2. Which 4-tuples are in the relation $\{(a, b, c, d) \mid a, b, c,$ and $d$ are positive integers with $abcd = 6\}$?

6 can be divide into $2\times3$ or $6\times1$.

① $6\times1\times1\times1$. 4 cases.

(6,1,1,1) (1,6,1,1) (1,1,1,6) (1,1,6,1)

② $2\times3\times1\times1$ 12 cases.

(2 3 1 1) (2 1 3 1) (2 1 1 3)

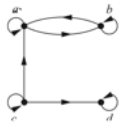
(3 2 1 1) (3 1 2 1) (3 1 1 2)

(1 3 1 2) (1 3 2 1) (1 2 1 3) (1 2 3 1)

(1 1 3 2) (1 1 2 3)

32. Determine whether the relations represented by the directed graphs shown in Exercises 26–28 are reflexive, irreflexive, symmetric, antisymmetric, asymmetric, and/or transitive.

26.




reflexive each node has a loop

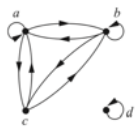
asymmetric: having only one edge in opposite directions

since a c→a a→b not c→b

not transitive

reflexive, asymmetric. not transitive not symmetric not antisymmetric

27.




28.


27. not reflexive. c doesn't have loop. not antisymmetric

symmetric. each edge has oppsite directions not asymmetric

not transitive (a,c)∈R (c,a)∈R (c,c)∉R

28. reflexive. each node has loop

symmetric not antisymmetric

transitive not asymmetric

26. Use Algorithm 1 to find the transitive closures of these relations on $\{a, b, c, d, e\}$.

a) $\{(a, c), (b, d), (c, a), (d, b), (e, d)\}$

$MR^* = MR \vee MR^2 \vee MR^3 \vee MR^4 \vee MR^5$

$$MR = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$MR^2 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$MR^3 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

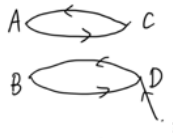
$$MR^4 = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix}$$

$$MR^5 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$M_{R^*} = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

28. Use Warshall's algorithm to find the transitive closures of the relations in Exercise 26.




$$W_0 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_1 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad W_2 = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad W_3 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$W_4 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix} \quad W_5 = \begin{bmatrix} 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$$

36. What is the congruence class $[4]_m$ when $m$ is
a) 2? b) 3? c) 6? d) 8?

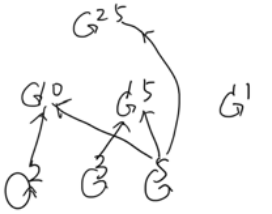
a. $[4]_2 = \{\ldots, -2, 0, 2, 4, 6, \ldots\}$

b. $[4]_3 = \{\ldots, -2, 1$

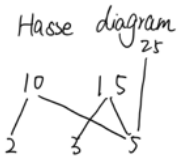
c. $[4]_6 =$

d. $[4]_8 =$

22. Draw the Hasse diagram for divisibility on the set
a) {1, 2, 3, 4, 5, 6}. b) {3, 5, 7, 11, 13, 16, 17}.
c) {2, 3, 5, 10, 11, 15, 25}. d) {1, 3, 9, 27, 81, 243}.

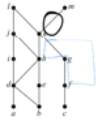



Hasse diagram




11

32. Answer these questions for the partial order represented by this Hasse diagram.




a) Find the maximal elements.
b) Find the minimal elements.
c) Is there a greatest element?
d) Is there a least element?
e) Find all upper bounds of $\{a, b, c\}$.
f) Find the least upper bound of $\{a, b, c\}$, if it exists.
g) Find all lower bounds of $\{f, g, h\}$.
h) Find the greatest lower bound of $\{f, g, h\}$, if it exists.

(a) l m

(b) a b c

(c) no., l and m are not comparable

(d) no. abc are not comparable

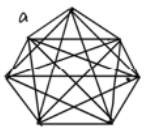
(e) k. l m

(f) k

(g) no lower bounds (h) no the greatest lower bound.

20. Draw these graphs.
a) $K_7$ b) $K_{1,8}$ c) $K_{4,4}$
d) $C_7$ e) $W_7$ f) $Q_4$

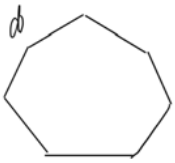
(a)



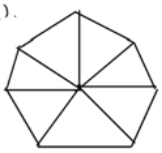
(b)

(c)

d



e).



f.