8.3

ex.36. $a=8$ $b=2$ $c=1$ $d=2$

Using the conclusion of ex.31.

$$C_1 = \frac{b^d \times c}{b^d - a} = \frac{2^2 \times 1}{2^2 - 8} = -1$$

$$C_2 = \frac{b^d \times c}{a - b^d} + f(1) = \frac{2^2 \times 1}{8 - 2^2} + f(1) = 2$$

$$f(n) = -1 \times n^2 + 2 \cdot n^3$$

ex.37.

$$\because a > b^d \quad O(n) = n^{\log_b a}$$

$$= n^3$$

8.4

ex.12

a). $f(x) = \sum_{k=0}^{\infty} -3^k x^k$

$k=12$

Coefficient equals to 3^{12}

b) $\frac{1}{(1-2x)^2} = \frac{1}{1-2x} \cdot \frac{1}{1-2x}$

$f(x) = g(x) = \sum_{k=0}^{\infty} 2^k x^k$

$$f(x)g(x) = \sum_{k=0}^{\infty} \left(\sum_{j=0}^{k} 2^{j} \cdot 2^{k-j} \right) x^{k}$$

$$= \sum_{k=0}^{\infty} 2^{k} (k+1) x^{k}$$

$k=12$ coefficient equals to 13×2^{12}

c)

$$\frac{1}{(1+x)^{8}} = (1+x)^{-8} = \sum_{k=0}^{\infty} \binom{-8}{k} x^{k}$$

$k=12$.

$$\binom{-8}{12} = \frac{(-1)^{12}\, 19!}{12!\, 7!}$$

$$= \frac{13 \times 14 \times 15 \times 16 \times 17 \times 18 \times 19}{7!}$$

$$= 50388$$

ex24.

$(x^3 + x^4 + \cdots)(x^1 + x^2 + x^3 + x^4 + x^5)(1 + x + x^2 + x^3 + x^4)(x + x^2 + x^3 + \cdots)$

$$= \frac{x^5 (1+x+x^2+x^3+x^4)^2}{(1-x)^2}$$

b). we already have x^5 so we need to find the coefficient of x^2 in $(1+x+x^2+x^3+x^4)^2 \ldots = (1+2x+3x^2+\cdots)(1+2x+2x^2+\cdots)$

$(1-x)^2$

$a_7 = 1\times3 + 2\times2 + 3\times1 = 10$

ex 34

Assume $G(x) = \sum_{k=0}^{\infty} a_k x^k$

$a_k x^k = 3a_{k-1}x^k + 4^{k-1}x^k$

$G(x) - 1 = \sum_{k=1}^{\infty} a_k x^k = \sum_{k=1}^{\infty} (3a_{k-1}x^k + 4^{k-1}x^k$

$= 3x\sum_{k=1}^{\infty} a_{k-1}x^{k-1} + x\sum_{k=1}^{\infty} 4^{k-1}x^{k-1}$

$= 3x\,G(x) + x\frac{1}{1-4x}$

$(1-3x)G(x) = \frac{x}{1-4x} + 1$

$(1-3x)G(x) = \frac{1-3x}{1-4x}$

$G(x) = \frac{1}{1-4x} = \sum_{k=0}^{\infty} 4^k x^k$

$\therefore a_k = 4^k$