44: Fast and Accurate Uncertainty Quantification for the ECG With Random Electrodes Location

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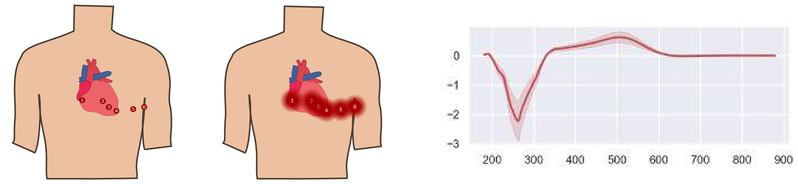
Motivations

The standard **surface ECG** is a valuable tool for assessing the electric activity of the heart. Routinely acquired and **non-invasive**, the ECG is also the perfect candidate for individualizing **patient-specific models**. As most clinical data, however, the ECG is unavoidably subject to **uncertainty**.

A simulated ECG is a point-wise evaluation of the potential on the chest. Therefore, when the **location of the electrodes is not precise**, the signal is a random variable. Here we show how to **estimate the statistical moments** of the ECG in a **very efficient** manner and suitable for model personalization.



Methods

Problem statement

▶ **Source of uncertainty**
The electrode locations on the chest. Their position is a random vector $\Xi = \{\xi_1, \dots, \xi_L\}$ with density $\rho(\mathbf{X})$.

▶ **Quantity of interest**
The statistics of the ECG $V(t, \Xi)$, specifically the expectation $\mathbb{E}[V](t)$ and correlation $\mathrm{Cor}[V](t, s)$.

▶ **Forward model**
Given the transmembrane potential $V_\mathrm{m}(\mathrm{x}, t)$ on the heart Ω_H, we compute the ECG with the forward bidomain model

$$-\mathrm{div}(\mathbf{G}\nabla u) = \begin{cases} \mathrm{div}(\mathbf{G}_\mathrm{i}\nabla V_\mathrm{m}), & \text{on } \Omega_\mathrm{H}, \\ 0, & \text{otherwise.} \end{cases}$$

An ECG lead is a zero-sum linear combination of $u(\xi_\ell, t)$.

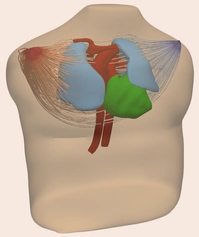
▶ **Isn't it a simple integration problem?**
Yes, but a rather expensive one as the cost grows linearly with the number of evaluations of the transmembrane potential $V_\mathrm{m}(\mathrm{x}, t)$, each requiring the solution of the elliptic PDE.

Lead field formulation and expectation

▶ **Equivalent formulation**
The ECG is exactly represented by the formula

$$V(t, \Xi) = \int_{\Omega_\mathrm{H}} \mathbf{G}_\mathrm{i}(\mathrm{x})\nabla V_\mathrm{m}(\mathrm{x}, t) \cdot \nabla Z(\mathrm{x}, \Xi)\, \mathrm{dx},$$

where $Z(\mathrm{x}, \Xi)$ is the lead field.
The formula is linear in Z, the only random variable. Hence, we focus on statistics of Z.


Lead Field

▶ **Lead field problem**

$$\begin{cases} -\mathrm{div}(\mathbf{G}\nabla Z) = 0, & \text{in torso,} \\ -\mathbf{G}\nabla Z \cdot \mathbf{n} = \sum_i^L a_i \delta(\mathrm{x} - \xi_i), & \text{on chest.} \end{cases}$$

Note: the function $Z(\mathrm{x}, \Xi)$ is computed once!

▶ **Expectation**
By linearity, we solve as above by replacing

$$\begin{aligned} V(\mathrm{x}, \Xi) &\rightsquigarrow \mathbb{E}[V](t) \\ Z(\mathrm{x}, \Xi) &\rightsquigarrow \mathbb{E}[Z](\mathrm{x}) \\ \delta(\mathrm{x} - \xi_i) &\rightsquigarrow \rho_i(\mathrm{x}) \quad \text{(marginal distribution)} \end{aligned}$$

Computation of ECG time correlation

▶ **Tensor product**
The correlation of the ECG is obtained by tensor product:

$$\mathrm{Cor}[V](t, s) = \int_{\Omega_\mathrm{H}^2} (\mathbf{G}_\mathrm{i}\nabla \otimes \mathbf{G}_\mathrm{i}\nabla) V_\mathrm{m}(\mathrm{x}, t) V_\mathrm{m}(\mathrm{x}', s) : (\nabla \otimes \nabla)\, \mathrm{Cor}[Z]\, \mathrm{dx dx}'.$$

The correlation $\mathrm{Cor}[Z](\mathrm{x}, \mathrm{x}')$ of the lead field will satisfy a problem obtained again from the tensor product of the lead field equation. The problem size is squared.

▶ **Low-rank approximation**
Since electrodes distribution is highly localized, we approximate the correlation with a low-rank tensor

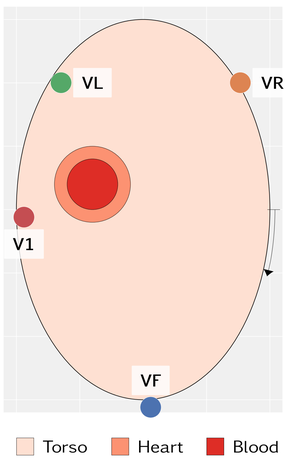
$$\mathrm{Cor}[Z](\mathrm{x}, \mathrm{x}') \approx \sum_{k=1}^{K} \zeta_k(\mathrm{x})\zeta_k(\mathrm{x}'), \qquad K \text{ small.}$$

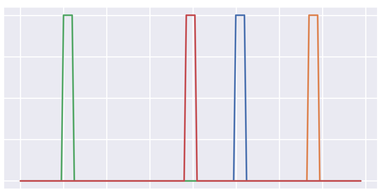
▶ **Numerical solution**
In total, computing $\mathrm{Cor}[Z]$ amounts to the solution of K elliptic problems equivalent to the lead field problem (with different Neumann data.)

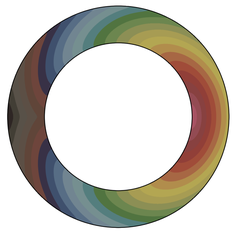
Numerical assessment

Problem setup: Idealized 2-D torso and an 2-lead ECG:

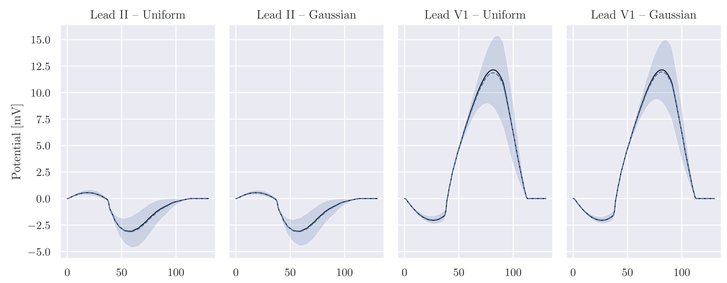
$$\text{Lead II} = \text{VF} - \text{VL}, \qquad \text{Lead V1} = \text{V1} - \tfrac{1}{3}\big(\text{VL} + \text{VR} + \text{VF}\big)$$




▲ Pdfs along curvilinear coordinate


▲ Activation map via eikonal

ECG results for all tests. In the plots, the dashed black curve is the deterministic ECG, the solid blue curve is the average ECG, and the shaded blue area corresponds to the 95 % confidence interval, that is $\mathbb{E}[V](t) \pm 1.96\sqrt{\mathrm{Var}[V](t)}$.



▶ **Low-rank modes** were 17 (lead II) and 33 (lead V1).
▶ **Max std deviation** was 0.83 mV (lead II) and 2.07 mV (lead V1).
▶ **High accuracy** to bidomain (error < 0.01 mV).
▶ **Computational cost** was 2- to 8-fold lower than bidomain.

Final remarks

▶ We have solved the problem of quantifying the uncertainty in the ECG when electrode positions is uncertain.
▶ Our method recasts the problem into a **fully deterministic setting** by using the **lead field** theory and a **low-rank approximation** for the correlation.
▶ **Computationally cheaper** than bidomain model, cost independent from number of evaluations of V_m.
▶ Suitable to compute ECGs for **long simulations**, e.g., arrhythmic events, and **inverse problems** involving the ECG.

Try yourself on Jupyter!
https://github.com/pezzus/fimh2021.
(Full paper also available.)

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