NOT PART OF THE MANUSCRIPT

— Online Appendix —

Included for reference convenience of the referees/editors

Appendix

In Part A, we show the original nonlinear model and derive the linear-quadratic representation, which is also done in Benigno and Benigno (2006).[31] In Part B, we derive the equilibrium conditions (presented in the main paper) for each alternative policy regime. In Part C, we revisit the traditional Cooperation versus Non-Cooperation comparisons and provide additional analyses and explanations for the results reported in the paper. In particular, we also discuss an alternative unconditional welfare decomposition, which allows us to have additional insights into how important each variable in the system contributes to absolute and unconditional welfare within each of the Cooperation and Non-cooperation regimes. In Part D, we derive the characterization of the main focus of the paper: the Sustainable Cooperation equilibrium. We also provide its computation procedure in more detail here. As in the other two regimes, here we also perform an alternative unconditional welfare decomposition exercise for the Sustainable Cooperation regime. Part E provides the derivations underlying the Pareto set analyzed in paper. Part F considers another experiment where we vary country size while holding all else equal.

A A Microfoundation and Welfare Derivations

In this appendix, we provide the microfoundations and second-order welfare derivations of the well-known model of Benigno and Benigno (2006) (BB). We will also discuss on a particular type of Ramsey policy-making assumption (i.e., with timeless-perspective commitment) underlying all three policy regimes considered in the paper, and the role this plays in a correct quadratic approximation of the social welfare functions in the LQ framework.

A.1 Model microfoundation

The model underlying the competitive equilibrium characterization is as follows: There are two countries—Home and Foreign. In each country, there is a representative household. Each household consumes bundles of differentiated goods pro-

[31] Benigno and Benigno (2006) considered more general cases than ours since the authors were interested in studying open-economy targeting rules in the face of many shocks. These additional complications are set aside here: We consider a setting with onlymarkup shocks and symmetric country size in the baseline case, as this is sufficient to understand the policy incentive problems at heart in the model.

duced in Home and Foreign countries. Each household also provides firm-specific labor to firms within the country. Firms in each country produce differentiated goods under monopolistic competition and sticky prices, given the demand function of the households in both countries. For clarity of exposition in the main text of this paper, we will focus mainly on the case where both countries are symmetric in size.[32]

There are internationally complete markets for state-contingent consumption claims and the law of one price holds for all goods. As in BB, these two assumptions help to simplify the equilibrium descriptions later: The real exchange rate is unity and consumption is equalized between the two countries. Each country also has a monetary policymaker who maximizes a social welfare function, given the equilibrium conditions of the whole economy. In order to isolate our focus on incentive-feasibility problems in terms of international monetary policy cooperation, we abstract from time-consistency issues within each country. In particular, we assume that each country's policymaker commits to maximizing its own citizen's ex-ante welfare.[33]

To discipline our analyses, we restrict attention to equilibria under the following settings: Consumption is the only component of GDP, the two countries are symmetric in terms of taste, technology and market sizes, and the steady state markup is unity.[34] Without loss of generality, we assume that the two countries are also symmetric in terms of their initial levels of assets. We further assume that the elasticity of substitution between domestic and foreign goods is equal to one. Under these assumptions, BB showed that in response to technology shocks, (i) the flexible-price allocation is constrained optimal under cooperation, and (ii) there are no gains by deviating from cooperation to non-cooperation. These results also hold in our model. Given this insight, we will only focus on inefficient markup shocks as the only sources of policy incentive to cooperate or not.[35]

In Section 2.2, we present and discuss the relevant social welfare criteria relevant to the three policy regimes to be considered—Cooperation, Non-cooperation,

[32]In a more general setting of this model (see Section F), Home's country size will be indexed by, and is increasing in, a parameter $\gamma \in (0, 1)$. On the flipside, Foreign's size is $1 - \gamma$. The two countries are symmetric in size if $\gamma = 1/2$.

[33]We adopt the timeless perspective to derive the LQ approximation (Benigno and Woodford 2005, 2012). This allows us to restrict attention to equilibrium (policy trade-off) conditions that are time-invariant.

[34]This is achieved by assuming subsidies that eliminate positive rents in the steady state. Thus, a markup shock in this paper can also be interpreted as a structural shock to this subsidy.

[35]As in the case of the closed economy, markup shocks generate a trade-off between inflation and the output gap represented in the NK Phillips curve.

and Sustainable Cooperation.[36] The criterion function will turn out to be the same in the Cooperation and Sustainable Cooperation problems. The social welfare in each setting considered will be representable by a purely quadratic function, which accurately approximates the indirect utility of the representative household in each country up to second order.[37]

A.1.1 Household

There is a representative household in each country. We focus on the household in the Home country. By symmetry, the same applies to the household in the Foreign country. The domestic household minimizes its total expenditure

$$P_t C_t = P_{Ht} C_{Ht} + P_{Ft} C_{Ft},$$

subject to the aggregator $C_t = \gamma^{-\gamma}(1-\gamma)^{-(1-\gamma)} (C_{H,t})^{\gamma} (C_{F,t})^{1-\gamma}$, where C_t is total consumption (per capita), C_H and C_F are bundles of consumption goods produced in Home and Foreign countries. The consumer price index is denoted as P_t. The price indices, respectively, P_H and P_F, relate to goods produced in the Home and Foreign countries. The parameter $\gamma \in [0, 1]$ is the size of Home country. Therefore, $(1-\gamma)$ is the size of Foreign country. By inspecting the consumption index above, we can see that a larger γ implies a "smaller" Home country in that its agents are more dependent on Foreign-produced goods. The FONCs are

$$C_{H,t} = \gamma \left(\frac{P_{H,t}}{P_t} \right)^{-1} C_t,$$
$$C_{F,t} = (1-\gamma) \left(\frac{P_{F,t}}{P_t} \right)^{-1} C_t.$$

They yield the demand function for each bundle of consumption of the domestic household. By substituting them into the aggregator, the price index is can be derived as $P_t = P_{H,t}^{\gamma} P_{F,t}^{1-\gamma}$.

The household minimizes expenditure on bundles of Home and Foreign goods subject to the aggregators $C_{H,t} = \left[\left(\frac{1}{\gamma} \right)^{\frac{1}{\sigma}} \int_0^{\gamma} C_{H\,t}(h)^{\frac{\sigma-1}{\sigma}} \mathrm{d}h \right]^{\frac{\sigma}{\sigma-1}}$ and $C_{F,t} = \left[\left(\frac{1}{1-\gamma} \right)^{\frac{1}{\sigma}} \int_{\gamma}^{1} C_{F\,t}(f)^{\frac{\sigma-1}{\sigma}} \mathrm{d}f \right]^{\frac{\sigma}{\sigma-1}}$, where σ is the elasticity of substitution between differentiated products à la Dixit

[36] Note that we will use upper-case letters when referring to the names of these regimes.

[37] The competitive equilibrium conditions are approximate linear constraints, representing the optimizing behavior of households and firms, hence the LQ approach. However, when we consider the case of Sustainable Cooperation, the problem is no longer a standard LQ problem, since the sustainability constraints, albeit involving quadratic forms, will only be occasionally binding.

and Stiglitz (1977). There is an infinite number of firms indexed by $i \in [0,1]$, and index $i = h \in [0,\gamma)$ are for the domestic firms and $i = f \in [\gamma, 1.0]$ are for the Foreign firms. Each good, $C_H(i)$ or $C_F(i)$, is produced by firms in both countries. The FONCs to the expenditure minimization problem are

$$C_{H,t}(h) = \frac{1}{\gamma}\left[\frac{P_{H,t}(h)}{P_{H,t}}\right]^{-\sigma} C_{H,t},$$
$$C_{F,t}(f) = \frac{1}{1-\gamma}\left[\frac{P_{F,t}(f)}{P_{F,t}}\right]^{-\sigma} C_{F,t}.$$

Plugging these into the consumption aggregators, we have $P_{H,t} = \left\{\frac{1}{\gamma}\int_0^\gamma [P_{H,t}(h)]^{1-\sigma} \mathrm{d}h\right\}^{\frac{1}{1-\sigma}}$ and $P_{F,t} = \left\{\frac{1}{1-\gamma}\int_\gamma^1 [P_{F,t}(f)]^{1-\sigma} \mathrm{d}f\right\}^{\frac{1}{1-\sigma}}$ as the price indices of Home and Foreign goods, respectively.

Given the above market structure, the domestic household maximizes its lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{1}{\gamma}\int_0^\gamma \frac{h_t(h)^{1+\eta}}{1+\eta}\mathrm{d}h\right],$$

subject to

$$\mathbb{E}_t [m_{t,t+1}A_{t+1}] + D_t + P_t C_t = A_t + (1+i_{t-1})D_{t-1} + \frac{1}{\gamma}\int_0^\gamma W_t(h)h_t(h)\mathrm{d}h + \Pi_t,$$

where $m_{t,t+1}A_{t+1}$ is the purchase of state-contingent securities by the household, which pays A_{t+1} for each state realized in the next period. D_t is the amount of one-period bond, which pays $(1+i_t)D_t$ for any state in the next period. $W_t(h)$ and $h_t(h)$ are firm-specific nominal wage and hours worked. Π_t is the transfer from firms owned by the household. β is the discount factor, ρ is the coefficient of relative risk aversion, η is Frisch elasticity of labor disutility. The FONCs are given by

$$h_t(h)^\eta = \frac{W_t(h)}{P_t} C_t^{-\rho},$$
$$m_{t,t+1} = \beta \frac{C_{t+1}^{-\rho} P_t}{C_t^{-\rho} P_{t+1}},$$
$$C_t^{-\rho} = \beta(1+i_t)\,\mathbb{E}_t\left\{\frac{P_t}{P_{t+1}} C_{t+1}^{-\rho}\right\}.$$

By symmetry, the same results described above also apply for the household in the Foreign country. We denote the variables in the Foreign country with asterisk $(*)$.

A.1.2 Law of one price, complete risk sharing and the terms of trade

As in BB, we assume that the law of one price $P_{H,t}(i) = E_t P^*_{H,t}(i)$ and $P_{F,t}(i) = E_t P^*_{F,t}(i)$ hold for each good $i \in [0,1]$ produced in both of the Home and Foreign countries, where E_t is the nominal exchange rate. This implies $P_t = E_t P^*_t$, $P_{Ht}/P_t = P^*_{Ht}/P^*_t$ and $P_{Ft}/P_t = P^*_{Ft}/P^*_t$. Also, from the international trade of state-contingent securities,

$$
\begin{aligned}
m_{t,t+1} = \frac{C_{t+1}^{-\rho}}{C_t^{-\rho}} \frac{P_t}{P_{t+1}} &= \frac{\left(C^*_{t+1}\right)^{-\rho}}{\left(C^*_t\right)^{-\rho}} \frac{E_t P^*_t}{E_{t+1} P^*_{t+1}}, \\
\Leftrightarrow \quad \left(\frac{C_t}{C^*_t}\right)^{-\rho} \frac{E_t P^*_t}{P_t} &= \left(\frac{C_{t+1}}{C^*_{t+1}}\right)^{-\rho} \frac{E_{t+1} P^*_{t+1}}{P_{t+1}}.
\end{aligned}
$$

Without loss of generality, we assume that countries are initially symmetric. This implies that $(C_t/C^*_t)^{-\rho} E_t P^*_t/P_t = 1$ holds for all states and dates. Combined with the assumption of the law of one price, $C_t = C^*_t$ holds; i.e., complete risk sharing of consumption among countries. Note that in the international economics literature this setting is synonymous with the notion of producer currency pricing (PCP).

Terms of trade for the Home country is defined as $S_t \equiv P_{Ft}/P_{Ht} = P^*_{Ft}/P^*_{Ht}$, which implies $P_t/P_{Ht} = (P_{Ht}/P_{Ft})^{\gamma-1} = S_t^{1-\gamma}$ and $P_t/P_{Ft} = (P_{Ht}/P_{Ft})^{\gamma} = S_t^{-\gamma}$. The market clearing conditions for both countries imply[38]

$$
\begin{aligned}
\gamma Y_{H,t} &\equiv \gamma C_{Ht} + (1-\gamma) C^*_{Ht}, \\
&= \gamma^2 \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + (1-\gamma)\gamma \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} C^*_t, \\
&= \gamma \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t, \\
&= \gamma S_t^{1-\gamma} C_t,
\end{aligned}
$$

and

$$
\begin{aligned}
(1-\gamma) Y^*_{F,t} &\equiv (1-\gamma) C_{Ft} + \gamma C^*_{Ft}, \\
&= (1-\gamma)^2 \left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t + \gamma(1-\gamma) \left(\frac{P^*_{F,t}}{P^*_t}\right)^{-1} C^*_t, \\
&= (1-\gamma) \left(\frac{P_{F,t}}{P_t}\right)^{-1} C_t, \\
&= (1-\gamma) S_t^{-\gamma} C_t.
\end{aligned}
$$

[38] We define $Y_{H,t}$ and $Y^*_{F,t}$ as per capita variables following BB in pp. 478-479.

Then we have

$$S_t = \frac{Y_{H,t}}{Y^*_{F,t}},$$

That is, the terms of trade is determined by the relative output (in terms of per capita) only.

A.1.3 Firms

There is a continuum of firms indexed by $i \in [0,1]$. We focus on the domestic firms $i = h \in [0,\gamma)$. The same results apply for the Foreign firms $i = f \in [\gamma, 1.0]$. Each firm has a linear production technology which transfers firm-specific labor into differentiated good, $Y_t(h) = h_t(h)$. The period-by-period profit for firm producing good h is given by

$$\begin{aligned}
\Pi_t(h) &= P_t(h) Y_t(h) - (1-\tau_t) W_t(h) Y_t(h), \\
&= [P_t(h) - (1-\tau_t) W_t(h)] Y_t(h),
\end{aligned}$$

where τ_t is a subsidy to each firm, which is necessary to eliminate the distortion stemming from monopolistic competition. Note that the market clearing condition for good h implies:

$$\begin{aligned}
Y_t(h) &= \gamma C_{H,t}(h) + (1-\gamma) C^*_{H,t}(h), \\
&= \left[\frac{P_{Ht}(h)}{P_{H,t}}\right]^{-\sigma} C_{H,t} + \left[\frac{P^*_{Ht}(h)}{P^*_{H,t}}\right]^{-\sigma} C^*_{H,t}, \\
&= \gamma \left[\frac{P_{Ht}(h)}{P_{H,t}}\right]^{-\sigma} \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t + (1-\gamma) \left[\frac{P^*_{Ht}(h)}{P^*_{H,t}}\right]^{-\sigma} \left(\frac{P^*_{H,t}}{P^*_t}\right)^{-1} C^*_t, \\
&= \left[\frac{P_{Ht}(h)}{P_{H,t}}\right]^{-\sigma} \left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t.
\end{aligned}$$

Given the demand function, the firm h chooses $\bar{P}_{H,t} = P_{H,t+i}(h)$ for $i > 0$ so as to maximize

$$\mathbb{E}_t \sum_{i=0}^{\infty} \alpha^i m_{t,t+i} [\bar{P}_{H,t} - (1-\tau_{t+i}) W_{t+i}(h)] \underbrace{\left[\frac{\bar{P}_{H,t}}{P_{H,t+i}}\right]^{-\sigma} \left(\frac{P_{H,t+i}}{P_{t+i}}\right)^{-1} C_{t+i}}_{=Y_{t+i}(h)},$$

where α is the probability of fixing prices a la Calvo (1983). The random variable $m_{t,t+i} = \beta^i C^{-\rho}_{t+i} P_t / (C_t^{-\rho} P_{t+i})$ is the stochastic discount factor. Note that $W_{t+i}(h)$ is given for the firm. The optimality condition is

$$\mathbb{E}_t \sum_{i=0}^{\infty} \alpha^i m_{t,t+i} \left[\frac{\bar{P}_{H,t}}{P_{H,t+i}}\right]^{-\sigma} \left(\frac{P_{H,t+i}}{P_{t+i}}\right)^{-1} C_{t+i} \left[\bar{P}_{H,t} - \frac{\sigma}{\sigma-1}\left(1-\tau_{t+i}\right) W_{t+i}\left(h\right)\right] = 0.$$

This can be further transformed in a recursive fashion

$$\begin{aligned}
\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{1+\eta\sigma} F_t &= K_t, \\
F_t &= \mathcal{M}_t^{-1} C_t^{1-\rho} + \alpha\beta\mathbb{E}_t \Pi_{H,t+1}^{\sigma-1} F_{t+1}, \\
K_t &= Y_{H,t}^{1+\eta} + \alpha\beta\mathbb{E}_t \Pi_{H,t+1}^{\sigma(1+\eta)} K_{t+1},
\end{aligned}$$

where $\mathcal{M}_t = (1-\tau_t)\,\sigma/(\sigma-1)$ is a markup shock as a function of exogenous variations τ in the subsidy to firms; and $\Pi_{H,t} = P_{H,t}/P_{H,t-1}$ is the gross domestic inflation rate. Furthermore, the Home price index $P_{H,t} = \left\{\frac{1}{\gamma}\int_0^\gamma \left[P_{H,t}\left(h\right)\right]^{1-\sigma} \mathrm{d}h\right\}^{\frac{1}{1-\sigma}}$ can be written as

$$P_{H,t}^{1-\sigma} = \alpha P_{H\,t-1}^{1-\sigma} + (1-\alpha)\bar{P}_{H,t}^{1-\sigma}.$$

That is, only the $1-\alpha$ fraction of the domestic firms can set the new price $\bar{P}_{H,t}$. It can be further arranged as

$$\frac{\bar{P}_{H,t}}{P_{H,t}} = \left[\frac{1-\alpha\left(\frac{P_{H\,t-1}}{P_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1}{1-\sigma}}.$$

Using the demand function of good h, $Y_t(h) = \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma}\left(\frac{P_{H,t}}{P_t}\right)^{-1} C_t = \left[\frac{P_{H\,t}(h)}{P_{H,t}}\right]^{-\sigma} Y_{H,t}$ and the linear production technology of firm h, $h_t(h) = Y_t(h)$, the disutility from working is

$$\begin{aligned}
& \frac{1}{\gamma}\int_0^\gamma \frac{h_t\left(h\right)^{1+\eta}}{1+\eta}\mathrm{d}h, \\
= \; & \frac{1}{\gamma}\frac{Y_{H,t}^{1+\eta}}{1+\eta}\int_0^\gamma \left[\frac{P_{H,t}\left(h\right)}{P_{H,t}}\right]^{-\sigma(1+\eta)} \mathrm{d}h, \\
= \; & \frac{Y_{H,t}^{1+\eta}}{1+\eta}\Delta_t.
\end{aligned}$$

Also, the Home price dispersion $\Delta_t \equiv \frac{1}{\gamma}\int_0^\gamma \left[\frac{P_{H,t}(h)}{P_{H,t}}\right]^{-\sigma(1+\eta)} \mathrm{d}h \geq 1$ can be further transformed into

$$\begin{aligned}\Delta_t &= \alpha\frac{1}{\gamma}\int_0^\gamma \left[\frac{P_{H,t-1}(h)}{P_{H,t}}\right]^{-\sigma(1+\eta)} \mathrm{d}h + (1-\alpha)\left(\frac{\bar{P}_{H,t}}{P_{H,t}}\right)^{-\sigma(1+\eta)} \\ &= \alpha\left(\frac{P_{H,t-1}}{P_{H,t}}\right)^{-\sigma(1+\eta)} \Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{P_{H\,t-1}}{P_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}.\end{aligned}$$

A.1.4 Equilibrium conditions

The equilibrium conditions in the model described above are

$$Y_{H,t} = S_t^{1-\gamma} C_t, \tag{17}$$

$$\frac{Y_{H,t}}{Y^*_{F,t}} = S_t, \tag{18}$$

$$\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1+\eta\sigma}{1-\sigma}} F_t = K_t, \tag{19}$$

$$F_t = \mathcal{M}_t^{-1} C_t^{1-\rho} + \alpha\beta\mathbb{E}_t \Pi_{H,t+1}^{\sigma-1} F_{t+1}, \tag{20}$$

$$K_t = Y_{H,t}^{1+\eta} + \alpha\beta\mathbb{E}_t \pi \Pi_{H,t+1}^{\sigma(1+\eta)} K_{t+1}, \tag{21}$$

$$\Delta_t = \alpha\left(\frac{1}{\Pi_{H,t}}\right)^{-\sigma(1+\eta)} \Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}, \tag{22}$$

$$\left[\frac{1-\alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{1+\eta\sigma}{1-\sigma}} F^*_t = K^*_t, \tag{23}$$

$$F^*_t = \mathcal{M}_t^{*-1} C_t^{1-\rho} + \alpha\beta\mathbb{E}_t \left(\Pi^*_{F,t+1}\right)^{\sigma-1} F^*_{t+1}, \tag{24}$$

$$K^*_t = \left(Y^*_{F,t}\right)^{1+\eta} + \alpha\beta\mathbb{E}_t \left(\Pi^*_{F,t+1}\right)^{\sigma(1+\eta)} K^*_{t+1}, \tag{25}$$

$$\Delta^*_t = \alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{-\sigma(1+\eta)} \Delta^*_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi^*_{F,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta)}{\sigma-1}}. \tag{26}$$

We have 12 endogenous variables $\{F_t, K_t, \Delta_t, F_t^*, K_t^*, \Delta_t^*, C_t, Y_{H,t}, Y_{F,t}^*, \Pi_{H,t}, \Pi_{F,t}^*, S_t\}$ and 10 Eqs. (17)-(26). Note that consumption Euler equation is redundant in the equilibrium characterization once monetary policies are given. Note that the equilibrium is indeterminate without any policies, due to the lack of 2 ($= 12 - 10$) equilibrium conditions. Monetary policy must be defined to pin down the equilibrium. Policy is then determined by resolving the optimal policy trade-offs under one of the three regimes we consider in the paper: Cooperation, Non-cooperation, or Sustainable Cooperation.

A.1.5 Cooperation and Non-cooperation policies

The policymakers under Cooperation jointly maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \gamma \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_{H,t}^{1+\eta}}{1+\eta} \Delta_t \right] + (1-\gamma) \left[\frac{C_t^{*1-\rho}}{1-\rho} - \frac{Y_{F,t}^{*1+\eta}}{1+\eta} \Delta_t^* \right] \right\}$$

where $\frac{1}{\gamma} \int_0^\gamma \frac{h_t(h)^{1+\eta}}{1+\eta} \mathrm{d}h = \frac{Y_{H,t}^{1+\eta}}{1+\eta} \Delta_t$ and $\frac{1}{1-\gamma} \int_\gamma^1 \frac{h_t(f)^{1+\eta}}{1+\eta} \mathrm{d}f = \frac{Y_{F,t}^{*1+\eta}}{1+\eta} \Delta_t^*$, subject to the equilibrium conditions above. Under Non-cooperation, the domestic policymaker maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_{H,t}^{1+\eta}}{1+\eta} \Delta_t \right],$$

subject to the model above, given $\Pi_{F,t}^*$; On the other hand, the Foreign policymaker maximizes

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{C_t^{*1-\rho}}{1-\rho} - \frac{Y_{F,t}^{*1+\eta}}{1+\eta} \Delta_t^* \right],$$

subject to the model above, given $\Pi_{H,t}$.

We need to compute the steady state under Cooperation and Non-cooperation. We know that $\Pi_H = \Pi_F^* = 1$ in the steady state. We also assume that $\mathcal{M} = \mathcal{M}^* = 1$. Therefore, the model must be approximated around the steady state: $K = F = K^* = F^* = (1 - \alpha\beta)^{-1}$ and $\Delta = \Delta^* = Y_H = Y_F^* = C = S = 1$.

A.1.6 Log-linearization

The log-linearized equilibrium conditions around the steady state are

$$y_{H,t} - (1-\gamma) s_t = c_t,$$

$$s_t = y_{H,t} - y_{F,t}^*,$$

$$\frac{\alpha\left(1+\eta\sigma\right)}{1-\alpha}\pi_{H,t}+f_t=k_t,$$

$$f_t=\left(1-\alpha\beta\right)\left(1-\rho\right)c_t-\left(1-\alpha\beta\right)\mu_t+\alpha\beta\left(\sigma-1\right)\mathbb{E}_t\pi_{H,t+1}+\alpha\beta\mathbb{E}_tf_{t+1},$$

$$k_t \;=\; \left(1-\alpha\beta\right)\left(1+\eta\right)y_{H,t}+\alpha\beta\sigma\left(1+\eta\right)\mathbb{E}_t\pi_{H,t+1}+\alpha\beta\mathbb{E}_tk_{t+1},$$

$$\frac{\alpha\left(1+\eta\sigma\right)}{1-\alpha}\pi^*_{F,t}+f^*_t=k^*_t,$$

$$f^*_t=\left(1-\alpha\beta\right)\left(1-\rho\right)c^*_t-\left(1-\alpha\beta\right)\mu^*_t+\alpha\beta\left(\sigma-1\right)\mathbb{E}_t\pi^*_{F,t+1}+\alpha\beta\mathbb{E}_tf^*_{t+1},$$

$$k^*_t \;=\; \left(1-\alpha\beta\right)\left(1+\eta\right)y^*_{F,t}+\alpha\beta\sigma\left(1+\eta\right)\mathbb{E}_t\pi^*_{F,t+1}+\alpha\beta\mathbb{E}_tk^*_{t+1},$$

Note that the log deviation of a variable X_t from the steady state X is defined in lowercase as $x_t \equiv \log(X_t/X)$ and the Taylor approximation of X_t up to the first order is $X_t \approx X(1+x_t)$.[39] Given $\Pi_H = \Pi_F = 1$ in the steady state, $\pi_t \equiv \pi_{H,t} = \log(\Pi_{H,t}) \approx \Pi_{H,t}-1$ and $\pi^*_t \equiv \pi^*_{F,t} = \log(\Pi_{F,t}) \approx \Pi_{F,t}-1$ are the net domestic and Foreign inflation rates. Note that $\delta_t = \delta^*_t = 0$, i.e., the price dispersion terms have no effect at the first order. We also define $y_t \equiv y_{H,t}$ and $y^*_t \equiv y^*_{F,t}$. These equations are summarized as follows:

$$\pi_t=\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha\left(1+\eta\sigma\right)}\left[\mu_t+\left(\rho+\eta\right)y_t+\left(1-\gamma\right)\left(1-\rho\right)s_t\right]+\beta\mathbb{E}_t\pi_{t+1}, \tag{27}$$

$$\pi^*_t=\frac{\left(1-\alpha\beta\right)\left(1-\alpha\right)}{\alpha\left(1+\eta\sigma\right)}\left[\mu_t+\left(\rho+\eta\right)y^*_t-\gamma\left(1-\rho\right)s_t\right]+\beta\mathbb{E}_t\pi^*_{t+1}, \tag{28}$$

and

$$s_t=y_t-y^*_t,$$

which corresponds to Eqs. (1)-(2) (with $\gamma=1/2$) or (40)-(41) in Section 5.

A.2 Welfare approximation for LQ framework

Instead of the nonlinear Cooperation and Non-cooperation policies explained above, we will consider the LQ framework, following BB. For that purpose, the objective functions must be correctly approximated so that the welfare ordering of policies (under various regimes) in the LQ framework preserves that in the original non-

[39] We will also use the same lowercase Greek convention for Greek-lettered variables—e.g., x is to X , as δ is to Δ , or, as μ is to $\mathcal{M}$.

linear setting of the model (see e.g., Benigno and Woodford, 2012; Debortoli and Nunes, 2006; Levine et al., 2008; Bodenstein et al., 2014).

We note that these welfare functions are derived by taking a second-order Taylor expansion of household lifetime utility functions and of relevant competitive equilibrium conditions. As part of the derivation, a "timeless perspective" assumption is required: From each country's domestic perspective, the policymakers are assumed to be able to commit to implementing their particular policy plan that is assumed to have been in place in some infinite past leading up to an arbitrary date 0 (see e.g., Benigno and Benigno, 2006; Benigno and Woodford, 2005, 2012).[40] This is taken to hold in each policy regime—Cooperation, Non-cooperation, or what we will term Sustainable Cooperation—that we study.

Let x_t denote the percentage deviation of the level of a variable X_t from its deterministic steady-state point X. Note that for a variable X_t, the Taylor approximation up to the second order is $X_t - X \approx X\left(x_t + \frac{1}{2}x_t^2\right)$. Thus,

$$
\begin{aligned}
\frac{C_t^{1-\rho}}{1-\rho} &\approx \frac{C^{1-\rho}}{1-\rho} + C^{-\rho}(C_t - C) - \frac{\rho C^{-\rho-1}}{2}(C_t - C)^2, \\
&\approx C^{1-\rho}\left(\frac{C_t - C}{C}\right) - \frac{\rho C_t^{1-\rho}}{2}\left(\frac{C_t - C}{C}\right)^2 + \text{t.i.p.}, \\
&= c_t + \frac{1-\rho}{2}c_t^2 + \text{t.i.p.}
\end{aligned}
$$

where t.i.p. stands for terms independent of policy. Similarly,

$$
\frac{Y_t^{1+\eta}}{1+\eta} \approx y_t + \frac{1+\eta}{2}y_t^2 + \text{t.i.p.}
$$

Note that $C^{1-\rho} = Y^{1+\eta} = 1$.

[40]Note that by an arbitrary date 0, we mean the time at which one (i.e., the observer) begins recording the history of event in the model economy. This is different to the date at which the policy plan was designed and put in place (which was in some infinite past). Further discussion on this technicality can be found in Appendix A.2. See especially the ensuing remarks in Appendix A.3.

A.2.1 Cooperation

$$
\begin{aligned}
& \frac{C_t^{1-\rho}}{1-\rho} - \gamma \frac{Y_{H,t}^{1+\eta}}{1+\eta}\Delta_t - (1-\gamma)\frac{\left(Y_{F,t}^*\right)^{1+\eta}}{1+\eta}\Delta_t^*, \\
= \quad & -\gamma\left(y_t - c_t + \frac{1+\eta}{2}y_t^2 - \frac{1-\rho}{2}c_t^2 + \delta_t\right) \\
& -(1-\gamma)\left(y_t^* - c_t + \frac{1+\eta}{2}\left(y_t^*\right)^2 - \frac{1-\rho}{2}c_t^2 + \delta_t^*\right) + \text{t.i.p.}, \\
= \quad & -\gamma\frac{1+\eta}{2}y_t^2 - (1-\gamma)\frac{1+\eta}{2}\left(y_t^*\right)^2 \\
& +\gamma\frac{1-\rho}{2}\left(y_t - (1-\gamma)s_t\right)^2 + (1-\gamma)\frac{1-\rho}{2}\left(y_t^* + \gamma s_t\right)^2 \\
& -\gamma\delta_t - (1-\gamma)\delta_t^* + \text{t.i.p.}, \\
= \quad & -\gamma\frac{\rho+\eta}{2}y_t^2 - (1-\gamma)\frac{\rho+\eta}{2}\left(y_t^*\right)^2 - \gamma(1-\gamma)\frac{1-\rho}{2}s_t^2 \\
& -\gamma\delta_t - (1-\gamma)\delta_t^* + \text{t.i.p.},
\end{aligned}
$$

where we use $s_t = y_t - y_t^*$, and $y_t - c_t = (1-\gamma)s_t$, $y_t^* - c_t = -\gamma s_t$, noting that $s_t = -s_t^*$. Note that a second-order approximation of the price-dispersion term

$$
\Delta_t = \alpha\left(\frac{1}{\Pi_{H,t}}\right)^{-\sigma(1+\sigma\eta)}\Delta_{t-1} + (1-\alpha)\left[\frac{1-\alpha\left(\frac{1}{\Pi_{H,t}}\right)^{1-\sigma}}{1-\alpha}\right]^{\frac{\sigma(1+\eta\sigma)}{\sigma-1}}
$$

leads to

$$
\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Delta_t \approx \mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\frac{\sigma}{2k}\pi_t^2,
$$

which is approximately a second order transform of inflation. Thus, the joint/global welfare function used for studying the Cooperation regime and also the Sustainable Cooperation regime, can be approximated up to the second order as

$$
\begin{aligned}
& -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\gamma\delta_t + (1-\gamma)\delta_t^* + \gamma\frac{\rho+\eta}{2}y_t^2 + (1-\gamma)\frac{\rho+\eta}{2}\left(y_t^*\right)^2 + \gamma(1-\gamma)\frac{1-\rho}{2}s_t^2\right], \\
= \quad & -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\left[\begin{array}{c} \frac{\gamma\sigma}{k}\pi_t^2 + \frac{(1-\gamma)\sigma}{k}\left(\pi_t^*\right)^2 \\ +\gamma\left(\rho+\eta\right)y_t^2 + (1-\gamma)\left(\rho+\eta\right)\left(y_t^*\right)^2 \\ +\gamma(1-\gamma)(1-\rho)\left(y_t - y_t^*\right)^2 \end{array}\right].
\end{aligned}
$$

This corresponds to (6) in Section 3 (if we set countries to the same size,$\gamma = 1/2$) or (42) in Section F.

A.2.2 Non-cooperation

Domestic instantaneous welfare is

$$
\begin{aligned}
& \frac{C_t^{1-\rho}}{1-\rho} - \frac{Y_{H,t}^{1+\eta}}{1+\eta}\Delta_t, \\
\approx \quad & c_t + \frac{1-\rho}{2}c_t^2 - y_t - \frac{1+\eta}{2}y_t^2 - \delta_t + \text{t.i.p.}, \\
= \quad & -(1-\gamma)s_t + \frac{1-\rho}{2}c_t^2 - \frac{1+\eta}{2}\left(c_t + (1-\gamma)s_t\right)^2 - \delta_t + \text{t.i.p.}, \qquad (29)
\end{aligned}
$$

Similarly, for the Foreign country we have

$$
\begin{aligned}
& \frac{\left(C_t^*\right)^{1-\rho}}{1-\rho} - \frac{\left(Y_{F,t}^*\right)^{1+\eta}}{1+\eta}\Delta_t^*, \\
\approx \quad & c_t^* + \frac{1-\rho}{2}\left(c_t^*\right)^2 - y_t^* - \frac{1+\eta}{2}\left(y_t^*\right)^2 - \delta_t^* + \text{t.i.p.}, \\
= \quad & \gamma s_t + \frac{1-\rho}{2}c_t^2 - \frac{1+\eta}{2}\left(c_t - \gamma s_t\right)^2 - \delta_t^* + \text{t.i.p.}, \qquad (30)
\end{aligned}
$$

Note that each approximation includes the log-linear term of s_t. The linear terms in the approximated welfare induce spurious welfare evaluation in the LQ framework. The correct LQ approximation must be derived with a purely quadratic welfare function (Kim and Kim, 2003, 2007; Benigno and Woodford, 2005, 2012; Benigno and Benigno, 2006; Fujiwara and Teranishi, 2013). We need to substitute out the linear terms of s_t. For this purpose, we approximate the NK Phillips curve up to the second order.

Second order approximation of the NK Phillips Curve. By following Benigno and Benigno (2006), the second order approximation of the NK Phillips curve leads to

$$
\begin{aligned}
& k\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t - \tilde{p}_{H,t} + \mu_t\right) \\
\approx \quad & K_0 - \frac{k}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t - \tilde{p}_{H,t} + \mu_t\right)\left((2-\rho)c_t + \eta y_t - \tilde{p}_{H,t} + \mu_t\right) \\
& - \frac{\sigma(1+\eta)}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2.
\end{aligned}
$$

where $\tilde{p}_{H,t} = \log(P_{H,t}/P_t) = \log P_{H,t}^{1-\gamma} P_{F,t}^{\gamma-1} = (\gamma-1)s_t$ and K_0 is given and under the timeless perspective, assumed to be zero (Benigno and Woodford, 2005, 2012).

Therefore, we can have the approximated condition:

$$
\begin{aligned}
& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t + (1-\gamma) s_t + \mu_t \right) \\
= & -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t + \rho c_t + (1-\gamma) s_t + \mu_t \right) \left((2-\rho) c_t + \eta y_t + (1-\gamma) s_t + \mu_t \right) \\
& - \frac{\sigma (1+\eta)}{2k} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2.
\end{aligned}
$$

Similarly for the Foreign Phillips curve, we have

$$
\begin{aligned}
& \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t^* + \rho c_t - \gamma s_t + \mu_t^* \right) \\
= & -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\eta y_t^* + \rho c_t - \gamma s_t + \mu_t^* \right) \left((2-\rho) c_t + \eta y_t^* - \gamma s_t + \mu_t^* \right) \\
& - \frac{\sigma (1+\eta)}{2k} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^* \right)^2.
\end{aligned}
$$

From these approximations, we have the linear terms replaced by quadratic terms:

$$
\begin{aligned}
\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t s_t = & -\frac{1}{2(1+\eta)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(\eta+\rho) c_t + (1-\gamma)(1+\eta) s_t + \mu_t \right] \times \\
& \left[(2-\rho+\eta) c_t + (1-\gamma)(1+\eta) s_t + \mu_t \right] - \frac{\sigma}{2k} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \pi_t^2 \\
& + \frac{1}{2(1+\eta)} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left((\eta+\rho) c_t - \gamma (1+\eta) s_t + \mu_t^* \right) \times \\
& \left[(2-\rho+\eta) c_t - \gamma (1+\eta) s_t + \mu_t^* \right] + \frac{\sigma}{2k} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^* \right)^2 . \qquad (31)
\end{aligned}
$$

By substituting (31) into (29), and after tedious calculations, we have

$$
\begin{aligned}
V_0 = & -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t \\
= & -\frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \gamma (\eta+\rho) \left(y_t - \frac{1-\gamma}{\gamma(\eta+\rho)} \mu_t \right)^2 + \frac{\gamma\sigma}{k} \pi_t^2 \\ + \gamma (1-\gamma)(1-\rho)(y_t - y_t^*)^2 \\ + (1-\gamma)(\eta+\rho) \left(y_t^* + \frac{1}{\eta+\rho} \mu_t^* \right)^2 + \frac{(1-\gamma)\sigma}{k^*} \left(\pi_t^* \right)^2 \end{array} \right].
\end{aligned}
$$

This corresponds to (4) in Section 2. Similarly we can derive (5) as well.

A.2.3 Terms-of-trade externality channel and Non-cooperation regime

One way to expound on the externality problem considered in section 2.2.1 in the paper, when considering each country's policy problem separately, is as follows: Consider an intermediate step in arriving at the approximating per-period social welfare functions U_t and U_t^*, respectively, found in equations (4) and (5). This intermediate step would produce the following pair of expressions:

$$U_t = \frac{1}{2}s_t - \frac{1-\rho}{2}(y_t - \frac{1}{2}s_t)^2 + \frac{1+\eta}{2}y_t^2 + \frac{\sigma}{k}\pi_t^2,$$
$$U_t^* = -\frac{1}{2}s_t - \frac{1-\rho}{2}(y_t^* + \frac{1}{2}s_t)^2 + \frac{1+\eta}{2}\left(y_t^*\right)^2 + \frac{\sigma}{k}\left(\pi_t^*\right)^2.$$

Of course, one cannot stop at this point in deriving the approximate welfare functions.[41] For our exposition here, this break-down of the steps is nevertheless instructive. Note that a naïve addition of these two terms, $U_t + U_t^*$, yields an expression equivalent to the per-period loss in the global welfare function (6), as the linear terms of s_t are canceled out. That means that under global cooperation of monetary policy, there is no terms of trade externality problem. However, if the social welfare in each country is considered separately under independent national monetary-policy making, s_t is substitutable for a quadratic approximation of the Home and Foreign Phillips curves. This results in the non-zero output targets in Eqs. (4) and (5).

A.3 Time-0 vs. timeless welfare measures

In the *cross-regime comparative* welfare analyses and in the solution of the Sustainable Cooperation equilibrium in the paper, we need to work with time-0 *conditional* welfare measures—i.e., the functions derived earlier as (4) and (5).

Alternatively, if we are only interested in a decomposition into the components comprising the welfare outcome *within a particular policy regime*, then we evaluate welfare with respect to the ergodic and unconditional distribution of a given policy regime's equilibrium. We call this measure the *unconditional welfare measure*, or, elsewhere this is often interpreted as a *welfare metric under the timeless perspective*.[42] This alternative exercise is separately taken up in this Online Appendix in Section

[41] If taken at face value, the linear terms in the intermediate welfare approximation step may induce spurious welfare evaluation in the LQ framework (see e.g., Kim and Kim, 2003, 2007).

[42] By construction, the measured welfare outcome from this metric encodes an infinitely (in practice, sufficiently) long history of (within-country) commitment policy outcomes. Thus, we can also call this welfare metric one under the timeless perspective. See also its usage earlier by McCallum and Nelson (2004) and Sauer (2010).

C.1 (Cooperation and Non-cooperation) and in Section D.2 (Sustainable Cooperation).

By taking unconditional expectation of (4) and (5), the unconditional welfare functions for Home and Foreign, respectively, are

$$\begin{aligned} EV &= -(1-\beta)^{-1}\frac{1}{4}\mathbb{E}\left[(\eta+\rho)\,x_t^2+(\eta+\rho)\left(\tilde{x}_t^*\right)^2\right. \\ &\quad \left.+\frac{1-\rho}{2}s_t^2+\frac{\sigma}{k}\pi_t^2+\frac{\sigma}{k}\left(\pi_t^*\right)^2\right], \end{aligned} \tag{32}$$

$$\begin{aligned} EV^* &= -(1-\beta)^{-1}\frac{1}{4}\mathbb{E}\left[(\eta+\rho)\left(x_t^*\right)^2+(\eta+\rho)\,\tilde{x}_t^2\right. \\ &\quad \left.+\frac{1-\rho}{2}s_t^2+\frac{\sigma}{k}\pi_t^2+\frac{\sigma}{k}\left(\pi_t^*\right)^2\right], \end{aligned} \tag{33}$$

where $x_t = y_t - \frac{1}{\eta+\rho}\mu_t$, $\tilde{x}_t = y_t + \frac{1}{\eta+\rho}\mu_t$, $x_t^* = y_t^* - \frac{1}{\eta+\rho}\mu_t^*$, $\tilde{x}_t^* = y_t^* + \frac{1}{\eta+\rho}\mu_t^*$, and $\mathbb{E}[\cdot]$ denotes an expectations operator with respect to a regime's equilibrium (unconditional) distribution of state variables.

The convenient feature of these measures is that they are decomposable into the unconditional volatility (variance) of each welfare function argument from (4) and (5), respectively. This facilitates additional insights into what account for welfare outcomes. (See the discussions around Figure 7 on page OA.30 — C, and, 8 on page OA.40 — E.)

It should be noted that the unconditional welfare metric above cannot be used, if our question is one of comparing relative welfare outcomes across different policy regimes. That is because given each regime's equilibrium outcome, the corresponding implementation of an unconditional welfare measure would be with respect to an ergodic distribution likely to be different from another regime's or equilibrium's outcome—hence any welfare comparison using that metric would be meaningless, if not misleading as it involves arbitrary orderings of welfare numbers. Thus, we can only compare welfare outcomes across different equilibria or regimes if these welfare measures are *conditioned on the same initial states,* using the functions derived as (4) and (5). This, we label the time-0 *conditional welfare measure*. Later, we also call this the *stochastic steady-state welfare measure*.[43]

[43] In the numerical exercises below, we condition these welfare measures on the same initial natural states, where the initial lagged output states are $y_{-1} = y_{-1}^* = 0$. (See Figure 1 on page 14, Figure 9 on page OA.44 — F, Figure 4 on page 22 and the discussions surrounding them in Sections 3.1.1 and 3.4.) However, there will be another auxiliary state variable ν, interpretable as the (relative) pseudo-Pareto weight, when one considers the Sustainable Cooperation regime later. (This variable is non-existent in the other two regimes of Cooperation and Non-cooperation.) In this case, we also need to start the Sustainable Cooperation equilibrium outcome off at some point for the initial auxiliary state ν_{-1}, apart from setting the same initial natural state, $y_{-1} = y_{-1}^* = 0$, when comparing

Timeless within-country policy commitment and welfare. In the last section, when deriving the second-order accurate approximation of the original welfare functions, we had followed Benigno and Benigno (2006) and assumed the *timeless perspective* on policy commitments (see also Benigno and Woodford, 2005, 2012). Specifically, this perspective refers to the commitment of each country's policymaker to its policy plan (including its initial policy), and with respect to all agents' expectations. This is assumed in each regime that we consider: Cooperation, Non-cooperation, and Sustainable Cooperation.

Algebraically, this assumption was imbedded in the steps when we eliminated linear terms earlier, in order to combine second-order approximations of equilibrium conditions with second-order utility-function derivations. The resulting welfare function for each country would have contained a linearly separable term involving agents' date-0 expected total welfare conditional on particular transitions of past outcomes that would taken their beliefs to that state and point in time. We had subsumed such a term as K_0 in the previous section. The reason we can do so is that—as detailed in Benigno and Benigno (2006)—each policymaker ties its hands with respect to his date-0 policy and merely continues it from the history generated by the same plan (from some infinite past) preceding the current policymaker. Thus, we can focus on time-invariant policy functions enforcing each regime's policy plan, or equivalently, their respective time-independent characterizations of policy trade-offs in (34)-(35) for the case of Cooperation, (9)-(10) for the regime of Non-cooperation, and, (13)-(14) for that of Sustainable Cooperation.

This notion of timeless within-country commitment by policymakers to continue with past plans, while being used to derive the date-0 and state-contingent welfare functions (4) and (5), should not be confused with another notion of timelessness when *evaluating* these welfare functions. Next, we discuss two ways of evaluating the welfare functions (4) and (5), depending on the purpose that these methods will serve.

---

this regime's *stochastic steady-state welfare measure* with the other regimes'. In the paper we consider two possible cases: In the baseline setting we have $v_{-1} = 1/2$, which is the deterministic steady state of the model and is the same as the time-invariant Pareto weight in the Cooperation regime. Also, this is the convention in defining a Ramsey equilibrium steady state value (King and Wolman, 1999; Khan et al., 2003). We also consider v_{-1} being equal to the stochastic steady state of the model, i.e., its asymptotic upper bound (in short we called it "u.b.") in a particular Sustainable Cooperation equilibrium, but this does not change our results. Note that even though we adopt the LQ approximation, the deterministic and stochastic steady state values are different due to the occasionally binding sustainability constraints.

B Alternative Policy Regimes

In this appendix, we derive equilibrium policy trade-offs for the three regimes considered. We also discuss the computation procedure for the Sustainable Cooperation equilibrium's functional operator problem. This is a problem solving for nonlinear functions despite the underlying model is the LQ framework presented in Section 2. For comparability we also solve the Cooperation equilibrium and the Non-cooperation equilibrium using the same technique, despite this being a regular LQ Markov-perfect equilibrium problem.[44]

B.1 Equilibrium policy trade-off characterizations

In this section, we will derive the FONCs in each Cooperation, Non-cooperation and Sustainable Cooperation regime.

B.1.1 Cooperation

We consider the following maximization problem:

$$\max\left[\lambda V_0 + (1-\lambda)V_0^*\right],$$

where

$$\begin{aligned}
V_0 &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t \\
&= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} \gamma\left(\eta+\rho\right)\left(y_t - \frac{1-\gamma}{\gamma(\eta+\rho)}\mu_t\right)^2 + \frac{\gamma\sigma}{k}\pi_t^2 \\ +\gamma(1-\gamma)(1-\rho)(y_t - y_t^*)^2 \\ +(1-\gamma)\left(\eta+\rho\right)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}\left(\pi_t^*\right)^2 \end{array} \right],
\end{aligned}$$

and

$$\begin{aligned}
V_0^* &= -\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U_t^* \\
&= -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\begin{array}{c} (1-\gamma)\left(\eta+\rho\right)\left(y_t^* - \frac{\gamma}{(1-\gamma)(\eta+\rho)}\mu_t^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}\left(\pi_t^*\right)^2 \\ +\gamma(1-\gamma)(1-\rho)(y_t^* - y_t)^2 \\ +\gamma\left(\eta+\rho\right)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\gamma\sigma}{k}\pi_t^2 \end{array} \right],
\end{aligned}$$

[44] Computing the Cooperation equilibrium is a straight forward application of computing the Sustainable equilibrium; therefore we did not show it here.

and $\lambda \in [0,1]$ is a constant parameter, subject to

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + k\mu_t + k\left[(\rho+\eta)y_t + (1-\gamma)(1-\rho)(y_t - y_t^*)\right],$$

and

$$\pi_t^* = \beta \mathbb{E}_t \pi_{t+1}^* + k^*\mu_t^* + k^*\left[(\rho+\eta)y_t^* - \gamma(1-\rho)(y_t - y_t^*)\right],$$

where $k = k^* = (1-\alpha)(1-\alpha\beta) / [\alpha(1+\sigma\eta)]$. When $\lambda = \gamma$, it reduces to the Cooperation regime ($\gamma = 1/2$ in the case of symmetric country size). When $\lambda \neq \gamma$, it reduces to the λ-Pareto-cooperation regime considered in Section 3.4.

The FONCs are

$$\begin{aligned}
\partial y_t : \quad & -2\lambda\gamma(\eta+\rho)\left(y_t - \frac{1-\gamma}{\gamma(\eta+\rho)}\mu_t\right) - 2\lambda\gamma(1-\gamma)(1-\rho)(y_t - y_t^*) \\
& -2(1-\lambda)\gamma(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right) + 2(1-\lambda)\gamma(1-\gamma)(1-\rho)(y_t^* - y_t) \\
& +k(\rho+\eta+(1-\gamma)(1-\rho))\phi_t - k^*\gamma(1-\rho)\phi_t^* = 0, \\
\partial y_t^* : \quad & -2\lambda(1-\gamma)(\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right) + 2\lambda\gamma(1-\gamma)(1-\rho)(y_t - y_t^*) \\
& -2(1-\lambda)(1-\gamma)(\eta+\rho)\left(y_t^* - \frac{\gamma}{(1-\gamma)(\eta+\rho)}\mu_t^*\right) - 2(1-\lambda)\gamma(1-\gamma)(1-\rho)(y_t^* - y_t) \\
& +k^*(\rho+\eta+\gamma(1-\rho))\phi_t^* - k(1-\gamma)(1-\rho)\phi_t = 0, \\
\partial \pi_t : \quad & -\frac{2\gamma\sigma}{k}\pi_t - \phi_t + \phi_{t-1} = 0, \\
\partial \pi_t^* : \quad & -\frac{2(1-\gamma)\sigma}{k^*}\pi_t^* - \phi_t^* + \phi_{t-1}^* = 0.
\end{aligned}$$

We define $\tilde{\mu}_t = (\lambda-\gamma)\mu_t$ and $\tilde{\mu}_t^* = (\lambda-\gamma)\mu_t^*$. Then, The first two equations are solved for

$$\begin{aligned}
\phi_t &= \frac{2\gamma}{k}y_t - \frac{2\gamma}{k}\frac{[\eta+\rho+\gamma(1-\rho)]\tilde{\mu}_t - \gamma(1-\rho)\tilde{\mu}_t^*}{\gamma(1+\eta)(\rho+\eta)}, \\
\phi_t^* &= \frac{2(1-\gamma)}{k^*}y_t^* + \frac{2(1-\gamma)}{k^*}\frac{[\eta+\rho+(1-\gamma)(1-\rho)]\tilde{\mu}_t^* - (1-\gamma)(1-\rho)\tilde{\mu}_t}{(1-\gamma)(\rho+\eta)(1+\eta)}.
\end{aligned}$$

The *Cooperation equilibrium policy trade-off* conditions are summarized as

$$\begin{aligned}
-\sigma\pi_t &= y_t - \tilde{\zeta}_t - (y_{t-1} - \tilde{\zeta}_{t-1}), \\
-\sigma\pi_t^* &= y_t^* - \tilde{\zeta}_t^* - (y_{t-1}^* - \tilde{\zeta}_{t-1}^*),
\end{aligned}$$

where $\tilde{\zeta}_t = \frac{[\eta+\rho+\gamma(1-\rho)]\tilde{\mu}_t - \gamma(1-\rho)\tilde{\mu}_t^*}{\gamma(1+\eta)(\rho+\eta)}$ and $\tilde{\zeta}_t^* = -\frac{[\eta+\rho+(1-\gamma)(1-\rho)]\tilde{\mu}_t^* - (1-\gamma)(1-\rho)\tilde{\mu}_t}{(1-\gamma)(\rho+\eta)(1+\eta)}$.

Note that, when $\lambda = \gamma$, $\tilde{\mu}_t = \tilde{\mu}_t^* = \tilde{\zeta}_t = \tilde{\zeta}_t^* = 0\ \forall t$ holds and the equilibrium conditions reduce to:

$$
\begin{aligned}
-\sigma\pi_t &= y_t - y_{t-1}, \\
-\sigma\pi_t^* &= y_t^* - y_{t-1}^*.
\end{aligned}
$$

B.1.2 Non-cooperation

For the Home country, the FONCs are

$$
\begin{aligned}
\partial y_t: \quad & -2\gamma\,(\eta+\rho)\left(y_t - \frac{1-\gamma}{\gamma(\eta+\rho)}\mu_t\right) - 2\gamma(1-\gamma)(1-\rho)(y_t - y_t^*) \\
& + k(\rho+\eta+(1-\gamma)(1-\rho))\varphi_{1,t} - k^*\gamma(1-\rho)\varphi_{2,t} = 0, \\
\partial y_t^*: \quad & -2(1-\gamma)\,(\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right) + 2\gamma(1-\gamma)(1-\rho)(y_t - y_t^*) \\
& + k^*(\rho+\eta+\gamma(1-\rho))\varphi_{2,t} - k(1-\gamma)(1-\rho)\varphi_{1,t} = 0, \\
\partial \pi_t: \quad & -\frac{2\gamma\sigma}{k}\pi_t - \varphi_{1,t} + \varphi_{1,t-1} = 0,
\end{aligned}
$$

The first two equations are solved for

$$
\begin{aligned}
\varphi_{1,t} &= \frac{2\gamma}{k}y_t - \frac{2\gamma}{k}\frac{1-\gamma}{\gamma}\frac{(\eta+\rho+\gamma(1-\rho))\,\mu_t - \gamma(1-\rho)\mu_t^*}{(\eta+\rho)(1+\eta)}, \\
\varphi_{2,t} &= \frac{2(1-\gamma)}{k^*}y_t^* + \frac{2(1-\gamma)}{k^*}\frac{(\eta+\rho+(1-\gamma)(1-\rho))\,\mu_t^* - (1-\gamma)(1-\rho)\mu_t}{(\eta+\rho)(1+\eta)}.
\end{aligned}
$$

For the Foreign country, the FONCs are

$$
\begin{aligned}
\partial y_t: \quad & -2\gamma\,(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right) + 2\gamma(1-\gamma)(1-\rho)(y_t^* - y_t) \\
& + k(\rho+\eta+(1-\gamma)(1-\rho))\varphi_{1,t}^* - k^*\gamma(1-\rho)\varphi_{2,t}^* = 0, \\
\partial y_t^*: \quad & -2(1-\gamma)\,(\eta+\rho)\left(y_t^* - \frac{\gamma}{(1-\gamma)(\eta+\rho)}\mu_t^*\right) - 2\gamma(1-\gamma)(1-\rho)(y_t^* - y_t) \\
& + k^*(\rho+\eta+\gamma(1-\rho))\varphi_{2,t}^* - k(1-\gamma)(1-\rho)\varphi_{1,t}^* = 0, \\
\partial \pi_t^*: \quad & -\frac{2(1-\gamma)\sigma}{k^*}\pi_t^* - \varphi_{2,t}^* + \varphi_{2,t-1}^* = 0.
\end{aligned}
$$

The first two equations are solved for

$$
\begin{aligned}
\varphi_{1,t}^* &= \frac{2\gamma}{k}y_t + \frac{2\gamma}{k}\frac{(\eta+\rho+\gamma(1-\rho))\,\mu_t - \gamma(1-\rho)\mu_t^*}{(\eta+\rho)(1+\eta)}, \\
\varphi_{2,t}^* &= \frac{2(1-\gamma)}{k^*}y_t^* - \frac{2(1-\gamma)}{k^*}\frac{\gamma}{1-\gamma}\frac{(\eta+\rho+(1-\gamma)(1-\rho))\,\mu_t^* - (1-\gamma)(1-\rho)\mu_t}{(\eta+\rho)(1+\eta)}.
\end{aligned}
$$

The *Non-cooperation equilibrium policy trade-off* conditions are obtained as

$$-\sigma\pi_t = y_t - \xi_t - y_{t-1} + \xi_{t-1},$$
$$-\sigma\pi_t^* = y_t^* - \xi_t^* - y_{t-1}^* + \xi_{t-1}^*,$$

where

$$\begin{aligned}
\xi_t &= \frac{1-\gamma}{\gamma}\frac{(\eta+\rho+\gamma(1-\rho))\,\mu_t - \gamma(1-\rho)\mu_t^*}{(\eta+\rho)(1+\eta)},\\
\xi_t^* &= \frac{\gamma}{1-\gamma}\frac{(\eta+\rho+(1-\gamma)(1-\rho))\,\mu_t^* - (1-\gamma)(1-\rho)\mu_t}{(\eta+\rho)(1+\eta)},
\end{aligned}$$

are the variables related to the terms of trade externality. When $\gamma = 1/2$ in the case of symmetric country size, we have

$$\begin{aligned}
\xi_t &= \frac{(1+\rho+2\eta)\,\mu_t - (1-\rho)\mu_t^*}{2(\eta+\rho)(1+\eta)},\\
\xi_t^* &= \frac{(1+\rho+2\eta)\,\mu_t^* - (1-\rho)\mu_t}{2(\eta+\rho)(1+\eta)}.
\end{aligned}$$

B.1.3 Sustainable Cooperation

Set up the Lagrangean in Period 0 as

$$\begin{aligned}
\mathcal{L}_0 &= -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\,\{\gamma U_t + (1-\gamma)U_t^*\\
&\quad +\phi_t\left(-\pi_t + \beta\mathbb{E}_t\pi_{t+1} + k\mu_t + k\left[(\rho+\eta)y_t + (1-\gamma)(1-\rho)(y_t - y_t^*)\right]\right)\\
&\quad +\phi_t^*\left(-\pi_t^* + \beta\mathbb{E}_t\pi_{t+1}^* + k^*\mu_t^* + k^*\left[(\rho+\eta)y_t^* - \gamma(1-\rho)(y_t - y_t^*)\right]\right)\\
&\quad +\psi_t\left[-\mathbb{E}_t\sum_{s=t}^{\infty}\beta^{s-t}U_s - W(y_{t-1}, y_{t-1}^*, \tau_t)\right] + \psi_t^*\left[-\mathbb{E}_t\sum_{s=t}^{\infty}\beta^{s-t}U_s^* - W^*(y_{t-1}, y_{t-1}^*, \tau_t)\right]\Bigg\},\\
&= -\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\,\{\Psi_t U_t + \Psi_t^* U_t^*\\
&\quad +\phi_t\left(-\pi_t + k\mu_t + k\left[(\rho+\eta)y_t + (1-\gamma)(1-\rho)(y_t - y_t^*)\right]\right) - \phi_{t-1}\pi_t\\
&\quad +\phi_t^*\left(-\pi_t^* + k^*\mu_t^* + k^*\left[(\rho+\eta)y_t^* - \gamma(1-\rho)(y_t - y_t^*)\right]\right) - \phi_{t-1}^*\pi_t^*\\
&\quad -\psi_t W(y_{t-1}, y_{t-1}^*, \tau_t) - \psi_t^* W^*(y_{t-1}, y_{t-1}^*, \tau_t)\}\,,
\end{aligned}$$

where $\Psi_t = \Psi_{t-1} + \psi_t$ and $\Psi_t^* = \Psi_{t-1}^* + \psi_t^*$ given $\Psi_{-1} = \gamma$ and $\Psi_{-1}^* = 1-\gamma$.

The FONCs are

$$
\begin{aligned}
\partial y_t: \quad & -2\Psi_t\gamma\left(\eta+\rho\right)\left(y_t-\frac{1-\gamma}{\gamma(\eta+\rho)}\mu_t\right)-2\Psi_t\gamma(1-\gamma)(1-\rho)(y_t-y_t^*)\\
& -2\Psi_t^*\gamma\left(\eta+\rho\right)\left(y_t+\frac{1}{\eta+\rho}\mu_t\right)+2\Psi_t^*\gamma(1-\gamma)(1-\rho)(y_t^*-y_t)\\
& +k(\rho+\eta+(1-\gamma)(1-\rho))\phi_t-k^*\gamma(1-\rho)\phi_t^*\\
& -\beta\mathbb{E}_t\left\{\psi_{t+1}D_1W(y_t,y_t^*,\tau_{t+1})+\psi_{t+1}^*D_1W^*(y_t,y_t^*,\tau_{t+1})\right\}=0,\\
\partial y_t^*: \quad & -2\Psi_t(1-\gamma)\left(\eta+\rho\right)\left(y_t^*+\frac{1}{\eta+\rho}\mu_t^*\right)+2\Psi_t\gamma(1-\gamma)(1-\rho)(y_t-y_t^*)\\
& -2\Psi_t^*(1-\gamma)\left(\eta+\rho\right)\left(y_t^*-\frac{\gamma}{(1-\gamma)(\eta+\rho)}\mu_t^*\right)-2\Psi_t^*\gamma(1-\gamma)(1-\rho)(y_t^*-y_t)\\
& +k^*(\rho+\eta+\gamma(1-\rho))\phi_t^*-k(1-\gamma)(1-\rho)\phi_t\\
& -\beta\mathbb{E}_t\left\{\psi_{t+1}D_2W(y_t,y_t^*,\tau_{t+1})+\psi_{t+1}^*D_2W^*(y_t,y_t^*,\tau_{t+1})\right\}=0,\\
\partial\pi_t: \quad & -\frac{2\gamma\sigma}{k}\left(\Psi_t+\Psi_t^*\right)\pi_t-\phi_t+\phi_{t-1}=0,\\
\partial\pi_t^*: \quad & -\frac{2(1-\gamma)\sigma}{k^*}\left(\Psi_t+\Psi_t^*\right)\pi_t^*-\phi_t^*+\phi_{t-1}^*=0.
\end{aligned}
$$

Normalizing with $\Psi_t+\Psi_t^*$, we have:

$$
\begin{aligned}
& -2\gamma\left(\eta+\rho\right)y_t-2\gamma(1-\gamma)(1-\rho)(y_t-y_t^*)\\
& +k(\rho+\eta+(1-\gamma)(1-\rho))\tilde{\phi}_t-k^*\gamma(1-\rho)\tilde{\phi}_t^*\\
& +2(\nu_t-\gamma)\mu_t-\beta\mathbb{E}_t\Xi_{t+1}=0,\\
& -2(1-\gamma)\left(\eta+\rho\right)y_t^*+2\gamma(1-\gamma)(1-\rho)(y_t-y_t^*)\\
& +k^*(\rho+\eta+\gamma(1-\rho))\tilde{\phi}_t^*-k(1-\gamma)(1-\rho)\tilde{\phi}_t\\
& -2(\nu_t-\gamma)\mu_t^*-\beta\mathbb{E}_t\Xi_{t+1}^*=0,\\
& -\frac{2\gamma\sigma}{k}\pi_t-\tilde{\phi}_t+z_t\tilde{\phi}_{t-1}=0,\\
& -\frac{2(1-\gamma)\sigma}{k^*}\pi_t^*-\tilde{\phi}_t^*+z_t\tilde{\phi}_{t-1}^*=0.
\end{aligned}
$$

where $\tilde{\phi}_t=\phi_t/(\Psi_t+\Psi_t^*)$, $\tilde{\phi}_t^*=\phi_t^*/(\Psi_t+\Psi_t^*)$, $\nu_t=\Psi_t/(\Psi_t+\Psi_t^*)$, $z_t=(\Psi_{t-1}+\Psi_{t-1}^*)/(\Psi_t+\Psi_t^*)$, and

$$
\begin{aligned}
\Xi_{t+1} &:= \frac{\psi_{t+1}}{\Psi_t+\Psi_t^*}D_1W(y_t,y_t^*,\tau_{t+1})+\frac{\psi_{t+1}^*}{\Psi_t+\Psi_t^*}D_1W^*(y_t,y_t^*,\tau_{t+1}),\\
\Xi_{t+1}^* &:= \frac{\psi_{t+1}}{\Psi_t+\Psi_t^*}D_2W(y_t,y_t^*,\tau_{t+1})+\frac{\psi_{t+1}^*}{\Psi_t+\Psi_t^*}D_2W^*(y_t,y_t^*,\tau_{t+1}).
\end{aligned}
$$

Then, the first two equations are solved for

$$\tilde{\phi}_t = \frac{2\gamma}{k} y_t - \frac{2\gamma}{k} \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta_t - \gamma(1-\rho)\vartheta_t^*}{2\gamma(1+\eta)(\rho+\eta)},$$
$$\tilde{\phi}_t^* = \frac{2(1-\gamma)}{k^*} y_t^* + \frac{2(1-\gamma)}{k^*} \frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta_t^* - (1-\gamma)(1-\rho)\vartheta_t}{2(1-\gamma)(\rho+\eta)(1+\eta)}.$$

where $\vartheta_t := 2(\nu_t - \gamma)\mu_t - \beta \mathbb{E}_t \Xi_{t+1}$, $\vartheta_t^* := 2(\nu_t - \gamma)\mu_t^* + \beta \mathbb{E}_t \Xi_{t+1}^*$. Note that either of the sustainability constraints binds at a time. Then we have

$$\begin{aligned}
\vartheta_t &= 2(\nu_t - \gamma)\mu_t \\
&\quad -\beta \mathbb{E}_t \underbrace{\left\{ (z_{t+1}^{-1} - 1) \left[I_{t+1} D_1 W(y_t, y_t^*, \tau_{t+1}) + I_{t+1}^* D_1 W^*(y_t, y_t^*, \tau_{t+1}) \right] \right\}}_{=\Xi_{t+1}}, \\
\vartheta_t^* &= 2(\nu_t - \gamma)\mu_t^* \\
&\quad +\beta \mathbb{E}_t \underbrace{\left\{ (z_{t+1}^{-1} - 1) \left[I_{t+1} D_2 W(y_t, y_t^*, \tau_{t+1}) + I_{t+1}^* D_2 W^*(y_t, y_t^*, \tau_{t+1}) \right] \right\}}_{=\Xi_{t+1}^*}.
\end{aligned}$$

The indicator function $I_t = 1$ when the sustainability constraint in Home country is binding in period t; $I_t = 0$ otherwise. The *Sustainable Cooperation equilibrium policy trade-off conditions* are summarized as

$$-\sigma \pi_t = y_t - \zeta_t - z_t(y_{t-1} - \zeta_{t-1}),$$
$$-\sigma \pi_t^* = y_t^* - \zeta_t^* - z_t(y_{t-1}^* - \zeta_{t-1}^*),$$

where

$$\begin{aligned}
\zeta_t &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta_t - \gamma(1-\rho)\vartheta_t^*}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta_t^* &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta_t^* - (1-\gamma)(1-\rho)\vartheta_t}{2(1-\gamma)(\rho+\eta)(1+\eta)}, \\
\zeta_{t-1} &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta_{t-1} - \gamma(1-\rho)\vartheta_{t-1}^*}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta_{t-1}^* &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta_{t-1}^* - (1-\gamma)(1-\rho)\vartheta_{t-1}}{2(1-\gamma)(\rho+\eta)(1+\eta)}.
\end{aligned}$$

Note that $\vartheta_t = 2(\nu_t - \gamma)\mu_t - \beta \mathbb{E}_t \Xi_{t+1}$, $\vartheta_t^* = 2(\nu_t - \gamma)\mu_t^* + \beta \mathbb{E}_t \Xi_{t+1}^*$, $\vartheta_{t-1} = 2(\nu_{t-1} - \gamma)\mu_{t-1} - \beta \Xi_t$ and $\vartheta_{t-1}^* = 2(\nu_{t-1} - \gamma)\mu_{t-1}^* + \beta \Xi_t^*$. There are no expectational operators in front of Ξ_t and Ξ_t^*, as they are the result of optimization in period t.

When $\gamma = 1/2$, we have

$$\begin{aligned}
\zeta_t &= \frac{(1+\rho+2\eta)\vartheta_t - (1-\rho)\vartheta_t^*}{2(1+\eta)(\eta+\rho)}, \\
\zeta_t^* &= -\frac{(1+\rho+2\eta)\vartheta_t^* - (1-\rho)\vartheta_t}{2(1+\eta)(\eta+\rho)}, \\
\vartheta_t &= (2\nu_t - 1)\mu_t - \beta\mathbb{E}_t\Xi_{t+1}, \\
\vartheta_t^* &= (2\nu_t - 1)\mu_t^* + \beta\mathbb{E}_t\Xi_{t+1}^*.
\end{aligned}$$

C Comparative Equilibria: Cooperation vs. Non-cooperation

Policymakers under an assumed Cooperation regime maximize the global social welfare function (6) subject to the Phillips curves (1) and (2). The first-order necessary conditions (FONCs) in the Cooperation regime are (1) and (2), appended with the optimal trade-offs for the policymakers for every state and date $t \geq 0$:[45]

$$-\sigma\pi_t = y_t - y_{t-1}, \tag{34}$$

$$-\sigma\pi_t^* = y_t^* - y_{t-1}^*. \tag{35}$$

Because of commitment to future policies inherent in both of the Cooperation and Non-cooperation regimes, lagged output appears in each countries' equation—i.e., the policymakers conduct history-dependent policies in both regimes. Also, as shown by BB, in the Cooperation regime, the optimal targeting rules are always inward-looking in the sense that the optimal trade-offs only involve each policymaker's own-country variables.

In contrast, a policymaker in the Non-cooperation regime maximizes the social welfare function in his country given the other country's outcome. The policymaker in the Home country maximizes (4) subject to the Philips curves (1) and (2), given π_t^*, and the policymaker in the Foreign country maximizes (5) subject to the Philips curves (1) and (2), given π_t. The FONCs, for every state and date $t \geq 0$, in the Non-cooperation regime are (1) and (2), along with

$$-\sigma\pi_t = y_t - \xi_t - (y_{t-1} - \xi_{t-1}), \tag{36}$$

$$-\sigma\pi_t^* = y_t^* - \xi_t^* - (y_{t-1}^* - \xi_{t-1}^*), \tag{37}$$

[45]See Appendix B.1 for the derivation of Eqs. (34)-(10).

where

$$\xi_t = \frac{(1+\rho+2\eta)\mu_t - (1-\rho)\mu_t^*}{2(1+\eta)(\eta+\rho)} \quad \text{and} \quad \xi_t^* = \frac{(1+\rho+2\eta)\mu_t^* - (1-\rho)\mu_t}{2(1+\eta)(\eta+\rho)}$$

stem from the Home and Foreign target outputs in Eqs. (4) and (5), and they reflect the terms of trade externality.[46] (Recall the previous discussion in Section 2.2).

In what follows, we dissect the differences between the equilibria under Cooperation and Non-cooperation. First, we describe and compare their induced dynamics. Then we provide counterexamples to the presumption that Cooperation is always and everywhere incentive feasible, and we explain the forces underlying such incentive infeasibility of a Cooperation regime.

C.1 Dynamics and welfare decomposition: Cooperation and Non-cooperation

Figure 6 depicts the impulse responses of endogenous variables to a one-time positive Home markup shock in the initial period with $(y_{-1}, y^*_{-1}) = (0, 0)$. When the countries are insular ($\rho = 1$), Home inflation and output gap responses under Cooperation and Non-cooperation, respectively, look alike in response to a Home markup shock. The terms of trade s_t responds differently under Non-cooperation. When the countries are more risk averse ($\rho > 1$), the terms of trade plays a role like a positive (negative) markup shock to Foreign (Home) inflation. Conditional on shocks, policymakers may have incentive to deviate from Cooperation to Non-cooperation because under Non-cooperation they can manipulate the terms of trade. We further investigate the intuition behind these results as below.

Dynamics when $\rho = 1$. Consider first the special case of $\rho = 1$. Recall from earlier that when $\rho = 1$, the two countries become insular in the sense that the exogenous shock in one country does not feedback onto the other. That is, only the Home-country variables respond to the shock. Home inflation and output gap responses in the Cooperation and Non-cooperation regimes, respectively, are qualitatively similar: The policymaker in either case commits to future deflation and mitigates the trade-off between current inflation and the output gap as in the well-studied closed economy setting. The terms of trade s_t responds differently to a markup shock in Home country under the Non-cooperation regime. Under Cooperation, the terms of trade responds negatively to markup shocks. Under

[46] Observe that when $\rho = 1$, the term ξ_t (or ξ_t^*) only depends on domestic markup shocks μ_t (or μ_t^*) which means that the optimal Non-cooperation policy is inward-looking. However, there remain gains from cooperation because the terms of trade externality channel is still present.

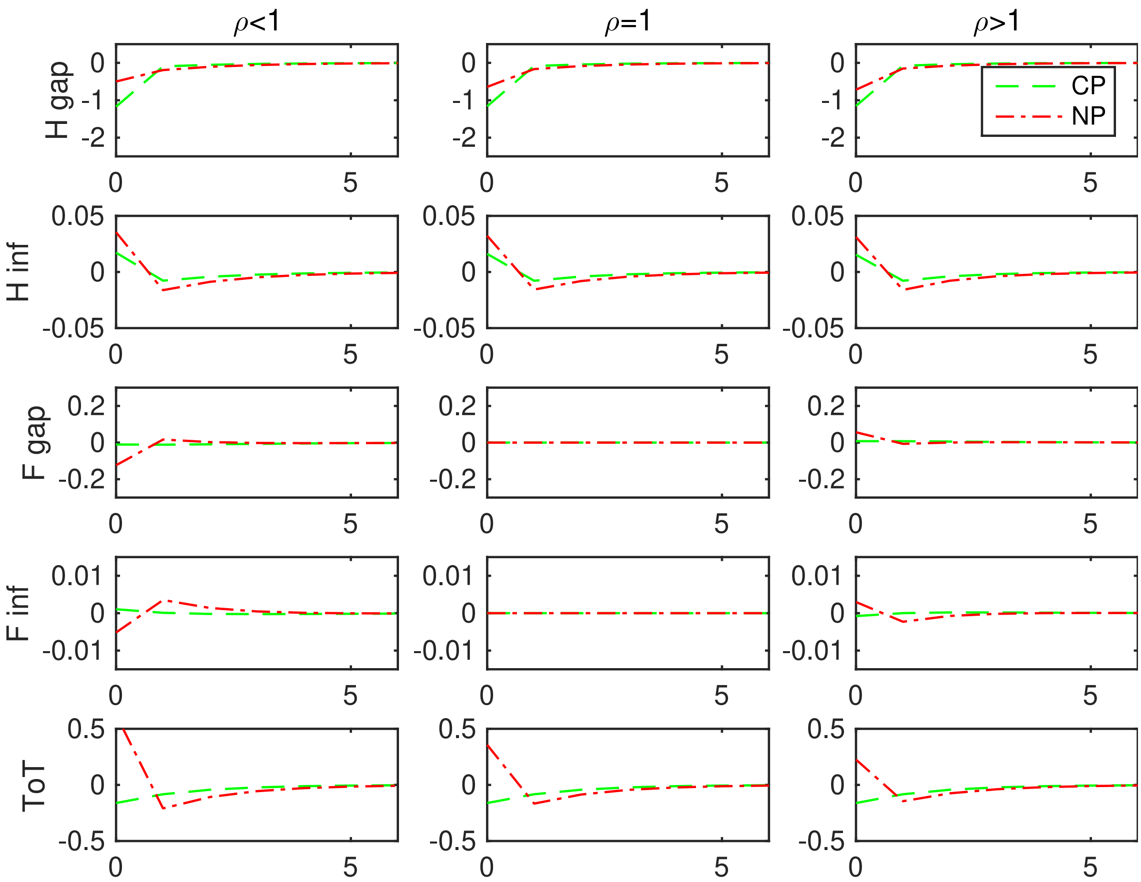
Non-cooperation, the terms of trade responds positively at the impact of the shock, so that the Home output gap ($x_t = y_t - \mu_t/(\eta + \rho)$) response is more attenuated; but this is traded off with a more aggressive response in Home inflation.

Under Non-cooperation, a positive response of the terms of trade can be deduced from the risk sharing first order condition (3). Note that when $\rho = 1$, the terms of trade s_t has no direct effect on the Phillips curve equilibrium restrictions on policy—i.e., Eqs. (1) and (2)—nor the policymakers' optimal trade-offs (9) and (10). Given its welfare trade-off with inflation, the output gap will not be completely closed, so then the shock will still imply a negative output gap outcome. Since the Foreign country is "insular", y_t^* remains unchanged in Foreign. Thus, the risk sharing condition implies that the terms of trade rises with the *rise* in y_t, upon impact of the shock (see Eq. (3)). Since Foreign still behaves in an insular manner and does not react to what Home does, it must absorb the marginal welfare loss involving $\tilde{x}_t = y_t + \mu_t/(\eta + \rho)$. The latter is the terms-of-trade externality effect discussed earlier (see Section 2.2.1). In this example, this is a spillover onto Foreign's welfare that is being exploited by Home.

In contrast, in the Cooperation regime, the consolidated policy maximizes the global social welfare function (6). As in the Non-cooperation regime, the shock will also yield a negative output gap, but the target output is zero in this case. Thus, the Cooperation regime's optimal plan ends up inducing a negative output in response to the Home markup shock. This translates as larger negative Home output gap (which is equal to output itself) and a negative terms of trade response upon impact. In return, Home inflation suffers a smaller fluctuation over time and output gap deviation from zero is shorter lived.

Dynamics when $\rho \neq 1$. When $\rho \neq 1$, the "non-insularity" channel (recall Section 2.1.1) against the terms-of-trade externality effect comes into consideration as well. When the countries are more risk averse ($\rho > 1$), the terms of trade plays a role akin to a positive markup shock to Foreign inflation. Since the Foreign country is no longer insular, under Non-cooperation Foreign will also react in order to offset Home's desire to manipulate the terms of trade externality. In a Markov perfect equilibrium of the Non-cooperation regime, this results in a positive Foreign output response which tends to weaken the positive terms of trade response that would have been if $\rho = 1$. Also, from Home's perspective, inducing a rise in the terms of trade acts as "negative markup shock" offsetting the incentive of Home to exploit the terms of trade externality itself. In other words, Home does not need to engineer such a large response in the terms of trade in order to absorb the original positive markup shock at home.

Figure 6: Impulse responses to a positive Home markup shock.



Notes: The left column is for the case of $\rho = 0.5$, the center column is for $\rho = 1.0$, and the right column is for $\rho = 1.5$.

Unconditional welfare decomposition. Figure 7 on page OA.30 — C depicts the (negative) unconditional expected welfare measures (i.e., welfare losses) for Home and Foreign, into their respective variance arguments in (32) and (33). This is done *separately* for each regime under Cooperation (panel *a*) and Non-cooperation (panel *b*), as a function of a sequence of economies indexed by the risk aversion parameter ρ.[47] This exercise reinforces the intuition developed above. Consider the Cooper-

[47]We should read this section with caution. In particular, note that when we consider the two regimes of Cooperation and Non-cooperation, we are looking at their respective equilibrium *unconditional* welfare outcomes separately. The discussion in this section makes no attempt to compare

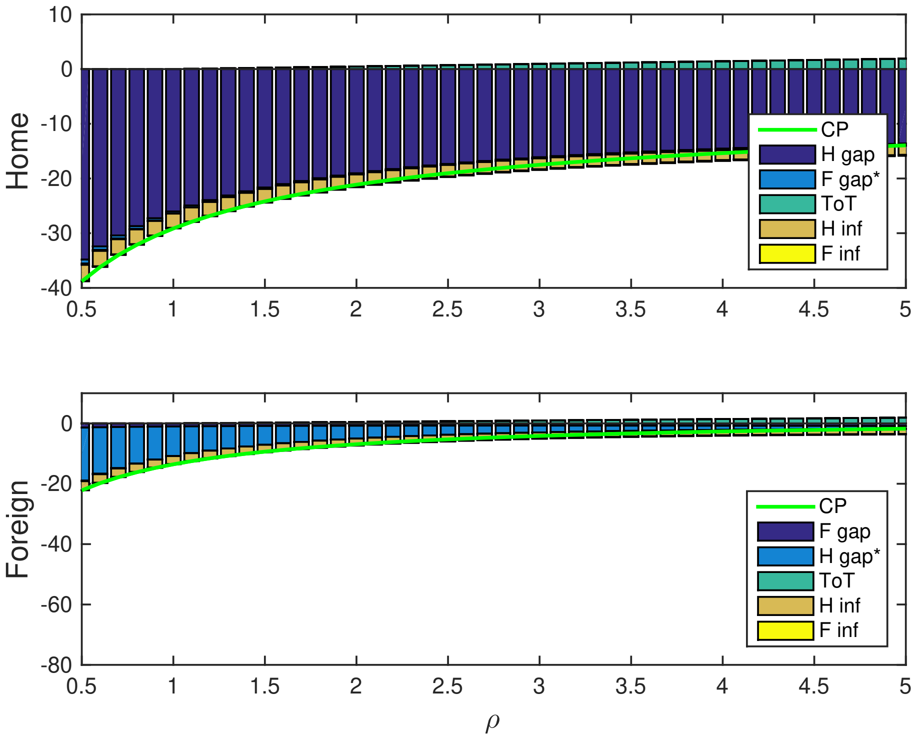
ation regime. Home country suffers from fluctuations in the Home output gap (labelled as "H gap" in the figure) in the face of the Home markup shock. (Foreign also faces a similar but more attenuated problem here as we are looking at stochastic simulations with all shocks active.) Because of the asymmetry of shock volatilities working relatively more against Home, the major component contributing to the unconditional welfare loss of Home shows up as Home output gap volatility. Also note that the direct terms of trade volatility contributes negatively (i.e. compounds losses) to Home welfare when $\rho < 1$, zero loss when $\rho = 1$, and positively when $\rho > 1$. This is an artifact of the third term in the welfare function (32) involving s_t—this term reflects the "non-insularity" channel we outlined in Section 2.1.1 earlier. This term switches sign from negative to positive as ρ is increased beyond unity, with its welfare effect taking on a zero value when $\rho = 1$.

These two observations, based on unconditional welfare decompositions, again echo the previously explained incentive problem for Home. If Home were in a Non-cooperation regime, it can reduce welfare loss associated with Home output-gap fluctuations by exploiting the terms of trade externality under Non-cooperation regime: Observe that in the contrasting Non-cooperation equilibrium in panel *b*, the contribution of Home output gap variance is smaller relative to the total welfare measured in the Non-cooperation regime itself. This is gained by sacrificing Foreign welfare in terms of noisier fluctuations in the second argument, $(y_t + \mu_t/(\eta + \rho))^2 > 0$, (labelled as "H gap*"). This term, as explained earlier, embodies the terms of trade externality effect from Home on Foreign, taking into account retaliation effects from Foreign to Home welfare in a Non-cooperation equilibrium. Consistent with the intuition developed earlier, a retaliation effect on Home's welfare (shown as "F gap*") becomes very small when $\rho = 1$.

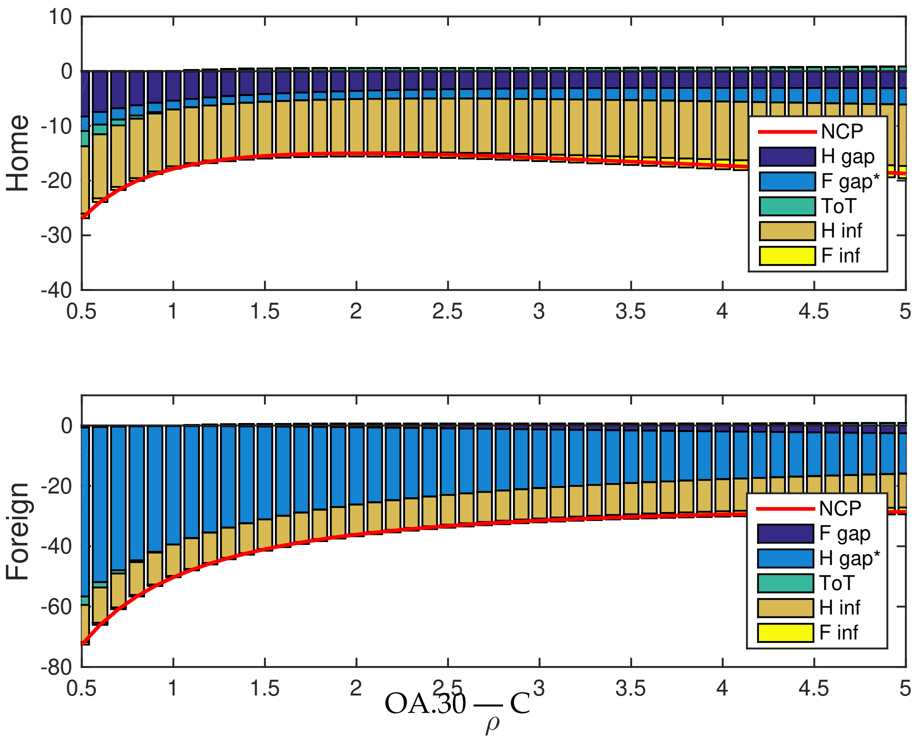
the welfare values across the two regime's equilibrium outcomes, as that would be a meaningless exercise.

Figure 7: Welfare (loss) decomposition under Cooperation and Non-cooperation: with varying ρ

a. Cooperation:



b. Non-cooperation:



Notes: These decompositions should only be read within the context of each regime only and comparisons between the top and the bottom panels should not be made.

D Recursive Sustainable Cooperation Plans

The preceding description of a Sustainable Cooperation equilibrium has a recursive representation. As a result, finding the equilibrium under a Sustainable Cooperation regime boils down to finding equilibrium policy functions that satisfy a modified Euler functional operator problem—i.e., one that is modified by a recursified set of history-dependent sustainability constraints. We describe what this recursive Sustainable Cooperation equilibrium operator looks like next, and then, we detail the nonlinear approximation schemes used to numerically compute the solution.

Let $s_{-1} = (y_{-1}, y^*_{-1}, \nu_{-1}) \in Y^2 \times N \equiv S \subset \mathbb{R}^2 \times (0,1)$ be a vector of endogenous state variables, and, $\tau = (u, u_{-1}) \in (M \times M^*)^2 \equiv T$ be a vector of exogenous state variables where $u = (\mu, \mu^*) \in M \times M^* \subset \mathbb{R}^2$. Note that the lagged markup shocks are included as state variables. Also let $x = h_x(s_{-1}, \tau)$, where $x \in \{y, y^*, \pi, \pi^*, z, \nu\}$, be unknown policy functions that induce a Sustainable Cooperation plans equilibrium. These functions are our objects of interest and they satisfy the following functional operator:

$$
\begin{aligned}
h_\pi(s_{-1}, \tau) &= \beta \sum_{u'} P(u'|u) h_\pi(s, u', u) + k\mu \\
&+ \frac{k}{2} \left[(1 + \rho + 2\eta)\, h_y(s_{-1}, \tau) - (1 - \rho)\, h_{y^*}(s_{-1}, \tau) \right], \\
h_{\pi^*}(s_{-1}, \tau) &= \beta \sum_{u'} P(u'|u) \pi^*(s, u', u) + k^* \mu^* \\
&+ \frac{k^*}{2} \left[(1 + \rho + 2\eta)\, h_{y^*}(s_{-1}, \tau) - (1 - \rho)\, h_y(s_{-1}, \tau) \right], \\
h_\pi(s_{-1}, \tau) &= -\sigma^{-1} \left[h_y(s_{-1}, \tau) - \zeta(s_{-1}, \tau) - h_z(s_{-1}, \tau)(y_{-1} - \zeta_{-1}(s_{-1}, \tau)) \right], \\
h_{\pi^*}(s_{-1}, \tau) &= -\sigma^{-1} \left[h_{y^*}(s_{-1}, \tau) - \zeta^*(s_{-1}, \tau) - h_z(s_{-1}, \tau)(y^*_{-1} - \zeta^*_{-1}(s_{-1}, \tau)) \right], \\
V(s_{-1}, \tau) &= -U(s_{-1}, \tau) + \beta \sum_{u'} P(u'|u) V(s, u', u) \geq W(y_{-1}, y^*_{-1}, \tau) \\
V^*(s_{-1}, \tau) &= -U^*(s_{-1}, \tau) + \beta \sum_{u'} P(u'|u) V^*(s, u', u) \geq W^*(y_{-1}, y^*_{-1}, \tau)
\end{aligned}
$$

where $U(s_{-1}, \tau)$ and $U^*(s_{-1}, \tau)$ are the per-period payoffs or losses for Home and Foreign, respectively, $W(y_{-1}, y^*_{-1}, \tau)$ and $W^*(y_{-1}, y^*_{-1}, \tau)$ are their respective values under the Non-cooperation regime, P is the joint Markov matrix for the independent markup shock process $\{\mu, \mu^*\}$. The functions (ζ, ζ^*) and $(\zeta_{-1}, \zeta^*_{-1})$ have been defined in the preceding section but now, it embeds a recursified structure with respect to the incentive compatibility requirements previously discussed in regard to (15) and (16).[48]

This system has a recursive structure with regard to $h_\pi(s_{-1},\tau)$, $h_{\pi^*}(s_{-1},\tau)$, $\Xi(s_{-1},\tau)$, $\Xi^*(s_{-1},\tau)$, $V(s_{-1},\tau)$, and $V(s_{-1},\tau)$.

Approximate solution scheme. The Sustainable Cooperation problem is nonlinear, despite involving quadratic and linear forms. This is because of the occasionally binding nature of the sustainability constraints. Thus, the solution for the Sustainable Cooperation equilibrium can only be obtained numerically.[49] We use a version of the policy function iteration method with occasionally binding constraints as in Kehoe and Perri (2002) and Sunakawa (2015). The occasionally binding constraints $V(s_{-1},\tau) \geq W(y_{-1},y^*_{-1},\tau)$ and $V^*(s_{-1},\tau) \geq W^*(y_{-1},y^*_{-1},\tau)$ must be addressed.[50] The functions need to be approximated by projection onto known families of basis functions, as continuation states $s = (h_y(s_{-1},\tau), h_{y^*}(s_{-1},\tau), h_v(s_{-1},\tau))$

[48] We repeat it here for convenience, and show how the problem has been recursified. Recall that:

$$
\begin{aligned}
\zeta(s_{-1},\tau) &= \frac{(1+\rho+2\eta)\vartheta(s_{-1},\tau)-(1-\rho)\vartheta^*(s_{-1},\tau)}{2(1+\eta)(\rho+\eta)}, \\
\zeta^*(s_{-1},\tau) &= -\frac{(1+\rho+2\eta)\vartheta^*(s_{-1},\tau)-(1-\rho)\vartheta(s_{-1},\tau)}{2(\rho+\eta)(1+\eta)}, \\
\zeta_{-1}(s_{-1},\tau) &= \frac{(1+\rho+2\eta)\vartheta_{-1}(s_{-1},\tau)-(1-\rho)\vartheta^*_{-1}(s_{-1},\tau)}{2(1+\eta)(\rho+\eta)}, \\
\zeta^*_{-1}(s_{-1},\tau) &= -\frac{(1+\rho+2\eta)\vartheta^*_{-1}(s_{-1},\tau)-(1-\rho)\vartheta(s_{-1},\tau)}{2(\rho+\eta)(1+\eta)},
\end{aligned}
$$

and

$$
\begin{aligned}
\vartheta(s_{-1},\tau) &= (2h_v(s_{-1},\tau)-1)\mu - \beta\sum_{u'} P(u'|u)\Xi(s,u',u), \\
\vartheta^*(s_{-1},\tau) &= (2h_v(s_{-1},\tau)-1)\mu^* + \beta\sum_{u'} P(u'|u)\Xi^*(s,u',u), \\
\vartheta_{-1}(s_{-1},\tau) &= (2v_{-1}-1)\mu_{-1} - \beta\Xi(s_{-1},\tau), \\
\vartheta^*_{-1}(s_{-1},\tau) &= (2v_{-1}-1)\mu^*_{-1} + \beta\Xi^*(s_{-1},\tau).
\end{aligned}
$$

Specifically, $\Xi(s_{-1},\tau)$ and $\Xi^*(s_{-1},\tau)$ recursify the problem as so:

$$
\begin{aligned}
\Xi(s_{-1},\tau) &= (z(s_{-1},\tau)^{-1}-1)\left[I(s_{-1},\tau)D_1W(y_{-1},y^*_{-1},\tau)+I^*(s_{-1},\tau)D_1W^*(y_{-1},y^*_{-1},\tau)\right], \\
\Xi^*(s_{-1},\tau) &= (z(s_{-1},\tau)^{-1}-1)\left[I(s_{-1},\tau)D_2W(y_{-1},y^*_{-1},\tau)+I^*(s_{-1},\tau)D_2W^*(y_{-1},y^*_{-1},\tau)\right],
\end{aligned}
$$

where $I(s_{-1},\tau)$ (and $I^*(s_{-1},\tau)$) are indicator functions that equal unity if the Home (Foreign) constraint is binding, and zero otherwise.

[49] Program codes are written in Fortran 90 with OpenMP shared-memory parallelization. We used a computer with 28 cores of 2.6 Ghz Intel Xeon E5-2680 v3 running on the Centos 6 GNU/Linux operating system.

[50] In general, these constraints may make the problem non-convex so that the numerical algorithm (based on assuming the existence of a unique functional fixed-point) may end up finding only one of multiple equilibria. We cannot prove the existence nor uniqueness of the equilibrium, but numerically we conduct robustness checks utilizing different initial guesses of the equilibrium policy functions that would have delivered the Sustainable Cooperation plan. Our numerical results do not seem to suffer from such problems of equilibrium multiplicity.

may not be on the grid points. Three-dimensional cubic spline bases are used for interpolation. We set $Y = [-3.0, 3.0]$ and $N = (0.0, 1.0)$ and divide them each into 5 knot points. Each element in $\tau = (\mu, \mu^*, \mu_{-1}, \mu^*_{-1})$ follows the Markov chain described earlier.[51]

D.1 Computational procedure

The initial guess of the functions is set as $h_x^{(0)}(s_{-1}, \tau)$ for $x = \{\pi, \pi^*, V, V^*, \Xi, \Xi^*\}$ on each grid point $(s_{-1}, \tau) \in Y^2 \times (0,1) \times M^2 \times (M^*)^2$. In each iteration $i = 1, 2, ...,$ given the functions $h_x^{(i-1)}(s_{-1}, \tau)$ whose values are defined on each grid point (s_{-1}, τ), first we assume the sustainability constraints are slack. That is, $z(s_{-1}, \tau) = 1$, $\nu(s_{-1}, \tau) = \nu_{-1}$ and $\Xi(s_{-1}, \tau) = \Xi^*(s_{-1}, \tau) = 0$. Then the relevant equations are solved

$$
\begin{aligned}
\pi &= \beta \hat{h}^{(i-1)}_{\pi,\mu}(y, y^*, \nu_{-1}) + k\mu \\
&+ k\left[(\rho + \eta + (1-\gamma)(1-\rho))\, y - (1-\gamma)\,(1-\rho)\, y^*\right], \\
\pi^* &= \beta \hat{h}^{(i-1)}_{\pi^*,\mu}(y, y^*, \nu_{-1}) + k^*\mu^* \\
&+ k^*\left[(\rho + \eta + (1-\gamma)(1-\rho))\, y^* - (1-\gamma)\,(1-\rho)\, y\right],
\end{aligned}
$$

where

$$
\begin{aligned}
\pi &= -\sigma^{-1}\left[y - \zeta - (y_{-1} - \zeta_{-1})\right], \\
\pi^* &= -\sigma^{-1}\left[y^* - \zeta^* - (y^*_{-1} - \zeta^*_{-1})\right], \\
\zeta &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta - \gamma(1-\rho)\vartheta^*}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^* &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta^* - (1-\gamma)(1-\rho)\vartheta}{2(1-\gamma)(\rho+\eta)(1+\eta)}, \\
\zeta_{-1} &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta_{-1} - \gamma(1-\rho)\vartheta^*_{-1}}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^*_{-1} &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta^*_{-1} - (1-\gamma)(1-\rho)\vartheta_{-1}}{2(1-\gamma)(\rho+\eta)(1+\eta)},
\end{aligned}
$$

[51] The number of the grid points for Y and N are increased to check the robustness of our result. As we have seven state variables, this kind of exercise is very time-consuming as it exponentially increases the total number of grid points.

and

$$
\begin{aligned}
\vartheta &= 2(\nu_{-1}-\gamma)\mu - \beta\hat{h}^{(i-1)}_{\Xi,u}(y,y^*,\nu_{-1}), \\
\vartheta^* &= 2(\nu_{-1}-\gamma)\mu^* + \beta\hat{h}^{(i-1)}_{\Xi^*,u}(y,y^*,\nu_{-1}), \\
\vartheta_{-1} &= 2(\nu_{-1}-\gamma)\mu_{-1}, \\
\vartheta^*_{-1} &= 2(\nu_{-1}-\gamma)\mu^*_{-1},
\end{aligned}
$$

for (y,y^*,π,π^*) using a non-linear optimization routine. Then the candidate values of welfare, (V,V^*) are also obtained by

$$
\begin{aligned}
V &= -U(y,y^*,\pi,\pi^*,\mu,\mu^*) + \beta\hat{h}^{(i-1)}_{V,u}(y,y^*,\nu_{-1}), \\
V^* &= -U^*(y,y^*,\pi,\pi^*,\mu,\mu^*) + \beta\hat{h}^{(i-1)}_{V^*,u}(y,y^*,\nu_{-1}),
\end{aligned}
$$

where

$$
\begin{aligned}
U(y,y^*,\pi,\pi^*,\mu,\mu^*) &= \gamma\,(\eta+\rho)\left(y - \frac{1-\gamma}{\gamma}\frac{1}{\eta+\rho}\mu\right)^2 + \frac{\gamma\sigma}{k}\pi^2 \\
&\quad +\gamma(1-\gamma)(1-\rho)(y-y^*)^2 \\
&\quad +(1-\gamma)\,(\eta+\rho)\left(y^* + \frac{1}{\eta+\rho}\mu^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}\,(\pi^*)^2, \\
U^*(y,y^*,\pi,\pi^*,\mu,\mu^*) &= (1-\gamma)\,(\eta+\rho)\left(y^* - \frac{\gamma}{1-\gamma}\frac{1}{\eta+\rho}\mu^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}\,(\pi^*)^2 \\
&\quad +\frac{1}{2}(1-\rho)(y^*-y)^2 \\
&\quad +\gamma\,(\eta+\rho)\left(y + \frac{1}{\eta+\rho}\mu\right)^2 + \frac{\gamma\sigma}{k}\pi^2.
\end{aligned}
$$

Also, $\hat{h}_{x,u}(s) = \sum_{u'} P(u'|u)h_x(s,u',u)$ where $x = \{\pi,\pi^*,\Xi,\Xi^*,V,V^*\}$, and the h_x functions are approximated using three-dimensional splines for $s \in \Upsilon^2 \times (0,1)$, which may be off the grid points.

Then we proceed to check if the sustainability constraints are binding with the candidate values of welfare, (V,V^*). $W(y_{-1},y^*_{-1},\tau)$ and $W^*(y_{-1},y^*_{-1},\tau)$ are also numerically obtained with the projection method (see the end of this section). Note that only the Home or Foreign constraint is binding at a time and there are two possible cases, (i) the Home constraint is binding or (ii) the Foreign constraint is binding.

- (i) When the Home constraint is binding, $V \leq W(y_{-1},y^*_{-1},\tau)$: The relevant equations are solved

$$
\begin{aligned}
\pi &= \beta \hat{h}^{(i-1)}_{\pi,\tau}(y, y^*, \nu) + k\mu \\
&+ k\left[(\rho+\eta+(1-\gamma)(1-\rho))\, y - (1-\gamma)\,(1-\rho)\, y^*\right], \\
\pi^* &= \beta \hat{h}^{(i-1)}_{\pi^*,\tau}(y, y^*, \nu) + k^*\mu^* \\
&+ k^*\left[(\rho+\eta+(1-\gamma)(1-\rho))\, y^* - (1-\gamma)\,(1-\rho)\, y\right], \\
W(y_{-1}, y^*_{-1}, \tau) &= -U(y, y^*, \pi, \pi^*, \mu, \mu^*) + \beta \hat{h}^{(i-1)}_{V,\tau}(y, y^*, \nu)
\end{aligned}
$$

where $\nu = 1 - z(1 - \nu_{-1}), z \in (0,1)$,

$$
\begin{aligned}
\pi &= -\sigma^{-1}\left[y - \zeta - z(y_{-1} - \zeta_{-1})\right], \\
\pi^* &= -\sigma^{-1}\left[y^* - \zeta^* - z(y^*_{-1} - \zeta^*_{-1})\right], \\
\zeta &= \frac{[\eta+\rho+\gamma(1-\rho)]\vartheta - \gamma(1-\rho)\vartheta^*}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^* &= -\frac{[\eta+\rho+(1-\gamma)(1-\rho)]\vartheta^* - (1-\gamma)(1-\rho)\vartheta}{2(1-\gamma)(\rho+\eta)(1+\eta)}, \\
\zeta_{-1} &= \frac{[\eta+\rho+\gamma(1-\rho)]\vartheta_{-1} - \gamma(1-\rho)\vartheta^*_{-1}}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^*_{-1} &= -\frac{[\eta+\rho+(1-\gamma)(1-\rho)]\vartheta^*_{-1} - (1-\gamma)(1-\rho)\vartheta_{-1}}{2(1-\gamma)(\rho+\eta)(1+\eta)},
\end{aligned}
$$

and

$$
\begin{aligned}
\vartheta &= 2(\nu-\gamma)\mu - \beta \hat{h}^{(i-1)}_{\Xi,\tau}(y, y^*, \nu), \\
\vartheta^* &= 2(\nu-\gamma)\mu^* + \beta \hat{h}^{(i-1)}_{\Xi^*,\tau}(y, y^*, \nu), \\
\vartheta_{-1} &= 2(\nu_{-1}-\gamma)\mu_{-1} - \beta(z^{-1}-1)D_1 W(y_{-1}, y^*_{-1}, \tau), \\
\vartheta^*_{-1} &= 2(\nu_{-1}-\gamma)\mu^*_{-1} + \beta(z^{-1}-1)D_2 W(y_{-1}, y^*_{-1}, \tau),
\end{aligned}
$$

for (y, y^*, π, π^*) and (z, ν). Note that the latter is now endogenously solved.

- (ii) When the Foreign constraint is binding, $V \leq W^*(y_{-1}, y^*_{-1}, \tau)$: The relevant equations are solved

$$\begin{aligned}
\pi &= \beta \hat{h}^{(i-1)}_{\pi,\tau}(y, y^*, \nu) + k\mu \\
&+ k\left[(\rho + \eta + (1-\gamma)(1-\rho))\, y - (1-\gamma)\,(1-\rho)\, y^*\right], \\
\pi^* &= \beta \hat{h}^{(i-1)}_{\pi^*,\tau}(y, y^*, \nu) + k^*\mu^* \\
&+ k^*\left[(\rho + \eta + (1-\gamma)(1-\rho))\, y^* - (1-\gamma)\,(1-\rho)\, y\right], \\
W^*(y_{-1}, y^*_{-1}, \tau) &= -U^*(y, y^*, \pi, \pi^*, \mu, \mu^*) + \beta \hat{h}^{(i-1)}_{V^*,\tau}(y, y^*, \nu)
\end{aligned}$$

where $\nu = z\nu_{-1}$, $z \in (0,1)$,

$$\begin{aligned}
\pi &= -\sigma^{-1}\left[y - \zeta - z(y_{-1} - \zeta_{-1})\right], \\
\pi^* &= -\sigma^{-1}\left[y^* - \zeta^* - z(y^*_{-1} - \zeta^*_{-1})\right], \\
\zeta &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta - \gamma(1-\rho)\vartheta^*}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^* &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta^* - (1-\gamma)(1-\rho)\vartheta}{2(1-\gamma)(\rho+\eta)(1+\eta)}, \\
\zeta_{-1} &= \frac{[\eta + \rho + \gamma(1-\rho)]\vartheta_{-1} - \gamma(1-\rho)\vartheta^*_{-1}}{2\gamma(1+\eta)(\rho+\eta)}, \\
\zeta^*_{-1} &= -\frac{[\eta + \rho + (1-\gamma)(1-\rho)]\vartheta^*_{-1} - (1-\gamma)(1-\rho)\vartheta_{-1}}{2(1-\gamma)(\rho+\eta)(1+\eta)},
\end{aligned}$$

and

$$\begin{aligned}
\vartheta &= 2(\nu - \gamma)\mu - \beta \hat{h}^{(i-1)}_{\Xi,\tau}(y, y^*, \nu), \\
\vartheta^* &= 2(\nu - \gamma)\mu^* + \beta \hat{h}^{(i-1)}_{\Xi^*,\tau}(y, y^*, \nu), \\
\vartheta_{-1} &= 2(\nu_{-1} - \gamma)\mu_{-1} - \beta(z^{-1} - 1)D_1 W^*(y_{-1}, y^*_{-1}, \tau), \\
\vartheta^*_{-1} &= 2(\nu_{-1} - \gamma)\mu^*_{-1} + \beta(z^{-1} - 1)D_2 W^*(y_{-1}, y^*_{-1}, \tau),
\end{aligned}$$

for (y, y^*, π, π^*) and (z, ν).

After checking binding constraints, calculate (Ξ, Ξ^*) following the binding pattern of the constraints:

$$\Xi = \begin{cases} (z^{-1}-1)D_1W(y_{-1}, y^*_{-1}, \tau) & \text{if } I(s_{-1}, \tau) = 1, \\ (z^{-1}-1)D_1W^*(y_{-1}, y^*_{-1}, \tau) & \text{if } I^*(s_{-1}, \tau) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

$$\Xi^* = \begin{cases} (z^{-1}-1)D_2W(y_{-1}, y^*_{-1}, \tau) & \text{if } I(s_{-1}, \tau) = 1, \\ (z^{-1}-1)D_2W^*(y_{-1}, y^*_{-1}, \tau) & \text{if } I^*(s_{-1}, \tau) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Once we finish solving the relevant equations at each grid point, the functions $h_x^{(i)}(s_{-1}, \tau) = \{x_{s_{-1},\tau}\}_{(s_{-1},\tau)\in S\times T}$ for $x = \{\pi, \pi^*, V, V^*, \Xi, \Xi^*\}$ are updated.

The algorithm is summarized as follows:

1. Set the initial guess of the functions $h_x^{(0)}(s_{-1}, \tau)$ where $x = \{\pi, \pi^*, V, V^*, \Xi, \Xi^*\}$ on each grid point $(s_{-1}, \tau) \in Y^2 \times (0, 1) \times M^2 \times (M^*)^2 = S \times T$.

2. In each iteration $i = 1, 2, ...,$ given the functions $h_x^{(i-1)}(s_{-1}, \tau)$ and on each grid point (s_{-1}, τ):

   (a) Assume the sustainability constraints are slack, and solve the equilibrium conditions for (y, y^*, π, π^*). Note that $z = 1$ and $\nu = \nu_{-1}$. Once the relevant equations are solved, the candidate values of welfare, (V, V^*), are also obtained.

   (b) Check if the sustainability constraints are binding with the candidate values of welfare. If the Home (or Foreign) constraint is binding, set $V = W(y_{-1}, y^*_{-1}, \tau)$ (or $V^* = W^*(y_{-1}, y^*_{-1}, \tau)$) and re-solve the equilibrium conditions for (y, y^*, π, π^*) and (z, ν).

   (c) Calculate (Ξ, Ξ^*) following the binding pattern of the constraints.

3. Update the functions $h_x^{(i)}(s_{-1}, \tau) = \{x_{s_{-1},\tau}\}_{(s_{-1},\tau)\in S\times T}$ for $x = \{\pi, \pi^*, V, V^*, \Xi, \Xi^*\}$.

4. Iterate 2-4 until the functions converge at each grid point, i.e., $\left\| h_x^{(i)}(s_{-1}, \tau) - h_x^{(i-1)}(s_{-1}, \tau) \right\| < \epsilon$, where $\|\cdot\|$ is the uniform norm and ϵ is a very small real number.

Non-cooperation $W(y_{-1}, y^*_{-1}, \tau)$ and $W^*(y_{-1}, y^*_{-1}, \tau)$ are also numerically obtained with the policy function iteration method. Let $s_{-1} = (y_{-1}, y^*_{-1})$, $\tau = (\mu, \mu^*, \mu_{-1}, \mu^*_{-1})$ be a vector of endogenous and exogenous variable each and $x = h_x(s_{-1}, \tau)$ be the policy functions where $x = \{y, y^*, \pi, \pi^*\}$. A similar policy function iteration algorithm above (but the state space is different as there is no ν_{-1}) is used to compute the policy functions. In Step 2 in the above algorithm, the equilibrium conditions are solved

$$\begin{aligned}
\pi &= \beta\hat{h}^{(i-1)}_{\pi,\tau}(y, y^*) + k\mu \\
&+ k\left[(\rho+\eta+(1-\gamma)(1-\rho))\,y - (1-\gamma)\,(1-\rho)\,y^*\right], \\
\pi^* &= \beta\hat{h}^{(i-1)}_{\pi^*,\tau}(y, y^*) + k^*\mu^* \\
&+ k^*\left[(\rho+\eta+(1-\gamma)(1-\rho))\,y^* - (1-\gamma)\,(1-\rho)\,y\right],
\end{aligned}$$

where

$$\begin{aligned}
\pi &= -\sigma^{-1}\left[y - \xi - (y_{-1} - \xi_{-1})\right], \\
\pi^* &= -\sigma^{-1}\left[y^* - \xi^* - (y^*_{-1} - \xi^*_{-1})\right], \\
\xi &= \frac{1-\gamma}{\gamma}\frac{(\eta+\rho+\gamma(1-\rho))\,\mu - \gamma(1-\rho)\mu^*}{(\eta+\rho)(1+\eta)}, \\
\xi^* &= \frac{\gamma}{1-\gamma}\frac{(\eta+\rho+(1-\gamma)(1-\rho))\,\mu^* - (1-\gamma)(1-\rho)\mu}{(\eta+\rho)(1+\eta)}, \\
\xi_{-1} &= \frac{1-\gamma}{\gamma}\frac{(\eta+\rho+\gamma(1-\rho))\,\mu_{-1} - \gamma(1-\rho)\mu^*_{-1}}{(\eta+\rho)(1+\eta)}, \\
\xi^*_{-1} &= \frac{\gamma}{1-\gamma}\frac{(\eta+\rho+(1-\gamma)(1-\rho))\,\mu^*_{-1} - (1-\gamma)(1-\rho)\mu_{-1}}{(\eta+\rho)(1+\eta)},
\end{aligned}$$

for (y, y^*, π, π^*). Once the relevant equations are solved, the value functions are also obtained by

$$\begin{aligned}
W &= -U(y, y^*, \pi, \pi^*, \mu, \mu^*) + \beta\hat{h}^{(i-1)}_{W,\tau}(y, y^*), \\
W^* &= -U^*(y, y^*, \pi, \pi^*, \mu, \mu^*) + \beta\hat{h}^{(i-1)}_{W^*,\tau}(y, y^*).
\end{aligned}$$

Note that $\hat{h}_{x,\tau}(s) = \sum_{\tau'} P(\tau'|\tau)h_x(s,\tau')$ where $x = \{\pi, \pi^*, W, W^*\}$ are approximated by using two-dimensional splines for $s \in Y^2$ conditioned on τ.

D.2 Unconditional Welfare Decompositions

From another perspective, Figure 8 on page OA.40 — E displays the unconditional expected welfare values under a Sustainable Cooperation regime. We will summarize the main points of this complementary analysis.[52]

[52]Please see Online Appendix A.3 for details of this welfare measure. This measure is useful for the decomposition of welfare into the contributions of its constituent variables. A similar exercise to

Home suffers from the loss associated with its output gap fluctuations under Cooperation (labelled as "H gap" in Figure 8 on page OA.40 — E). Under Non-cooperation, Home can reduce the loss dramatically by worsing Foreign's loss by the spillover effect from stabilizing Home's output gap onto Foreign's welfare (labelled as "H gap*" in Figure 8 on page OA.40 — E).

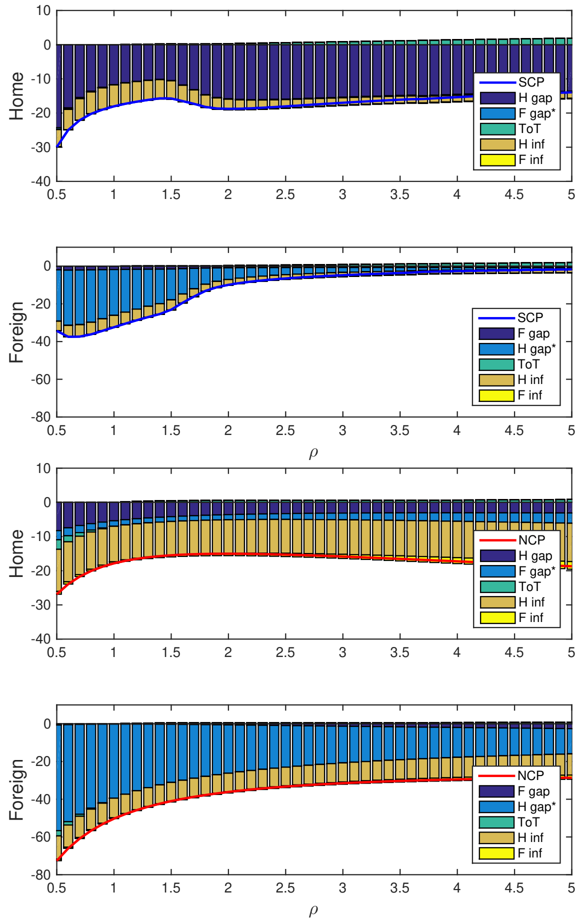
The history contingent contract under Sustainable Cooperation balances the incentive for Home to deviate from Cooperation to Non-cooperation, whereas in the Non-Cooperation regime Foreign bears more of the brunt of the terms-of-trade externality. We can deduce that a Sustainable Cooperation plan can alleviate Home's loss associated with the H gap fluctuations with a relatively smaller sacrifice of Foreign's welfare with the H gap* fluctuations.[53] These relatively more attenuated welfare loss spillovers to Foreign under Sustainable cooperation are supported by the observation earlier that under a Sustainable Cooperation regime, the contract ensures that Foreign retaliation-inducing incentives do not arise as much in the first place, as under a Non-cooperation regime. That is, under the Sustainable Cooperation plan, welfare must be redistributed to the Home country that experiences a relatively more volatile shock, but just enough and in a history contingent way, so that overall, welfare for both countries are no worse than under a Non-cooperation regime.

---

this was also done for the Cooperation versus the Non-cooperation regimes in the Online Appendix C (Figure 7).

[53]Note that we must not compare the absolute levels of welfare outcomes across the two figures, as the welfare approximation is based on specific assumptions under different cooperation regimes.

Figure 8: Welfare decomposition under Sustainable Cooperation (*left*) and Non-cooperation (*right*) as functions of ρ



Notes: These decompositions should only be read within the context of each regime only and comparisons between the left and the right panels should not be made.

E Efficiency and Redistribution Properties

To construct the Pareto set in Section 3.4 of the paper, we solve the family of problems:

$$\max \left[\lambda V_0 + (1-\lambda) V_0^*\right],$$

subject to Eqs. (1) and (2), where $\lambda \in (0,1)$ is an arbitrary Pareto weight assigned to Home country. The baseline Cooperation regime we considered was the case of $\lambda = 1/2$. The short-hand notations V_0 and V_0^* refer to Home and Foreign's ex-ante welfare defined in (4)-(5).

The equilibrium conditions are summarized as

$$-\sigma\pi_t = y_t - \tilde{\zeta}_t - (y_{t-1} - \tilde{\zeta}_{t-1}),$$
$$-\sigma\pi_t^* = y_t^* - \tilde{\zeta}_t^* - (y_{t-1}^* - \tilde{\zeta}_{t-1}^*),$$

where $\tilde{\zeta}_t = \frac{(1+\rho+2\eta)\tilde{\vartheta}_t-(1-\rho)\tilde{\vartheta}_t^*}{2(1+\eta)(\rho+\eta)}$, $\tilde{\zeta}_t^* = -\frac{(1+\rho+2\eta)\tilde{\vartheta}_t^*-(1-\rho)\tilde{\vartheta}_t}{2(1+\eta)(\rho+\eta)}$, $\tilde{\vartheta}_t = (2\lambda-1)\mu_t$ and $\tilde{\vartheta}_t^* = (2\lambda-1)\mu_t^*$. There are similarities between these equations and the trade-off equations with the shifters (Eqs. (13)-(16)) under Sustainable Cooperation. Note that at the limit $\lambda = 0$, i.e., the corresponding Cooperation solution effectively cares only about Foreign's welfare, so that $\tilde{\zeta}_t = -\zeta_t$ and $\tilde{\zeta}_t^* = \zeta_t^*$ hold, but this is equivalent to the limiting case of the Sustainable Cooperation regime (with history-dependent pseudo-Pareto weights) in Proposition 2(i). Conversely, when we are at the limit $\lambda = 1$, $\tilde{\zeta}_t = \zeta_t$ and $\tilde{\zeta}_t^* = -\zeta_t^*$ hold. Also, when $\lambda = 1/2$, $\tilde{\zeta}_t = \tilde{\zeta}_t^* = 0$ holds, so the equilibrium conditions are the same as in the baseline Cooperation regime.

F Asymmetric Country Size

We have considered the case of asymmetric shocks when the countries are symmetric in all other aspects. Now we generalize this to the setting with asymmetric country sizes. To keep the insights clean, we now render the country-specific shock processes symmetric. Benigno (2002) also considered the case of asymmetric country size and discussed when countries have incentive to deviate from Cooperation to Non-cooperation. Here we revisit this question with the addition of the Sustainable Cooperation regime.

When the countries are asymmetric in their size, the welfare functions for Home and Foreign countries are given by

$$V_0 = -\frac{1}{2}\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \begin{bmatrix} \gamma(\eta+\rho)\left(y_t - \frac{1-\gamma}{\gamma(\eta+\rho)}\mu_t\right)^2 + \frac{\gamma\sigma}{k}\pi_t^2 \\ +\gamma(1-\gamma)(1-\rho)(y_t - y_t^*)^2 \\ +(1-\gamma)(\eta+\rho)\left(y_t^* + \frac{1}{\eta+\rho}\mu_t^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}(\pi_t^*)^2 \end{bmatrix}, \quad (38)$$

and

$$V_0^* = -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t \left[\begin{array}{c} (1-\gamma)(\eta+\rho)\left(y_t^* - \frac{\gamma}{(1-\gamma)(\eta+\rho)}\mu_t^*\right)^2 + \frac{(1-\gamma)\sigma}{k^*}(\pi_t^*)^2 \\ +\gamma(1-\gamma)(1-\rho)(y_t^* - y_t)^2 \\ +\gamma(\eta+\rho)\left(y_t + \frac{1}{\eta+\rho}\mu_t\right)^2 + \frac{\gamma\sigma}{k}\pi_t^2 \end{array}\right] \quad (39)$$

where $\gamma \in [0,1]$ is the (Cobb-Douglas) share of Home-produced goods in the consumption index of Home consumers—i.e., it measures the size of Home relative to Foreign.[54] The countries are also subject to the Phillips curves,

$$\pi_t = \beta\mathbb{E}_t\pi_{t+1} + k\mu_t + k\left[(\rho+\eta)y_t + (1-\gamma)(1-\rho)(y_t - y_t^*)\right], \quad (40)$$

and

$$\pi_t^* = \beta\mathbb{E}_t\pi_{t+1}^* + k\mu_t^* + k\left[(\rho+\eta)y_t^* - \gamma(1-\rho)(y_t - y_t^*)\right]. \quad (41)$$

The equilibrium policy trade-off conditions in the Cooperation, Non-cooperation and Sustainable Cooperation regimes can be derived as respective generalizations of equations (34)-(35) (Cooperation), (9)-(10) (Non-cooperation), and (13)-(14) (Sustainable Cooperation) in which $\gamma \neq 1/2$. These generalized conditions are shown in Appendix B.1. Note that the global welfare function is now given by

$$\begin{aligned} V_0^W &= \gamma V_0 + (1-\gamma)V_0^*, \\ &= -\frac{1}{2}\mathbb{E}_0\sum_{t=0}^{\infty}\beta^t\Big[\gamma(\eta+\rho)y_t^2 + (1-\gamma)(\eta+\rho)(y_t^*)^2 \\ &\qquad + \gamma(1-\gamma)(1-\rho)(y_t - y_t^*)^2 + \frac{\gamma\sigma}{k}\pi_t^2 + \frac{(1-\gamma)\sigma}{k}(\pi_t^*)^2\Big] \quad (42) \end{aligned}$$

Observe that $\gamma \neq 1/2$ now shifts the weights of each variable in the welfare functions as shown in Eqs. (38)-(39) and (42). Besides that, γ changes the target output for Home and Foreign in the welfare functions. For example, the Home output target is $[(1-\gamma)/\gamma]\mu_t/(\eta+\rho)$; a lower γ implies a higher target output given μ_t. Also, note that γ enters as a part of coefficient on the terms of trade in the NKPCs (40) and (41). If γ is low, the feedback effect of Home markup shock via the terms of trade on Foreign NKPC is diminished. These effects of $\gamma \neq 1/2$ may temper the policymakers' incentive to manipulate the terms of trade, relative to the symmetric country size case considered earlier.

Consider again our first question within this more general environment: Can

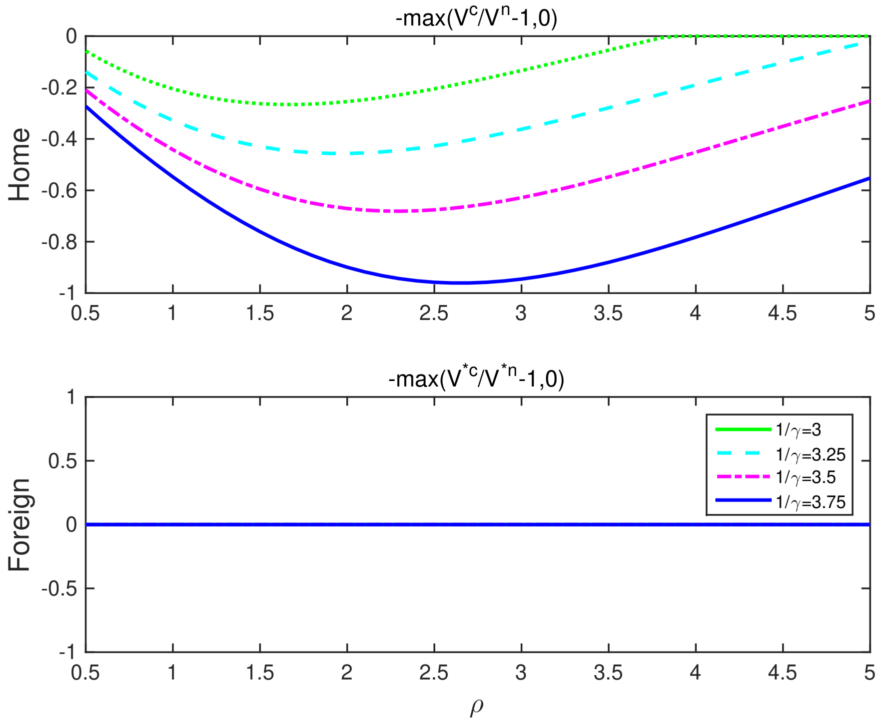
[54] See Appendix A.2 for the derivations.

Cooperation be an incentive infeasible regime? As in Section 3.1.1, we compute the statistic $R_0 \equiv -\max\{V_0^c/V_0^n - 1, 0\}$, which will be negative if the conditional welfare under Cooperation is dominated by that under Non-cooperation, $V_0^c < V_0^n$. Figure 9 on page OA.44 — F shows the statistics of Home (R_0) and Foreign (R_0^*) under the regimes of Cooperation and Non-cooperation. Even if the shock is symmetric among the countries, a relatively smaller Home country ($\gamma < 1/2$) has incentive to deviate from Cooperation to Non-cooperation. The Cooperation regime is not incentive-feasible when the countries differ in size: The lower is Home's size, the more Home has incentive to deviate from a given Cooperation regime. (Conversely, Foreign has incentive to deviate when $\gamma > 1/2$.)

Now consider R_0 as a function of risk aversion ρ in Figure 9 on page OA.44 — F, as in the case of asymmetric shocks.[55] Compared to the case of $\gamma = 1/2$ with asymmetric shocks, the economy where ρ is such that the Home policymaker has the largest incentive to deviate, shifts toward the right and away from unity. This is merely an artifact of $\gamma \neq 1/2$ (c.f., Figure 7 on page OA.30 — C earlier where the peak temptation occurs at unity). To illustrate the effect $\gamma \neq 1/2$ has on the location of peak temptation (as a function of risk aversion ρ), we consider the graph of the statistic $R_0(\rho, \gamma)$ for various values of $\gamma \in \{1/3.75, 1/3.5, 1/3.25, 1/3\}$. (We rig this example to consider the case that the temptation to deviate from Cooperation is solely on the side of Home.)

[55]When ρ is too high, V^n takes a positive value and the second-order conditions may be violated so that no equilibrium can exist. See Footnote 20.

Figure 9: Welfare comparison under Cooperation vs. Non-cooperation: Asymmetric country size.



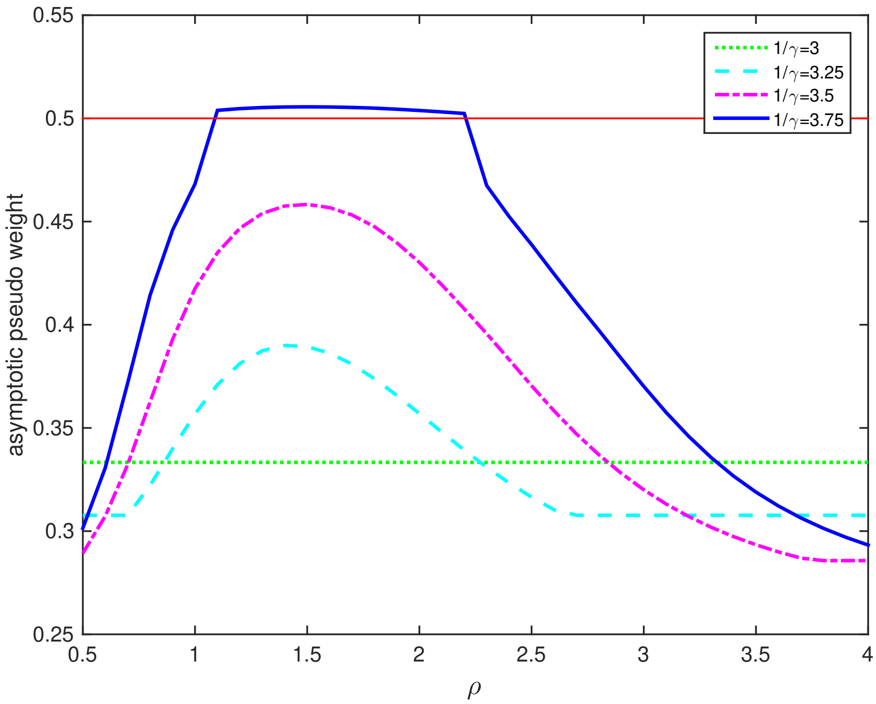
If γ is lower (Home is smaller in size), the terms of trade externality problem diminishes, since the target output gap is higher—i.e., Home has to stabilize output gap more given a markup shock. Moreover, a lower γ dampens the non-insularity channel of the terms of trade (for any given ρ), and its attendant feedback and Foreign retaliation effects onto Home welfare (should home attempt to exploit the terms of trade externality in a deviation to Non-cooperation). With these effects, we should see that, all else equal, a lower γ implies an environment in which Home has a relatively stronger temptation to walk away from a given Cooperation regime. Figure 9 shows the net effect of this tension with respect to γ, which resolves in the latter direction.

Finally, consider the question of what a Sustainable Cooperation equilibrium would be like in this setting. Figure 10 on page OA.45 — F shows the asymptotic upper bound of the pseudo-Pareto weight ν_t, for different environments indexed by (ρ, γ). When γ is close enough to $1/2$, Home's sustainability constraint never binds and $\nu_t = \gamma$ holds. As γ is away from $1/2$, the constraint kicks in and ν_t has to be larger to keep Home within the Sustainable Cooperation regime. When γ

is too small, ν_t is bounded around $1/2$.[56] In other words, the smaller is Home in size, the greater is the need for welfare redistribution from Foreign to Home, hence asymptotically a larger pseudo Pareto weight in favor of Home. Similar to the insight from the comparative welfare analysis between the two exogenous regimes of Cooperation and Non-cooperation in Figure 9 earlier, here endogenously, a Sustainable Cooperation regime will need to balance the terms of trade externality threat against the non-insularity feedback effect. Hence, for each γ economy, there will be a ρ that defines the point where the most (asymptotically) welfare redistribution has to be promised to Home. This point shifts toward a higher ρ as γ becomes smaller, again echoing a similar property in Figure 9 earlier.

In short, even though two countries are symmetric but for their size, the smaller (i.e., Home when $\gamma < 1/2$) country has an incentive to deviate from an assumed Cooperation to a Non-cooperation regime. In contrast, a Sustainable Cooperation plan takes this incentive problem into account and thus has to endogenously place more weight on Home's welfare over time, the smaller is Home in size.

Figure 10: The asymptotic values of pseudo-Pareto weight across economies with different ρ and various asymmetric country size.



[56] That is, as the countries are symmetric in all other respects, the Foreign country's sustainability constraint becomes relevant when ν_t exceeds $1/2$.