

- ✓ 8. In a survey of 270 college students, it is found that 64 like brussels sprouts, 94 like broccoli, 58 like cauliflower, 26 like both brussels sprouts and broccoli, 28 like both brussels sprouts and cauliflower, 22 like both broccoli and cauliflower, and 14 like all three vegetables. How many of the 270 students do not like any of these vegetables?

$$\begin{aligned}
 |A_1 \cup A_2 \cup A_3| &= |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| \\
 &\quad + |A_1 \cap A_2 \cap A_3| \\
 &= 64 + 94 + 58 - 26 - 28 - 22 + 14 \\
 &= 154
 \end{aligned}$$

student don't like any of these vegetable = $270 - 154 = 116$

16. A group of n students is assigned seats for each of two classes in the same classroom. How many ways can these seats be assigned if no student is assigned the same seat for both classes?

$$\begin{aligned}
 D_n &= n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right] \\
 \text{So } D_n &= n! \cdot n! \left[1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \dots + (-1)^n \frac{1}{n!} \right]
 \end{aligned}$$

6. Determine whether the relation R on the set of all real numbers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R$ if and only if

- | | |
|----------------------------------|------------------|
| a) $x + y = 0$. | b) $x = \pm y$. |
| c) $x - y$ is a rational number. | |
| d) $x = 2y$. | e) $xy \geq 0$. |
| f) $xy = 0$. | g) $x = 1$. |
| h) $x = 1$ or $y = 1$. | |

a) $1+1 \neq 0$. not reflexive

$x+y = y+x$. symmetric

$(1, -1) (-1, 1)$ are both in the set not antisymmetric

$(1, -1) (-1, 1)$ are in the set, but $(1, 1) \notin R$ not transitive

b) $x = \pm x$. reflexive (chose the plus sign)

$x = \pm y$. $y = \pm x$ symmetric

$(1, -1) (-1, 1)$ both in R , not antisymmetric

transitive. $(1, -1) (-1, 1) (1, 1)$ all in R

c) if $x-y=0$. reflexive.

only

not antisymmetric. since $x \neq y$.

symmetric. $\textcircled{1} x_1 - y_1 = c$. $\textcircled{2} x_2 - y_2 = -c$. $(x, y)(y, x) \in R$

transitive $(2, 1) \in R$. $(3, 2) \in R$ $(3, 1) \in R$

d) not reflexive

not symmetric. since $(2, 1) \in R$, $(1, 2) \notin R$

only if $x=y=0$ $x=0$. antisymmetric

not transitive

e) reflexive.

symmetric $x \cdot y = y \cdot x$.

not antisymmetric $(1, 2), (2, 1)$ both in R

not transitive $(1, 0) \in R$. $(0, -2) \in R$ but $(1, -2) \notin R$

f) only if $x=0$. $y=0$. reflexive otherwise not reflexive

symmetric. $(2, 0) \in R$ $(0, 2) \in R$

not antisymmetric. $(1, 0) \in R$ $(0, 2) \in R$ but $1 \neq 2$

not transitive. since $(2, 0), (0, 2), (0, 0)$ in R but $(2, 2) \notin R$

g) only if $y=1$ reflexive.

not symmetric. $(1, 2) \in R$. $(2, 1) \notin R$.

not antisymmetric.

not transitive.

1. $x \in R$ $1 \in R$ $0 \in R$ $1 \in R$ $0 \in R$

n) if and only if $x=1$ and $y=1$ reflexive
symmetric. $(1,2) \in R$. $(2,1) \in R$

not antisymmetric since $(2,1) \in R$, $(1,2) \in R$ $1 \neq 2$

not transitive since $(4,1) \in R$, $(1,2) \in R$ $(4,2) \notin R$

32. Let R be the relation $\{(1, 2), (1, 3), (2, 3), (2, 4), (3, 1)\}$,
and let S be the relation $\{(2, 1), (3, 1), (3, 2), (4, 2)\}$.
Find $S \circ R$.

$$S \circ R = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$$

Exercises 34–37 deal with these relations on the set of real numbers:

$R_1 = \{(a, b) \in \mathbf{R}^2 \mid a > b\}$, the “greater than” relation,

$R_2 = \{(a, b) \in \mathbf{R}^2 \mid a \geq b\}$, the “greater than or equal to” relation,

$R_3 = \{(a, b) \in \mathbf{R}^2 \mid a < b\}$, the “less than” relation,

$R_4 = \{(a, b) \in \mathbf{R}^2 \mid a \leq b\}$, the “less than or equal to” relation,

$R_5 = \{(a, b) \in \mathbf{R}^2 \mid a = b\}$, the “equal to” relation,

$R_6 = \{(a, b) \in \mathbf{R}^2 \mid a \neq b\}$, the “unequal to” relation.

34. Find

a) $R_1 \cup R_3$.

b) $R_1 \cup R_5$.

c) $R_2 \cap R_4$.

d) $R_3 \cap R_5$.

e) $R_1 - R_2$.

f) $R_2 - R_1$.

g) $R_1 \oplus R_3$.

h) $R_2 \oplus R_4$.

35. Find

a) $R_2 \cup R_4$.

b) $R_3 \cup R_6$.

c) $R_3 \cap R_6$.

d) $R_4 \cap R_6$.

e) $R_3 - R_6$.

f) $R_6 - R_3$.

g) $R_2 \oplus R_6$.

h) $R_3 \oplus R_5$.

36. Find

a) $R_1 \circ R_1$.

b) $R_1 \circ R_2$.

$$34. a) R_1 \cup R_2 = R_6$$

$$b) R_1 \cup R_5 = R_2$$

$$36. a) R_1 \cap R_1 = R_1$$

$$b) R_1 \cap R_2 = R_1$$

Your score is:
15 out of 17