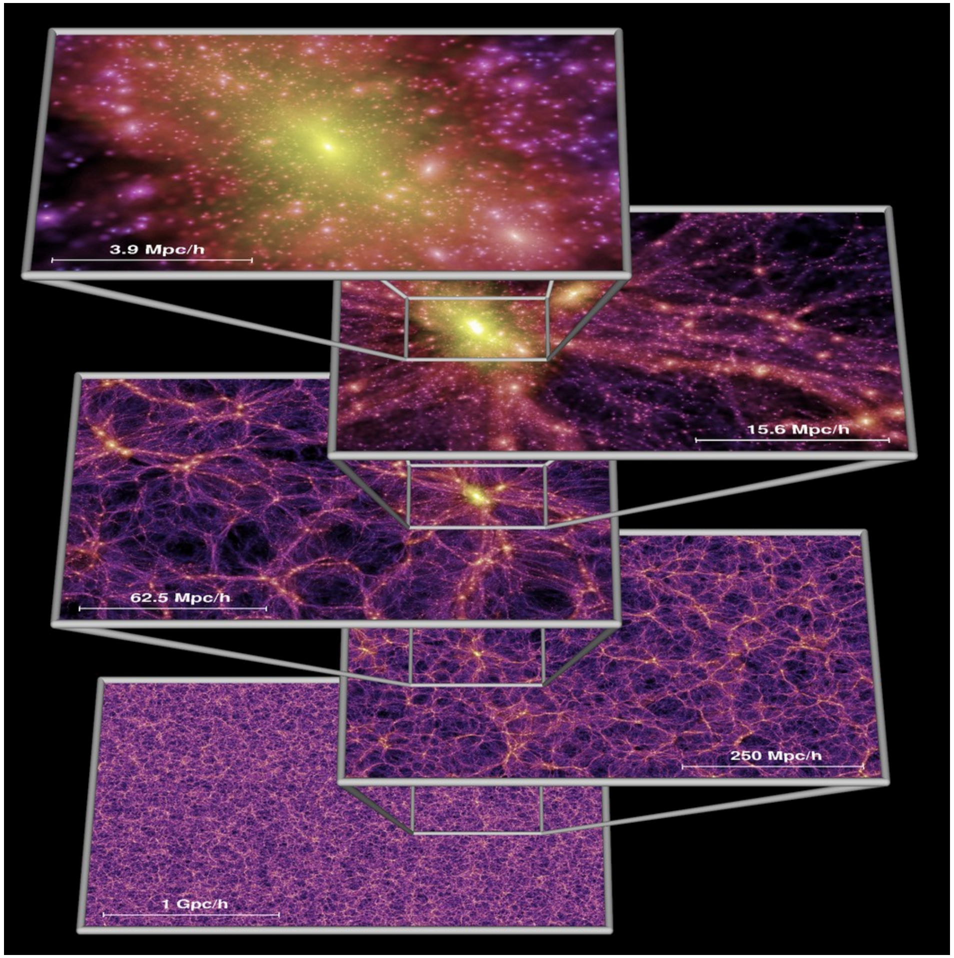
Supercomputer Simulations of the Emergence of Cosmic Structure

Volker Springel

- Introduction: The ΛCDM universe, and what we know about it through simulations
- Towards small scales: High-resolution dark matter simulations of invidual halos
- Towards large volumes: One way to falsify ΛCDM, and a handle on dark energy



Heidelberg Institute for Theoretical Studies
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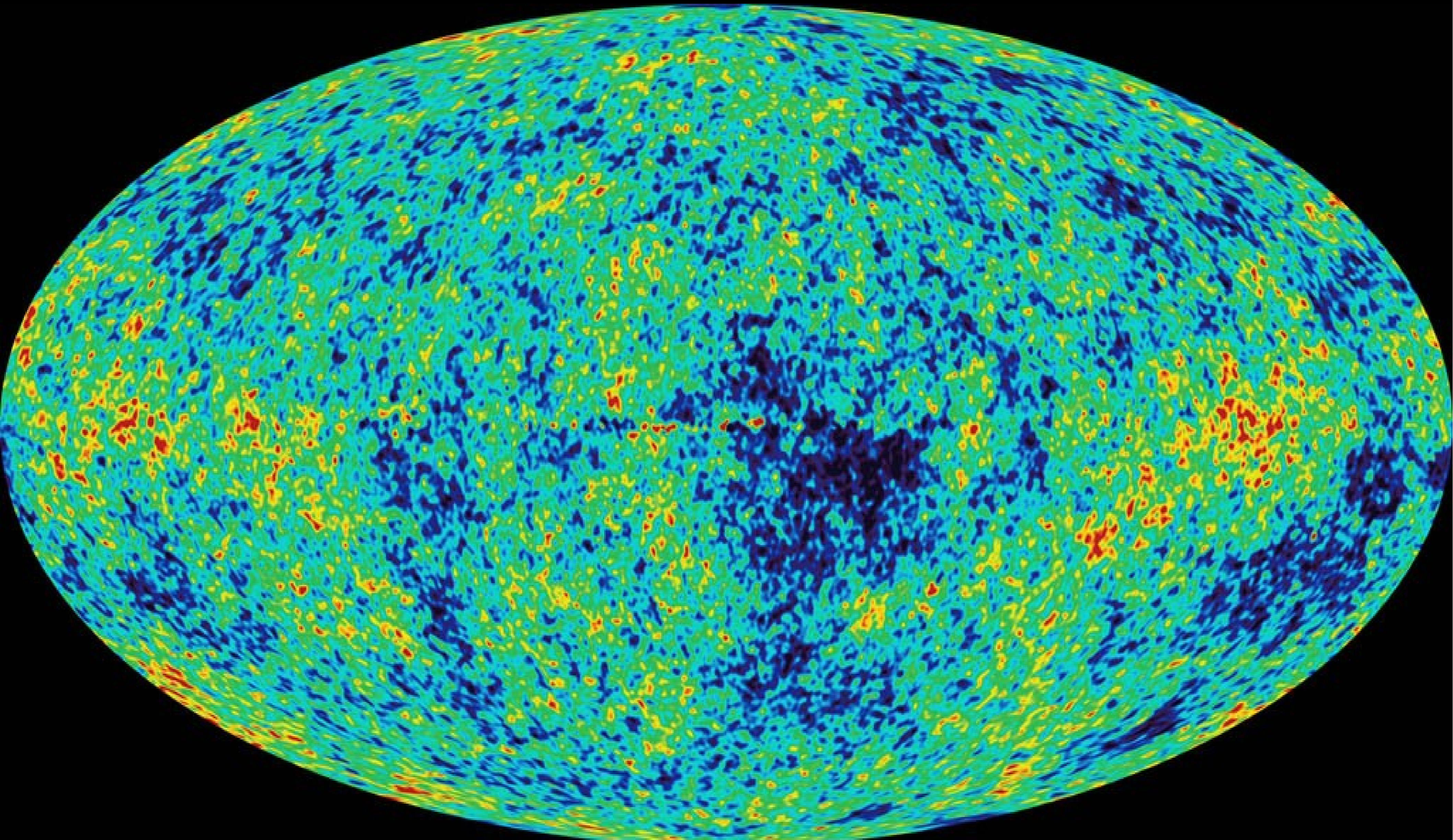
VIA Lecture
Paris/Internet, June 2011

The initial conditions for cosmic structure formation are directly observable

THE MICROWAVE SKY



WMAP Science Team (2003, 2006, 2008, 2010)

The most important cosmological parameters are well constrained

WMAP-5 CONSTRAINTS, INLCUDING TYPE-IA AND BAO DATA

Minimal, 6-parameter ΛCDM model is a great fit

$$\Omega_c = 0.233 \pm 0.013$$

$$\Omega_b = 0.0462 \pm 0.0015$$

$$\sigma_8 = 0.817 \pm 0.026$$

$$n_s = 0.960^{+0.014}_{-0.013}$$

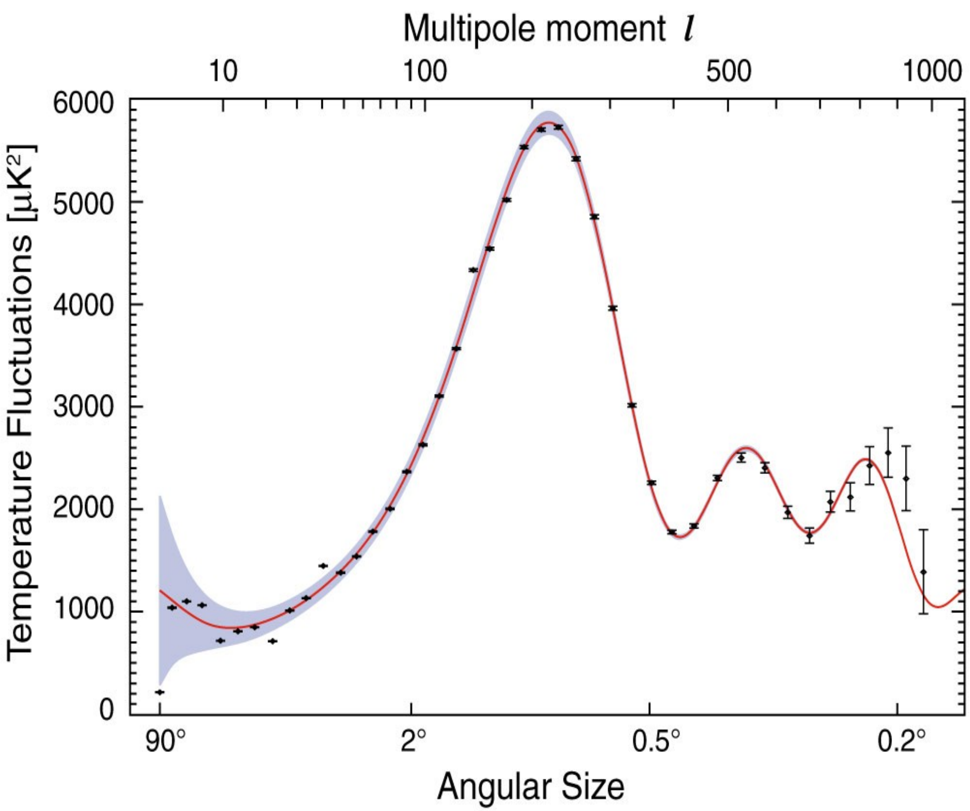
$$H_0 = 70.1 \pm 1.3 \text{ km s}^{-1} \text{ Mpc}^{-1}$$

$$\tau = 0.084 \pm 0.016$$

$$\Omega_\Lambda = 0.721 \pm 0.015$$

$$t_0 = 13.73 \pm 0.12 \text{ Gyr}$$

$$z_{\rm reion} = 10.8 \pm 1.4$$



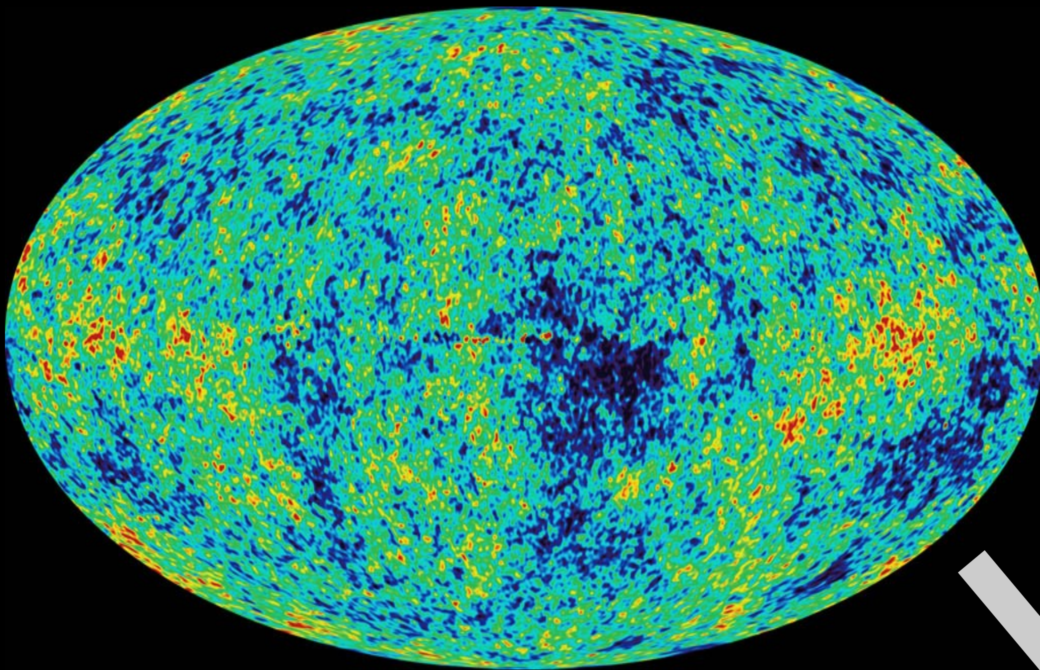
Constraints on dark energy equation of state:

$$-0.11 < 1 + w < 0.14$$

$$-0.0175 < \Omega_k < 0.0085$$

(95% CF, assuming a constant w)

Komatsu et al. (2008)



In principle, based on the initial conditions of the ΛCDM cosmology one should be able to *predict* the variety in shapes and sizes of observed galaxies

A SIMULATION PROBLEM
PAR EXCELLENCE

?

M87 – Anglo-Australian Observatory

NGC1332 – ESO, VLT

Currently the fastest supercomputers carry out about ~1 Petaflop, which are one thousand billion floating point operations per second

JUGENE IN JUELICH

The basic dynamics of structure formation in the dark matter

BASIC EQUATIONS AND THEIR DISCRETIZATION

Gravitation
General theory of relativity (Newtonian approximation in an expanding space-time)

Dark matter is collisionless

Monte-Carlo integration as **N-body System**

3N **coupled**, non-linear differential equations of second order

Friedmann-Lemaitre model

$$H(a) = H_0 \sqrt{a^{-3}\Omega_0 + a^{-2}(1 - \Omega_0 - \Omega_\Lambda) + \Omega_\Lambda}$$

Collisionless Boltzmann equation with self-gravity

$$\frac{\mathrm{d}f}{\mathrm{d}t} \equiv \frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} - \frac{\partial \Phi}{\partial \mathbf{r}}\frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \Phi(\mathbf{r}, t) = 4\pi G \int f(\mathbf{r}, \mathbf{v}, t)\mathrm{d}\mathbf{v}$$

Hamiltonian dynamics in expanding space-time

$$H = \sum_i \frac{\boldsymbol{p}_i^2}{2\, m_i\, a(t)^2} + \frac{1}{2}\sum_{ij} \frac{m_i m_j\, \varphi(\boldsymbol{x}_i - \boldsymbol{x}_j)}{a(t)}$$

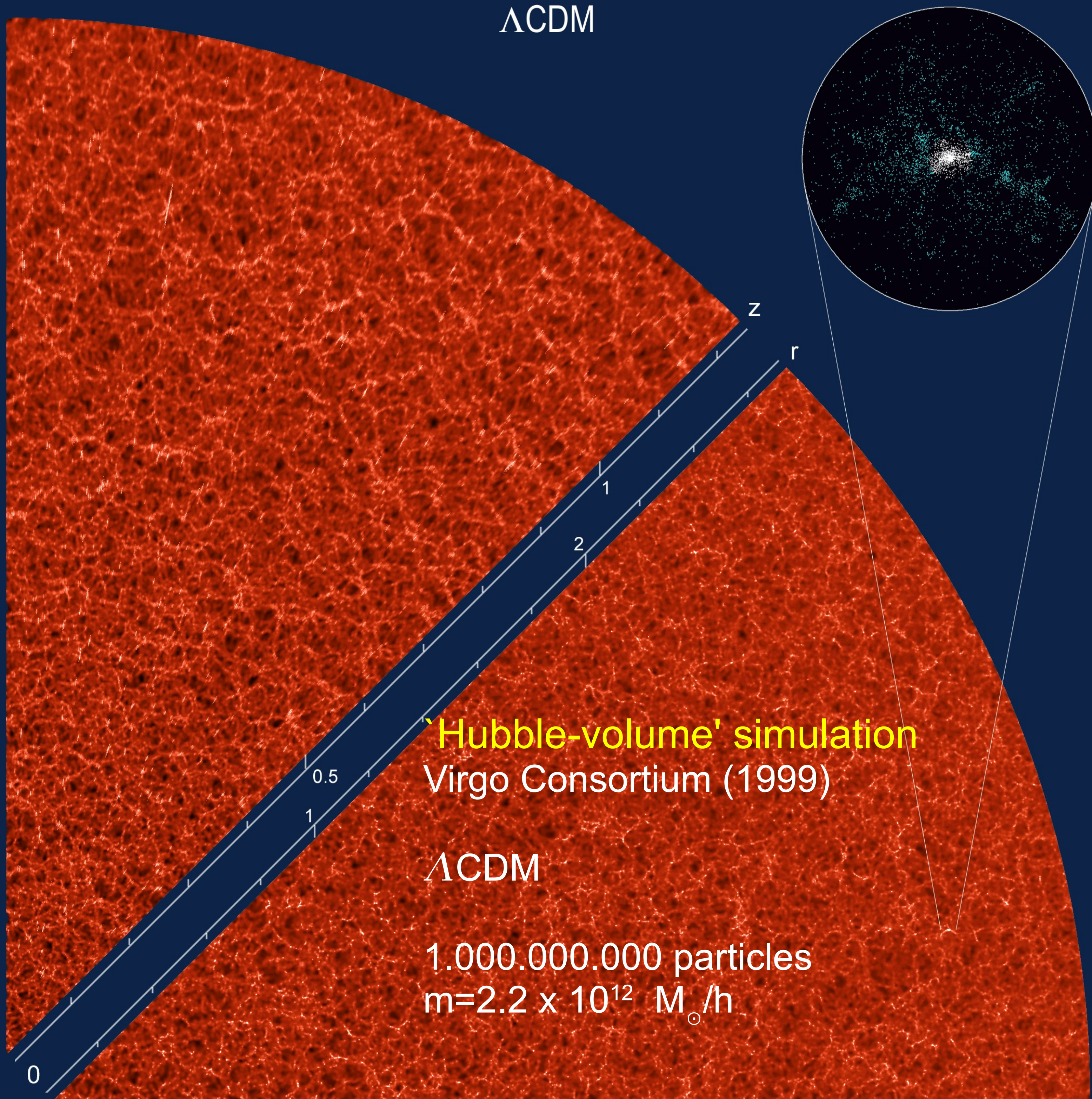
$$\nabla^2 \varphi(\boldsymbol{x}) = 4\pi G \left[-\frac{1}{L^3} + \sum_{\boldsymbol{n}} \tilde{\delta}(\boldsymbol{x} - \boldsymbol{n}L) \right]$$

Problems:
N ist very large
All equations are coupled with each other

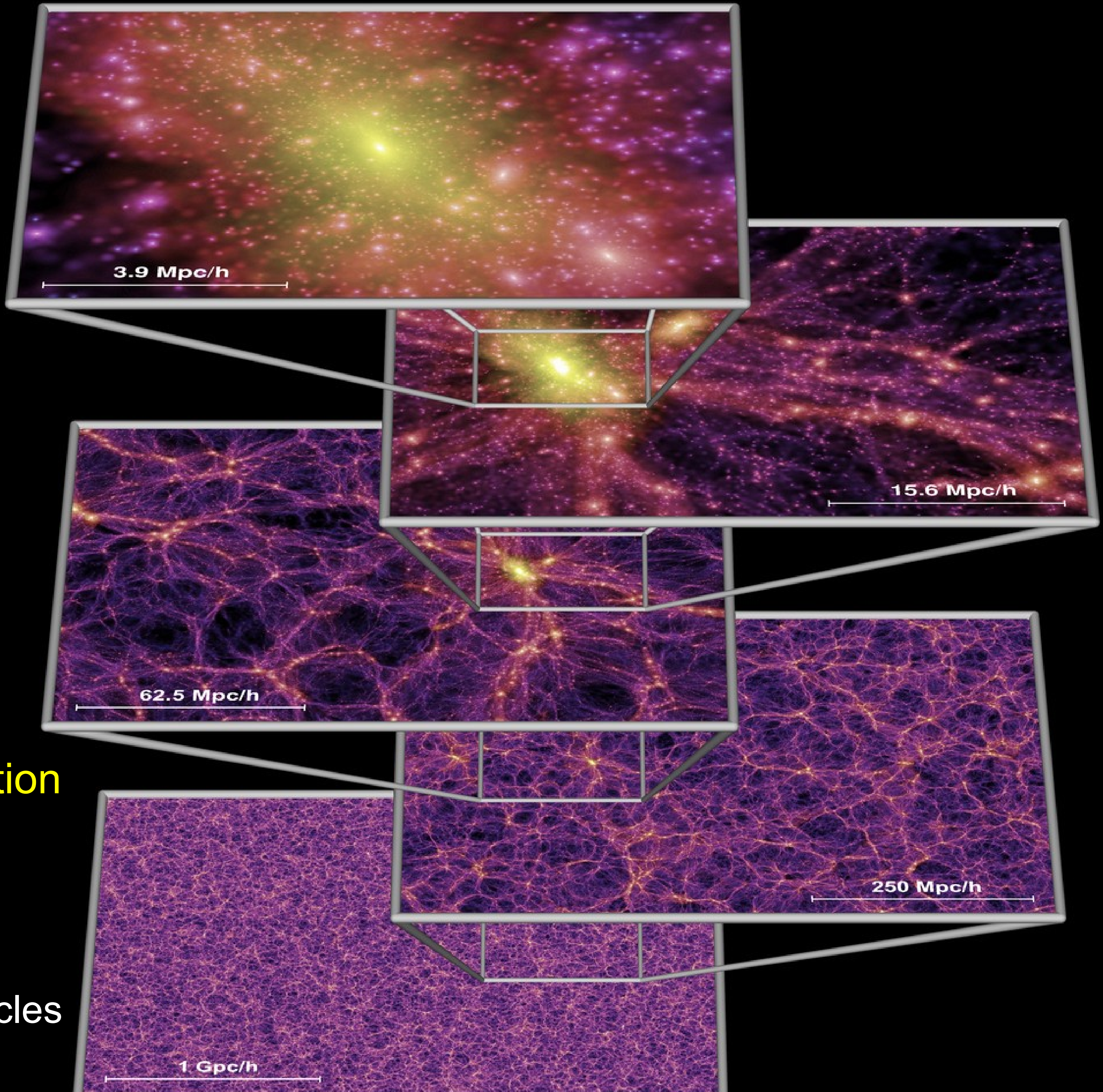
Large-scale structure

The largest N-body simulations cover a sizable fraction of the observable universe

DARK MATTER CLUSTERING ALONG THE PAST LIGHT-CONE OF THE HUBBLE SIMULATION



Evrard et al. (2001)



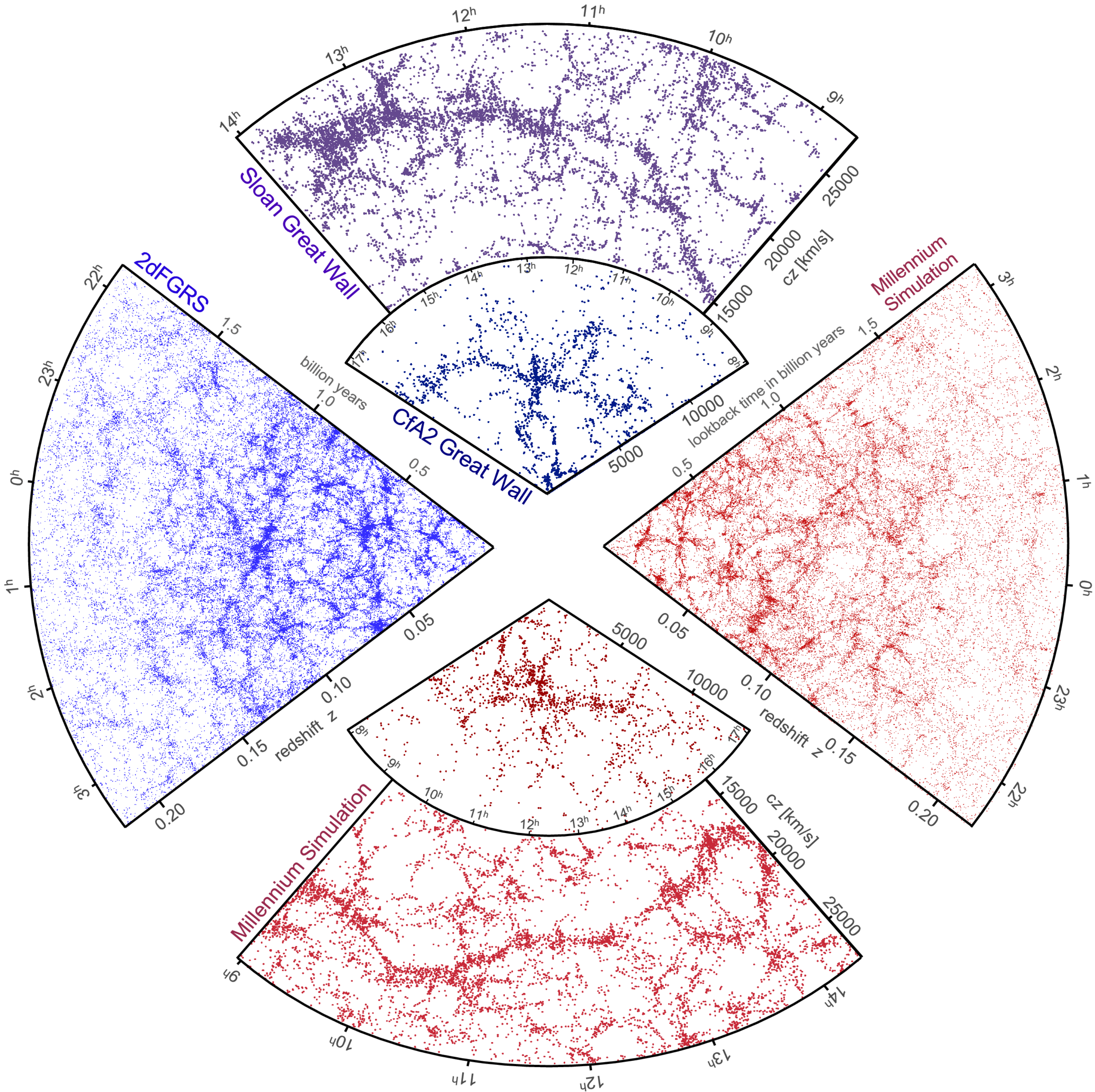
'Millennium' simulation
Springel et al. (2005)

ΛCDM

10.077.696.000 particles
m=8.6 x 10^8 $M_\odot$/h

Simulated and observed large-scale structure in the galaxy distribution

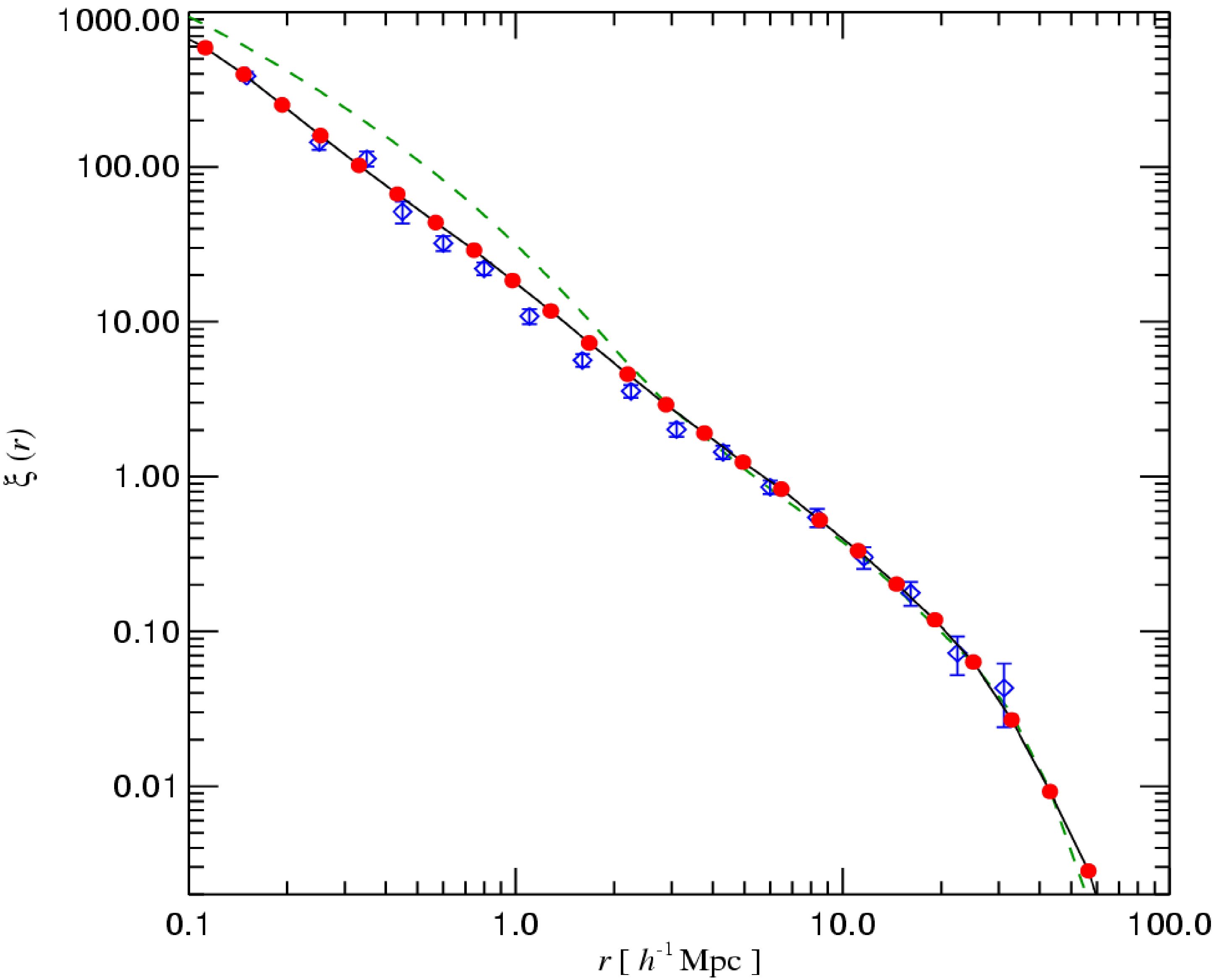
MOCK PIE DIAGRAMS COMPARED TO SDSS, 2DFGRS, AND CFA-2



Springel et al. (2006)

The two-point correlation function of galaxies in the Millennium run is a very good power law

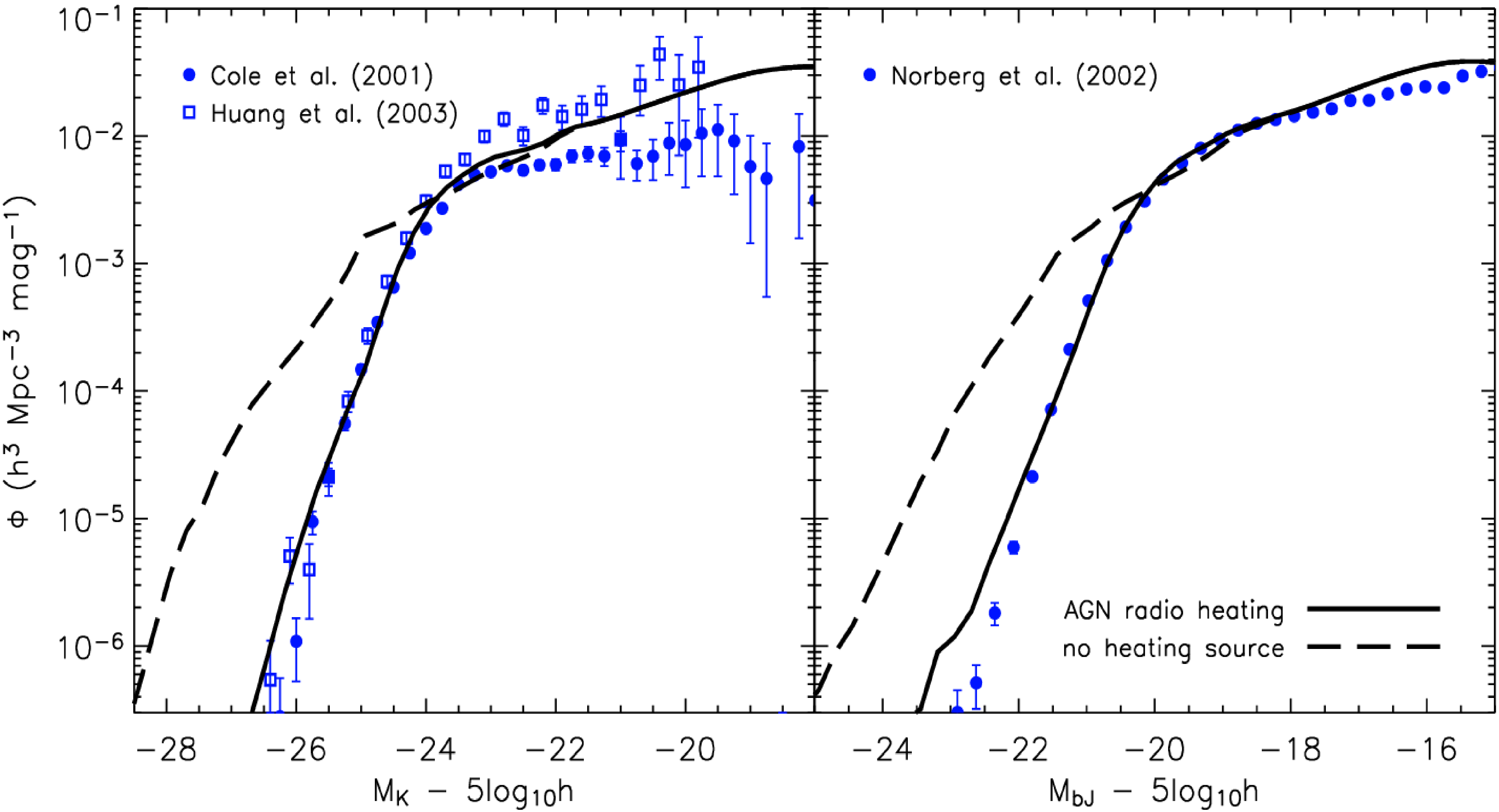
GALAXY TWO-POINT FUNCTION COMPARED WITH **2dFGRS**

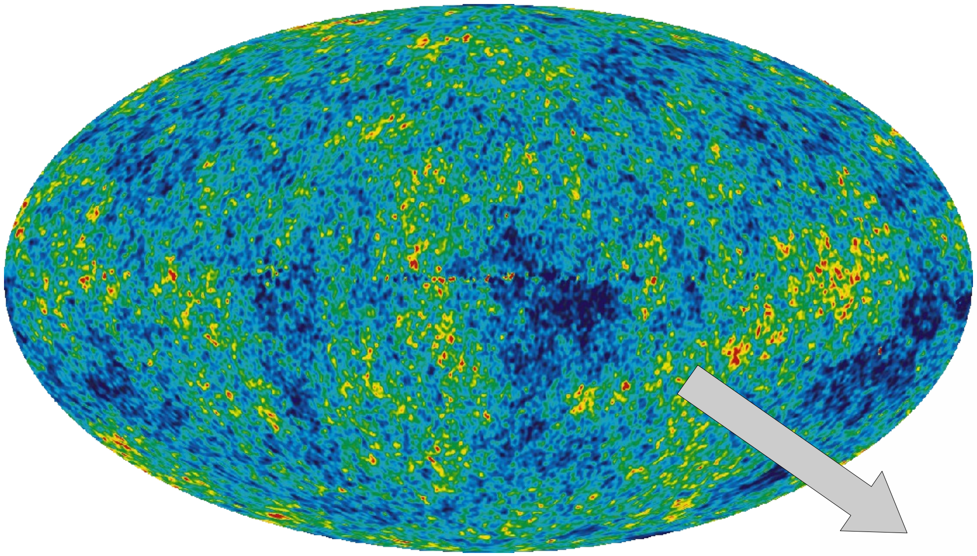


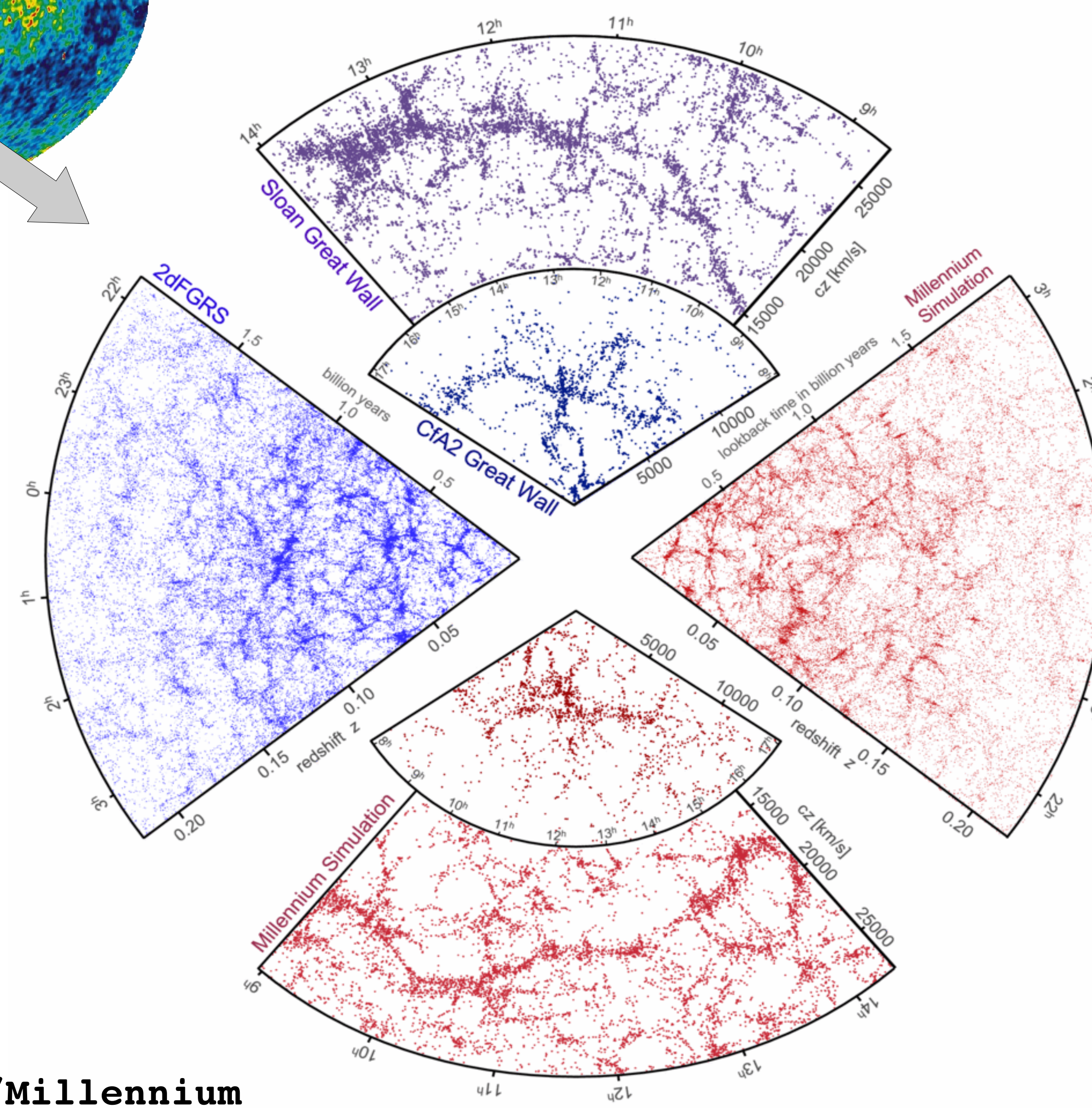
The inclusion of AGN feedback allows the semi-analytic model to reproduce a multitude of observational data

K-BAND AND Bj-BAND LUMINOSITY FUNCTIONS

Croton et al. (2006)







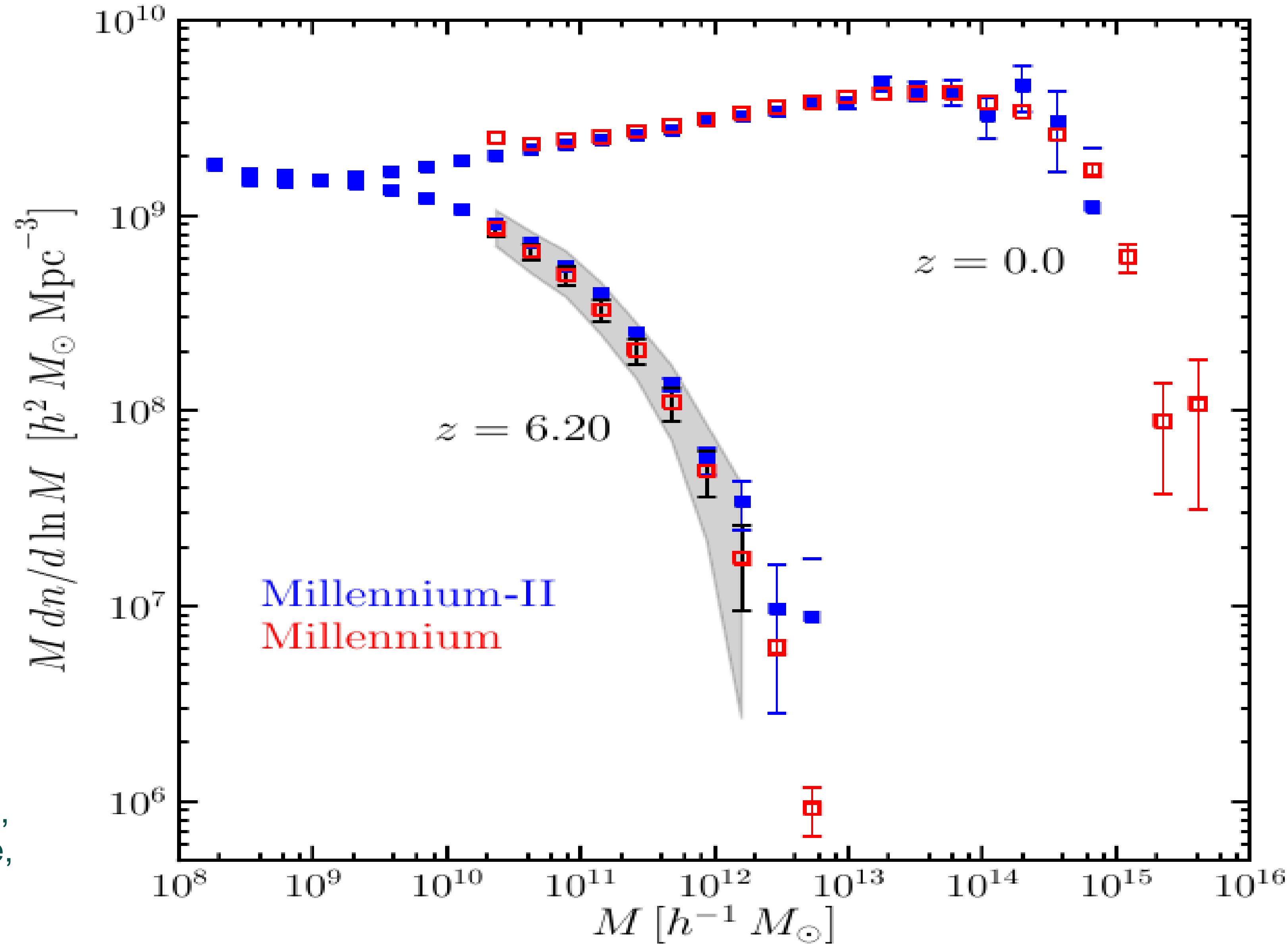
public data release of
the Millennium galaxies
on Aug 1st 2006

(~380 papers that used the data thus far)

http://www.mpa-garching.mpg.de/Millennium

Simulations provide accurate measurements for halo abundance as a function of time

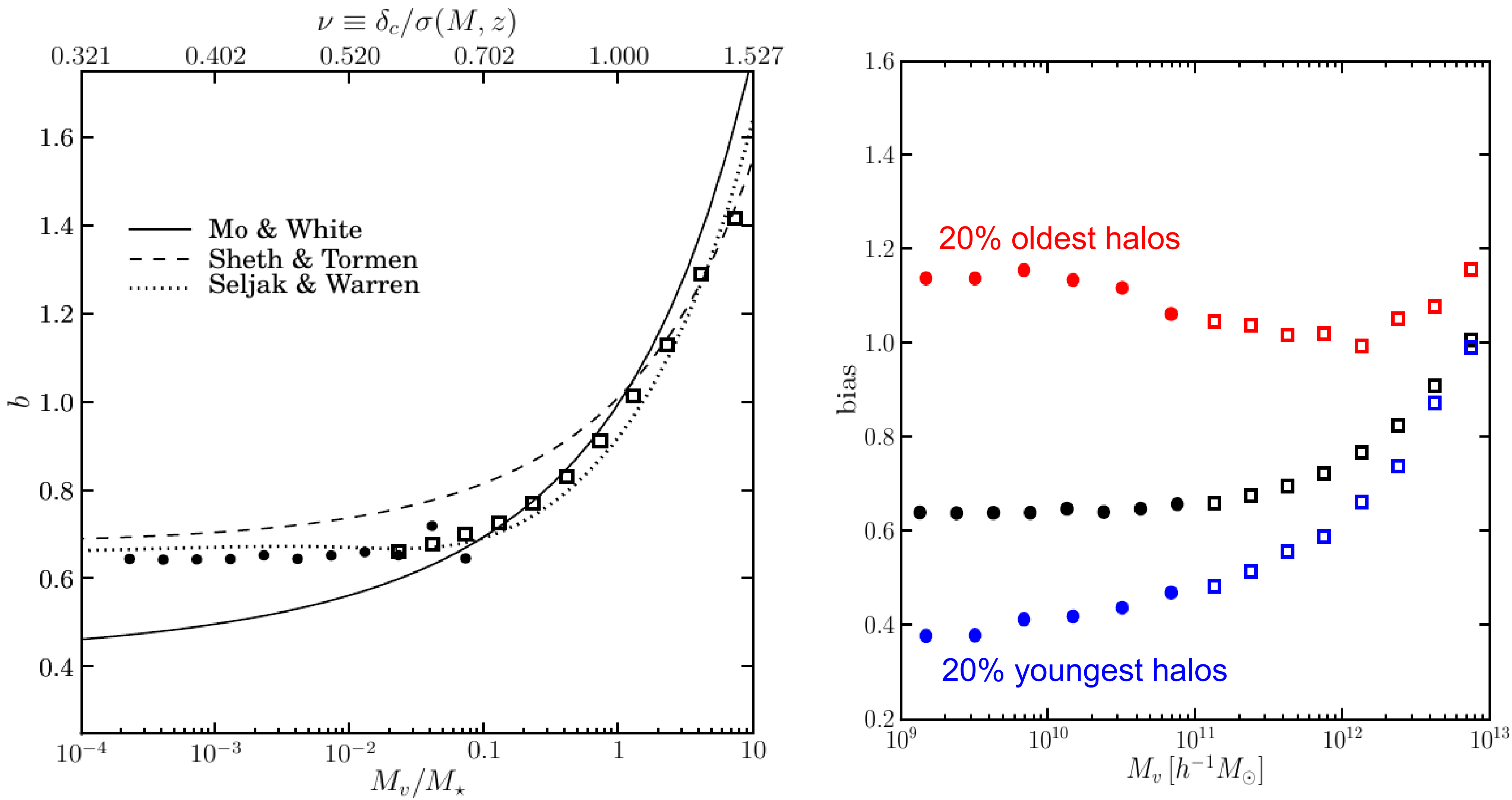
CONVERGENCE RESULTS FOR HALO ABUNDANCE



Boylan-Kolchin, Springel, White, et al. (2009)

Simulations provide accurate measurements for halo bias

BIAS AS A FUNCTION OF MASS AND FORMATION TIME



Gao et al. (2005)

***"Assembly bias"**: This dependence is inconsistent both with excursion set theory and HOD*

Small-scale structure

Millennium-II Simulation

BETTER MASS RESOLUTION BY A FACTOR OF 125 COMPARED TO THE MILLENNIUM SIMULATION

10^{10} particles

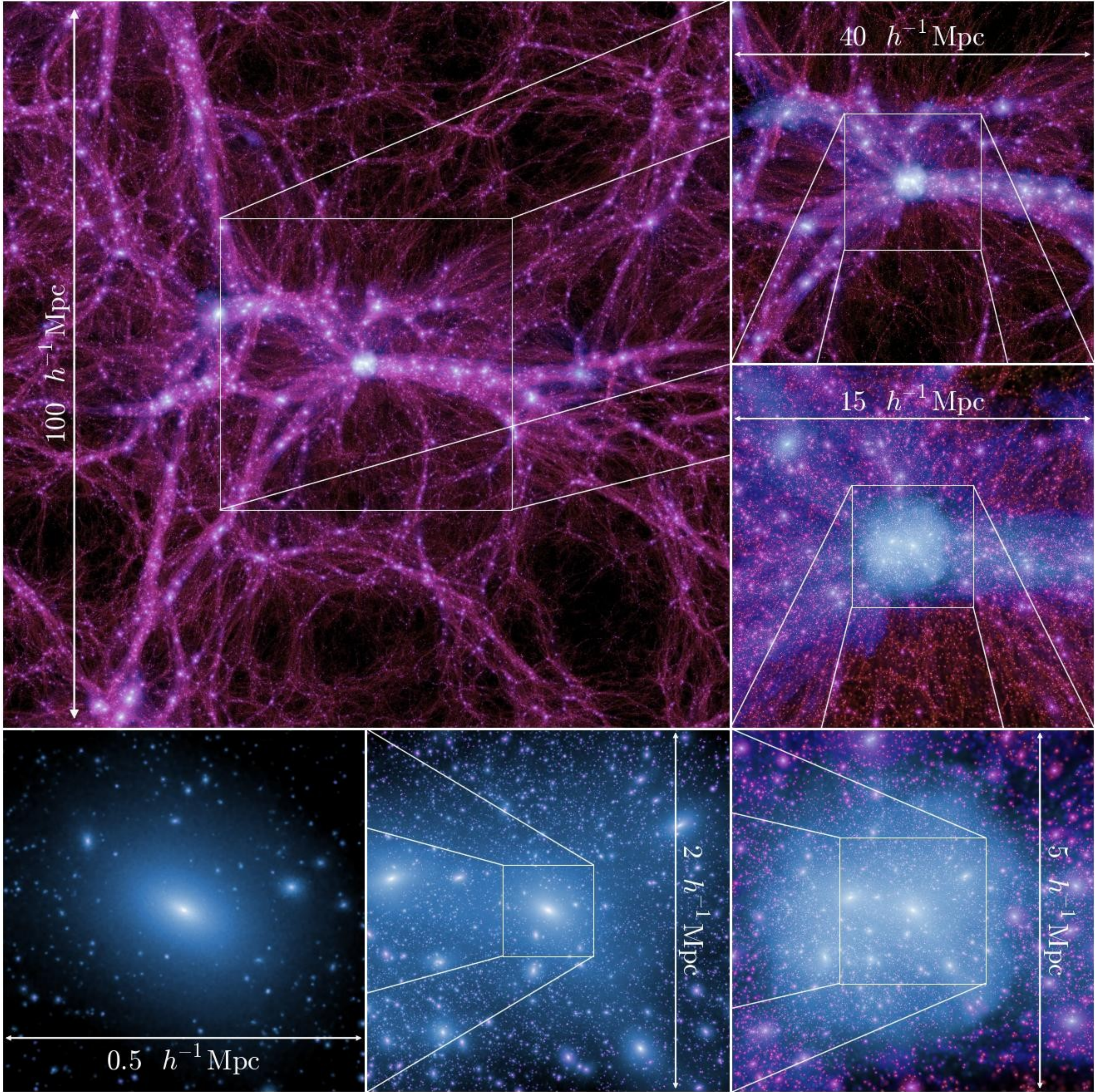
100 Mpc/h box

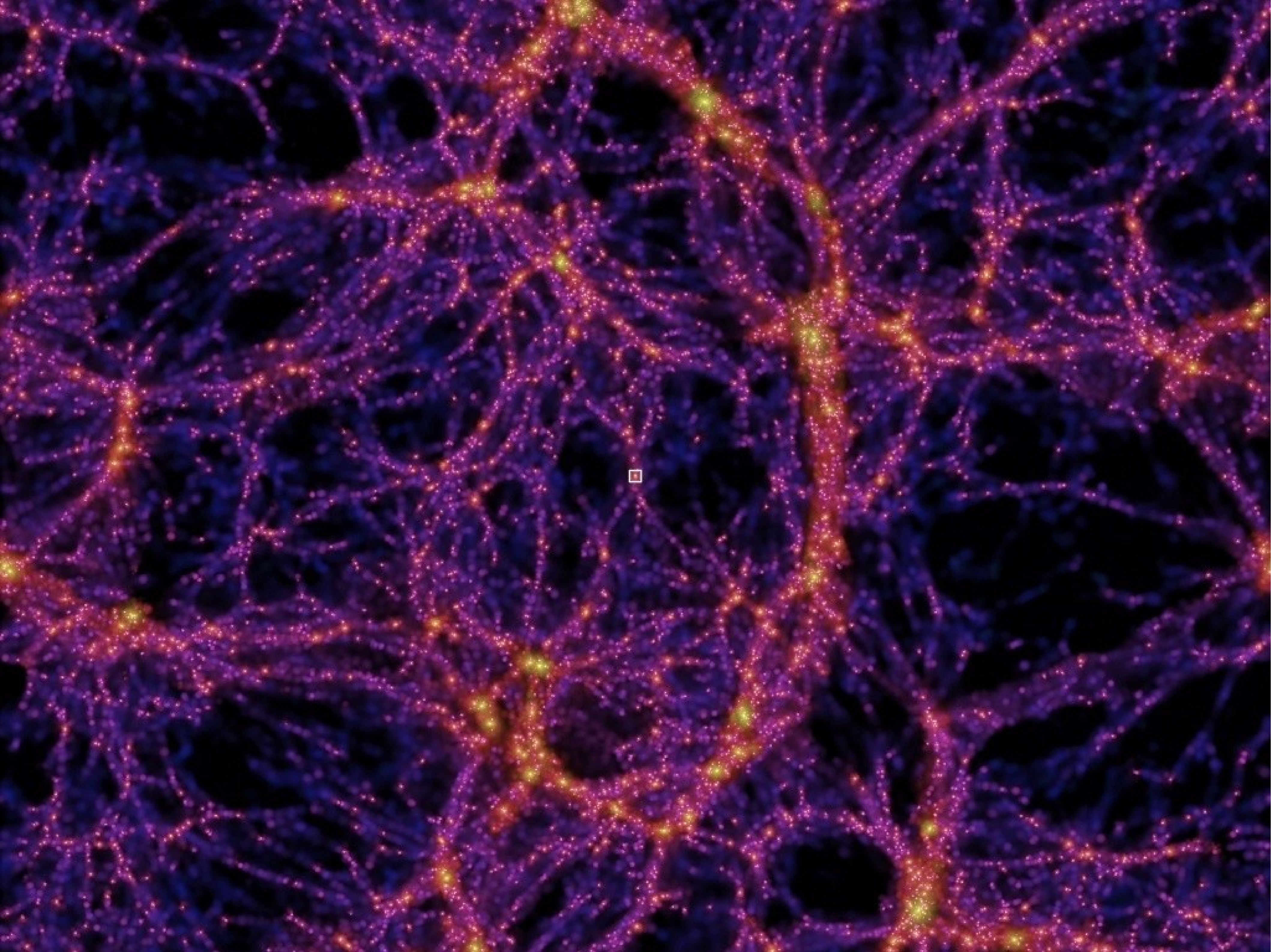
$m_p = 8.6 \times 10^6$ Msun/h

$\varepsilon = 1$ kpc/h

> 60% of particles in groups at z=0

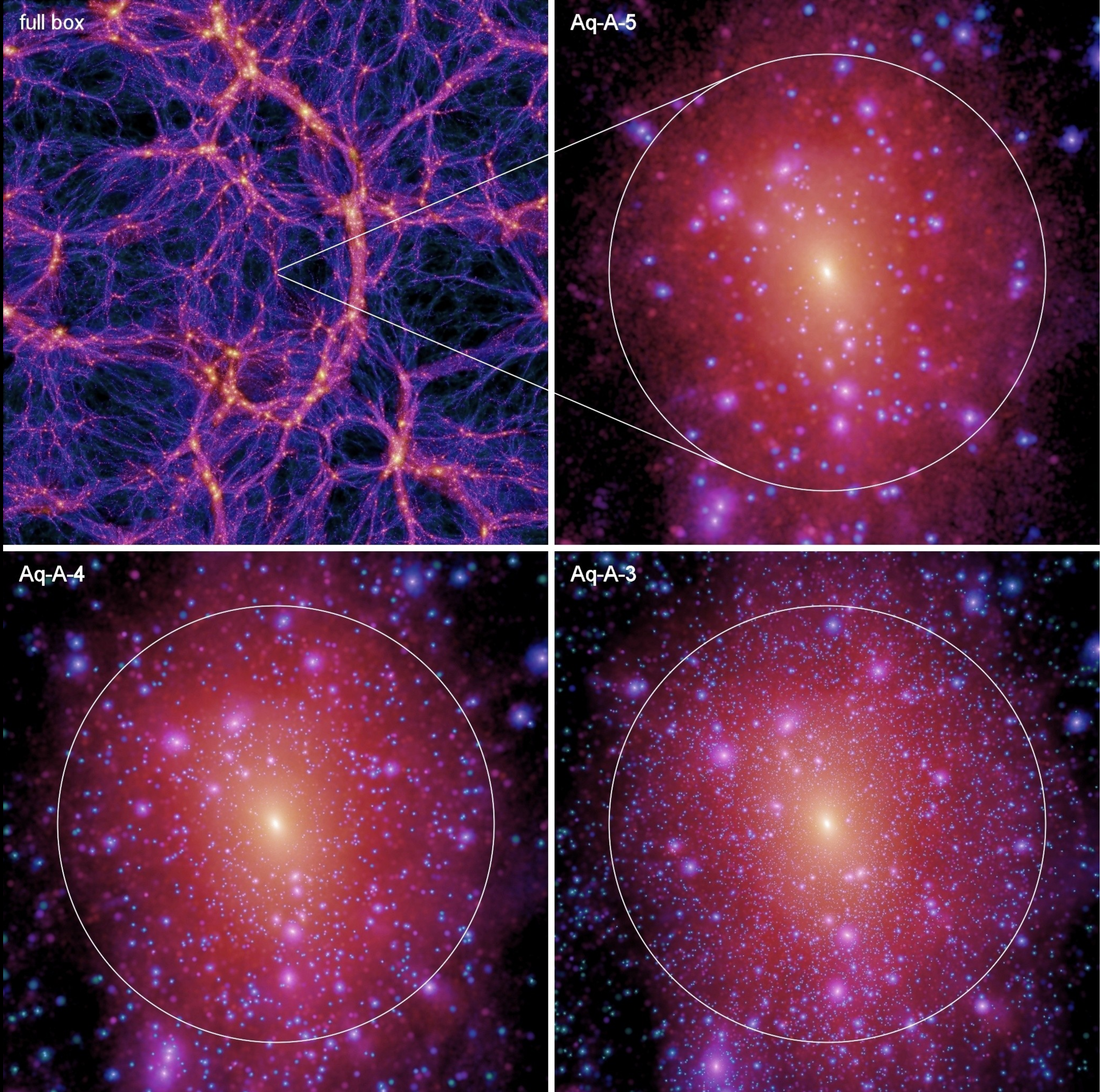
Boylan-Kolchin, Springel, White, et al. (2009)





Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

DARK MATTER DISTRIBUTION IN A MILKY WAY SIZED HALO AT DIFFERENT RESOLUTION




The movie...

The Aquarius project varies the resolution systematically and studies different halos in order to assess **convergence** and **cosmic variance**

NUMERICAL PARAMATERS OF AQUARIUS HALO A

Springel et al. (2008)

Numerical resolution	Particle number in halo (N_{50})	# of substructures	mass resolution
Aq-A-5	808,479	299	$3.14 \times 10^6\ M_\odot$
Aq-A-4	6,424,399	1,960	$3.92 \times 10^5\ M_\odot$
Aq-A-3	51,391,468	13,854	$4.91 \times 10^4\ M_\odot$
Aq-A-2	184,243,536	45,024	$1.37 \times 10^4\ M_\odot$
Aq-A-1	1,473,568,512	297,791	$1.71 \times 10^3\ M_\odot$ (20 pc softening)

N_{tot} = 4,400,000,000

Diemand et al. (2007, 2008)

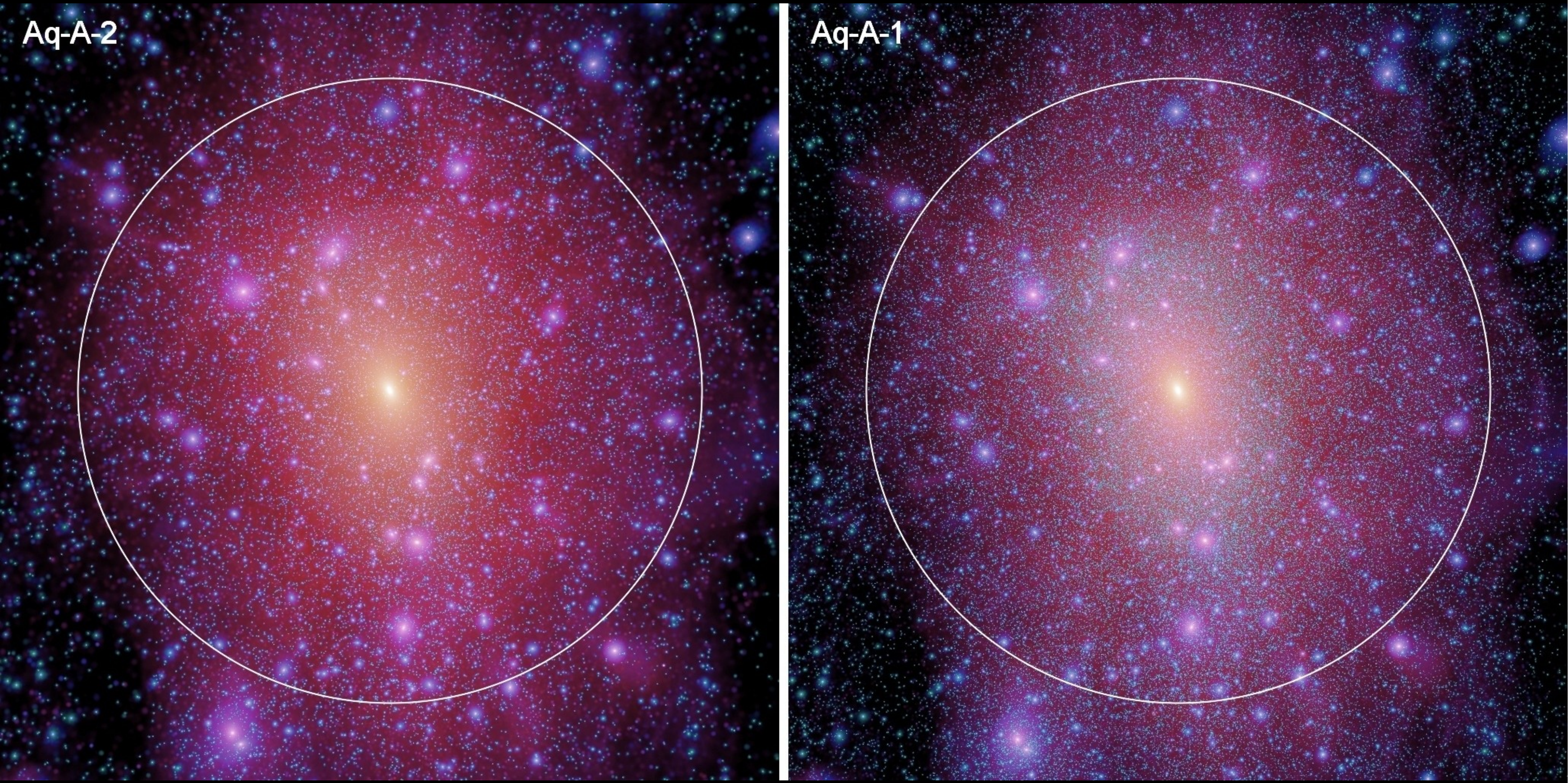
Via Lactea I simulation	84,700,000	~10,000	$2.18 \times 10^4\ M_\odot$
Via Lactea II simulation	470,000,000	~50,000	$4.10 \times 10^3\ M_\odot$

Stadel et al. (2009)

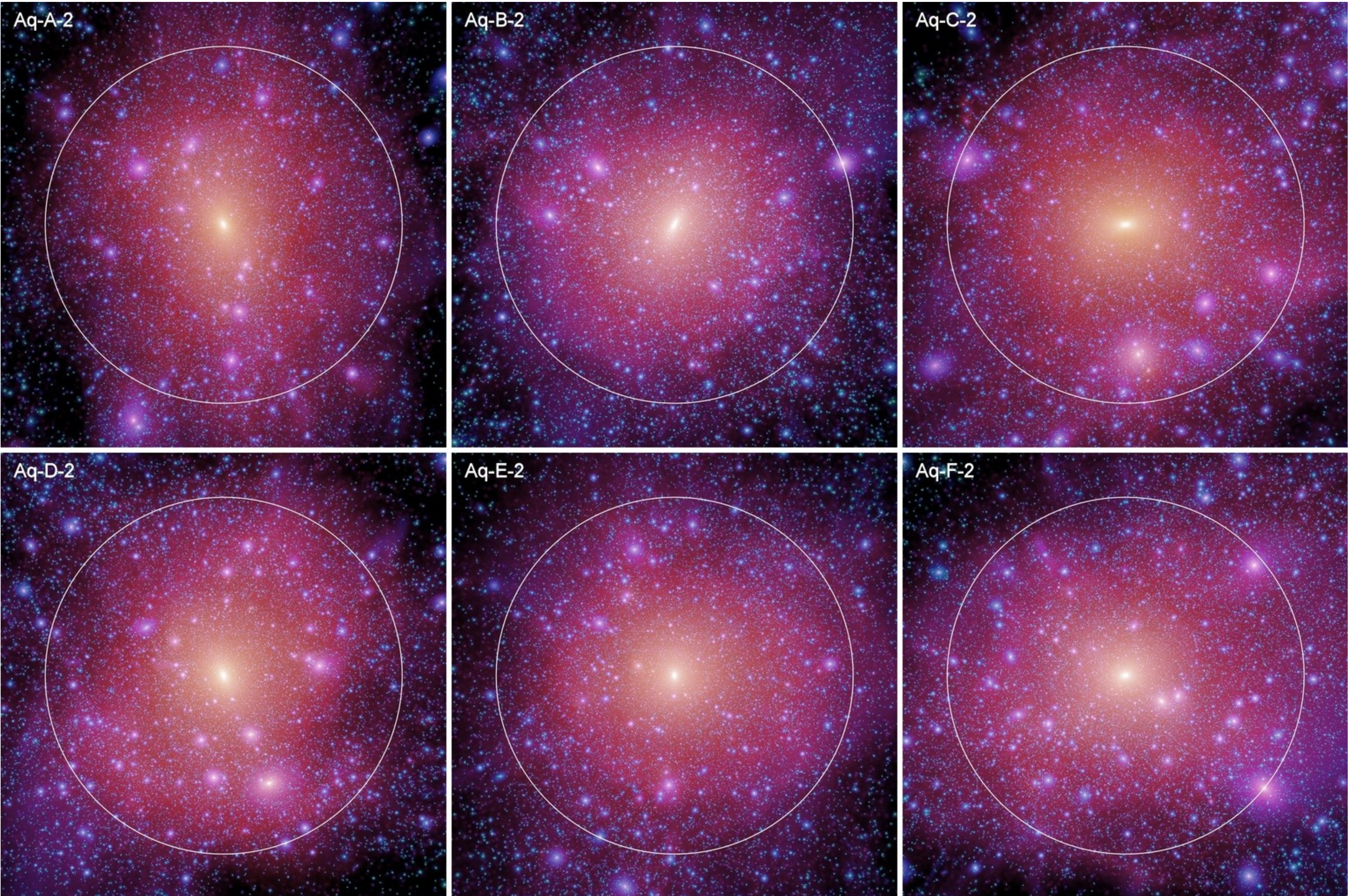
GHALO simulation	1,243,000,000		$1.00 \times 10^3\ M_\odot$

Zooming in on dark matter halos reveals a huge abundance of dark matter substructure

DARK MATTER IN A MILKY WAY SIZED HALO AT ULTRA-HIGH RESOLUTION

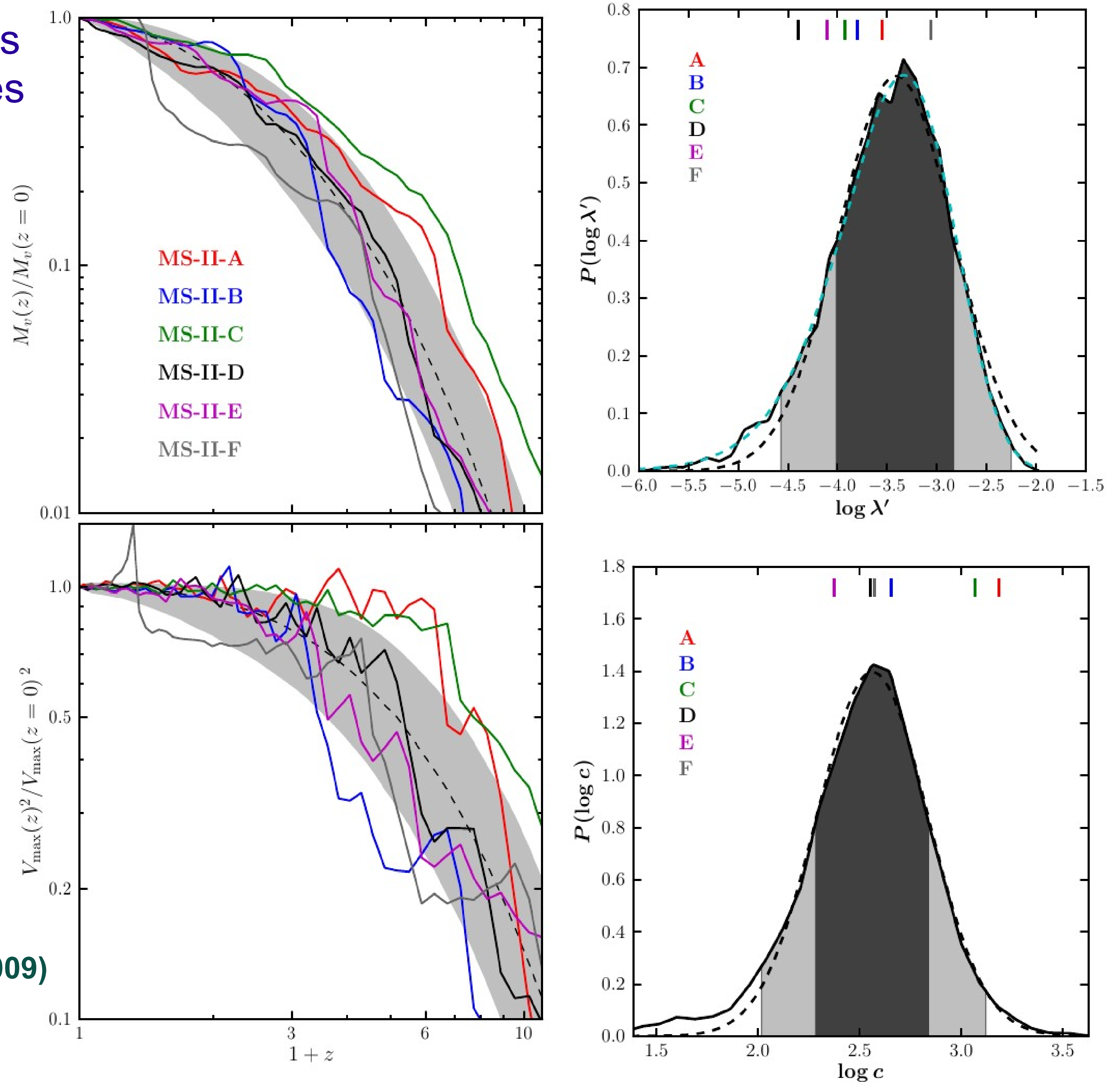


A sample of 6 simulated halos allows study of halo-to-halo scatter



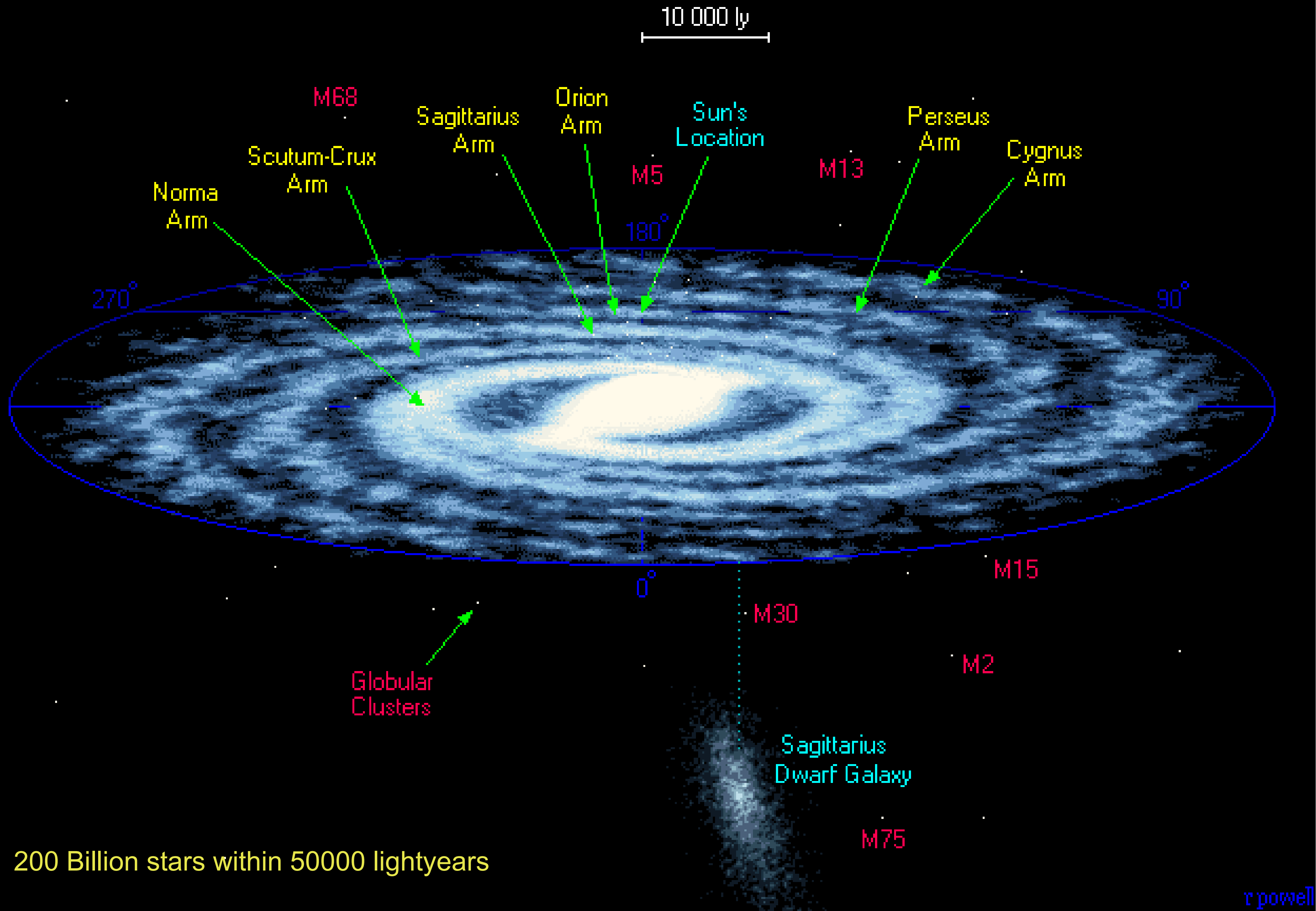
The set of Aquarius halos fairly samples the properties of Milky Way sized halos

MASS ACCRETION HISTORIES, SPIN AND CONCENTRATION DISTRIBUTION FROM THE MILLENNIUM-II SIMULATION



Boylan-Kolchin et al. (2009)

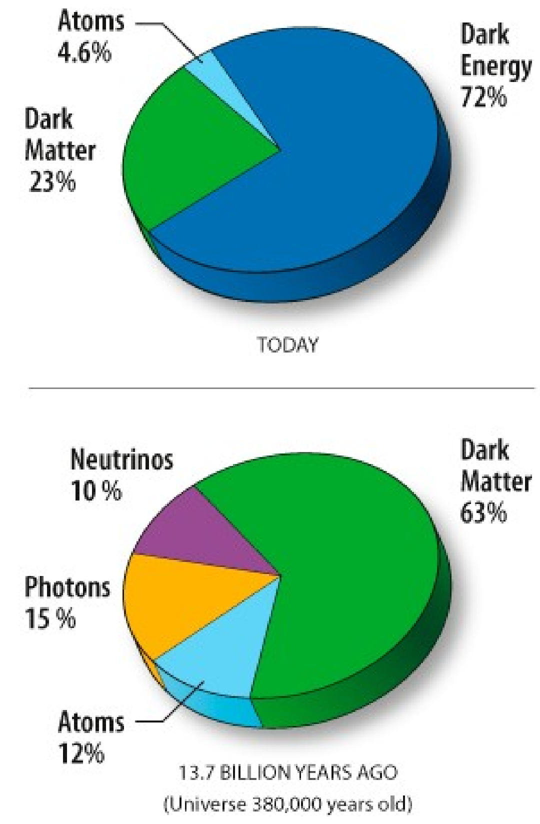
We need predictions for the dark matter structure at the position of the Sun in the Milky Way



200 Billion stars within 50000 lightyears

What is the Cold Dark Matter made of?

THERE ARE PROMISING CANDIDATES WITH MOTIVATION IN PARTICLE PHYSICS



Neutralino?

lightest supersymmetric particle
expected mass ~100 GeV
annihilation possibly observable
detectable through elastic scattering with nucleus

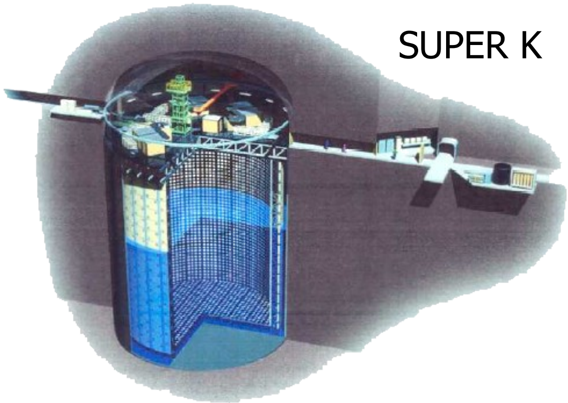
Axion?

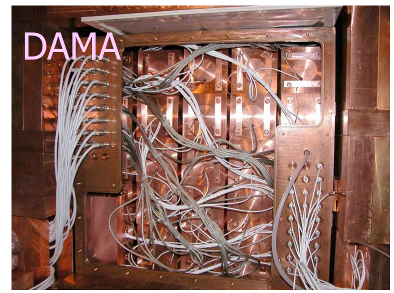
mass ~ 10^{-5} eV
detectable through photon-conversion in strong magnetic fields

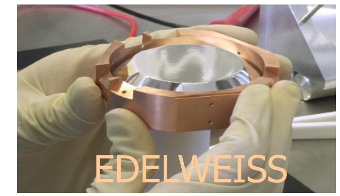
Large efforts have been started to experimentally detect dark matter

A FEW OF THE CURRENT EXPERIMENTS THAT SEARCH FOR DARK MATTER

SUPER K



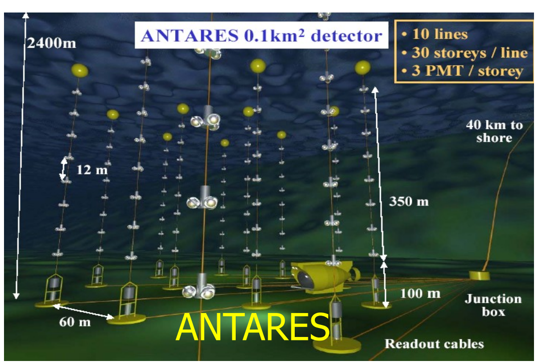




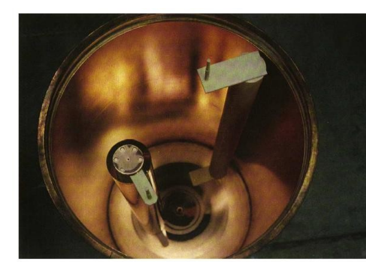
ZEPLIN

Indirect searches:
search for annihilation products

Direct searches:
measure nuclear recoil due to elastic scattering with nucleus

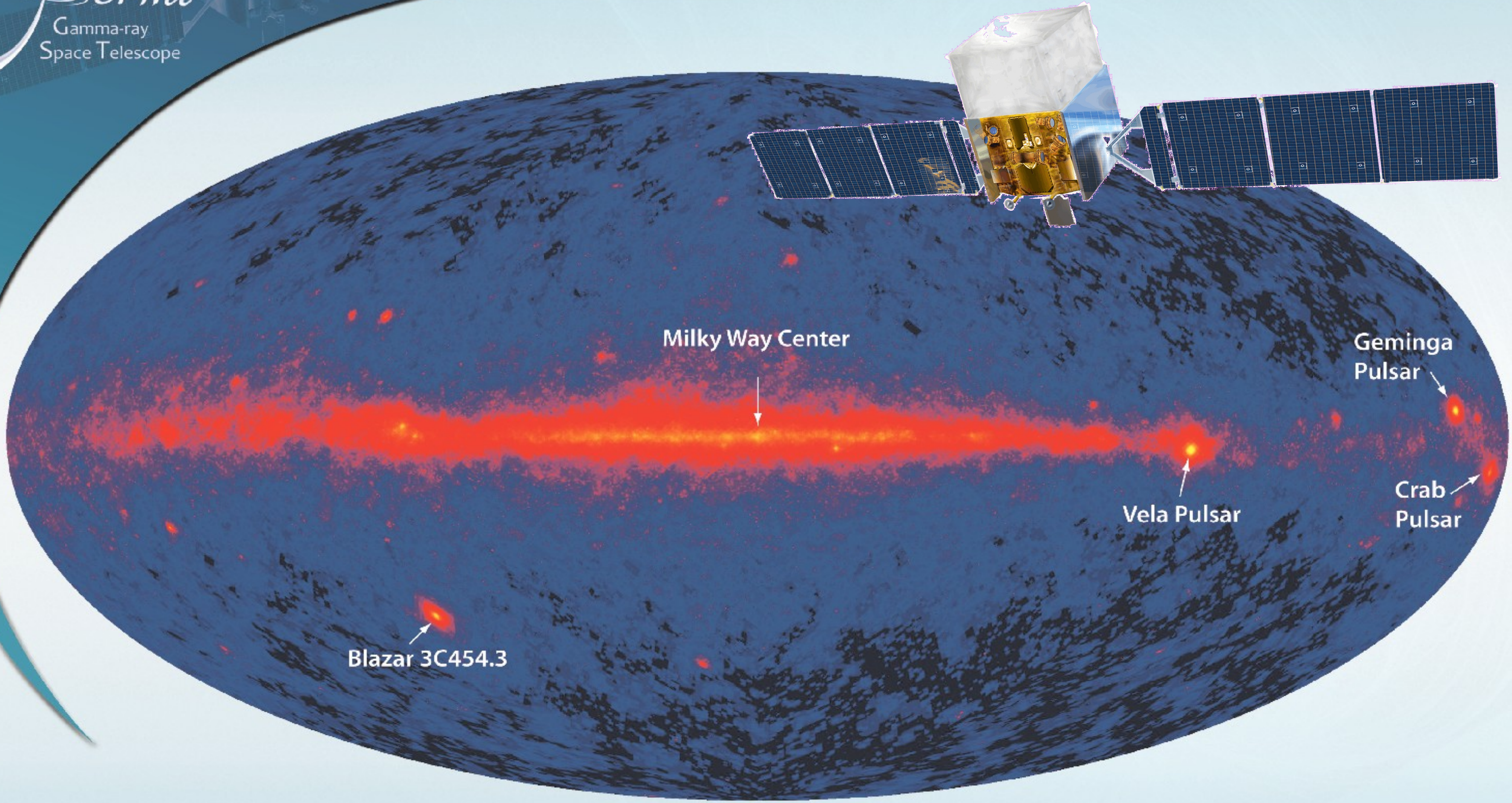


ADMX



Typical cross sections of WIMPS: **~10^{-42}** cm^2 (10^{-6} pb)

Fermi
Gamma-ray Space Telescope



Fermi First Light Skymap

95 hours of data

Dark matter could be self-annihilating, in which case the presence of subhalos should **boost** the expected flux

THE ANNIHILATION SIGNAL DUE TO SUBSTRUCTURES

Annihilation flux:

$$F = \boxed{\frac{N_\gamma \langle \sigma v \rangle}{2\, m_{\rm DM}^2}} \boxed{\int_V \frac{\rho_{\rm DM}^2(\mathbf{x})}{4\pi\, d^2(\mathbf{x})}\, \mathrm{d}^3 x}$$

Particle physics Astrophysics

Luminosity of a halo with maximum circular velocity $V_c(r_{max}) = V_{max}$: $L = \int \rho_{\rm DM}^2(\mathbf{x})\, \mathrm{d}^3 x$

NFW-Profile: $L = 1.23 \frac{V_{\rm max}^4}{G^2\, r_{\rm max}}$

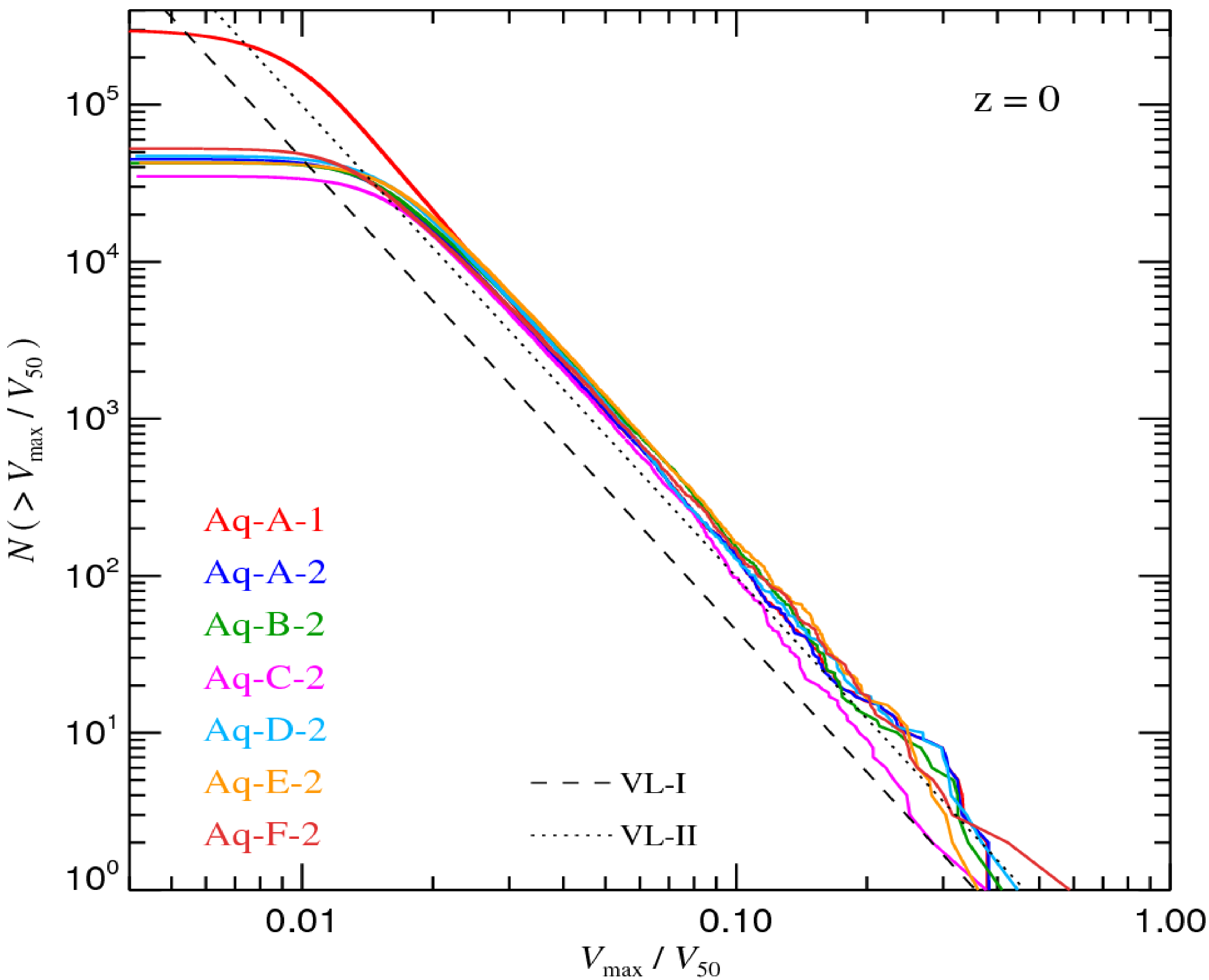
α = -1.4 Profile: $L = 3.97 \frac{V_{\rm max}^4}{G^2\, r_{\rm max}}$

Einasto-Profile: $L = 1.87 \frac{V_{\rm max}^4}{G^2\, r_{\rm max}}$

Moore-Profile: $L = \infty$

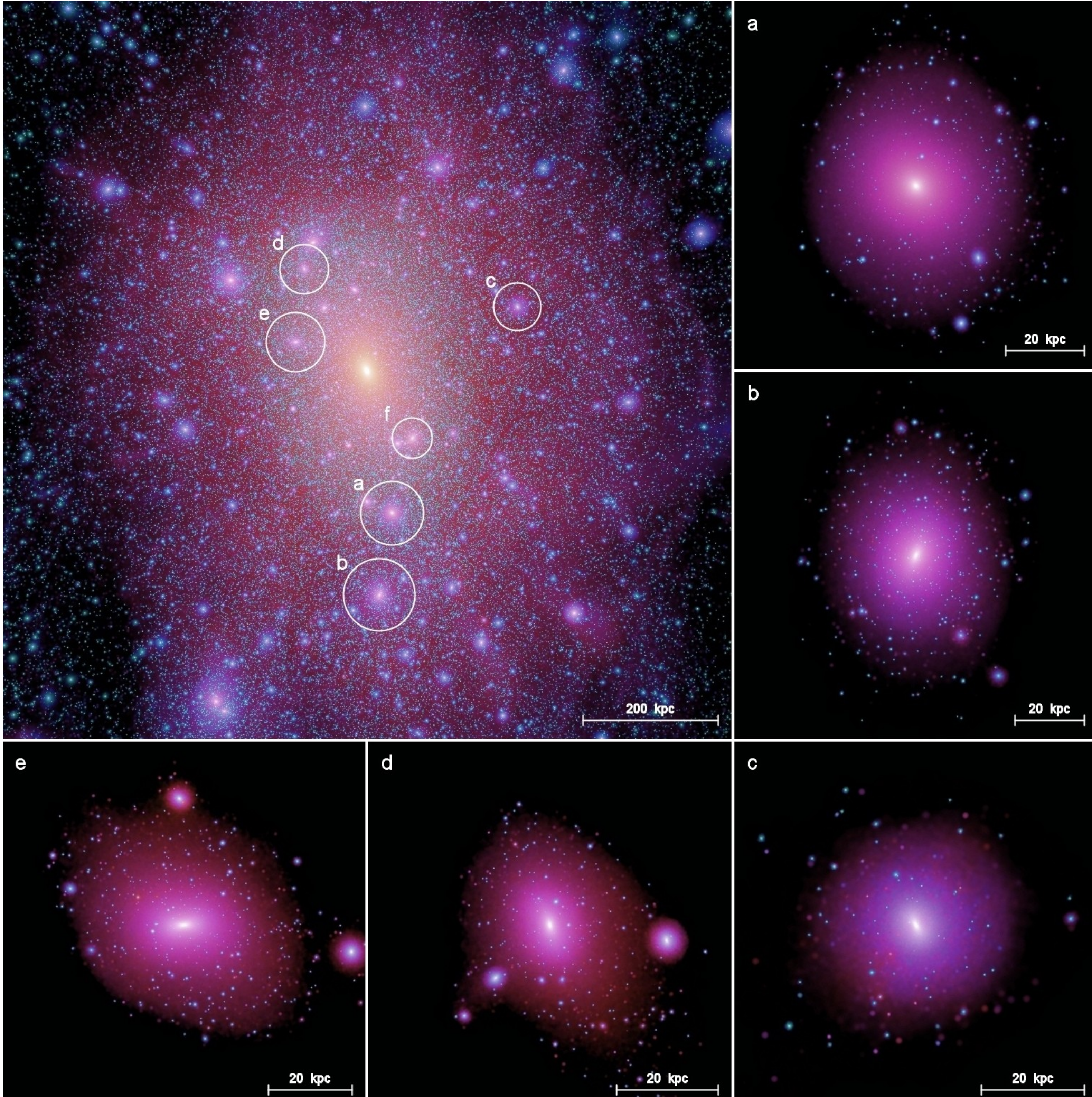
The subhalo abundance per unit halo mass is surprisingly uniform

VELOCITY FUNCTION IN OUR DIFFERENT HALOS



We detect up to four generations of substructures within substructures

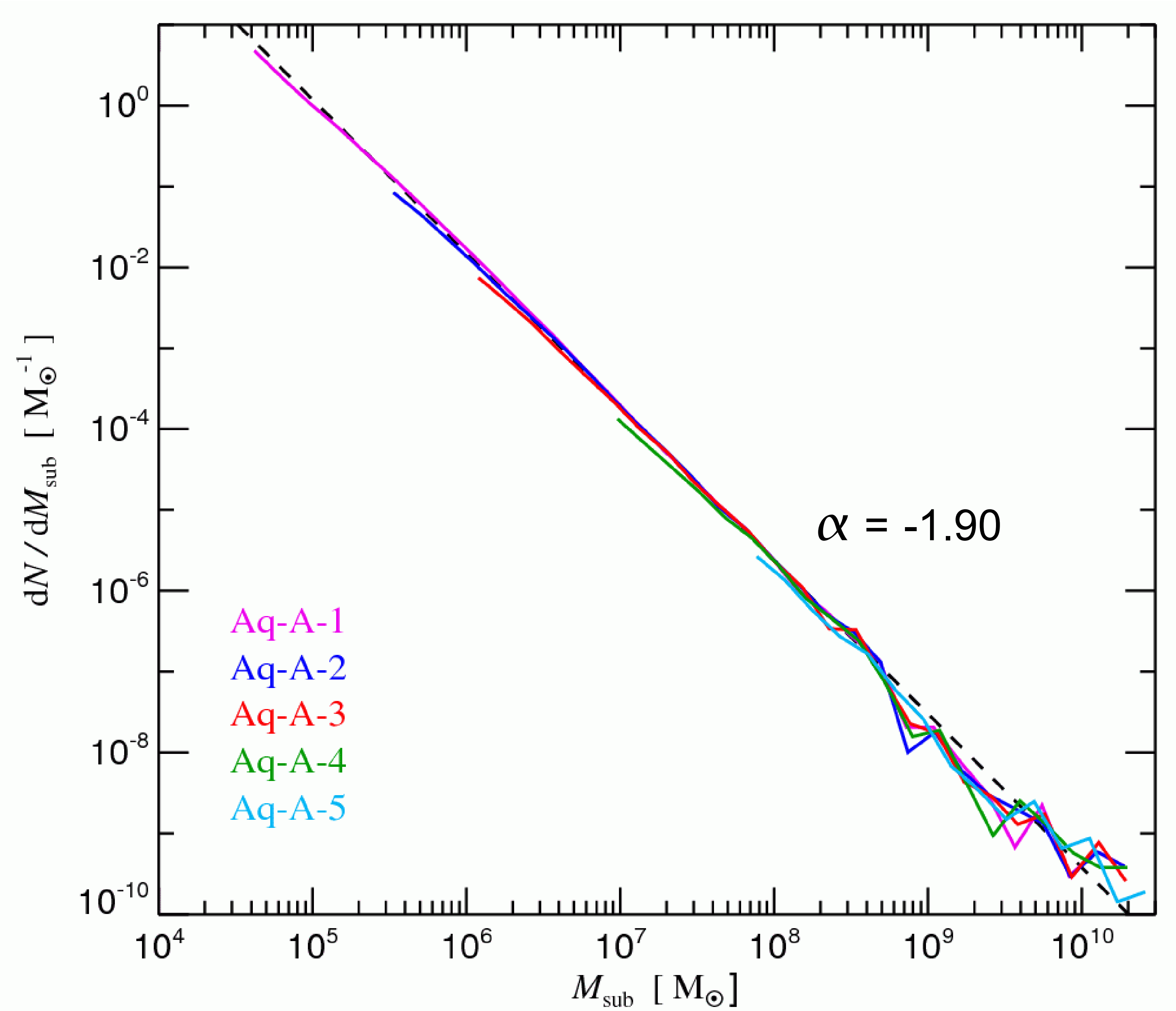
HALO IN HALOS IN HALOS IN THE AQUARIUS SIMULATIONS



The hierarchy does not appear to be strictly self-similar – we find somewhat fewer substuctures in subhalos than in field halos within the same overdensity.

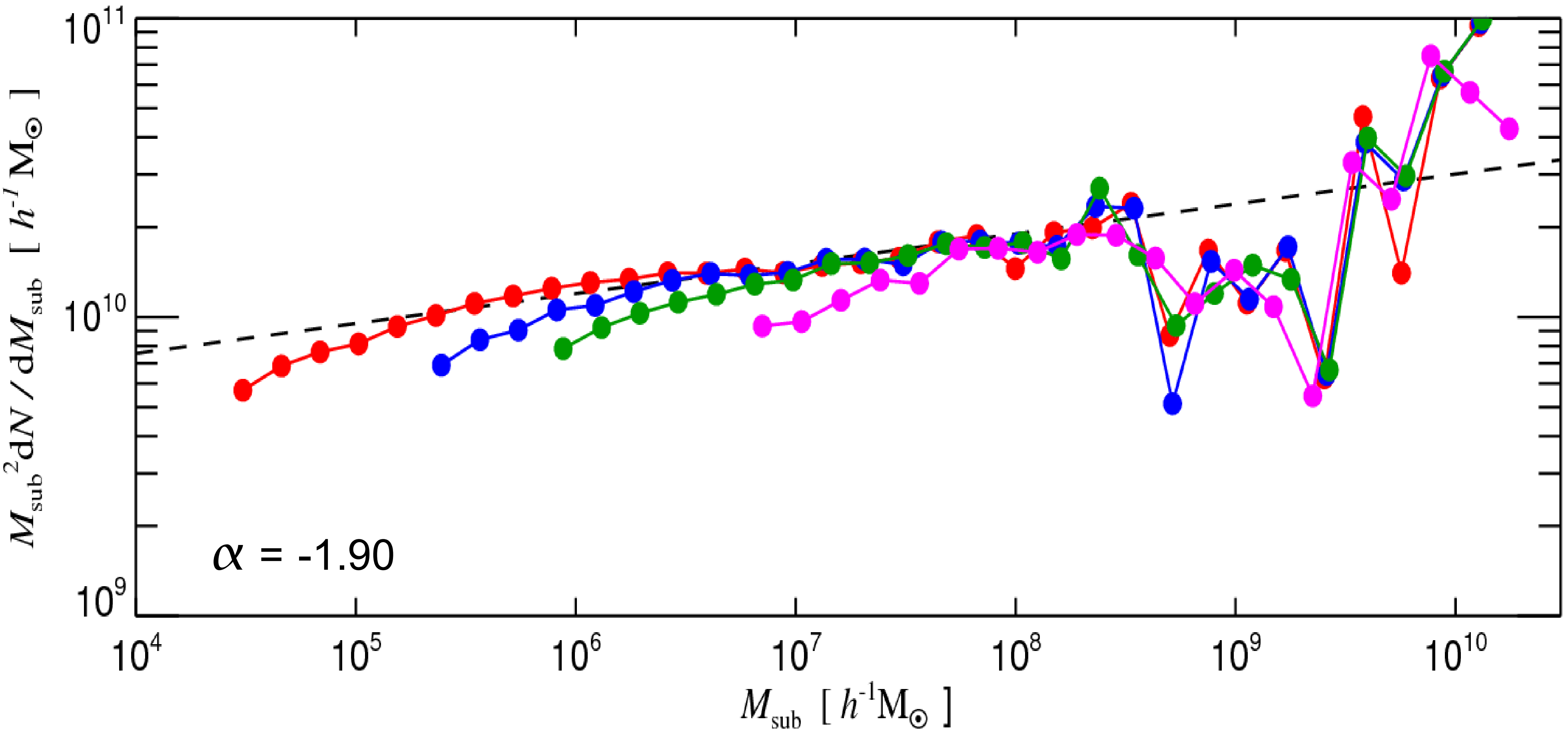
The differential subhalo mass function converges quite well to a power-law

RESULTS FROM A RESOLUTION STUDY OF AQUARIUS HALO AQ-A



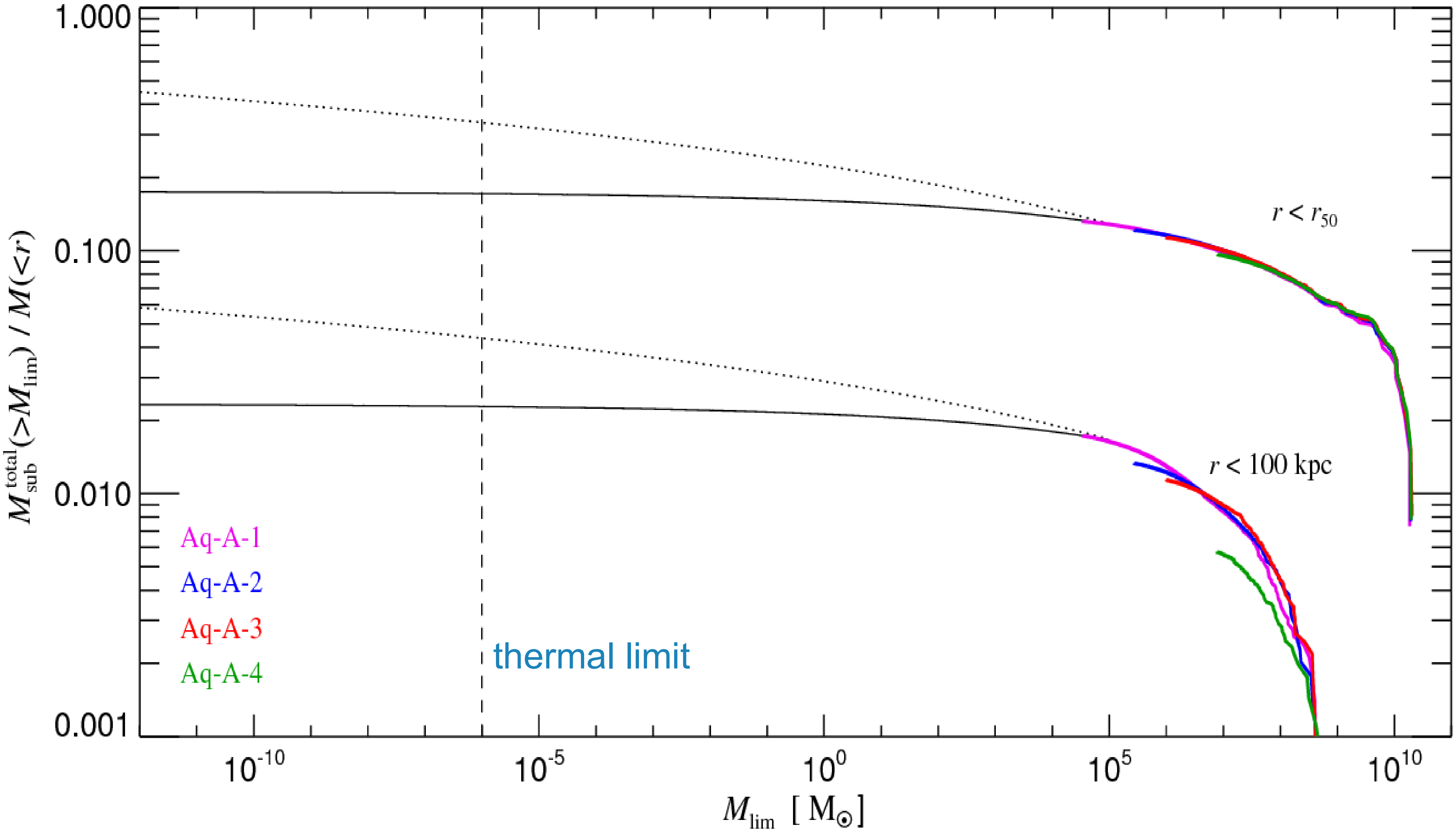
The subcritical slope of the differential subhalo mass function implies that the total mass in substructures converges at the faint end

THE DIFFERENTIAL SUBHALO MASS FUNCTION



The cumulative mass fraction in **resolved substructures** reaches about 12-13%, we expect up to ~18% down to the thermal limit

FRACTION OF MASS IN SUBSTRUCTURES AS A FUNCTION OF MASS LIMIT



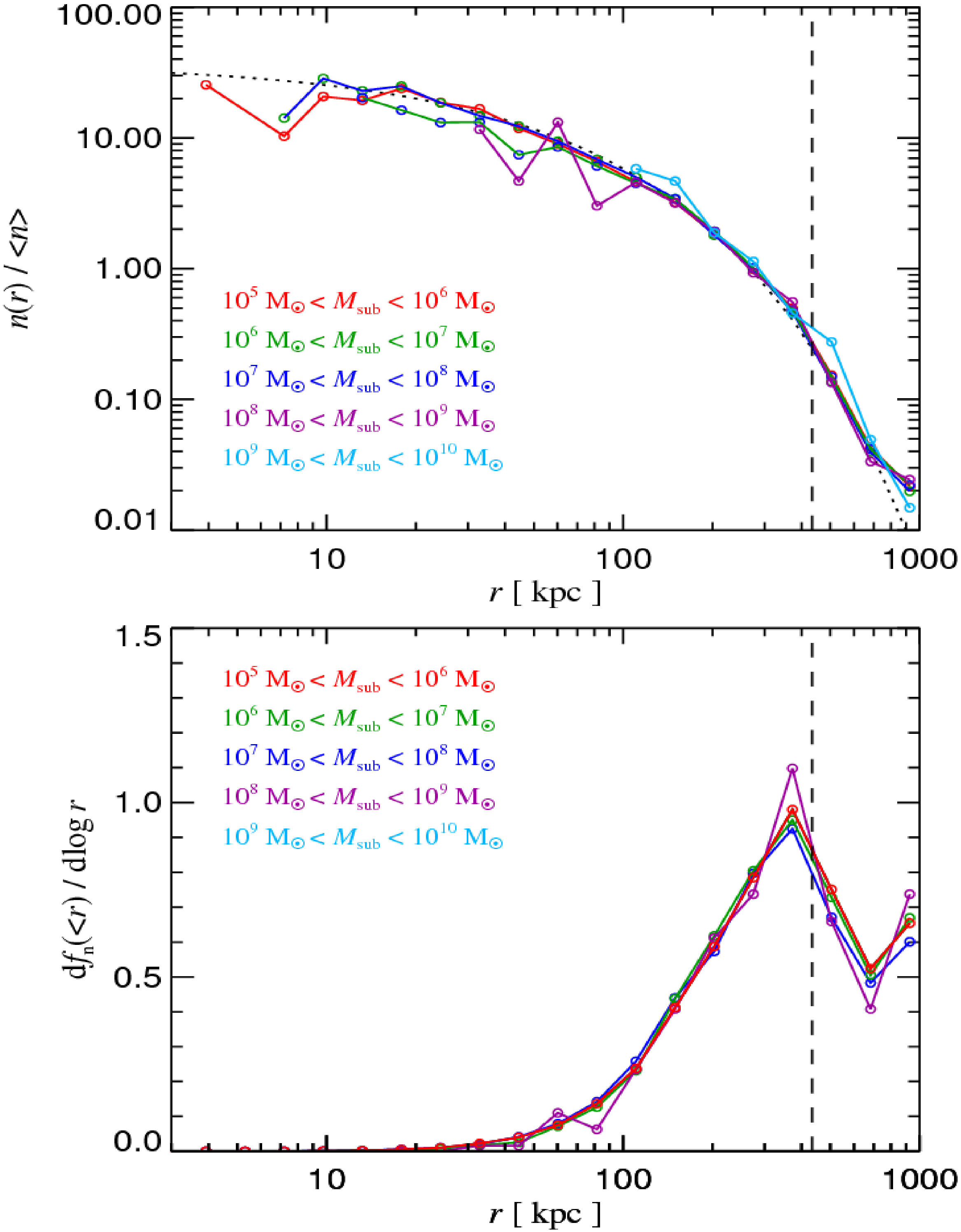
The radial distribution of substructures is strongly antibiased relative to all dark matter, and independent of subhalo mass

RADIAL SUBSTRUCTURE DISTRIBUTION IN Aq-A-1

Most subhalos are at large radii, subhalos are more effectively destroyed near the centre

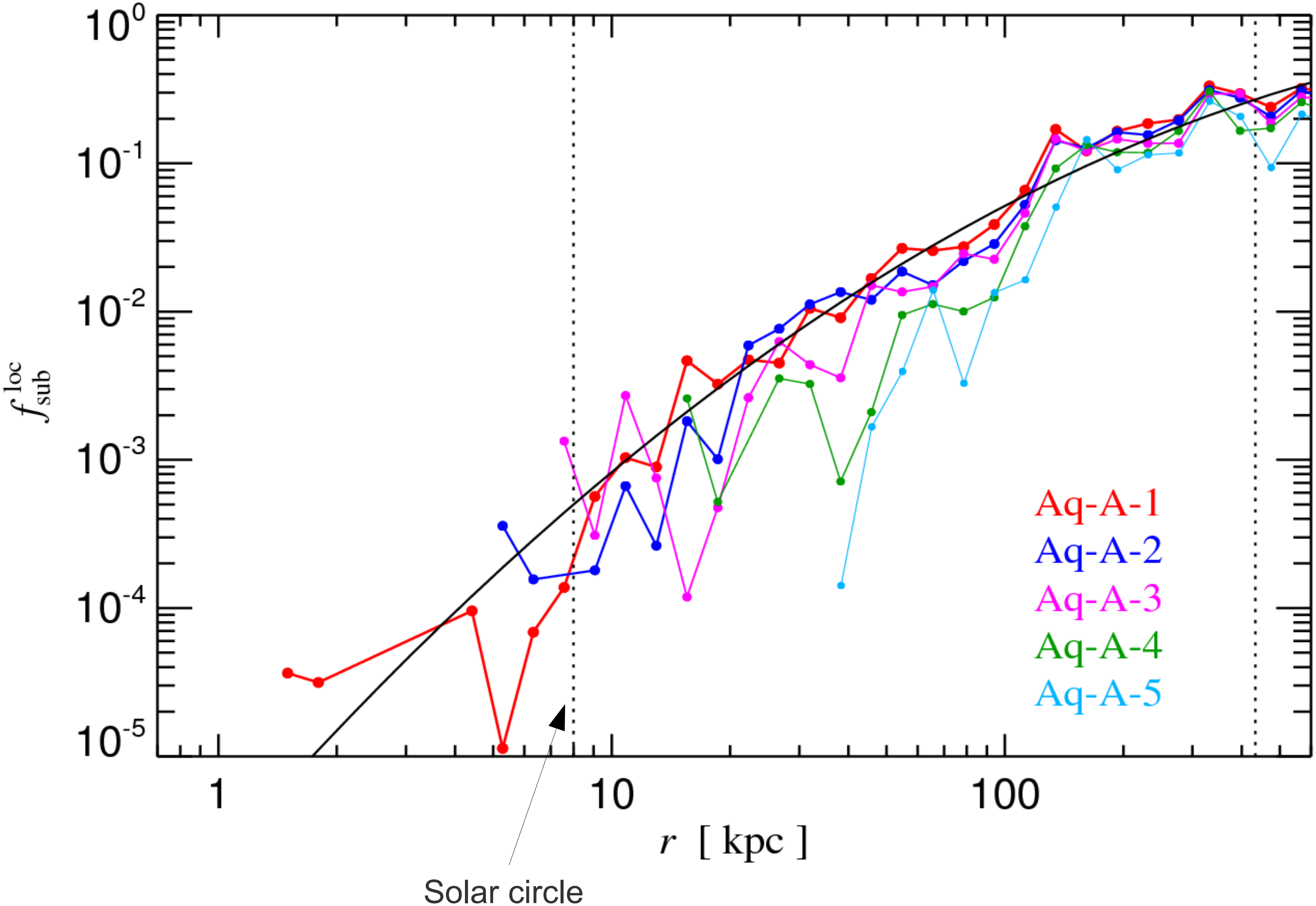
Subhalos are far from the Sun

see also Diemand et al. (2007, 2008)



The local mass fraction in substructures is a strong function of radius

MASS FRACTION IN SUBSTRUCTURES AS A FUNCTION OF RADIUS IN HALO AQ-A

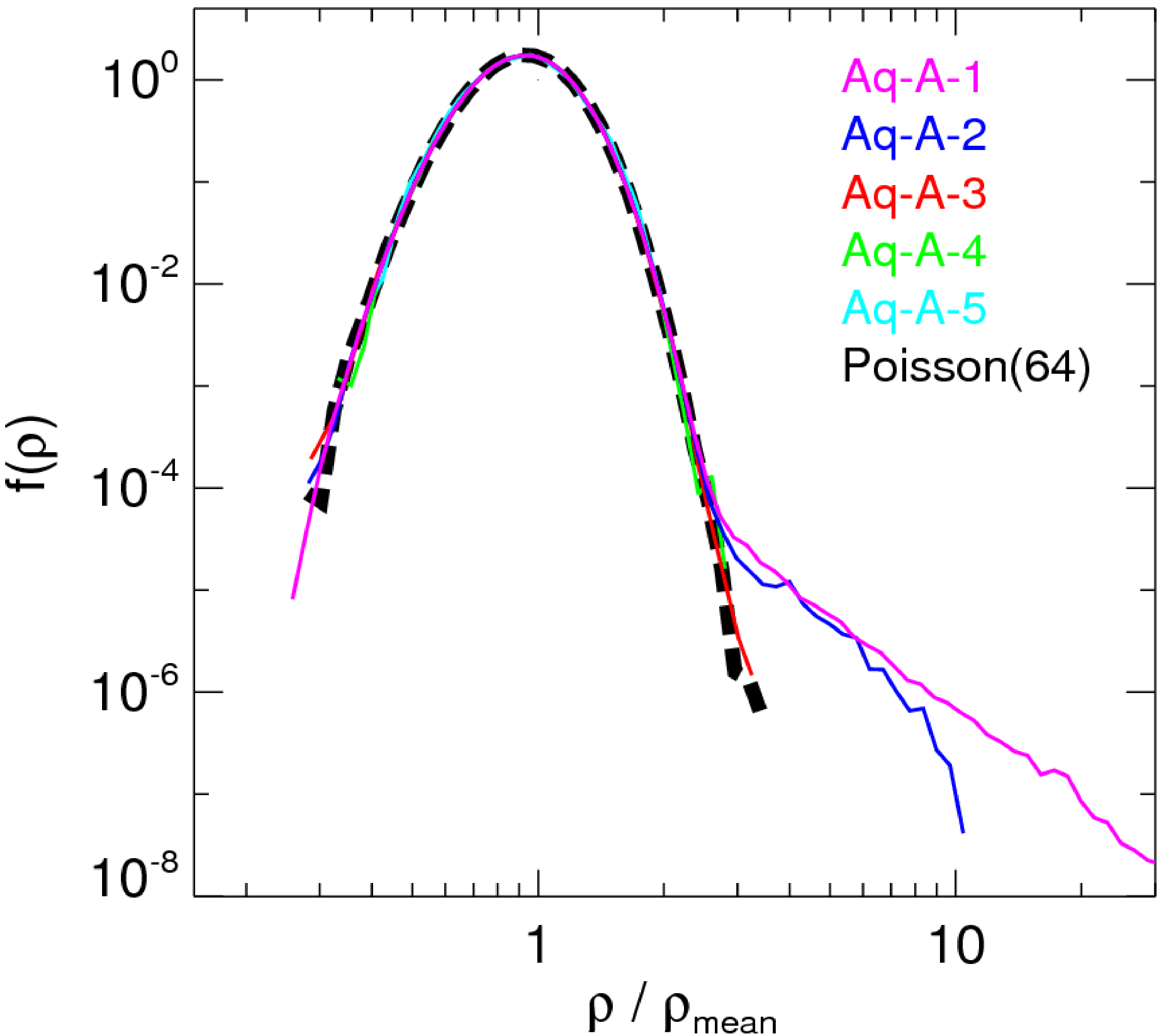


The dark matter distribution at the Solar circle is surprisingly smooth

THE DENSITY PDF AT THE POSITION OF THE SUN

Essentially all of the scatter is due to particle sampling noise.

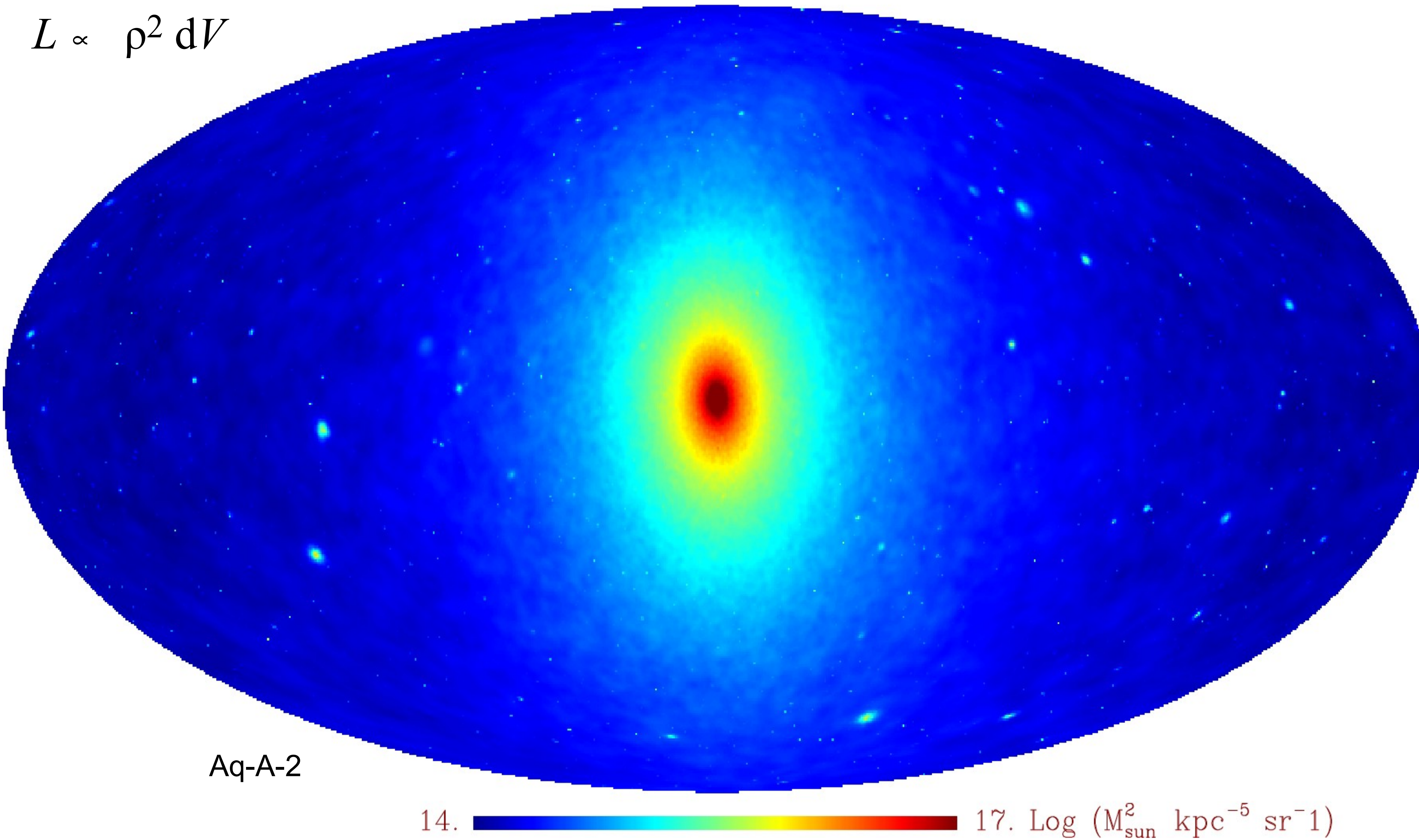
In fact, the **residual intrinsic rms scatter is only 4%** around the smooth model



Is the dark matter annihilation flux boosted significantly by dark matter substructures?

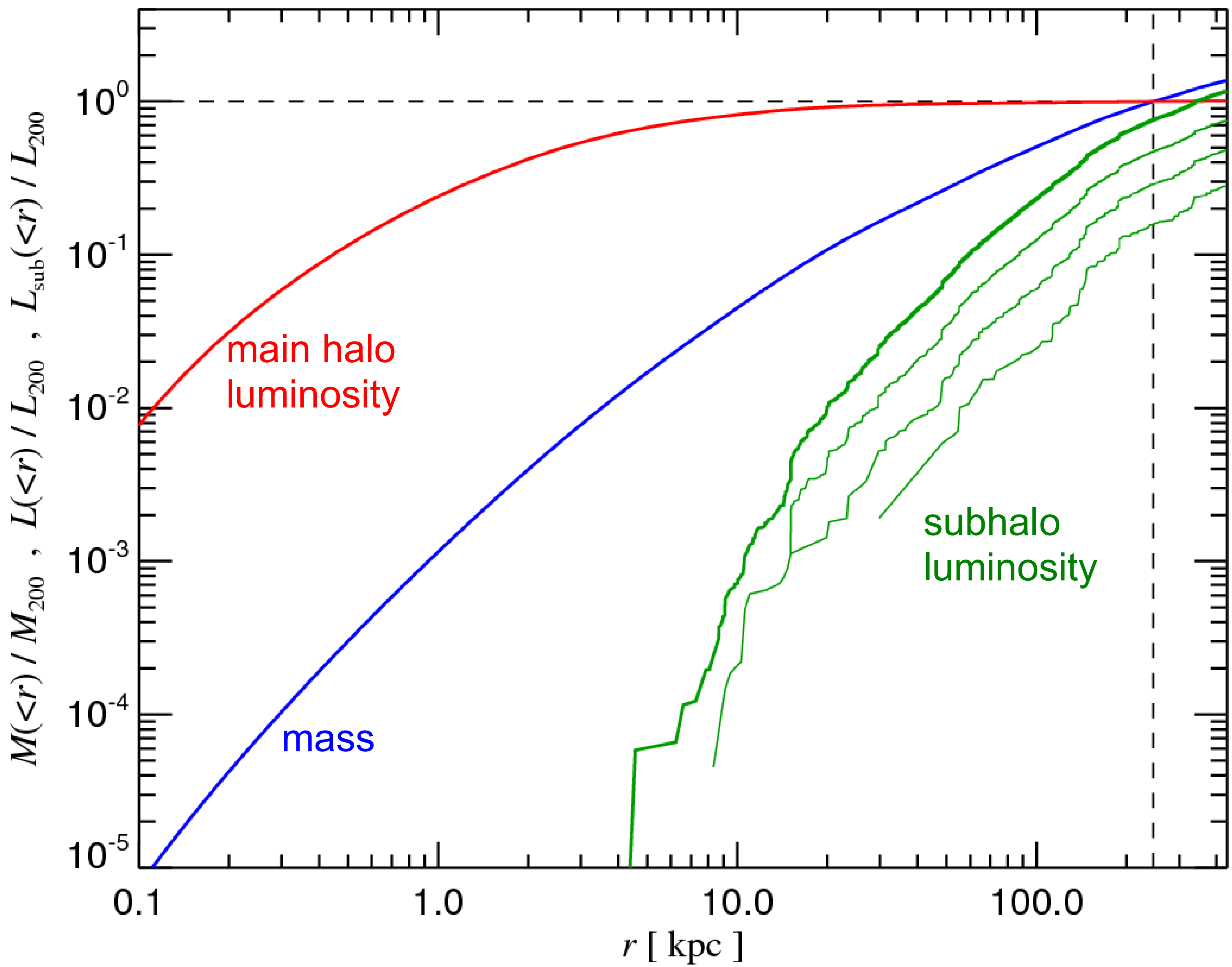
SIMULATED ALL-SKY MAP OF THE DM ANNIHILATION FLUX AROUND THE SUN IN THE MILKY WAY

$L \propto \rho^2 \, \mathrm{d}V$



The annihilation luminosity from main halo and subhalos has a very different radial distribution

THE RELATIVE DISTRIBUTION OF MASS, MAIN HALO, AND SUBHALO LUMINOSITY



Lower mass limits of

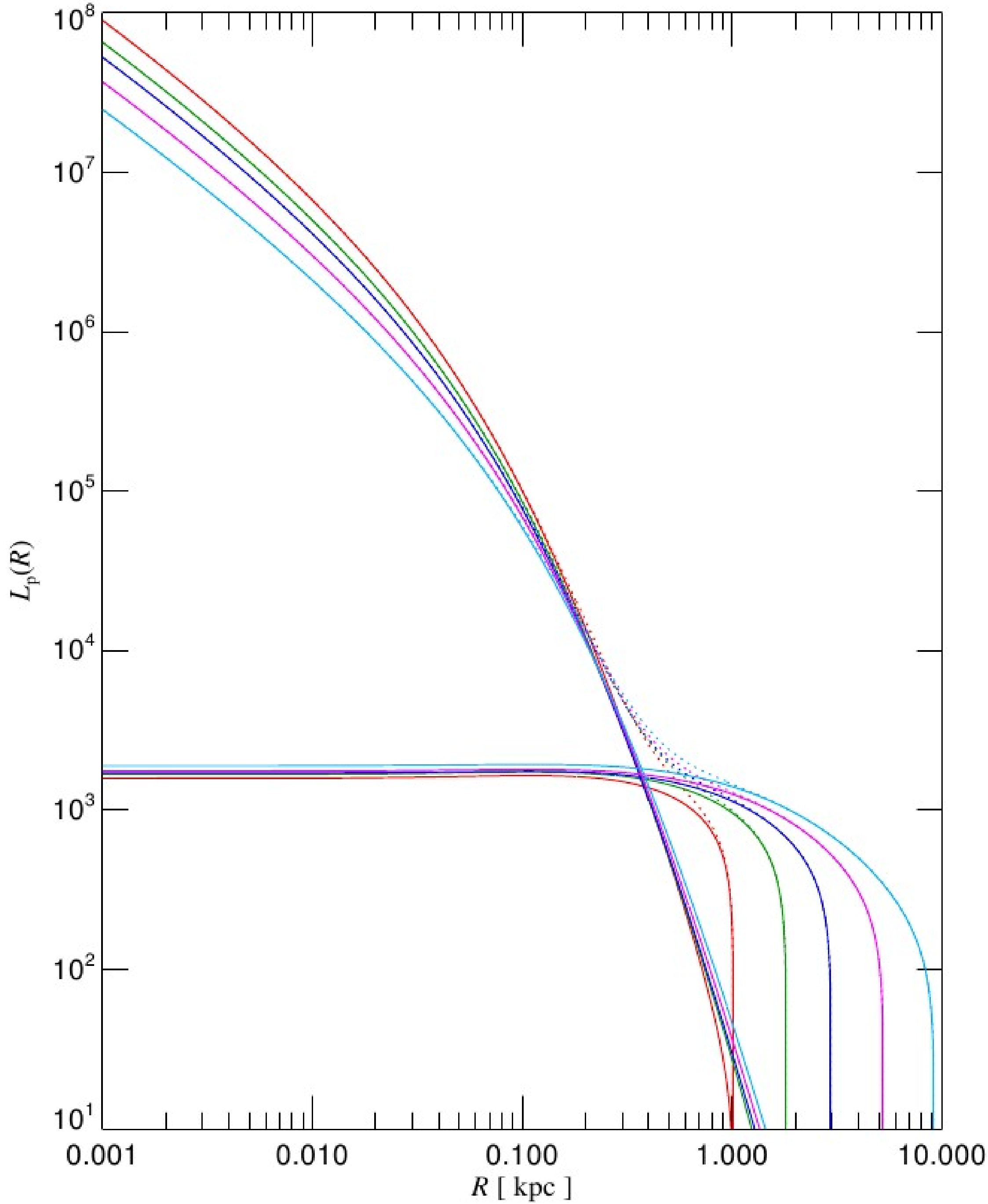
$10^5\ M_\odot$
$10^6\ M_\odot$
$10^7\ M_\odot$
$10^8\ M_\odot$

Surface brightness profile of a typical subhalo with V_{max}=10 km/s at different distances from the galactic center

SURFACE BRIGHTNESS PROFILE OF DIFFERENT SUBHALO COMPONENTS

The sub-sub component appears as a (extended) “disk” on the sky

The central surface brightness of the smooth component actually increases with smaller distance (because the concentration increases)



Dark matter annihilation can be best discovered with an optimal filter against a bright background

THE SIGNAL-TO-NOISE FOR DETECTION WITH AN OPTIMAL FILTER

The optimal filter is proportional to the signal

$$S/N = \sqrt{\tau A_{\text{eff}}} \left[\int \frac{n_\gamma^2(\theta,\phi)}{n_\gamma(\theta,\phi) + b_\gamma(\theta,\phi)} \mathrm{d}\Omega \right]^{1/2}$$

signal

background noise

The background dominates, then:

Main halo's smooth component:

$$(S/N)_{\text{MainSm}} = f_{\text{MainSm}} \left[\frac{\tau A_{\text{eff}}}{b_\gamma} \right]^{1/2} \frac{F}{\theta_h}$$

Subhalo's smooth component:

$$(S/N)_{\text{SubSm}} = f_{\text{SubSm}} \left(\theta_h/\theta_{\text{psf}} \right) \left[\frac{\tau A_{\text{eff}}}{b_\gamma(\vec{\alpha})} \right]^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}$$

Sub-substructure of a subhalo:

$$(S/N)_{\text{SubSub}} = f_{\text{SubSub}} \left(\theta_h/\theta_{\text{psf}} \right) \left[\frac{\tau A_{\text{eff}}}{b_\gamma(\vec{\alpha})} \right]^{1/2} \frac{F}{(\theta_h^2 + \theta_{\text{psf}}^2)^{1/2}}$$

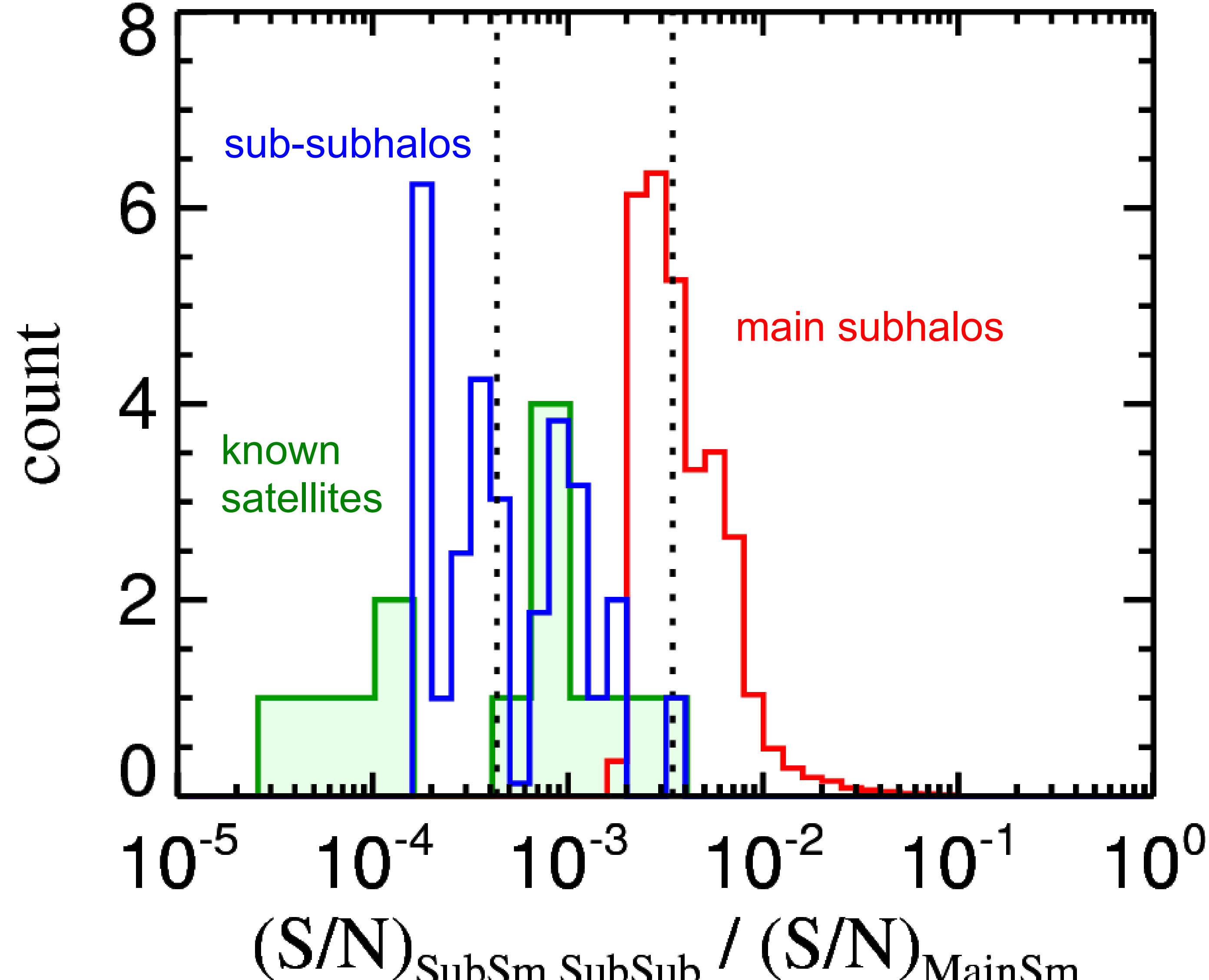
$$\boxed{\begin{array}{l} S/N \sim F / \theta \\ S/N \sim L / \; r_h \, d \end{array}}$$

Detectability of different annihilation emission components in the Milky Way

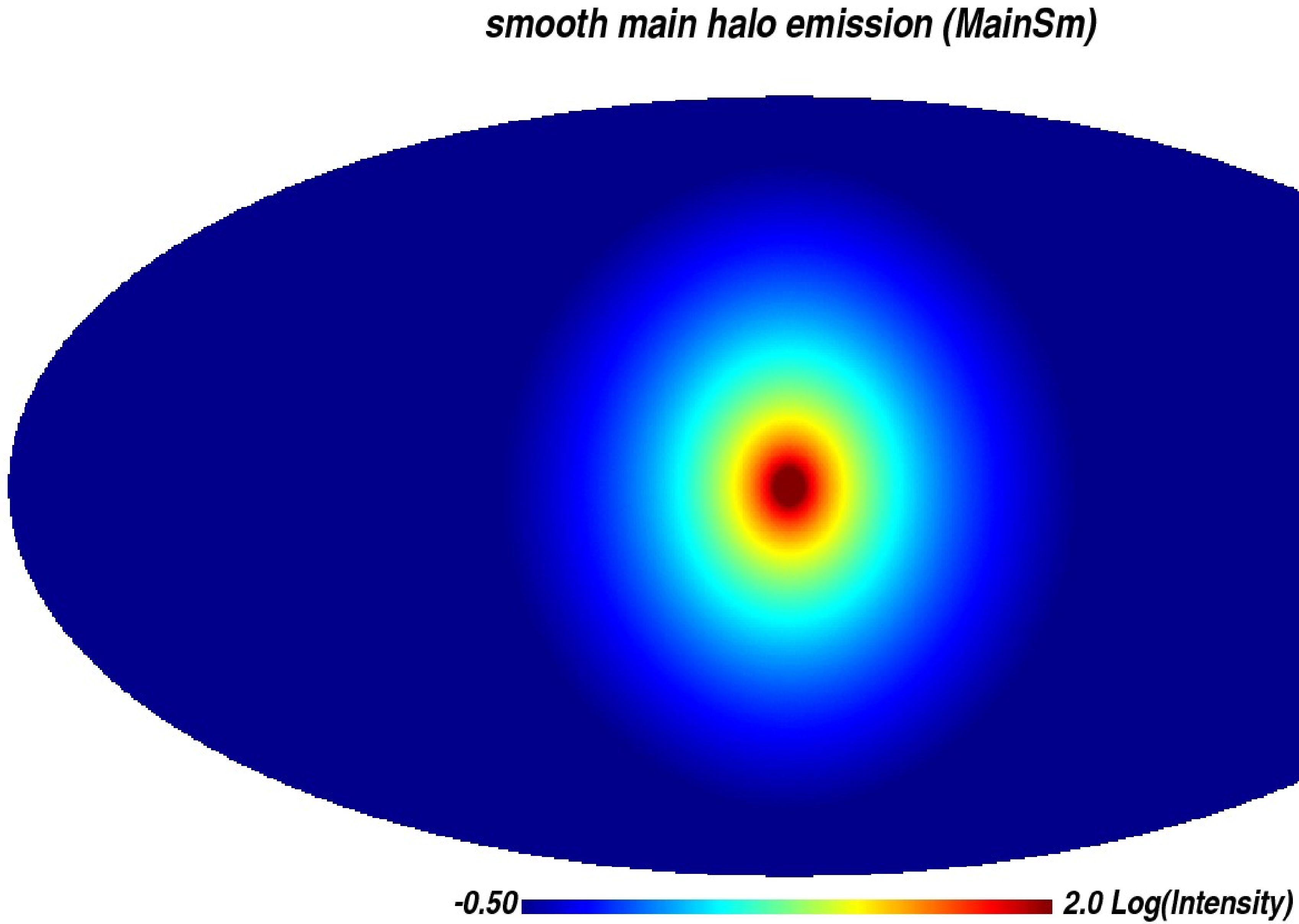
S/N for detecting subhalos in units of that for the main halo

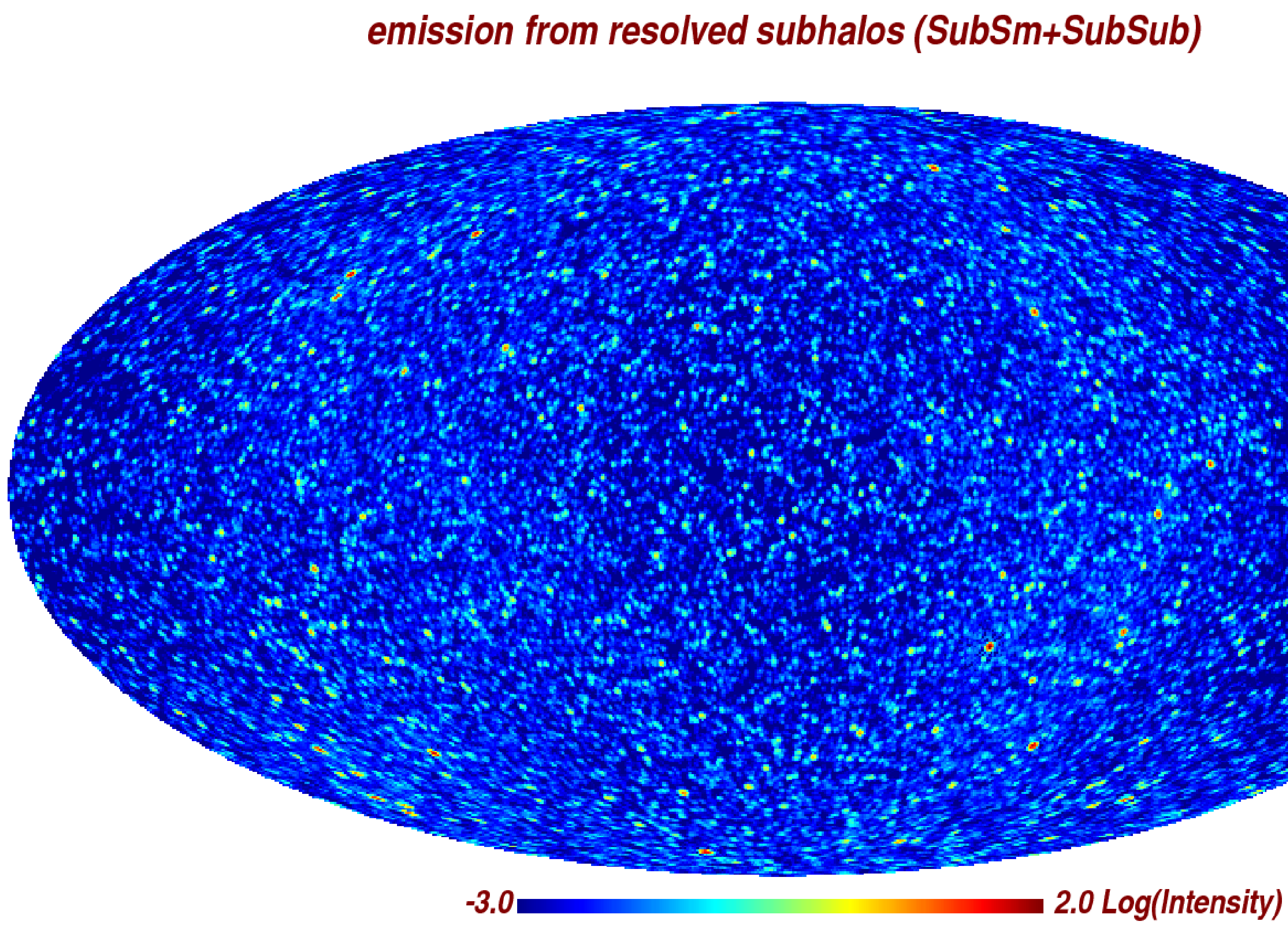
30 highest S/N objects, assuming the use of optimal filters

$$S/N \propto C V_{\max}^4 / (r_{\rm half}^2 d)$$

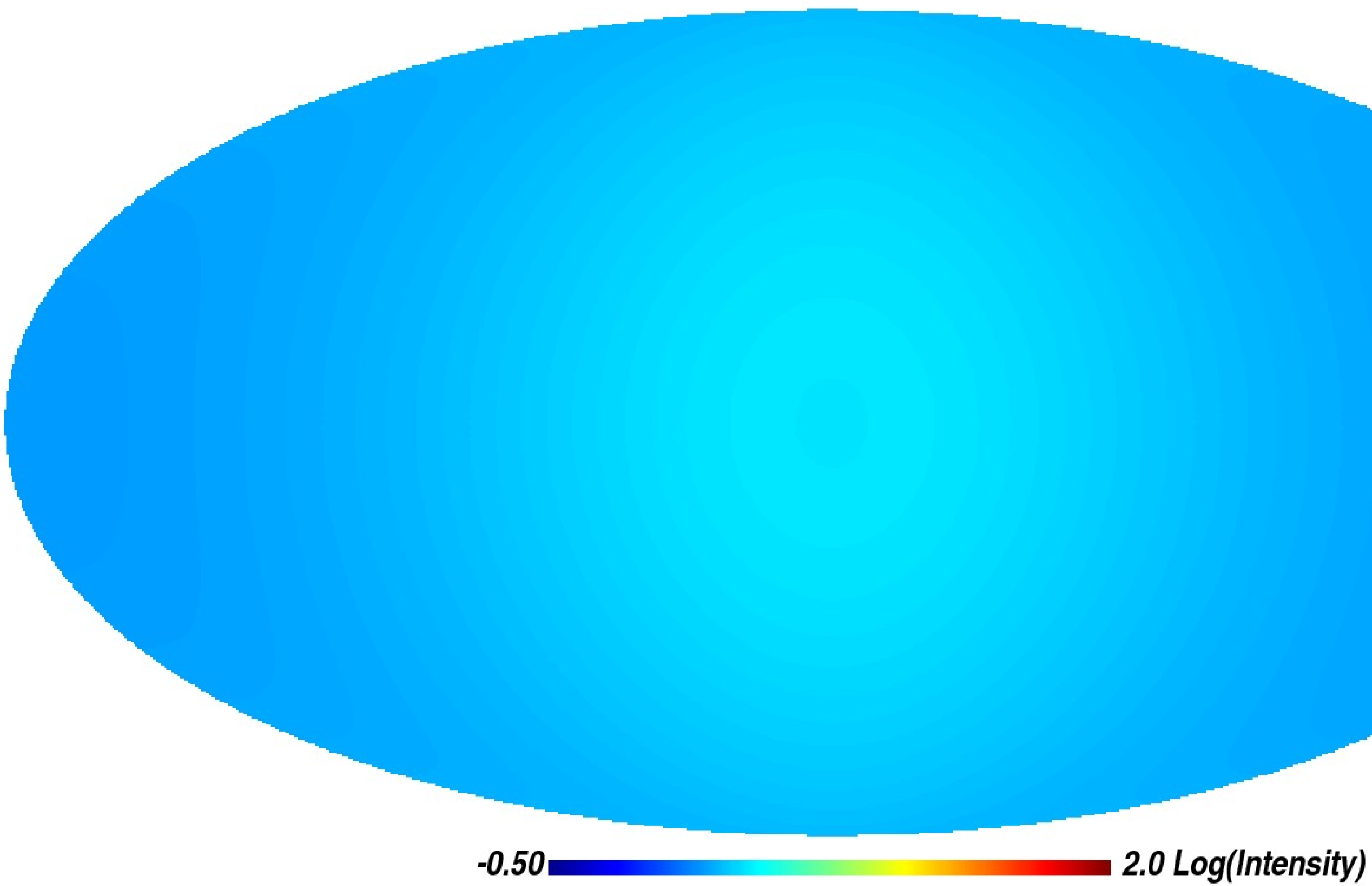


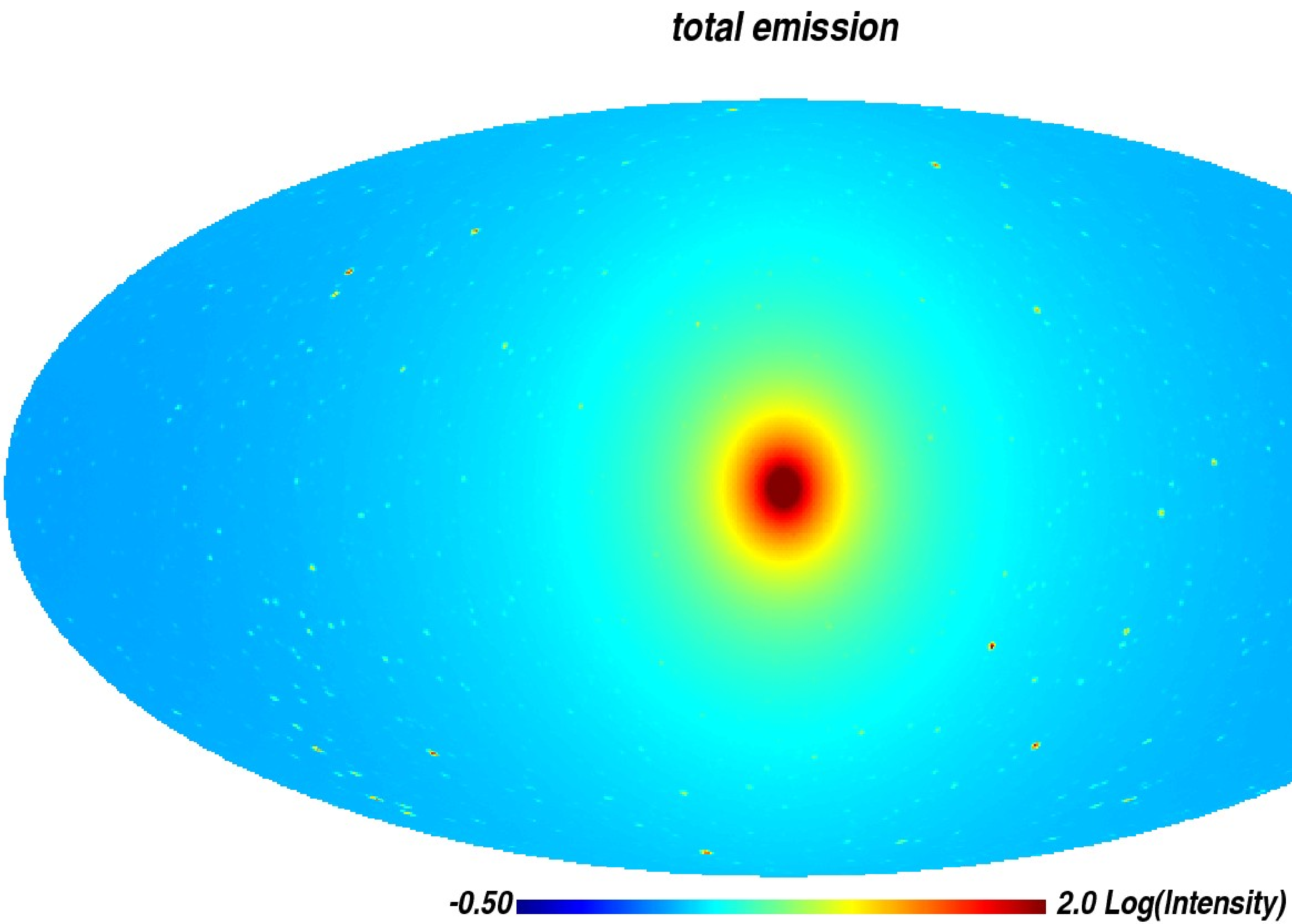
Highest S/N subhalos have 1% of S/N of main halo
Highest S/N subhalos have 10 times S/N of known satellites
Substructure of subhalos has no influence on detectability


smooth main halo emission (MainSm)
-0.50
2.0 Log(Intensity)


emission from resolved subhalos (SubSm+SubSub)
-3.0
2.0 Log(Intensity)

unresolved subhalo emission (MainUn)

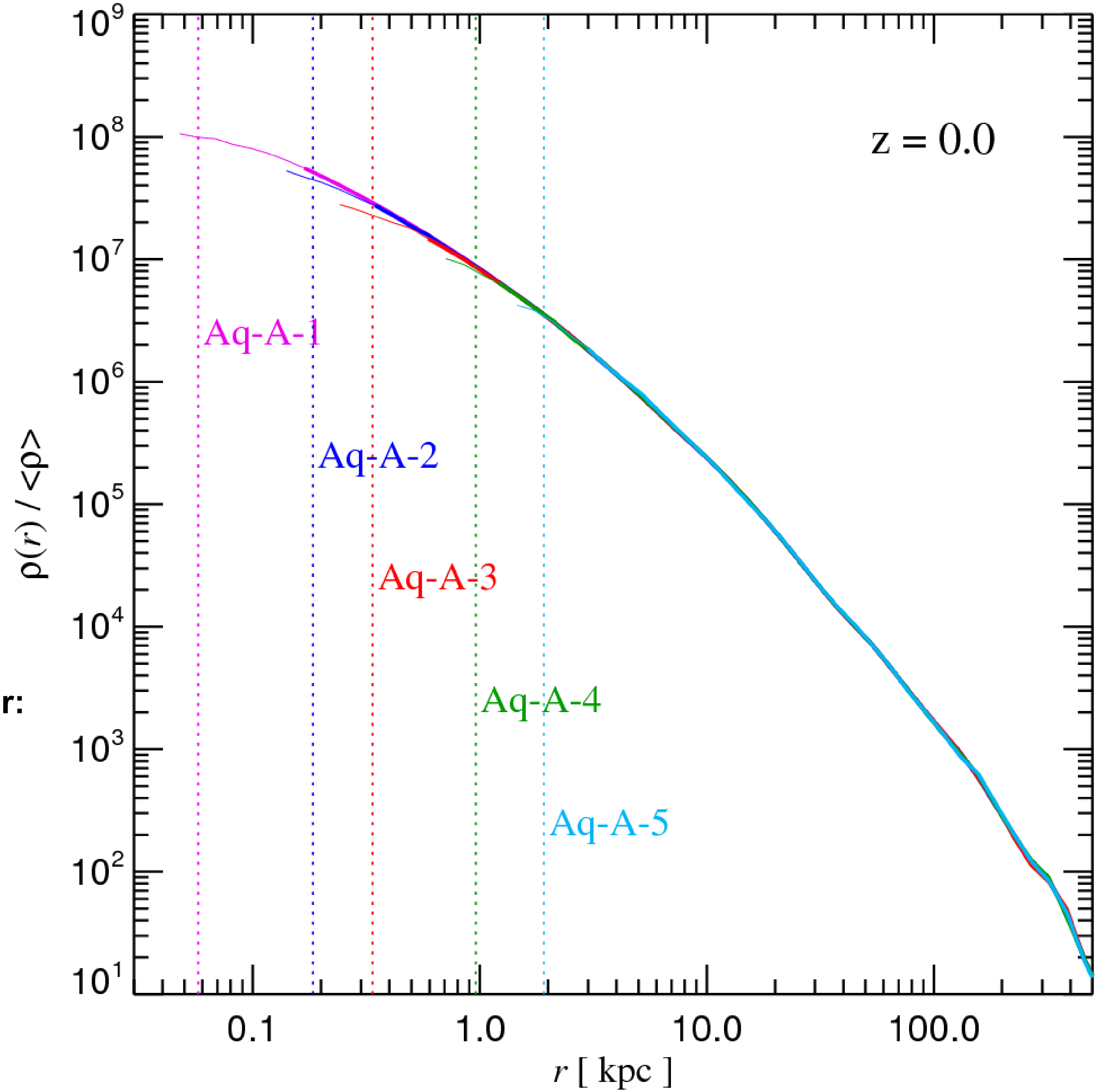



total emission
-0.50
2.0 Log(Intensity)

Structure of the central cusp

The spherically averaged density profiles at z = 0.0 show good convergence

DENSITY PROFILE AS A FUNCTION OF RADIUS



Fundamental importance for:

Rotation curve of galaxies

Internal structure of galaxy clusters

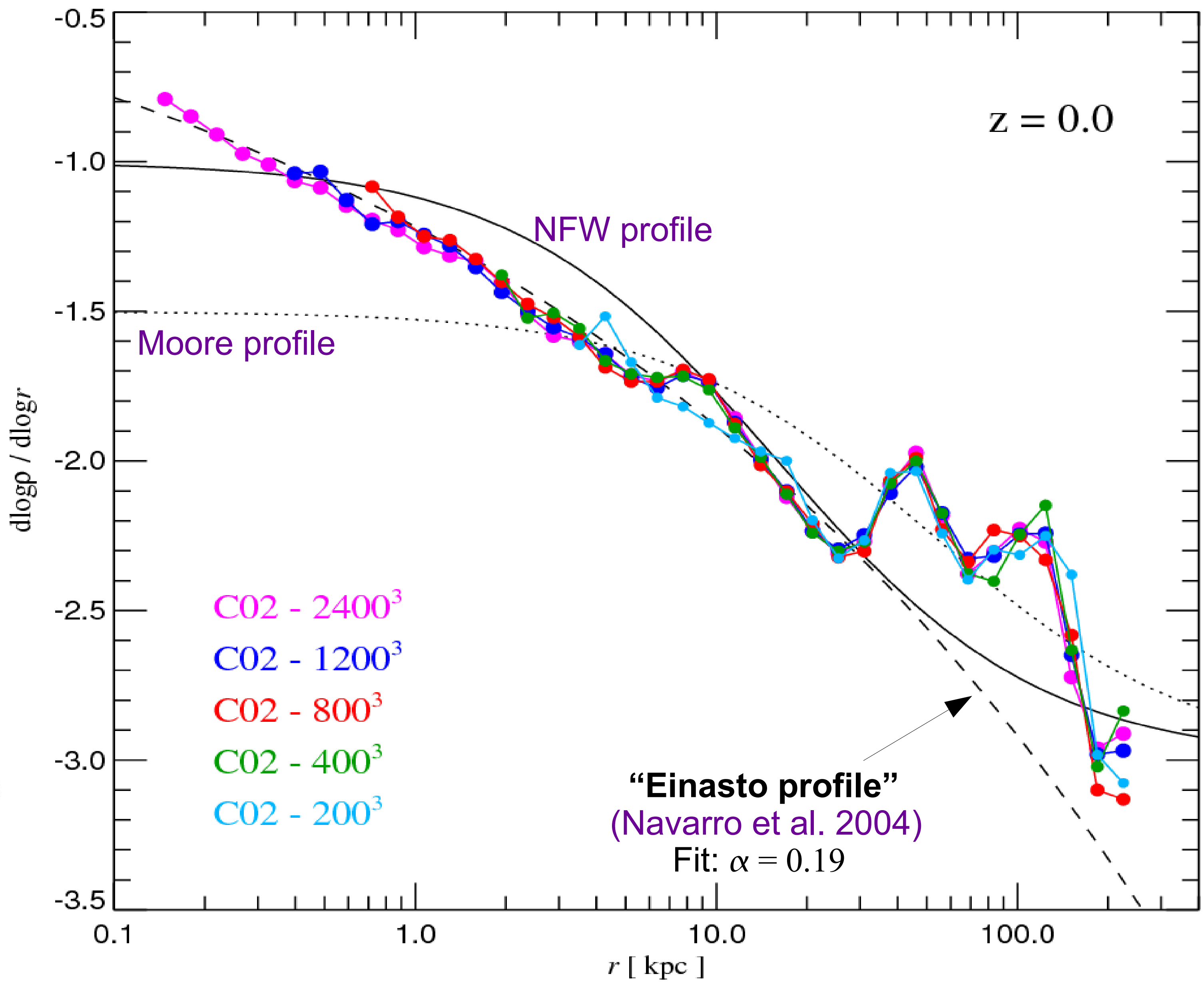
Gravitational lensing

DM annihilation

Galaxy mergers

The logarithmic slope of the density profile does not show asymptotic behavior towards the core

SLOPE OF THE DENSITY PROFILE AS A FUNCTION OF RADIUS



$$\frac{d\log\rho}{d\log r} = -2\left(\frac{r}{r_{-2}}\right)^{\alpha}$$

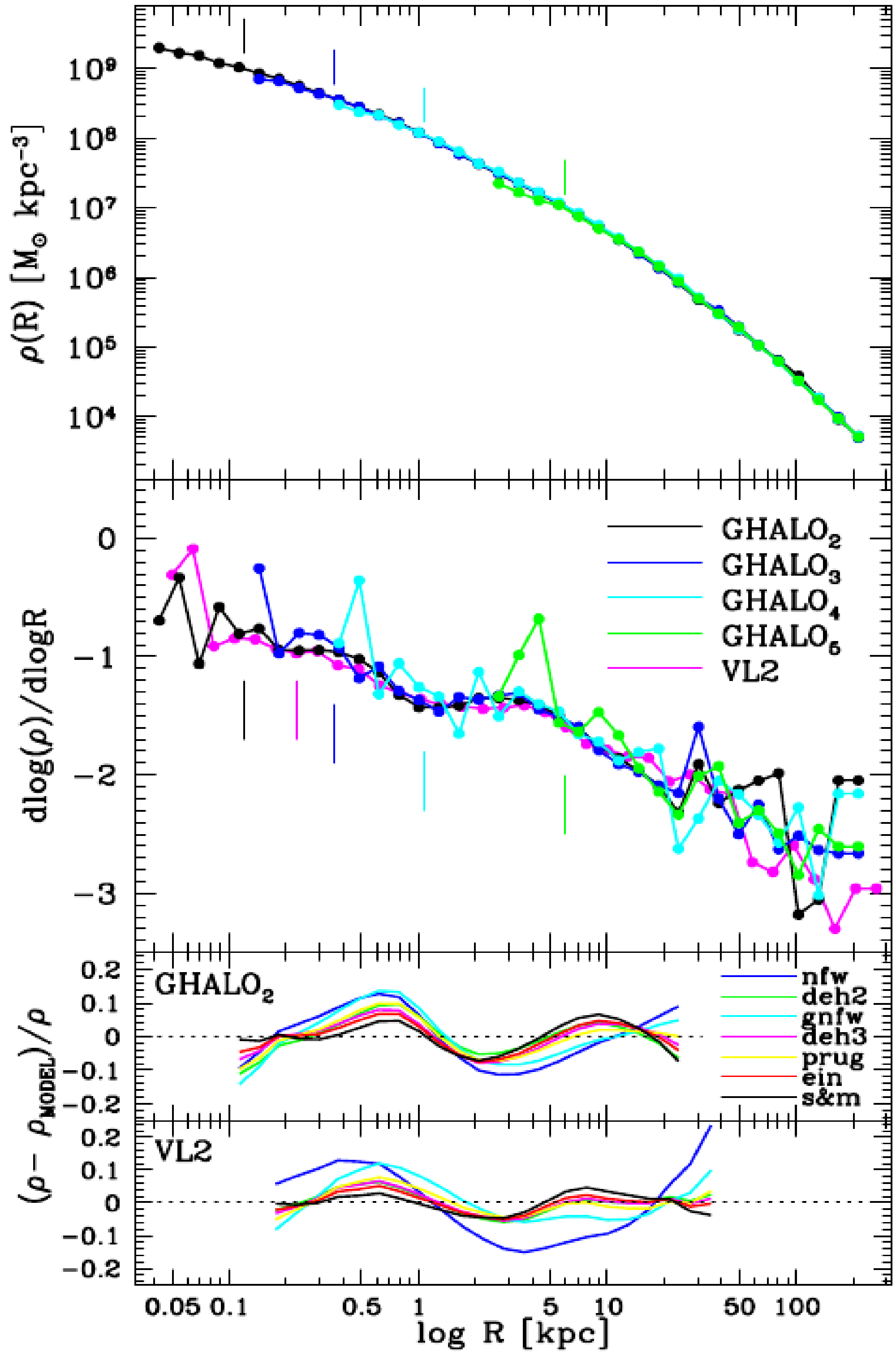
A consensus on the central structure of the cups seems to be emerging

RECENT RESULTS FROM THE 'GHALO' SIMULATION OF THE ZURICH GROUP

Stadel et al. (2009)

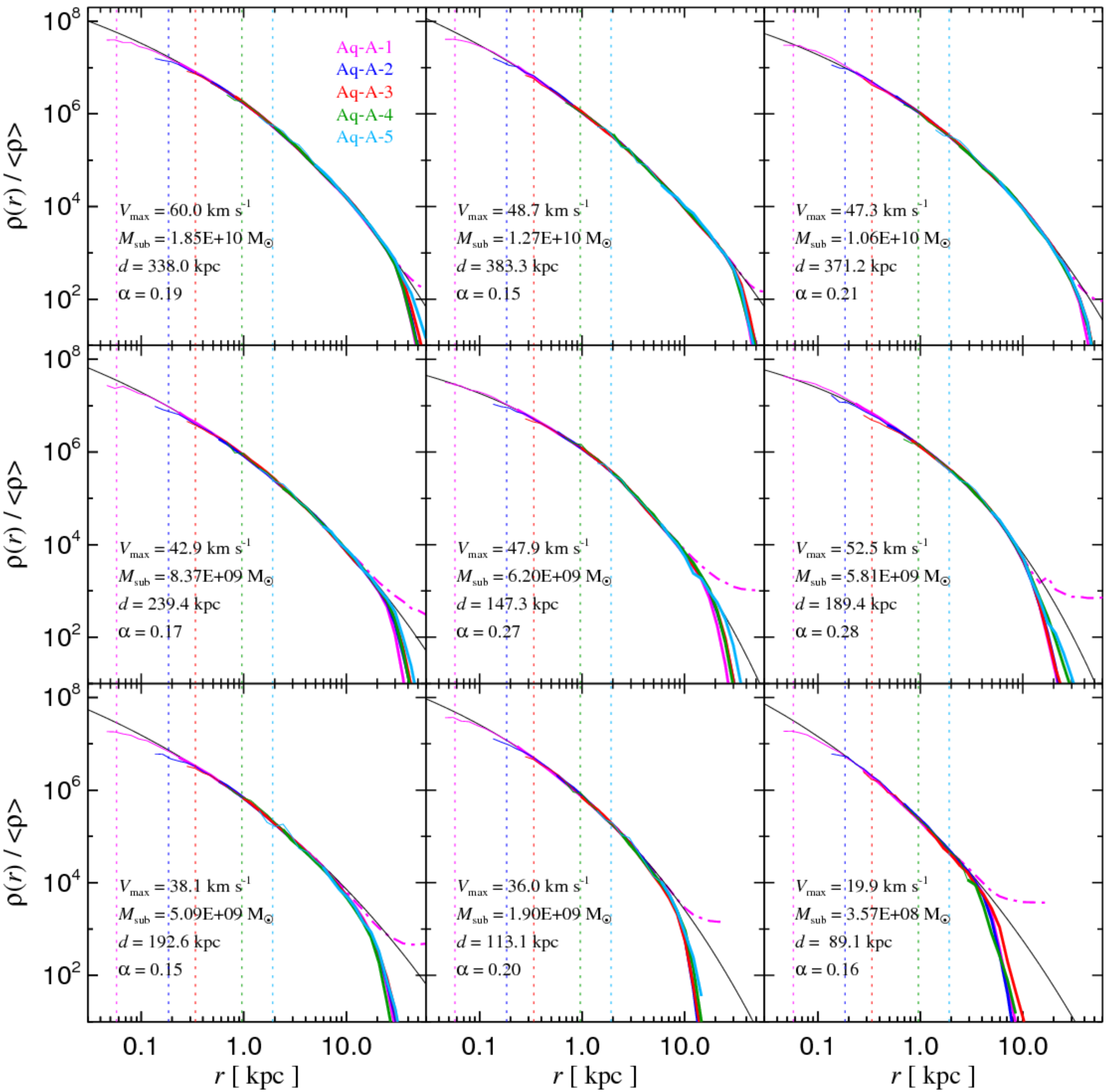
“The logarithmic slope of the radial density profile is close to a power law, gradually turning over to a slope of -0.8 at our innermost resolved point.”

“The Einasto profile also provides an excellent fit to the density profiles of the two simulations.”



Our simulations allow us to study the convergence of **subhalo density profiles**

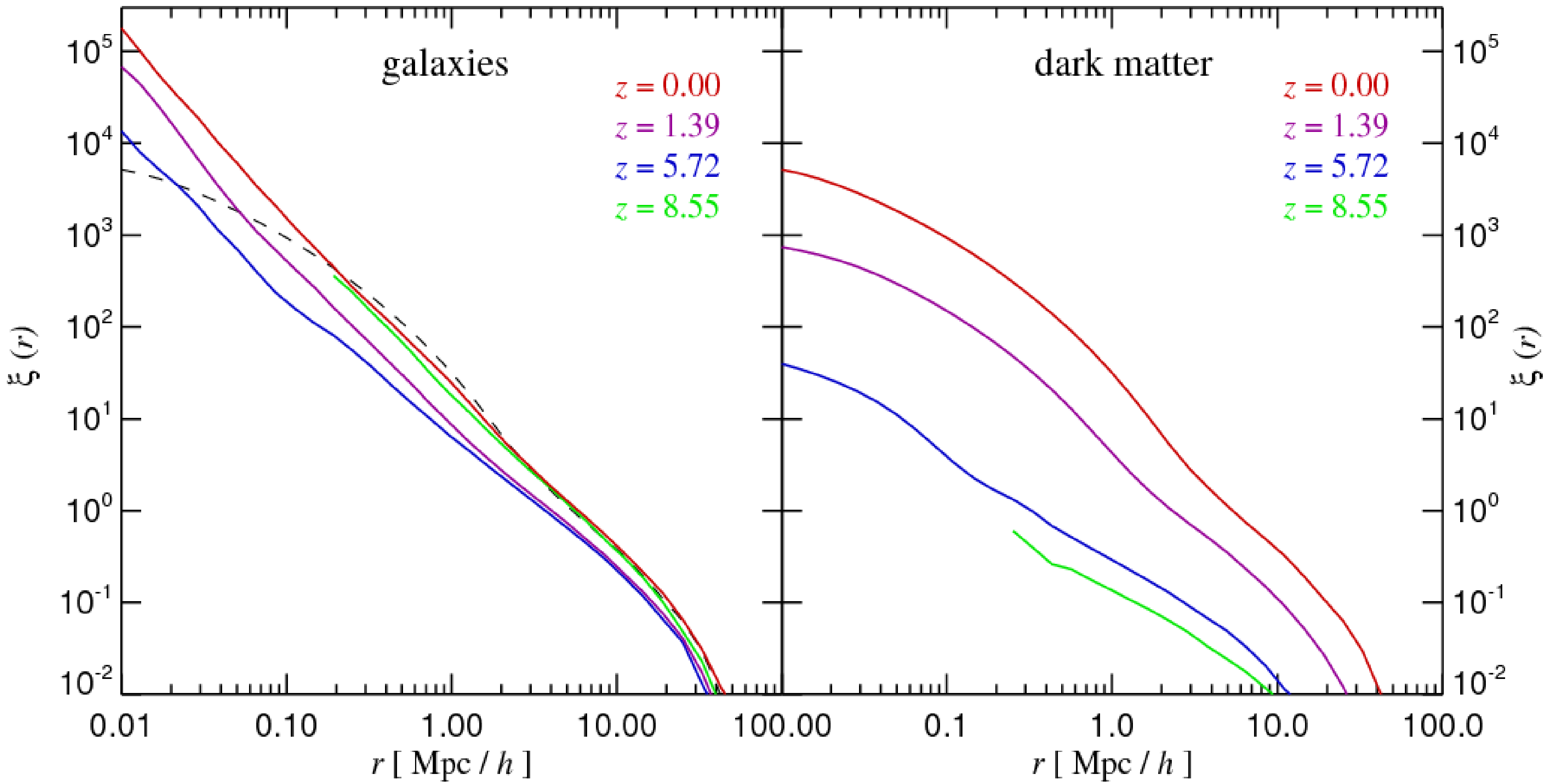
SPHERICALLY AVERAGED DENSITY PROFILES IN THE AQ-A HALO AT DIFFERENT RESOLUTION



The need for simulations with larger volume

The galaxy distribution is **biased** with respect to the mass distribution

GALAXY AND MASS CLUSTERING AT DIFFERENT EPOCHS



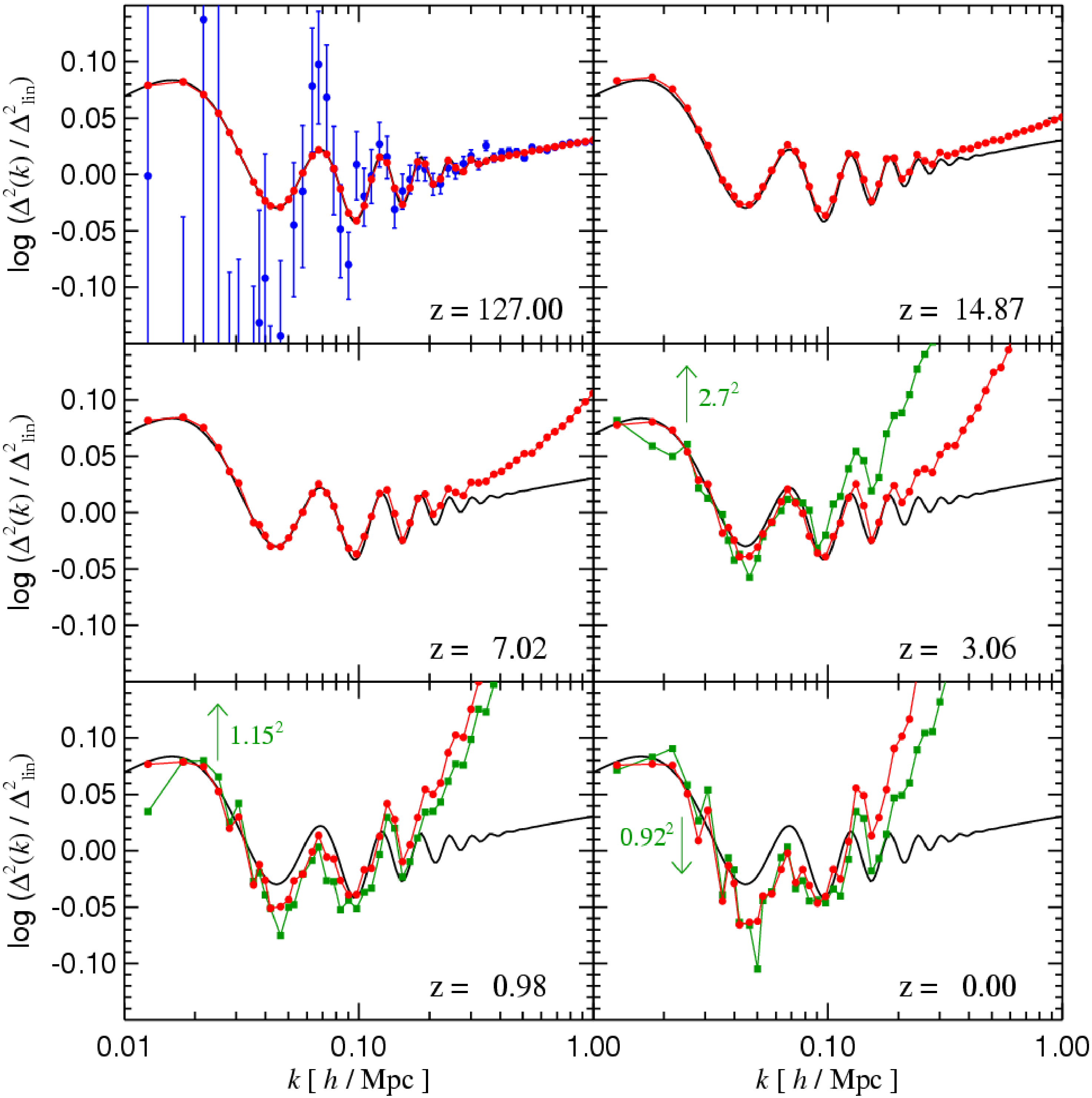
evolves little in time

evolves strongly in time

Springel, Frenk & White (2006)

The baryonic wiggles remain visible in the galaxy distribution down to low redshift and may serve as a "standard ruler" to constrain dark energy

DARK MATTER AND GALAXY POWER SPECTRA FROM THE MILLENNIUM SIMULATION IN THE REGION OF THE WIGGLES



Springel et al. (2005)

Large volume simulations are needed for ongoing and future surveys

GOALS FOR A "MILLENNIUM-XXL" CALCULATION

Science goals

- Impact of galaxy physics on BAO/growth factor measurements
- Realistic mock catalogs for Pan-STARRS, SDSS-III/BOSS, BigBOSS, etc.
- Exploring galaxy physics in different cosmologies
- Integrated Sachs-Wolfe effect
- Clustering up to ~500 Mpc/h, universality of halo bias and mass function, **rare events**, environmental effects, first quasars, etc.

Desired simulation characteristics

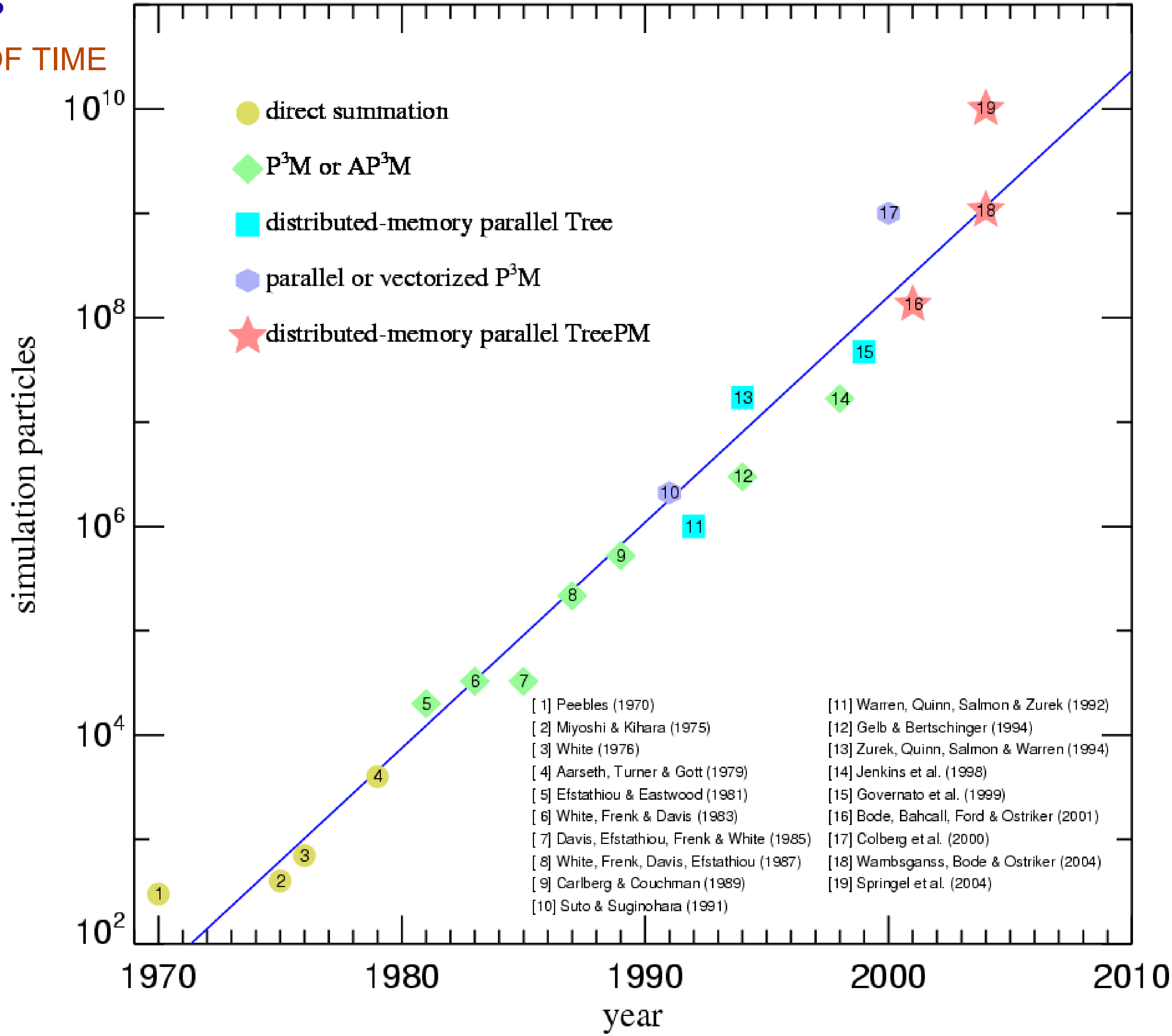
- Box-Size: $>$ 3000 Mpc/h
- Particle mass: $< \sim 6 \times 10^9\ M_\odot/h$
- Particle number: $>$ 300 billion particles

Cosmological N-body simulations have grown rap last four decades

Millennium-XXL Project

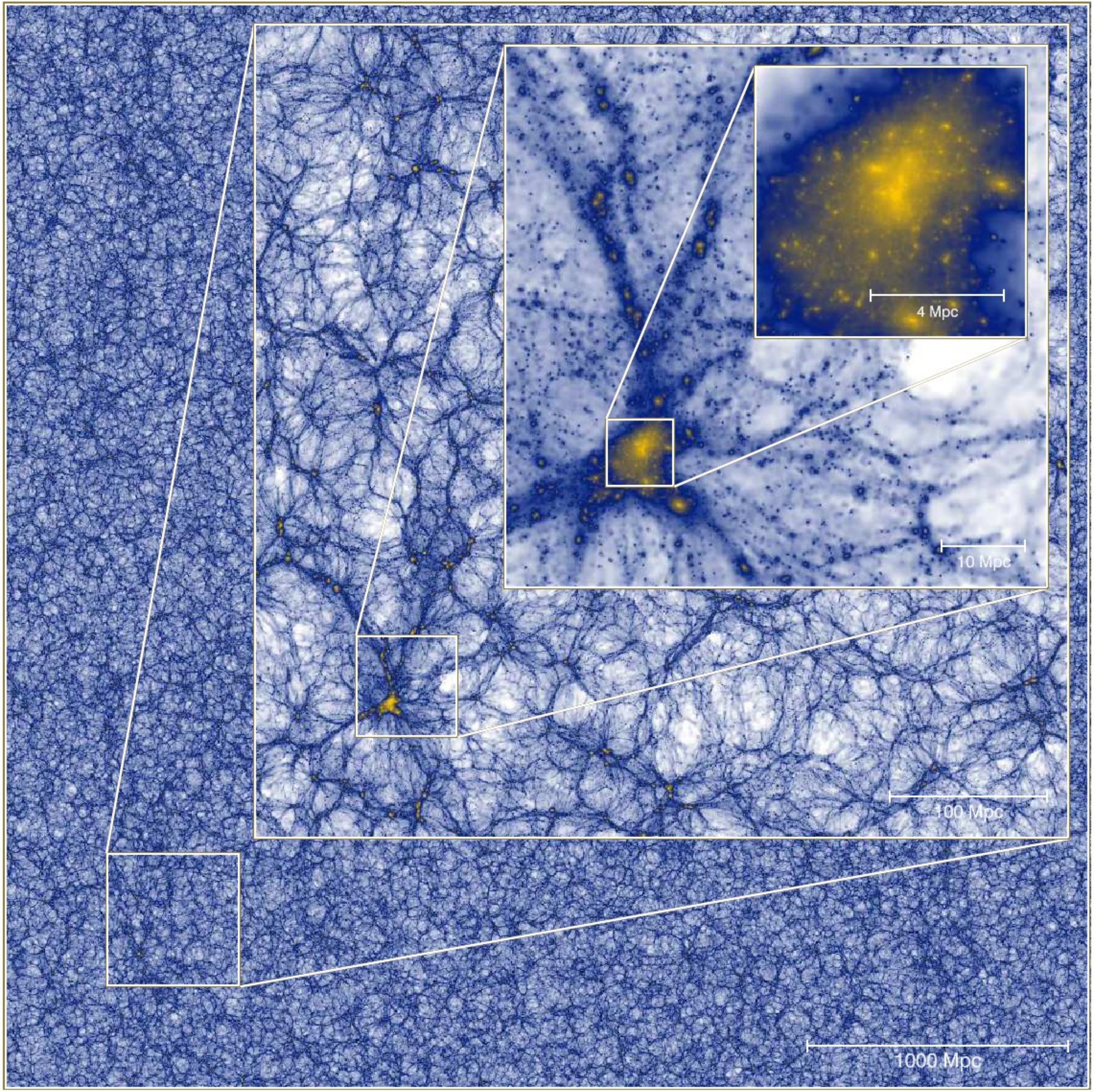
"**N**" AS A FUNCTION OF TIME

- Computers double their speed every 18 months (Moore's law)
- N-body simulations have doubled their size every 16-17 months
- Recently, growth has accelerated further.
The Millennium Run should have become possible in 2010 – we have done it in 2004 !



Millennium-XXL

The World's largest cosmological simulation



Angulo, Springel, White et al. (2011)

Millennium-XXL was successfully executed on JUROPA in Summer 2010

PARAMETERS OF FINAL RUN

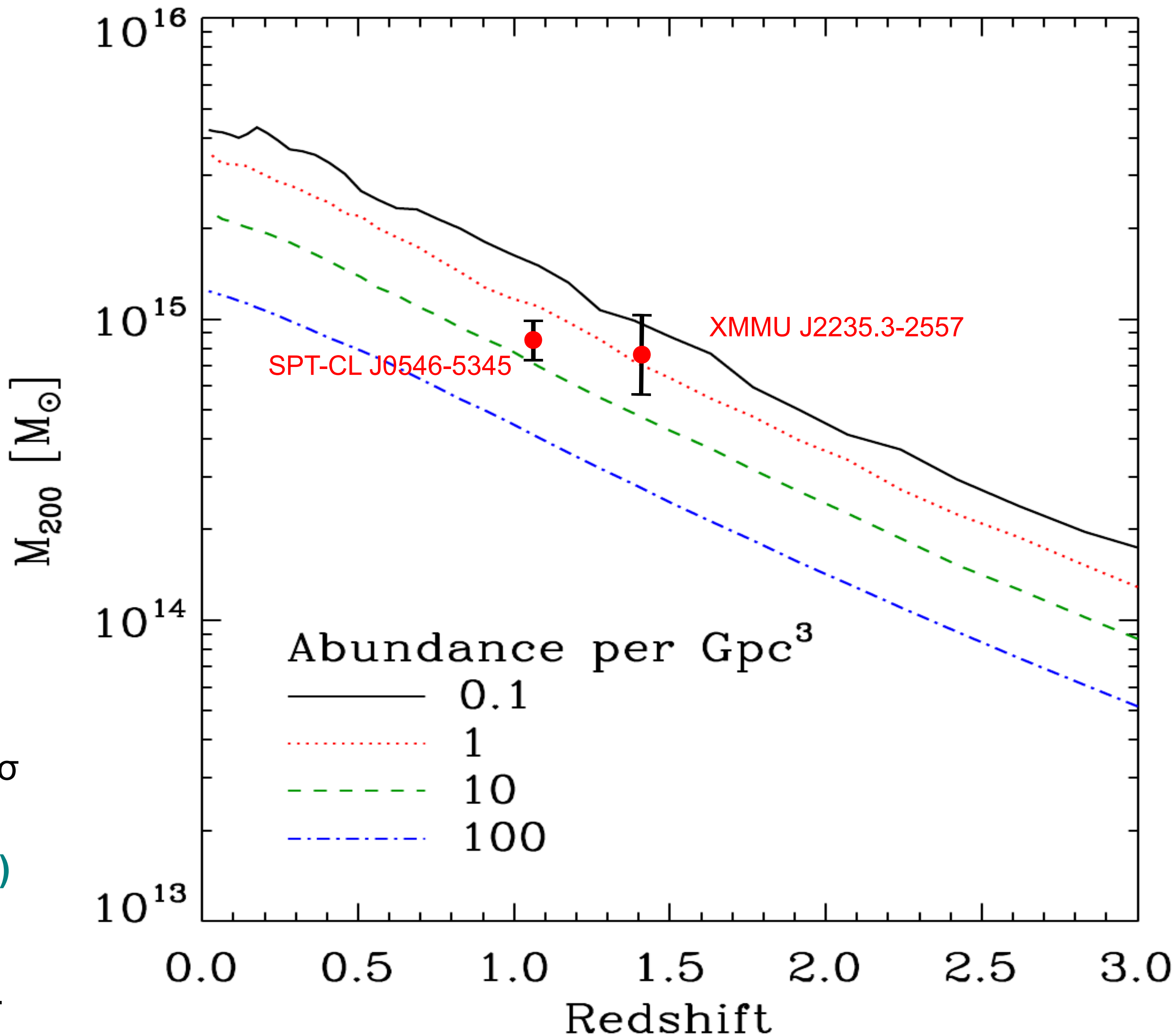
- 6720^3 ~ 303 billion particles
- 3000 Mpc/h box, Millennium cosmology
- 12288 cores: 3072 MPI-task / 4 threads (70% of Juropa)
- 9216^3 FFT mesh
- 86 trillion force calculations
- Cost: 2.7 million CPU hours (~300 years), corresponding to 9.3 days wallclock time (including FOF+SUBFIND)
- Peak memory usage: 29 TB (105 bytes/particle)
- 700 million halos at z=0 (44% of particles)
- About 25 billion (sub)halos in merger trees
- Largest cluster has $9 \times 10^{15}\ M_\odot$
- Size of a full snapshot: ~10 TB

JUROPA
Jülich
Forschungszentrum

Are the presently known high-mass clusters still consistent with ΛCDM?

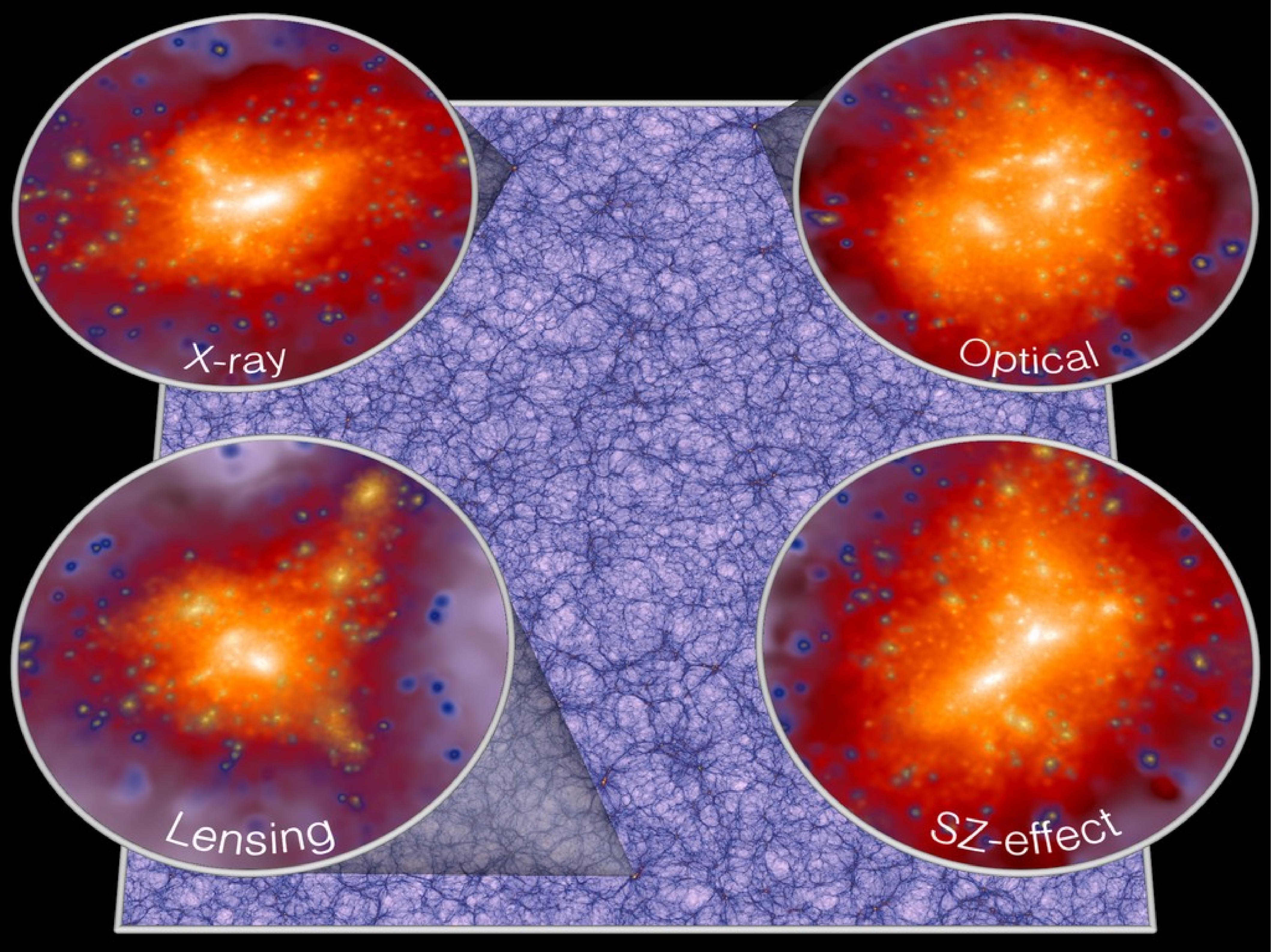
CLUSTER MASS FOR GIVEN ABUNDANCE AS A FUNCTION OF TIME

Detection of one violating cluster would invalidate ΛCDM



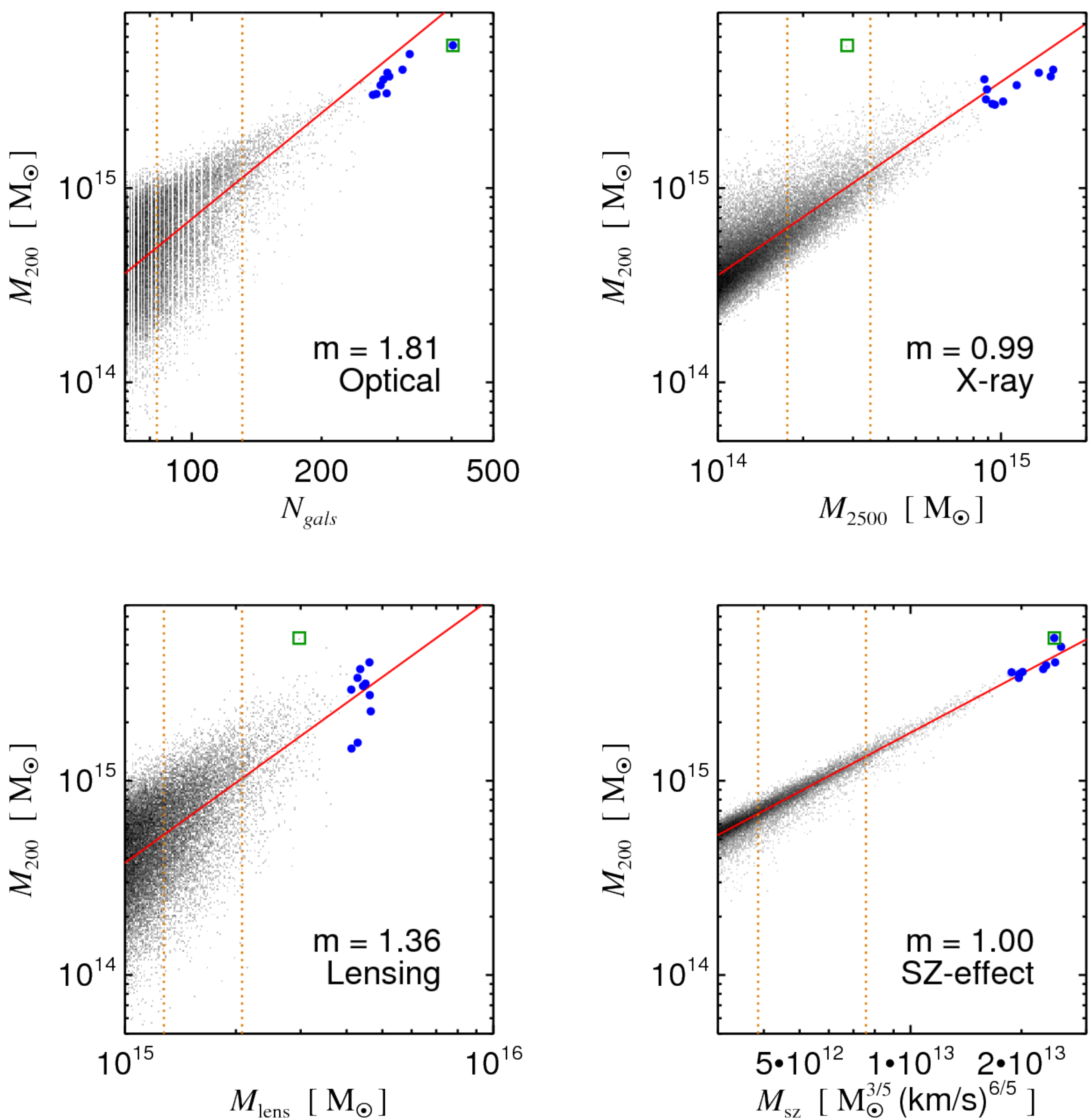
Holz & Perlmutter (2010)
argued XMMU J2235.3-2557 to be inconsistent with ΛCDM at 3σ

Boyle, Jiminez & Verde (2010)
argue that σ_8 would have to be ~4σ higher to accommodate massive clusters. Suggest non-Gaussian ICs as a solution.


X-ray
Optical
Lensing
SZ-effect

Diversity in the Extreme

THE MOST MASSIVE AND RAREST CLUSTERS FOLLOW THE SCALING RELATIONS EXPECTED FROM MORE ABUNDANT SMALLER SYSTEMS



So far reported massive clusters not in conflict with ΛCDM (yet)

Angulo et al. (2011)

Conclusions

Dark matter dynamics can be studied with high precision in cosmological N-body simulations

- Ultra high-resolution simulations begin to reliably resolve the inner dark matter cusp, both for the main halo and its subhalos.
  The inner profiles become gradually shallower – no sign of an asymptotic power-law slope.

- The phase-space structure of dark matter halos is extremely complex and lumpy. Less than ~18% of the mass of a typical Milky Way halo is expected to be in dark matter substructures. There *is* a smooth component in the halo, and the halo is very smooth at the position of the Sun.

- The smooth halo component shows residual structure in energy space, which affects direct detection experiments: WIMP recoil rates are changed at the 10% level, while axion experiments may even obtain clues about the formation history of the Milky Way.

- For dark matter annihilation radiation, the central cusp is the most promising target for detection if observed some angle away from the Galactic centre.

- If the central cusp is strongly detected, subhalos may also be detectable. The highest S/N subhalos are smaller and closer than the known satellites, and may contain no stars.

- At present, the most massive galaxy clusters known are not yet in serious tension with the LCDM model.