GPS-INS Multipath Accommodation using a Sliding Window Filter

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***Abstract*— Many highway vehicle applications require reliable, high precision navigation (error less than meter level) while using low-cost consumer-grade inertial and global navigation satellite systems (GNSS). The application environment causes numerous GNSS measurement outliers. Common implementations use a single epoch Extended Kalman Filter (EKF) combined with the Receiver Autonomous Integrity Monitoring (RAIM) for GNSS outlier detection. However, if the linearization point of the EKF is incorrect or if the number of residuals is too low, the outlier detection decisions may be incorrect. False alarms result in good information not being incorporated into the state and covariance estimates. Missed detections result in incorrect information being incorporated into the state and covariance estimates. Either case can cause subsequent incorrect decisions, possibly causing divergence, due to the state and covariance now being incorrect. This article formulates a sliding window estimator containing multiple GNSS epochs, and solves the full-nonlinear *Maximum A Posteriori* estimate in real-time. By leveraging the resulting window of residuals, an improved fault detection and removal strategy is implemented. Experimental sensor data is used to demonstrate the interval RAIM (iRAIM) performance improvement.**

I. Introduction

The past decade has seen the rapid rise and adoption of navigation systems on automobiles, unmanned vehicles, and personal mobile devices such as smartphones. These systems can exhibit very good accuracy (e.g. sub-meter error). However, further improvements in the reliability and continuity of this accuracy are required to fully support autonomous vehicle operations, especially in urban environments, where variations in the operating conditions and direct signal path can have critical effects. To design a reliable, high-performance system, it is critical to detect and remove outlier measurements before they degrade performance. In GNSS applications such outlier measurements can be caused by multi-path, non-line of sight signals, or overhead foliage.

In the urban canyon or similar natural structure, multipathing of the GPS signals can lead to position errors up to several meters. The standard EKF formulation cannot properly account for the errors. This article further develops the sliding window smoothing estimator of [7], [19], [20] by incorporating of 12 Satellite Vehicle (SV) correlated noise (e.g. multipath) states, one for each SV tracked. This is the first known literature report of a GPS-INS sliding window smoothing estimator with multipath state estimation.

II. Background and Notation

This section introduces Global Positioning System (GPS) aided inertial navigation system (INS) background [11].

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A. Aided Inertial Navigation

Let $\boldsymbol{x} \in \mathbb{R}^{n_s}$ denote the rover state vector, where

$$\boldsymbol{x}(t) = [\mathbf{p}^\intercal(t), \mathbf{v}^\intercal(t), \mathbf{q}^\intercal(t), \mathbf{b}_a^\intercal(t), \mathbf{b}_g^\intercal(t), \mathbf{b}_\nu^\intercal(t)]^\intercal \in \mathbb{R}^{n_s},$$

where $\mathbf{p}$, $\mathbf{v}$, $\mathbf{b}_a$, $\mathbf{b}_g$ each in $\mathbb{R}^3$ represent the position, velocity, accelerometer bias and gyro bias vectors, respectively, $\mathbf{q} \in \mathbb{R}^4$ represents the attitude quaternion, and $\mathbf{b}_\nu \in \mathbb{R}^{12}$ represents the bias due to correlated-noise ($n_s = 28$).

The kinematic equations for the rover state are

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t), \mathbf{u}(t)), \tag{1}$$

where $\boldsymbol{f} : \mathbb{R}^{n_s} \times \mathbb{R}^6 \mapsto \mathbb{R}^{n_s}$ represents the kinematics, and $\mathbf{u} \in \mathbb{R}^6$ is the vector of specific forces and angular rates. The function $\boldsymbol{f}$ is accurately known (see eqn. 11.31-11.33 in [11], derivations specific to this article are provided in [12]). Nature integrates eqn. (1) to produce $\boldsymbol{x}(t)$.

Let τ_i denote the time instants at which IMU measurements are valid. Assume there is a prior for the initial state: $\boldsymbol{x}(t_0) \sim \mathcal{N}(\boldsymbol{x}_0, \mathbf{P}_0)$. Given the initial condition $\boldsymbol{x}_0$ and the IMU measurements $\tilde{\mathbf{u}}(\tau_i) = \mathbf{u}(\tau_i) + \mathbf{b}(\tau_i) + \boldsymbol{\omega}_u(\tau_i)$, with additive stochastic errors $\boldsymbol{\omega}_u(\tau_i) \sim \mathcal{N}(\mathbf{0}, \mathbf{Qd})$ and $\mathbf{b} = [\mathbf{b}_a^\intercal, \mathbf{b}_g^\intercal]^\intercal$, a navigation system propagates an estimate of the vehicle state as the solution of

$$\dot{\hat{\boldsymbol{x}}}(t) = \boldsymbol{f}(\hat{\boldsymbol{x}}(t), \tilde{\mathbf{u}}(t)), \tag{2}$$

where $\hat{\boldsymbol{x}}(t)$ denotes the real-time estimate of $\boldsymbol{x}(t)$.

The solution of eqn. (2) over the interval $t \in [\tau_{i-1}, \tau_i]$ from the initial condition $\boldsymbol{x}_{i-1}$ is represented as the operator:

$$\phi(\boldsymbol{x}_{i-1}, \mathbf{u}_{i-1}) = \boldsymbol{x}_{i-1} + \int_{\tau_{i-1}}^{\tau_i} \boldsymbol{f}(\boldsymbol{x}(\tau), \mathbf{u}(\tau))\, d\tau \tag{3}$$

where $\hat{\boldsymbol{x}}_{i+1} = \phi(\hat{\boldsymbol{x}}_{i-1}, \hat{\mathbf{u}}_{i-1})$, with $\hat{\mathbf{u}}_{i-1} = \tilde{\mathbf{u}}_{i-1} - \hat{\mathbf{b}}_{i-1}$. Define $\mathbf{U}_{k-1} = \{\tilde{\mathbf{u}}(\tau_i) \text{ for } \tau_i \in [t_{k-1}, t_k]\}$. The integral operator in eqn. (3) can be iterated for all IMU measurements in $\mathbf{U}_k$ to propagate the state from t_{k-1} to t_k: $\hat{\boldsymbol{x}}_k = \boldsymbol{\Phi}(\hat{\boldsymbol{x}}_{k-1}, \mathbf{U}_{k-1})$. It is shown in [12] that $\hat{\boldsymbol{x}}_k - \boldsymbol{\Phi}(\hat{\boldsymbol{x}}_{k-1}, \mathbf{U}_{k-1}) = \mathbf{w}_k$ can be modeled with covariance $\mathbf{Q}_{\mathbf{D}k}$.

B. GPS Model

For notational simplicity, it is assumed that the double difference approach removes all common-mode errors (e.g., ionosphere, troposphere, satellite clock and ephemeris errors), as well as the receiver clock biases. Let $t_k = kT$ denote the time instants at which GPS measurements are valid, and $\boldsymbol{x}_k$ denote the state at $\boldsymbol{x}(kT)$. It is typically the case that $T \gg [\tau_i - \tau_{i-1}]$. Therefore, there are numerous IMU measurements available between GPS epochs.

For $(m+1)$ satellites, $\mathbf{y}_k$ represents the double-differenced code (pseudorange) measurement vector, as defined in Section 8.8 of [11]. The double-differenced measurement vector at t_k is modeled as

$$\mathbf{y}_k = \mathbf{h}_k(\boldsymbol{x}_k) + \boldsymbol{\nu}_k + \boldsymbol{\eta}_k \tag{4}$$

where $\mathbf{y}_k,\ \boldsymbol{\nu}_k,\ \boldsymbol{\eta}_k \in \mathbb{R}^m$. The symbol $\boldsymbol{\eta}_k \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\sigma}_\eta^2 \mathbf{I})$ represents the pseudorange measurement noise with $\boldsymbol{\sigma}_\eta = 0.1 \sim 3m$. The symbol $\boldsymbol{\nu}_k \sim \mathcal{N}(\mathbf{0}, \boldsymbol{\sigma}_\nu^2 \mathbf{I})$ represents the time-correlated measurement noise (e.g. multipath) with $\boldsymbol{\sigma}_\nu = 3 \sim 10m$. Depending on receiver design, environmental factors and the performance of multipath mitigation techniques, the noise level $\boldsymbol{\sigma}_\eta$ and correlated noise $\boldsymbol{\sigma}_\nu$ can vary for each available satellite. The symbol $\mathbf{R} = \text{blkdiag}(\text{Cov}(\boldsymbol{\eta}_k), \text{Cov}(\boldsymbol{\nu}_k))$. Using the state estimate, the GPS measurements at t_k are predicted to be

$$\hat{\mathbf{y}}_k = \mathbf{h}_k(\hat{\boldsymbol{x}}_k).$$

The GPS measurement residual vector is computed as $\delta\mathbf{y}_k = \mathbf{y}_k - \hat{\mathbf{y}}_k$.

III. Estimation Theory

For a known linear system with white, normally distributed, and mutually uncorrelated process and measurement noise vectors with known covariance, the Kalman filter (KF) is the optimal estimator. When the time propagation or measurement models are nonlinear, a variety of methods (e.g., the extended Kalman filter [13]) are available to solve the sensor fusion problem over a single GPS epoch.

This section reviews the *Maximum A Posteriori* estimator [14] solved over a sliding temporal window in real-time. This approach has been developed extensively in the Simultaneous Localization and Mapping (SLAM) research community [15], [16], [17], [18]. The approach developed for GNSS and IMU integration in [7] is referred to as a Contemplative Real Time (CRT) method due to its enhanced ability to accommodate outliers in real-time. That ability was demonstrated in [19], [20], however the experimental results demonstrated correlated noise in the GPS residuals. Accounting for GPS correlated noise is developed and demonstrated herein.

A. Theoretical Solution

Let $\mathbf{X}$ denote the vehicle trajectory over a sliding time window $\mathbf{X} = [\boldsymbol{x}(t_{k-L})^\intercal,\ \ldots\ ,\ \boldsymbol{x}(t_k)^\intercal]^\intercal$, where L is the length of the window, and contains L GPS measurement epochs, $[\mathbf{y}_{k-L+1}, \ldots, \mathbf{y}_k]$. We assume that the window will slide one epoch upon arrival of each new GPS measurement. For presentation purposes only, we assume that each GPS epoch aligns with an IMU measurement time. The results in the experimental section relax this assumption.

Estimation of the vehicle trajectory $\mathbf{X}$ can be formulated as a MAP problem (see Ch. 11.5 of [14]):

$$\hat{\mathbf{X}} = \underset{\mathbf{X}}{\text{argmax}} \left\{ p(\mathbf{X}, \mathbf{U}, \mathbf{Y}) \right\}, \tag{5}$$

where within the time window $\mathbf{U} = \{\mathbf{U}_i \mid i \in [k-L, k-1]\}$, and $\mathbf{Y} = \{\mathbf{y}_j \mid j \in [k-L+1, k]\}$ is the set of GPS measurements over the time window for satellites $1, ..., m$. The joint probability for the GPS-INS problem, $p(\mathbf{X}, \mathbf{U}, \mathbf{Y})$, can be factored as

$$\begin{aligned} & p(\mathbf{X}, \mathbf{U}, \mathbf{Y}) \\ &= p(\mathbf{X}, \mathbf{U}) p(\mathbf{Y} \mid \mathbf{X}, \mathbf{U}) \\ &= p(\mathbf{X}, \mathbf{U}) p(\mathbf{Y} \mid \mathbf{X}) \\ &= p(\mathbf{X}, \mathbf{U}) \prod_{j=k-L+1}^{k} p(\mathbf{y}_j | \boldsymbol{x}_j) \\ &= p(\boldsymbol{x}_{k-L}) \prod_{l=k-L}^{k-1} p(\boldsymbol{x}_{l+1} | \boldsymbol{x}_l, \mathbf{U}_l) \prod_{j=k-L+1}^{k} p(\mathbf{y}_j | \boldsymbol{x}_j), \end{aligned} \tag{6}$$

where $p(\boldsymbol{x}_{k-L})$ is the distribution of the initial condition for the time window, $p(\boldsymbol{x}_{l+1} | \boldsymbol{x}_l, \mathbf{U}_{l+1})$ is the distribution of the IMU measurement noise, $p(\mathbf{y}_j | \boldsymbol{x}_j)$ is the distribution of the GPS measurement noise.

B. Numerical Solution

Assume that $\boldsymbol{x}(t_k - L)$, $\boldsymbol{\omega}_u$, and $\boldsymbol{\eta}$ have Gaussian distributions with positive definite covariance matrices $\mathbf{P}_{(k-L)}$, $\mathbf{Q}_\mathbf{D}$, and $\mathbf{R}$, respectively. Let $\mathbf{W} = \text{blkdiag}(\mathbf{P}_{(k-L)}, \mathbf{Q}_\mathbf{D}, \mathbf{R})$. Then $\|\mathbf{v}\|_\mathbf{W}^2 = \mathbf{v}^\intercal \mathbf{W}^{-1} \mathbf{v}$ represents the squared Mahalanobis norm.

Finding $\mathbf{X}$ that maximizes eqn. (6) is identical to minimizing the negative of its natural logarithm. This yields the equivalent nonlinear cost function:

$$\begin{aligned} \|\mathbf{v}(\mathbf{X})\|_\mathbf{W}^2 = & \|\hat{\boldsymbol{x}}_{k-L} - \boldsymbol{x}(t_{k-L})\|_{\mathbf{P}_{(k-L)}}^2 \\ & + \sum_{l=k-L}^{k-1} \|\boldsymbol{\Phi}\big(\boldsymbol{x}(t_l), \mathbf{U}_l\big) - \boldsymbol{x}(t_{l+1})\|_{\mathbf{Q}_\mathbf{D}}^2 \\ & + \sum_{j=k-L+1}^{k} \|\mathbf{y}(t_j) - \mathbf{h}_j\big(\boldsymbol{x}(t_j)\big)\|_\mathbf{R}^2. \end{aligned} \tag{7}$$

The cost function can be normalized using Cholesky Decomposition. For the positive definite matrix $\mathbf{W}$, defining $\boldsymbol{\Sigma}_\mathbf{W}$, such that $\mathbf{W}^{-1} = \boldsymbol{\Sigma}_\mathbf{W}^\intercal \boldsymbol{\Sigma}_\mathbf{W}$. Then, for $\mathbf{r} \triangleq \boldsymbol{\Sigma}_\mathbf{W} \mathbf{v}$, $\|\mathbf{v}\|_\mathbf{W} = \|\mathbf{r}\|_2$. The minimization problem of eqn. (7) reduces to the standard nonlinear least squares optimization

$$\min_{\mathbf{X} \in \mathbb{R}^{n_s(L+1)}} \|\mathbf{r}(\mathbf{X})\|_2^2$$

which will be solved iteratively.

Consider the l^{th} iteration of the optimization, where l is a positive integer. Given an estimate of the solution

$$\hat{\mathbf{X}}^l = \left[\hat{\boldsymbol{x}}^l(t_{k-L})^\intercal,\ \ldots\ ,\ \hat{\boldsymbol{x}}^l(t_k)^\intercal\right]^\intercal,$$

which is treated as a vector in $\mathbb{R}^{n_s(L+1)}$. The optimization algorithm computes an error vector $\delta\mathbf{X}^l \in \mathbb{R}^{n_e(L+1)}$, which corrects $\hat{\mathbf{X}}^l$ to yield an improved solution $\hat{\mathbf{X}}^{l+1}$ to eqn. (7). The dimension of the error state vector is n_e. The error state vector is $\delta\boldsymbol{x} = [\delta\mathbf{p}^\intercal,\ \delta\mathbf{v}^\intercal,\ \delta\boldsymbol{\theta}^\intercal,\ \delta\mathbf{b}_a^\intercal,\ \delta\mathbf{b}_g^\intercal,\ \delta\mathbf{b}_\nu^\intercal]^\intercal \in \mathbb{R}^{27}$, where $\delta\mathbf{p}$, $\delta\mathbf{v}$, $\delta\boldsymbol{\theta}$, $\delta\mathbf{b}_a$, and $\delta\mathbf{b}_g$ each in $\mathbb{R}^3$ are the position, velocity, attitude, accelerometer bias and gyro bias error vectors, respectively. The symbol $\delta\mathbf{b}_\nu \in \mathbb{R}^{12}$ represents the error vector of the bias due to correlated-noise. The dynamics and stochastic properties of this estimation error vector are well understood, and can be found in Section 11.4 of [11]. The fact that $n_s = 28$ and $n_e = 27$ is discussed in [12]. The optimization approach is formulated in the following section.

C. Optimization: Iterated Solution

In the l-th iteration, the *residual* $\mathbf{r}$ linearized around the current estimate $\hat{\mathbf{X}}^l$ is

$$\mathbf{r}(\mathbf{X}) = \mathbf{J}(\hat{\mathbf{X}}^l)\delta\mathbf{X}^l + \boldsymbol{\eta}_{\mathbf{r}}, \tag{8}$$

where $\mathbf{J}(\hat{\mathbf{X}}^l)$ is the Jacobian of $\mathbf{r}(\mathbf{X})$ evaluated at $\hat{\mathbf{X}}^l$, and $\boldsymbol{\eta}_{\mathbf{r}} \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$. In the following, without loss of generality, we consider one iteration of the optimization method and drop the iteration counter 'l'.

With eqn. (8), a convex optimization problem can be formulated by a quadratic approximation $\mathbf{L}(\delta\mathbf{X})$ to the cost function $\mathscr{C}(\mathbf{X}) \triangleq \|\mathbf{r}(\hat{\mathbf{X}})\|_2^2$:

$$\mathbf{L}(\delta\mathbf{X}) = \frac{1}{2}\|\mathbf{r}(\hat{\mathbf{X}}) - \mathbf{J}(\hat{\mathbf{X}})\delta\mathbf{X}\|_2^2. \tag{9}$$

By minimizing $\mathbf{L}(\delta\mathbf{X})$, a candidate step for the estimation error $\delta\mathbf{X}$ is obtained. A line search in the direction of $\delta\mathbf{X}$ is used to update the state estimate.

The Gauss-Newton step is the solution of the normal equation,

$$\mathbf{J}^\intercal\mathbf{J}\delta\mathbf{X} = \mathbf{J}^\intercal\mathbf{b}, \tag{10}$$

where $\mathbf{b} \triangleq \mathbf{r}(\hat{\mathbf{X}})$ and $\mathbf{J} = \mathbf{J}(\hat{\mathbf{X}}^l)$ as defined in eqn. (8). Then eqn. (10) can be compactly expressed as

$$\boldsymbol{\Lambda}\delta\mathbf{X} = \boldsymbol{\xi}, \tag{11}$$

where $\boldsymbol{\Lambda} = \mathbf{J}^\intercal\mathbf{J}$ is the information matrix, $\boldsymbol{\xi} = \mathbf{J}^\intercal\mathbf{b}$ is the information vector. The matrix $\mathbf{J}$ is sparse; therefore, eqn. (11) can be solved efficiently by many methods, e.g. Cholesky, or QR. Further computational gains can be achieved by employing a sparse matrix library as discussed in [15], [18]. The computational complexity of the algorithm is discussed in [12].

IV. State Augmentation

V. Illustrative Example

Real-world performance is evaluated using data from a drive-test around University of California, Riverside campus using a consumer-grade GPS antenna (Antcomm ANN-MS-0-005) mounted on the vehicle roof. During driving, the sensor data is time-stamped and stored. The sensor data includes consumer-grade: Quartz-MEMS IMU data (Epson M-G320) at $250Hz$, and L1 GPS data (Ublox 6T) at $1Hz$. Differential corrections were obtained from the UCR base-station NTRIP caster (`ntrip.engr.ucr.edu`) in real-time via cellular connection. This trajectory contains a variety of real-world automotive conditions that adversely affect GPS receiver performance, e.g. tall buildings and trees.

The experimental data *ground truth* trajectory is found by solving a nonlinear optimization problem over the entire (600 second) trajectory, formulated in the maximum a posteriori perspective. This smoother uses integer resolved carrier phase DGPS and IMU measurements, to achieve centimeter level accuracy [21].

Synthetic-data performance is evaluated using a simulated 600 second automotive trajectory generated by a trajectory planner and a 6DOF kinematic model. A signal generator utilizes the trajectory to produce both ground truth and noise-corrupted sensor "measurements". The simulated trajectory approximated the real-world trajectory discussed above. The IMU signal generator models a triad gyroscope and a triad accelerometer. Tactical-grade IMU measurements were generated at $250Hz$ according to the sensor model in Ch. 11.6 of [11]. GPS signal generator models L1 & L2 C/A pseudoranges and Doppler for both the vehicle and base station. Measurements were generated at $1Hz$ according to the models in Ch. 8.2 and 8.3 of [11]. Satellite vehicle orbits were produced from Receiver Independent Exchange (RINEX) files downloaded on January 10, 2017 from a Continuously Operating Reference Station (CORS) server [22].

To allow direct comparison of the performance of various algorithms, using the identical input data, the results of this section are computed during post-processing. Even though running in post-processing for this evaluation, each algorithm is written in C++ to run in real-time, using only the data and prior as would be applicable for each approach. The navigation algorithms being compared only use L1 GPS with differential corrections, and IMU data.

Due to limited space, only 3D position performance is discussed herein. Velocity and attitude results are provided in [12], and are similar to position performance.

VI. Conclusion

This article presented two methods to enhance the level of redundancy in a GNSS and IMU based navigation system to facilitate the accommodation of outlier measurements. Over a multiple epoch sliding window of data the algorithm performs MAP estimation within a nonlinear optimization framework, while maintaining a real-time estimate as necessary for control and planning purposes. Increasing the duration L of the sliding window enhances redundancy at the expense of increased computation. Enhancing redundancy improves the reliability of achieving any given accuracy specification, by better outlier removal. The MAP framework, through real-time nonlinear optimization, achieves optimal state estimation without linearization assumptions. The enhanced performance of these methods is demonstrated through direct comparisons of both the accuracy and outlier detection abilities of various algorithms using experimental data from a challenging environment.

Related areas of interest for future research include accommodation of time correlated errors (e.g., multipath) either by augmented states or non-diagonal covariance matrices (e.g. $\mathbf{R}$), adaptation of the LSS threshold λ to minimize the risk of missing outliers while guaranteeing a desired level of expected accuracy, and adaptation of the window length L again trading off risk and performance.

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