Distributive property and binomial multiplication

The distributive property can be used even when there are two sets of parentheses with two terms each. It's called binomial multiplication (remember that a bicycle has two wheels and a binomial has two terms).

Binomial Multiplication:

$$(a+b)(c+d) = ac + ad + bc + bd$$

$$(a-b)(c-d) = ac - ad - bc + bd$$

Notice that a is multiplied by both terms in the second set of parentheses and then b is multiplied by both terms in the second set of parentheses.

We can also make a chart in which the terms a and b from the first set of parentheses go across the top, and the terms c and d from the second set of parentheses go along the left side. Then we multiply each row by each column to get a result. The four results all get added together to make the expanded polynomial.

	a	**b**
c	ac	bc
d	ad	bd

When we add all the results in the chart together, we get

$$ac + bc + ad + bd$$

When we have negative signs in the binomials, we keep the negative sign with the term that follows it, and our chart looks like

	a	-b
c	ac	-bc
-d	-ad	bd

When we add all the results in the chart together, we get

$$ac - bc - ad + bd$$

These charts are another way to keep track of the different multiplications that happen during binomial multiplication.

Example

Use the distributive property to expand the expression.

$$5(x - 2)(x + 3)$$

Start by distributing the 5 across $x - 2$.

$$[5(x) - 5(2)](x + 3)$$

$$(5x - 10)(x + 3)$$

Now distribute both of the terms in the first set of parentheses across both of the terms in the second set of parentheses. You may use a chart to help organize your work.

	5x	-10
x	$5x^2$	-10x
3	15x	-30

When we add all the results in the chart together, we get

$$5x^2 + 15x - 10x - 30$$

Combine like terms $15x - 10x$.

$$5x^2 + 5x - 30$$

Let's try another example of binomial multiplication.

Example

Use the distributive property to expand the expression.

$$3x(x + 4)(x + 1)(x - 2)$$

Start by distributing the $3x$ across $x + 4$.

$$(3x^2 + 12x)(x + 1)(x - 2)$$

Now distribute $3x^2 + 12x$ across $x + 1$. You may use a chart to help organize your work.

	$3x^2$	$12x$
x	$3x^3$	$12x^2$
1	$3x^2$	$12x$

When we add all the results in the chart together, we get

$$3x^3 + 3x^2 + 12x^2 + 12x$$

Combine like terms $3x^2 + 12x^2$.

$$3x^3 + 15x^2 + 12x$$

Then distribute $3x^3 + 15x^2 + 12x$ across $x - 2$. You may use a chart to help organize your work.

	$3x^3$	$15x^2$	$12x$
x	$3x^4$	$15x^3$	$12x^2$
-2	$-6x^3$	$-30x^2$	$-24x$

When we add all the results in the chart together, we get

$$3x^4 + 15x^3 + 12x^2 - 6x^3 - 30x^2 - 24x$$

Combine like terms $15x^3 - 6x^3$ and $12x^2 - 30x^2$.

$$3x^4 + 9x^3 - 18x^2 - 24x$$