Homework09

8. Design a circuit for a light fixture controlled by four switches, where flipping one of the switches turns the light on when it is off and turns it off when it is on.

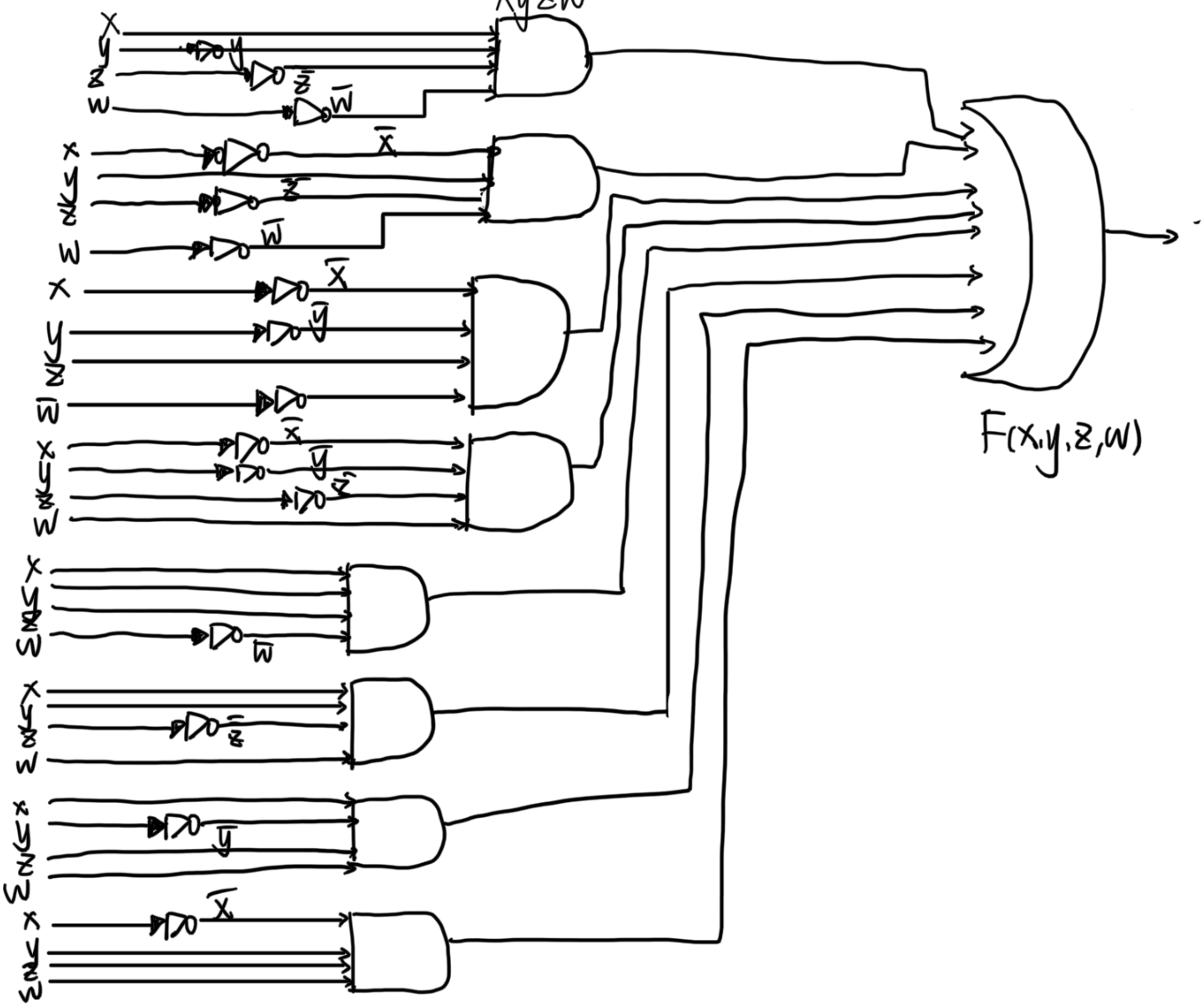
12.3 ex.8 Suppose the first switch is x second is y third is z fourth is w 1 represent the switch is open 0 represent the switch is off when light is on F(xyzw) = 1 when light is off. F(xyzw) = 0

suppose when four switch is off the light is off.

To turn on the light. you need to turn on one or three switch

x	y	z	w	F
0	0	0	0	0
1	0	0	0	1
0	1	0	0	1
0	0	1	0	1
0	0	0	1	1
1	1	0	0	0
1	0	1	0	0
1	0	0	1	0
0	1	1	0	0
0	1	0	1	0
0	0	1	1	0
1	1	1	0	1
1	1	0	1	1
1	0	1	1	1
0	1	1	1	1
1	1	1	1	0

$$F(x,y,z,w) = x\overline{yzw} + \bar{x}y\overline{zw} + \overline{xy}z\bar{w} + \overline{xyz}w + xyz\bar{w} + xy\bar{z}w + x\bar{y}zw + \bar{x}yzw$$



$x\bar{y}\bar{z}\bar{w}$
$F(x,y,z,w)$

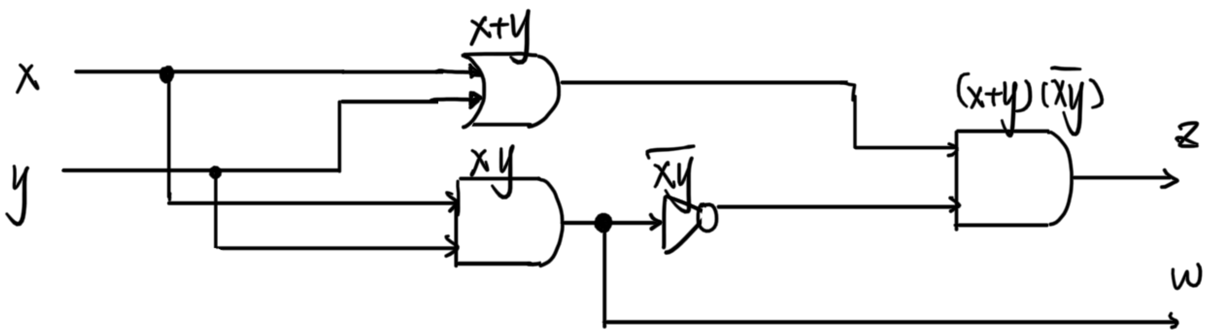
10. Construct a circuit for a half subtractor using AND gates, OR gates, and inverters. A **half subtractor** has two bits as input and produces as output a difference bit and a borrow.

12.3 ex.10 Suppose output bit as z, borrow is w

x	y	z	w
0	0	0	0
1	0	1	0
0	1	1	0
1	1	0	1

$z = (x+y)(\cdot\overline{xy})$

$w = \overline{xy}$



12. Use a K-map to find a minimal expansion as a Boolean sum of Boolean products of each of these functions in the variables x, y, and z.

a) $\overline{x}yz + \overline{x}\,\overline{y}z$

b) $xyz + xy\overline{z} + \overline{x}yz + \overline{x}y\overline{z}$

c) $xy\overline{z} + x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}\,\overline{y}z$

d) $xyz + x\overline{y}z + x\overline{y}\,\overline{z} + \overline{x}yz + \overline{x}y\overline{z} + \overline{x}\,\overline{y}\,\overline{z}$

12.4 ex.12

c)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x		1	1	1
$\bar{x}$	1			1

$F = \bar{x}z + \bar{y}z + x\bar{z}$

d)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
x	1		1	1
$\bar{x}$	1	1	1	

$F = x\bar{y} + \bar{x}\bar{z} + yz$

14. Use a K-map to find a minimal expansion as a Boolean sum of Boolean products of each of these functions in the variables w, x, y, and z.

a) $wxyz + wx\bar{y}z + wx\bar{y}\,\bar{z} + w\bar{x}y\bar{z} + w\bar{x}\,\bar{y}z$

b) $wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + \bar{w}x\bar{y}z + \bar{w}\,\bar{x}y\bar{z} + \bar{w}\,\bar{x}\,\bar{y}z$

c) ✓ $wxyz + wxy\bar{z} + wx\bar{y}z + w\bar{x}\,\bar{y}z + w\bar{x}\,\bar{y}\,\bar{z} + \bar{w}x\bar{y}z + \bar{w}\,\bar{x}y\bar{z} + \bar{w}\,\bar{x}\,\bar{y}z$

d) ✓ $wxyz + wxy\bar{z} + wx\bar{y}z + w\bar{x}yz + w\bar{x}y\bar{z} + \bar{w}xyz + \bar{w}\,\bar{x}yz + \bar{w}\,\bar{x}y\bar{z} + \bar{w}\,\bar{x}\,\bar{y}z$ ✓

12.4 ex.14.

c)

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	1	1		1
$w\bar{x}$			1	1
$\overline{wx}$		1		1
$\bar{w}x$				1

$$F = \bar{y}z + w\bar{x}\bar{y} + wxy + \overline{wx}y\bar{z}$$

d

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx	1	1		1
$w\bar{x}$	1	1		
$\overline{wx}$	1	1		1
$\bar{w}x$	1			

$$F = \overline{wx}z + wx\bar{z} + wxy + yz + \bar{x}y$$

24. Use the Quine–McCluskey method to simplify the sum-of-products expansions in Example 4.

EXAMPLE 4 Use K-maps to simplify these sum-of-products expansions.

(a) $wxyz + wxy\bar{z} + wx\bar{y}\,\bar{z} + w\bar{x}yz + w\bar{x}\,\bar{y}z + w\bar{x}\,\bar{y}\,\bar{z} + \bar{w}x\bar{y}z + \bar{w}\bar{x}yz + \bar{w}\bar{x}y\bar{z}$

(b) $wx\bar{y}\,\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\,\bar{y}\,\bar{z} + \bar{w}x\bar{y}\,\bar{z} + \bar{w}\bar{x}y\bar{z} + \bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$

(c) $wxy\bar{z} + wx\bar{y}\,\bar{z} + w\bar{x}yz + w\bar{x}y\bar{z} + w\bar{x}\,\bar{y}\,\bar{z} + \bar{w}xyz + \bar{w}xy\bar{z} + \bar{w}x\bar{y}\,\bar{z} + \bar{w}\bar{x}\bar{y}z + \bar{w}\bar{x}y\bar{z} + \bar{w}\,\bar{x}\,\bar{y}\,\bar{z}$

12.4 ex 24.

b)

1	$wx̄yz$	1011
2	$wx\bar{y}\bar{z}$	1100
3	$w\bar{x}y\bar{z}$	1010
4	$w\bar{x}\bar{y}\bar{z}$	1000
5	$\bar{w}x\bar{y}\bar{z}$	0100
6	$\bar{w}\bar{x}y\bar{z}$	0010
7	$\bar{w}\bar{x}\bar{y}\bar{z}$	0000

(1,3)	$w\bar{x}y$	101-
(2,4)	$w\bar{y}\bar{z}$	1-00
(2,5)	$x\bar{y}\bar{z}$	-100
(3,4)	$w\bar{x}\bar{z}$	10-0
(3,6)	$\bar{x}y\bar{z}$	-010
(4,7)	$\bar{x}\bar{y}\bar{z}$	-000
(5,7)	$\bar{w}\bar{y}\bar{z}$	0-00
(6,7)	$\bar{w}\bar{x}\bar{z}$	00-0

(2,5,4,7)	$\bar{y}\bar{z}$	--00
(3,6,4,7)	$\bar{x}\bar{z}$	-0-0

	$wx\bar{y}\bar{z}$	$w\bar{x}yz$	$w\bar{x}y\bar{z}$	$w\bar{x}\bar{y}\bar{z}$	$\bar{w}x\bar{y}\bar{z}$	$\bar{w}\bar{x}y\bar{z}$	$\bar{w}\bar{x}\bar{y}\bar{z}$
$w\bar{x}y$		X	X				
$\bar{y}\bar{z}$	X			X	X		X
$\bar{x}\bar{z}$			X	X		X	X

$$F = \bar{y}\bar{z} + \bar{x}\bar{z} + w\bar{x}y$$

In Exercises 30–32 find a minimal sum-of-products expansion, given the K-map shown with *don't care* conditions indicated with *d*s.

32.

	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}z$
wx		d	d	1
$w\bar{x}$	d	d	1	d
$\bar{w}\bar{x}$				
$\bar{w}x$	1	1	1	d

12.4 ex32.

$F = \bar{w}x + w\bar{y}$

***6.** Let $V = \{S, A, B, a, b\}$ and $T = \{a, b\}$. Find the language generated by the grammar (V, T, S, P) when the set P of productions consists of

a) $S \to AB$, $A \to ab$, $B \to bb$.
b) $S \to AB$, $S \to aA$, $A \to a$, $B \to ba$.
c) $S \to AB$, $S \to AA$, $A \to aB$, $A \to ab$, $B \to b$.
d) $S \to AA$, $S \to B$, $A \to aaA$, $A \to aa$, $B \to bB$, $B \to b$.
e) $S \to AB$, $A \to aAb$, $B \to bBa$, $A \to \lambda$, $B \to \lambda$.

13.1 ex.6

a) $S \Rightarrow AB \Rightarrow abB \Rightarrow abbb$

$L(G) = \{abbb\}$

b) $S \Rightarrow AB \Rightarrow aB \Rightarrow aba$

$S \Rightarrow aA \Rightarrow aa$

$L(G) = \{aba, aa\}$

16. Construct phrase-structure grammars to generate each of these sets.

a) $\{1^n \mid n \geq 0\}$ **b)** $\{10^n \mid n \geq 0\}$

c) $\{(11)^n \mid n \geq 0\}$

13.1 ex.16 a) $G = \{V, T, S, P\}$ where $V = \{S, 1\}$

$P = \{S \to 1S, S \to \lambda\}$ $T = \{1\}$

b) $G = \{V, T, S, P\}$ where $V = \{S, 0, 1, A\}$ $T = \{0, 1\}$

$P = \{S \to 1A, A \to 0A, A \to \lambda\}$

32. Give production rules in Backus–Naur form for the name of a person if this name consists of a first name, which is a string of letters, where only the first letter is uppercase; a middle initial; and a last name, which can be any string of letters.

13.1 ex.32

<name>::= <first name><middle initial><last name>

<first name>::=<uppercase letter><remainder letter>|<uppercase letter>

<remainder lettle>::= <lowercase letter><remainder letter>|<lowercase letter>

<uppercase letter>::= A|B|·····|Z

<lower case letter>::= a|b|····|y|z

<middle initial>::= A|B|····|Z

<last name>::=<lowercase letter><last name>|<lowercase letter>