Technical Note: INS Integration Before and After GNSS Epoch

Paul F. Roysdon, Ph.D.

I. Numerical Integration

Between GPS epochs, it is computationally efficient to use the 1^{st} order Euler integration from $\tilde{u}_1, \tilde{u}_2, ..., \tilde{u}_{i-1}$. However at the time event of the GPS epoch, both a forward integration from $\tilde{u}_i$ to the epoch ρ_k, and backward integration from $\tilde{u}_0$ to the epoch ρ_k must be performed. A numerically stable and efficient method is the Runge-Kutta 4^{th} order (RK4) solver.

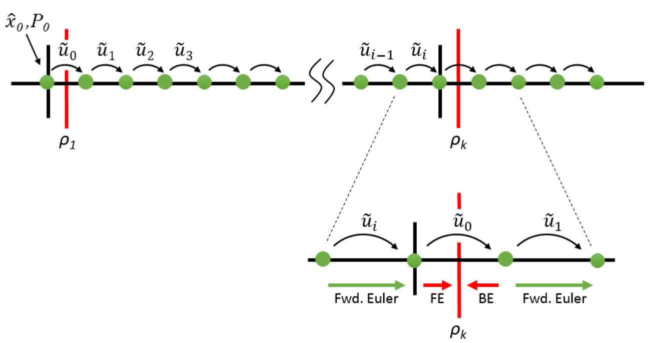

Fig. 1. GPS-INS integration **to** the GPS measurement epoch, and **from** the GPS measurement epoch.

In the Euler method, the local truncation error (error per step) is proportional to the square of the step size, and the total accumulated error (error at a given time) is proportional to the step size. The 1^{st} order Euler method solves an ordinary differential equation (ODE) for an initial value problem (IVP). For an unknown function y and time t, we are told that $\dot{y}$ converges at the rate at which y changes, given an initial time t_0 and y-value y_0, e.g.

$$\dot{y}(t) = f(t, y(t)), \quad y(t_0) = y_0 \tag{1}$$

if a value $h > 0$ is chosen for the step size, then set $t_n = t_0 + nh$. Then, one step n of the Euler method from t_n to $t_n + 1 = t_n + h$ is

$$y_{n+1} = y_n + f(t_n, y_n), \tag{2}$$

where the value of y_n is an approximate solution to the ODE at time t_n.

Alternatively, the RK4 is a 4^{th} order solver, meaning that the local truncation error is on the order of $\mathcal{O}(h^5)$ while the total accumulated error is of order $\mathcal{O}(h^4)$.

From the initial value problem specified in eqn. (1)

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4), \tag{3}$$

$$t_{n+1} = t_n + h \tag{4}$$

for $n = 0, 1, 2, 3, ...$ using

$$k_1 = f(t_n, y_n) \tag{5}$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1) \tag{6}$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2) \tag{7}$$

$$k_4 = f(t_n + h, y_n + hk_3) \tag{8}$$

Here y_{n+1} is the RK4 approximation of $y(t_{n+1})$, and the next value (y_{n+1}) is determined by the present value (y_n) plus the weighted average of four increments, where each increment is the product of the size of the interval, h, and an estimated slope specified by function f on the right-hand side of the differential equation.

- k_1 is the increment based on the slope at the beginning of the interval, using y, (Euler's method)
- k_2 is the increment based on the slope at the midpoint of the interval, using $y + \frac{h}{2}k_1$
- k_3 is again the increment based on the slope at the midpoint, but now using $y + \frac{h}{2}k_2$
- k_4 is the increment based on the slope at the end of the interval, using $y + hk_3$.