theta \in 1.^{\circ}Q)$$

$$\begin{cases} -2\sqrt{2}\sin\theta = 2 \\ \sqrt{2}\cos\theta = -1 \end{cases} \Leftrightarrow \begin{cases} \sin\theta = -\frac{\sqrt{2}}{2} \\ \cos\theta = -\frac{\sqrt{2}}{2} \end{cases} \Leftrightarrow \theta = \frac{5\pi}{4} \quad (\theta \in 3.^{\circ}Q)$$

$\vec{r}\left(\frac{\pi}{4}\right) = \left(2\sqrt{2}\cos\left(\frac{\pi}{4}\right), \sqrt{2}\sin\left(\frac{\pi}{4}\right)\right) = (2, 1) \qquad \vec{r}\left(\frac{5\pi}{4}\right) = \left(2\sqrt{2}\cos\left(\frac{5\pi}{4}\right), \sqrt{2}\sin\left(\frac{5\pi}{4}\right)\right) = (-2, -1)$

R: São os pontos $(2,1)$ e $(-2,-1)$

25 Seja β o plano que passa no ponto $P = (0,-1,1)$ e $\hat{B}_Q$ o vetor normal ao plano osculador da curva no ponto $Q = (-1, 0, 1)$.

Se $\beta \parallel$ plano osculador, então $\hat{B}$ é vetor normal de β.

$$Q = (-1,0,1) \Rightarrow \begin{cases} t^2 - 1 = -1 \\ \sin(2t) = 0 \\ e^{-t} = 1 \end{cases} \Leftrightarrow \begin{cases} t = 0 \\ t = 0 \\ t = 0 \end{cases} \qquad \hat{B}_Q = \frac{r'(0) \times r''(0)}{\|r'(0) \times r''(0)\|}$$

$\vec{r}\,'(t) = (2t, 2\cos(2t), -e^{-t}) \qquad \vec{r}\,'(0) = (0, 2, -1)$

$\vec{r}\,''(t) = (2, -4\sin(2t), e^{-t}) \qquad \vec{r}\,''(0) = (2, 0, 1)$

$$\vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 2 & -1 \\ 2 & 0 & 1 \end{vmatrix} = (2, -2, -4) \qquad \|\vec{r}\,'(0) \times \vec{r}\,''(0)\| = \sqrt{2^2 + 2^2 + (-4)^2} = \sqrt{24} = 2\sqrt{6}$$

$\hat{B}_Q = \frac{(2,-2,-4)}{2\sqrt{6}} = \frac{1}{\sqrt{6}}(1, -1, -2) \qquad \beta: x - y - 2z = D \qquad$ Como $P \in \beta$:

$0 + 1 - 2 = D \Leftrightarrow D = -1$

R: $\beta: x - y - 2z = -1$

$\sec(\arctan(x)) = \sqrt{1+x^2}$ $\quad$ $\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$

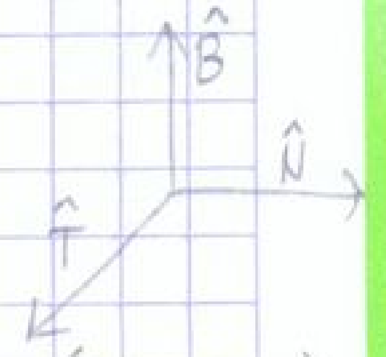
26 a) $O=(0,0,0) \Rightarrow \lambda = 0$ $\qquad \hat{T}_0 = \frac{\vec{r}\,'(0)}{\|\vec{r}\,'(0)\|}$

$\vec{r}\,'(\lambda) = (\cos(\lambda) - \lambda\sin(\lambda), \sin(\lambda) + \lambda\cos(\lambda), b)$ $\qquad \vec{r}\,'(0) = (1,0,b)$

$\|\vec{r}\,'(0)\| = \sqrt{1^2+b^2} = \sqrt{1+b^2}$ $\qquad \hat{T}_0 = \frac{1}{\sqrt{1+b^2}}(1,0,b)$ $\qquad (1,0,b)$ é vetor colinear com $\hat{T}_0$

$\hat{B}_0 = \frac{\vec{r}\,'(0) \times \vec{r}\,''(0)}{\|\vec{r}\,'(0) \times \vec{r}\,''(0)\|}$ $\qquad \vec{r}\,''(\lambda) = (-\sin(\lambda) - \sin(\lambda) - \lambda\cos(\lambda), \cos(\lambda) + \cos(\lambda) - \lambda\sin(\lambda), 0)$

$\vec{r}\,''(0) = (0,2,0)$ $\qquad \vec{r}\,'(0) \times \vec{r}\,''(0) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & b \\ 0 & 2 & 0 \end{vmatrix} = (-2b, 0, 2)$



$\|\vec{r}\,'(0) \times \vec{r}\,''(0)\| = \sqrt{(-2b)^2 + 2^2} = \sqrt{4b^2+4} = 2\sqrt{b^2+1}$

$\hat{B}_0 = \frac{1}{\sqrt{1+b^2}}(-b,0,1)$ $\qquad \hat{N}_0 = \hat{B}_0 \times \hat{T}_0 = \frac{1}{1+b^2}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -b & 0 & 1 \\ 1 & 0 & b \end{vmatrix} = \frac{1}{1+b^2}(0, 1+b^2, 0) = (0,1,0)$

$(-b,0,1)$ é um vetor colinear com $\hat{B}_0$

plano osculador: $\vec{r}(u,v) = O + u\hat{T}_0 + v\hat{N}_0 \Leftrightarrow \vec{r}(u,v) = (u, v, bu)$; $u,v \in \mathbb{R}$

plano normal: $\vec{r}(s,t) = O + t\hat{N}_0 + s\hat{B}_0 \Leftrightarrow \vec{r}(s,t) = (-bs, t, s)$; $t,s \in \mathbb{R}$

plano retificador: $\vec{r}(q,w) = O + q\hat{T}_0 + w\hat{B}_0 \Leftrightarrow \vec{r}(q,w) = (q-bw, 0, bq+w)$; $q,w \in \mathbb{R}$

b) O vetor $(-b,0,1)$, colinear com $\hat{B}_0$, é normal ao plano osculador.

$X \cdot (-b,0,1) = (0,0,0)\cdot(-b,0,1) \Leftrightarrow -bx + z = 0 \Leftrightarrow bx - z = 0 \leftarrow$ plano osculador

O vetor $(1,0,b)$, colinear com $\hat{T}_0$, é normal ao plano normal.

$X \cdot (1,0,b) = (0,0,0)\cdot(1,0,b) \Leftrightarrow x + bz = 0 \leftarrow$ plano normal

O vetor $(0,1,0)$, que corresponde a $\hat{N}_0$, é normal ao plano retificador.

$X \cdot (0,1,0) = (0,0,0)\cdot(0,1,0) \Leftrightarrow y = 0 \leftarrow$ plano retificador.

27 $y = \frac{x^2}{2} \Rightarrow \vec{r}(t) = \left(t, \frac{t^2}{2}\right), t \in \mathbb{R}$ $\qquad P(0,0) \Rightarrow t = 0$; $\; P\left(\frac{1}{2}, \frac{1}{8}\right) \Rightarrow t = \frac{1}{2}$

$s(t) = \int_0^{\frac{1}{2}} \|\vec{r}\,'(t)\|\, dt$ $\qquad \begin{cases} t = \tan(u) \\ dt = \sec^2(u)\,du \end{cases}$

$\vec{r}\,'(t) = (1,t)$ $\qquad \|\vec{r}\,'(t)\| = \sqrt{1^2+t^2} = \sqrt{1+t^2}$

$s(t) = \int_0^{\frac{1}{2}} \sqrt{1+t^2}\, dt = \int_0^{\frac{1}{2}} \sqrt{1+\tan^2(u)} \times \sec^2(u)\, du = \int_0^{\frac{1}{2}} \sec^3(u)\, du \overset{(1)}{=}$

Regra de redução de integrais: $\int \sec^m(x)\,dx = \frac{\sec^{m-1}(x)\sin(x)}{m-1} + \frac{m-2}{m-1}\int \sec^{m-2}(x)\,dx$

$\overset{(1)}{=} \frac{\sec^2(u)\sin(u)}{2} + \frac{1}{2}\int \sec(u)\,du = \frac{\sec^2(u)\sin(u)}{2} + \frac{1}{2}\ln|\tan(u) + \sec(u)| =$

$= \frac{\sec^2(\arctan(t))\sin(\arctan(t))}{2} + \frac{1}{2}\ln|\tan(\arctan(t)) + \sec(\arctan(t))| =$

$= \frac{(\sqrt{1+t^2})^2 \frac{t}{\sqrt{1+t^2}}}{2} + \frac{1}{2}\ln|t + \sqrt{1+t^2}| = \left[\frac{1}{2}t\sqrt{1+t^2} + \frac{1}{2}\ln|t+\sqrt{1+t^2}|\right]_0^{\frac{1}{2}}$

$= \frac{1}{4} \times \frac{\sqrt{5}}{2} + \frac{1}{2}\ln\left|\frac{1}{2} + \frac{\sqrt{5}}{2}\right| = \frac{\sqrt{5}}{8} + \frac{1}{2}\ln\left|\frac{1+\sqrt{5}}{2}\right|$ m.

28 $y^2 = x^3 \Rightarrow \vec{r}(t) = (t^2, t^3);\ t \in [-1,1]$

$\vec{r}'(t) = (2t, 3t^2),\ t \in [-1,1]$ $\quad \|\vec{r}'(t)\| = \sqrt{9t^4+4t^2} = \begin{cases} -t\sqrt{4+9t^2} & \text{se } t \in [-1,0] \\ t\sqrt{4+9t^2} & \text{se } t \in]0,1] \end{cases}$

$$\int_{-1}^{1} \|\vec{r}'(t)\|\,dt = \int_{-1}^{0} -t\sqrt{4+9t^2}\,dt + \int_0^1 t\sqrt{4+9t^2}\,dt = 2\int_0^1 t\sqrt{4+9t^2}\,dt =$$

$\begin{cases} u = 4+9t^2 \\ du = 18t\,dt \end{cases}$

$$= \frac{1}{9}\int_0^1 18t\sqrt{4+9t^2}\,dt = \frac{1}{9}\int_0^1 \sqrt{u}\,du = \frac{1}{9}\left[\frac{u^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^1 = \frac{1}{9} \times \left[\frac{2u^{\frac{3}{2}}}{3}\right]_0^1 =$$

$$= \frac{1}{9} \times \frac{2\sqrt{(4+9t^2)^3}}{3}\Bigg|_0^1 = \frac{1}{9} \times \frac{2 \times 13\sqrt{13}}{3} - \frac{1}{9} \times \frac{2 \times 4 \times 2}{3} = \frac{26\sqrt{13}}{27} - \frac{16}{27} =$$

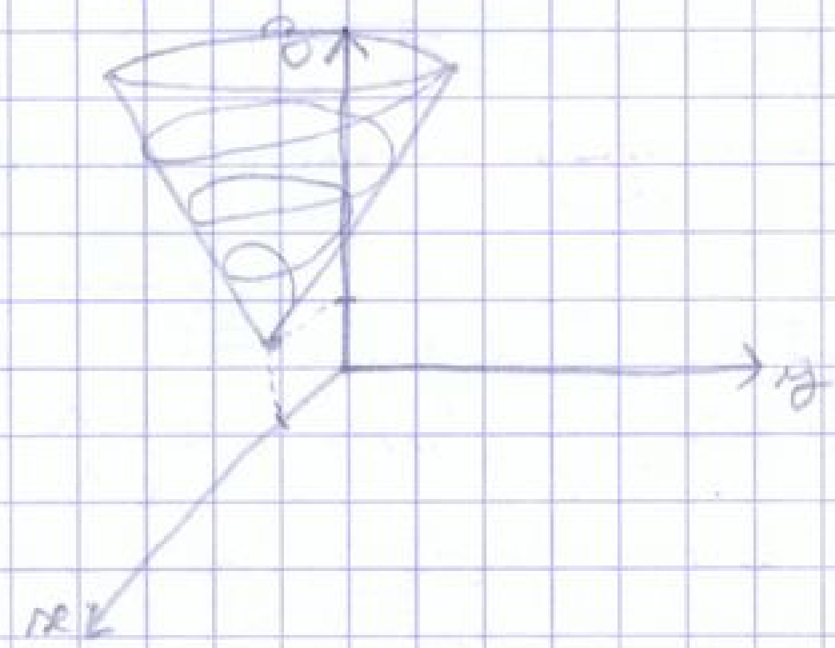
$$= \frac{2}{27}(13\sqrt{13} - 8)\ \text{m}$$

29 $\vec{r}(t) = \left(t, t^2, \frac{2t^3}{3}\right),\ t \geq 0$ $\quad \vec{r}'(t) = (1, 2t, 2t^2),\ t \geq 0$

$\|\vec{r}'(t)\| = \sqrt{1^2 + 4t^2 + 4t^4} = \sqrt{(1+2t^2)^2} = 1 + 2t^2$

$s(t) = \int_0^t \|\vec{r}'(t)\|\,dt = \int_0^t 1+2t^2\,dt = t + \frac{2t^3}{3}\ \text{m}$

30 a) $\vec{r}(t) = (e^t\cos(t), e^t\sin(t), e^t)$ $(t \geq 0)$ ponto inicial: $(1,0,1)$



b) $\hat{T}(t) = \frac{\vec{r}'(t)}{\|\vec{r}'(t)\|}$ $\quad \vec{r}'(t) = e^t(\cos(t)-\sin(t), \sin(t)+\cos(t), 1)$

$\|\vec{r}'(t)\| = e^t\sqrt{(\cos(t)-\sin(t))^2 + (\sin(t)+\cos(t))^2 + 1} = \sqrt{3}\,e^t$

$\hat{T}(t) = \frac{e^t(\cos(t)-\sin(t), \sin(t)+\cos(t), 1)}{\sqrt{3}\,e^t} = \frac{1}{\sqrt{3}}(\cos(t)-\sin(t), \sin(t)+\cos(t), 1)$

$\hat{N}(t) = \frac{\hat{T}'(t)}{\|\hat{T}'(t)\|}$ $\quad \hat{T}'(t) = \frac{1}{\sqrt{3}}(-\sin(t)-\cos(t), \cos(t)-\sin(t), 0)$

$\|\hat{T}'(t)\| = \frac{1}{\sqrt{3}}\sqrt{(-\sin(t)-\cos(t))^2 + (\cos(t)-\sin(t))^2} = \frac{\sqrt{2}}{\sqrt{3}}$

$\hat{N}(t) = \frac{1}{\sqrt{2}}(-\sin(t)-\cos(t), \cos(t)-\sin(t), 0)$ $\quad \hat{B}(t) = \hat{T} \times \hat{N} =$

$$= \frac{1}{\sqrt{6}}\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \cos(t)-\sin(t) & \sin(t)+\cos(t) & 1 \\ -\sin(t)-\cos(t) & \cos(t)-\sin(t) & 0 \end{vmatrix} = \frac{1}{\sqrt{6}}(\sin(t)-\cos(t), -\sin(t)-\cos(t), 2)$$

e) Um vetor perpendicular ao plano osculador é o versor $\hat{B}$.

$P(1,0,1) \in$ plano osculador $\quad R: (e^t\cos(t), e^t\sin(t), e^t)$

$\vec{RX} \cdot \hat{B} = 0 \Leftrightarrow (x - e^t\cos(t), y - e^t\sin(t), z - e^t) \cdot (\sin(t)-\cos(t), -\sin(t)-\cos(t), 2) = 0$

$\Leftrightarrow x(\sin(t)-\cos(t)) - e^t\cos(t)\sin(t) + e^t\cos^2(t) - y(\sin(t)+\cos(t)) + e^t\cos(t)\sin(t) + e^t\sin^2(t) + 2z - 2e^t = 0 \Leftrightarrow$

$\Leftrightarrow (\sin(t)-\cos(t))x - (\sin(t)+\cos(t))y + 2z - e^t = 0 \Leftrightarrow$

$\Leftrightarrow (\sin(t)-\cos(t))x - (\sin(t)+\cos(t))y + 2z = e^t \leftarrow$ eq. cartesiana plano osculador.

Um vetor perpendicular ao plano normal é o versor $\hat{T}$.

$R: (e^t\cos(t), e^t\sin(t), e^t) \quad \vec{RX} \cdot \hat{T} = 0 \Leftrightarrow$

$\Leftrightarrow (x - e^t\cos(t), y - e^t\sin(t), z - e^t) \cdot (\cos(t)-\sin(t), \sin(t)+\cos(t), 1) = 0 \Leftrightarrow$

$\Leftrightarrow (\cos(t)-\sin(t))x + e^t\cos(t)\sin(t) - e^t\cos^2(t) + y(\sin(t)+\cos(t)) - e^t\sin^2(t) - e^t\cos(t)\sin(t) + z - e^t = 0 \Leftrightarrow$

$\Leftrightarrow (\cos(t)-\sin(t))x + (\sin(t)+\cos(t))y + z = 2e^t \leftarrow$ eq. cartesiana do plano normal

Um vetor perpendicular ao plano retificador é o versor $\hat{N}$.

$R: (e^t\cos(t), e^t\sin(t), e^t) \quad \vec{RX} \cdot \hat{N} = 0 \Leftrightarrow$

$\Leftrightarrow (x - e^t\cos(t), y - e^t\sin(t), z - e^t) \cdot (-\sin(t)-\cos(t), \cos(t)-\sin(t), 0) = 0 \Leftrightarrow$

$\Leftrightarrow -(\sin(t)+\cos(t))x + e^t\cos^2(t) + e^t\cos(t)\sin(t) + (\cos(t)-\sin(t))y - e^t\sin(t)\cos(t) + e^t\sin^2(t) = 0 \Leftrightarrow$

$\Leftrightarrow -(\sin(t)+\cos(t))x + (\cos(t)-\sin(t))y + e^t = 0 \Leftrightarrow$

$\Leftrightarrow (\sin(t)+\cos(t))x + (\sin(t)-\cos(t))y = e^t \leftarrow$ eq. cartesiana do plano retificador.

d) $P = (1,0,1) \Rightarrow t = 0$

plano osculador: $(\sin(0)-\cos(0))x - (\sin(0)+\cos(0))y + 2z = e^0 \Leftrightarrow$

$\Leftrightarrow -x - y + 2z = 1 \Leftrightarrow x + y - 2z = -1$

plano normal: $(\cos(0)-\sin(0))x + (\sin(0)+\cos(0))y + z = 2e^0 \Leftrightarrow$

$\Leftrightarrow x + y + z = 2$

plano retificador: $(\sin(0)+\cos(0))x + (\sin(0)-\cos(0))y = e^0 \Leftrightarrow$

$\Leftrightarrow x - y = 1$

e) $K(t) = \dfrac{\|\hat{T}'(t)\|}{\|\vec{r}'(t)\|} \quad \|\hat{T}'(t)\| = \dfrac{\sqrt{2}}{\sqrt{3}}$ (alínea b)) Em $t=0$:

$\|\vec{r}'(t)\| = \sqrt{3}\, e^t$ (alínea b)) $\quad K(t) = \dfrac{\frac{\sqrt{2}}{\sqrt{3}}}{\sqrt{3}\, e^t} = \dfrac{\sqrt{2}}{3e^t} \quad K(0) = \dfrac{\sqrt{2}}{3}\, m^{-1}$

$\rho(0) = \dfrac{1}{K(0)} = \dfrac{1}{\frac{\sqrt{2}}{3}} = \dfrac{3\sqrt{2}}{2}\, m \quad C_0 = \vec{r}(0) + \rho(0) \times \hat{N}(0) =$

$= (1,0,1) + \dfrac{3\sqrt{2}}{2} \times \left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0\right) \Leftrightarrow$

$\Leftrightarrow C_0 = \left(-\frac{1}{2}, \frac{3}{2}, 1\right)$

f) $s(t)=\int_0^t \|\vec{r}'(t)\|\,dt$ $\quad \|\vec{r}'(t)\|=\sqrt{3}\,e^t$ (alínea b))

$s(t)=\int_0^t \sqrt{3}\,e^t\,dt=\sqrt{3}\,e^t\Big|_0^t=\sqrt{3}\,e^t-\sqrt{3}=\sqrt{3}\,(e^t-1)$ m

g) $s(2)=\sqrt{3}\,(e^2-1)$ $\quad s(0)=0$

$s(2)-s(0)=\sqrt{3}\,(e^2-1)-0=\sqrt{3}\,(e^2-1)$ m

h) $s(1)=\sqrt{3}\,(e-1)$ $\quad s(2)=\sqrt{3}\,(e^2-1)$

$s(2)-s(1)=\sqrt{3}\,(e^2-1)-\sqrt{3}\,(e-1)=\sqrt{3}\,e^2-\sqrt{3}-\sqrt{3}\,e+\sqrt{3}=\sqrt{3}\,e\,(e-1)$ m

31 a) $\vec{r}'(u)=(a\sin(u),\,a-a\cos(u))$ $\quad \|\vec{r}'(u)\|=\sqrt{a^2\sin^2(u)+a^2(1-\cos(u))^2}=$

$=a\sqrt{\sin^2(u)+1-2\cos(u)+\cos^2(u)}=a\sqrt{2-2\cos(u)}=\sqrt{2}\,a\sqrt{1-\cos(u)}$

$s(u)=\int_0^u \|\vec{r}'(u)\|\,du=\sqrt{2}\,a\int_0^u \sqrt{1-\cos(u)}\,du\overset{(1)}{=}$

CA: $\sin^2(t)=\frac{1}{2}-\frac{1}{2}\cos(2t)\Leftrightarrow 2\sin^2(t)=1-\cos(2u)$

$\overset{(1)}{=}\sqrt{2}\,a\int_0^u\sqrt{2\sin^2\left(\frac{u}{2}\right)}=2a\int_0^u \sin\left(\frac{u}{2}\right)=2a\times\frac{-\cos\left(\frac{u}{2}\right)}{\frac{1}{2}}=-4a\cos\left(\frac{u}{2}\right)$ m

$s(2\pi)-s(0)=-4a\cos(\pi)+4a\cos(0)=4a+4a=8a$ m

b) $\vec{r}'(u)=(e^u\cos(u)-e^u\sin(u),\,e^u\sin(u)+e^u\cos(u))$

$\|\vec{r}'(u)\|=\sqrt{e^{2u}(\cos(u)-\sin(u))^2+e^{2u}(\sin(u)+\cos(u))^2}\Leftrightarrow$

$\Leftrightarrow\|\vec{r}'(u)\|=e^u\sqrt{\cos^2(u)-2\sin(u)\cos(u)+\sin^2(u)+\sin^2(u)+2\sin(u)\cos(u)+\cos^2(u)}\Leftrightarrow$

$\Leftrightarrow\|\vec{r}'(u)\|=e^u\sqrt{2}=\sqrt{2}\,e^u$

$s(u)=\int_0^u\sqrt{2}\,e^u=\sqrt{2}\,e^u\Big|_0^u=\sqrt{2}\,e^u-\sqrt{2}=\sqrt{2}\,(e^u-1)$

$s(2)-s(0)=\sqrt{2}\,(e^2-1)-\sqrt{2}\,(1-1)=\sqrt{2}\,(e^2-1)$ m

e) $\vec{r}'(u)=(-a\sin(u)+a\sin(u)+au\cos(u),\,a\cos(u)-a\cos(u)+au\sin(u))\Leftrightarrow$

$\Leftrightarrow\vec{r}'(u)=(au\cos(u),\,au\sin(u))$ $\quad \|\vec{r}'(u)\|=\sqrt{a^2u^2\cos^2(u)+a^2u^2\sin^2(u)}\Leftrightarrow$

$\Leftrightarrow\|\vec{r}'(u)\|=au\sqrt{\cos^2(u)+\sin^2(u)}=au$

$s(u)=\int_0^u au\,du=a\times\frac{u^2}{2}\Big|_0^u=\frac{au^2}{2}$

$s(2\pi)-s(0)=\frac{a\times4\pi^2}{2}-0=2a\pi^2$ m

d) $\vec{r}'(u)=(\cos(u),\,1,\,\sin(u))$ $\quad \|\vec{r}'(u)\|=\sqrt{\cos^2(u)+1^2+\sin^2(u)}=\sqrt{2}$

$s(u)=\int_0^u\sqrt{2}\,du=\sqrt{2}\,u\Big|_0^u=\sqrt{2}\,u$

$s(2\pi)-s(0)=\sqrt{2}\times2\pi-\sqrt{2}\times0=2\sqrt{2}\,\pi$ m

R: $2\sqrt{2}\,\pi$ m

e) $\vec{r}'(u) = (1, 6u, 18u^2)$ $\quad \|\vec{r}'(u)\| = \sqrt{1^2+36u^2+(18u^2)^2} =$

$= \sqrt{(18u^2+1)^2} = 18u^2+1$

$s(u) = \int_0^u 18u^2+1 \, du = 6u^3+u\Big|_0^u = 6u^3+u$

$s(2)-s(0) = 6\times 2^3+2-(6\times 0^3+0) = 50$ m

f) $\vec{r}'(u) = (-a\omega \sin(\omega u), a\omega \cos(\omega u), b\omega)$

$\|\vec{r}'(u)\| = \sqrt{a^2\omega^2\sin^2(\omega u)+a^2\omega^2\cos^2(\omega u)+b^2\omega^2} =$

$= \sqrt{a^2\omega^2(\sin^2(\omega u)+\cos^2(\omega u))+b^2\omega^2} = |\omega|\sqrt{a^2+b^2}$

$s(u) = \int_0^u |\omega|\sqrt{a^2+b^2} = |\omega|\sqrt{a^2+b^2}\, u\Big|_0^u = |\omega|\sqrt{a^2+b^2}\, u$

$s(u_1)-s(u_0) = |\omega|\sqrt{a^2+b^2}\,(u_1-u_0)$ m

32 a) $P=(1,0,0) \Rightarrow t=0$ $\qquad Q=(1,0,2\pi) \Rightarrow t=2\pi$

$\vec{f}'(t) = (-\sin(t), \cos(t), 1)$ $\quad \|\vec{f}'(t)\| = \sqrt{(-\sin(t))^2+\cos^2(t)+1} = \sqrt{2}$

$s(t) = \int_0^{2\pi} \sqrt{2}\, dt = \sqrt{2}\, t\Big|_0^{2\pi} = 2\pi\sqrt{2} - 0 = 2\sqrt{2}\pi$ m

b) $s(t) = \int_0^t \sqrt{2}\, dt = \sqrt{2}\, t\Big|_0^t = \sqrt{2}\, t$ $\qquad s = \sqrt{2}\, t \Leftrightarrow t = \frac{s}{\sqrt{2}}$

$\vec{f}(s) = \left(\cos\left(\frac{s}{\sqrt{2}}\right), \sin\left(\frac{s}{\sqrt{2}}\right), \frac{s}{\sqrt{2}}\right), s \geq 0$

33 $r'(t) = (-3\cos^2(t)\sin(t), 3\sin^2(t)\cos(t))$

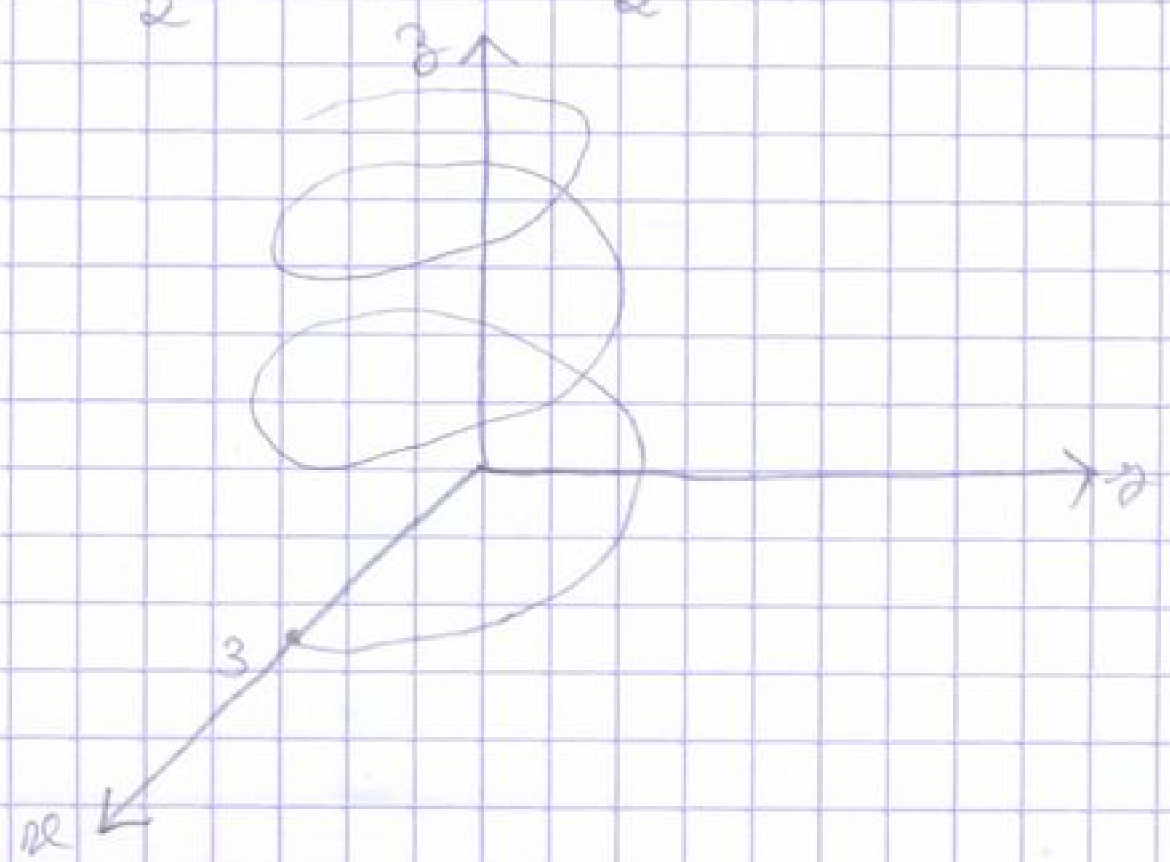
$\|\vec{r}'(t)\| = \sqrt{9\cos^4(t)\sin^2(t)+9\sin^4(t)\cos^2(t)} = \sqrt{9\cos^2(t)\sin^2(t)(\cos^2(t)+\sin^2(t))} =$

$= 3|\cos(t)\sin(t)| = 3\cos(t)\sin(t)$ $\qquad (t \in [0, \frac{\pi}{2}])$ $\qquad \begin{cases} u = \sin(t) \\ du = \cos(t)\, dt \end{cases}$

$s(t) = \int_0^{\frac{\pi}{2}} 3\cos(t)\sin(t)\, dt = \int_0^{\frac{\pi}{2}} 3u\, du = 3\left[\frac{u^2}{2}\right]_0^{\frac{\pi}{2}} =$

$= 3\times\left[\frac{\sin^2(t)}{2}\right]_0^{\frac{\pi}{2}} = 3\times\frac{1}{2} - 3\times 0 = \frac{3}{2}$ m

34 a)



b) $\vec{r}'(t) = (-3\sin(t), 3\cos(t), 4)$

$\|\vec{r}'(t)\| = \sqrt{9\sin^2(t) + 9\cos^2(t) + 16} = \sqrt{9(\sin^2(t) + \cos^2(t)) + 16} =$

$= \sqrt{9+16} = \sqrt{25} = 5 \qquad s(t) = \int_0^t 5\,dt \Leftrightarrow s(t) = 5t$ m

c) $s(t) = 5\pi \Leftrightarrow 5t = 5\pi \Leftrightarrow t = \pi \qquad Q(-3, 0, 4\pi)$

d) $s = 5t \Leftrightarrow t = \frac{s}{5} \qquad \vec{r}(s) = \left(3\cos\left(\frac{s}{5}\right), 3\sin\left(\frac{s}{5}\right), \frac{4s}{5}\right), s \geq 0$

e) $\vec{r}'(s) = \left(-\frac{3}{5}\sin\left(\frac{s}{5}\right), \frac{3}{5}\cos\left(\frac{s}{5}\right), \frac{4}{5}\right)$

$\|\vec{r}'(s)\| = \sqrt{\frac{9}{25}\left(\sin^2\left(\frac{s}{5}\right) + \cos^2\left(\frac{s}{5}\right)\right) + \frac{16}{25}} = \sqrt{\frac{9}{25} + \frac{16}{25}} = \sqrt{\frac{25}{25}} = \sqrt{1} = 1$

Logo, como $\|\vec{r}'(s)\| = 1$, $\vec{r}'(s)$ é versor.

35 a) $\vec{h}'(t) = (-a\omega\sin(\omega t), a\omega\cos(\omega t), b\omega)$

$\|\vec{h}'(t)\| = \sqrt{a^2\omega^2(\sin^2(\omega t) + \cos^2(\omega t)) + b^2\omega^2} = \sqrt{\omega^2(a^2+b^2)} =$

$= \omega\sqrt{a^2+b^2} \qquad \hat{T}(t) = \frac{\vec{h}'(t)}{\|\vec{h}'(t)\|} = \frac{(-a\sin(\omega t), a\cos(\omega t), b)}{\sqrt{a^2+b^2}}$

$\hat{T}(t) \cdot \hat{k} = \|\hat{T}(t)\| \times \|\hat{k}\| \times \cos\theta \Leftrightarrow \frac{b}{\sqrt{a^2+b^2}} = \cos\theta \Leftrightarrow \theta = \arccos\frac{b}{\sqrt{a^2+b^2}}$

b) $\vec{h}''(t) = (-a\omega^2\cos(\omega t), -a\omega^2\sin(\omega t), 0)$

$$\vec{h}'(t) \times \vec{h}''(t) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -a\omega\sin(\omega t) & a\omega\cos(\omega t) & b\omega \\ -a\omega^2\cos(\omega t) & -a\omega^2\sin(\omega t) & 0 \end{vmatrix} = (ab\omega^3\sin(\omega t), -ab\omega^3\cos(\omega t), a^2\omega^3)$$

$\|\vec{h}'(t) \times \vec{h}''(t)\| = \sqrt{a^2b^2\omega^6\sin^2(\omega t) + a^2b^2\omega^6\cos^2(\omega t) + a^4\omega^6} =$

$= \sqrt{a^2b^2\omega^6 + a^4\omega^6} = \sqrt{a^2\omega^6(b^2+a^2)} = a\omega^3\sqrt{a^2+b^2}$

$\|\vec{h}'(t)\|^3 = \left(\omega\sqrt{a^2+b^2}\right)^3 = \omega^3\left(\sqrt{a^2+b^2}\right)^3$

$k(t) = \frac{\|\vec{h}'(t) \times \vec{h}''(t)\|}{\|\vec{h}'(t)\|^3} = \frac{a\omega^3\sqrt{a^2+b^2}}{\omega^3\left(\sqrt{a^2+b^2}\right)^3} = \frac{a}{\left(\sqrt{a^2+b^2}\right)^2} = \frac{a}{a^2+b^2}$ m^{-1}

36