On-line Colouring of interval graphs

Introduction to research

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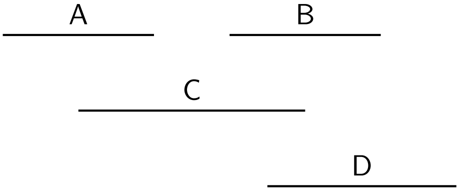
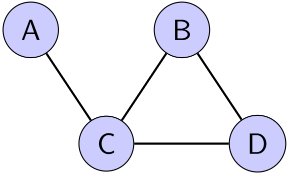
Overview

1. **Interval graph representation**
2. **Problem definition**
3. **Status of research**
4. **Using a computer**
5. **Our work**

Interval Representation and Corresponding Graph

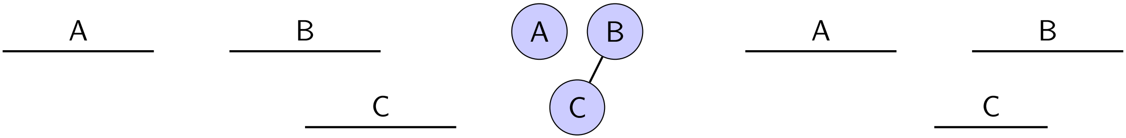


Interval Representation

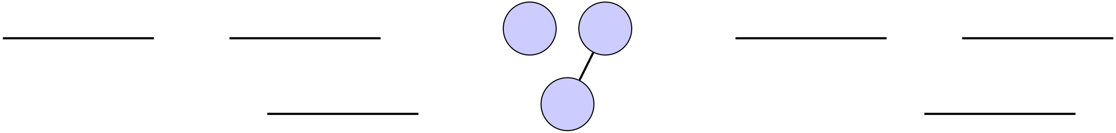


Corresponding Graph

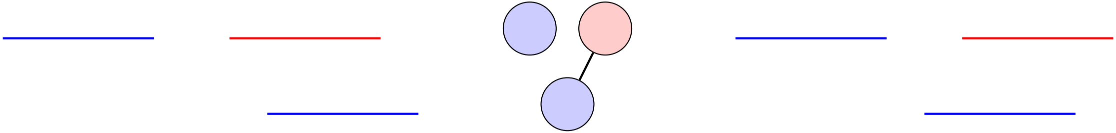
Graph with two Interval Representations

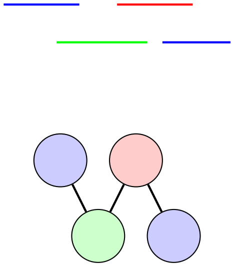


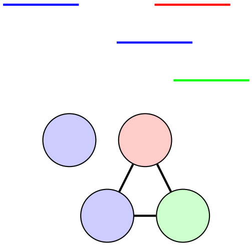
Are these representations equivalent?



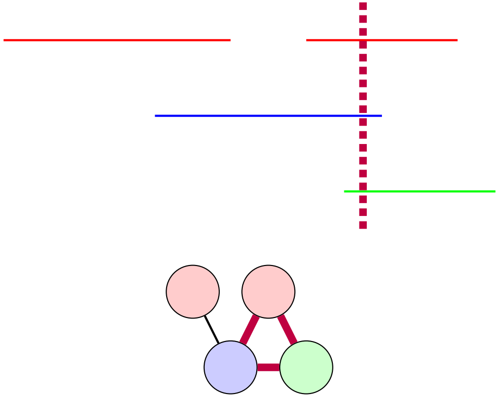
What if we introduce colours?



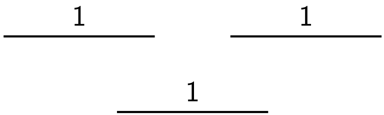




Max clicque size

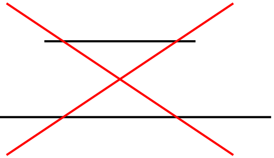


Classes of interval representations



Unit interval representation

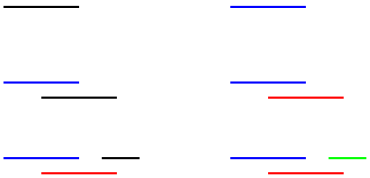
Proper interval representation



Problem definition

A **game** in rounds:

- The *builder* places an interval s.t. the max clicque size is not exceeded
- The colouring *algorithm* chooses a colour for that interval



$R(\omega)$:= what is the maximum number of colours the *builder* can force the *algorithm* to use without exceeding the max clicque size ω?

Where we are at?

$R(\omega) :=$ what is the maximum number of colours the *builder* can force the *algorithm* to use without exceeding the max clicque size ω?

- $R(\omega) \leq 2\omega - 1$ Chrobak and Ślusarek, 1981
- $R(\omega) \geq \lfloor 3\omega/2 \rfloor$ Epstein and Levy, 2005
- $R(1) = 1,\ R(2) = 3$
- $R(3) \in \{4, 5\}$
    - $R(3) = 5$ Biró and Curbelo, 2022
- $R(4) \in \{6, 7\}$
    - $R(4) = 7$ Curbelo and Malko, 2024 [1]
- $R(\omega)$ unsolved for $\omega \geq 5$

Minimax

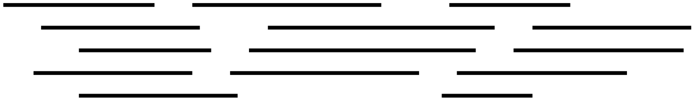
Maximin value

$$v_p = \max_{a_p} \min_{a_o} v_p(a_p, a_o)$$

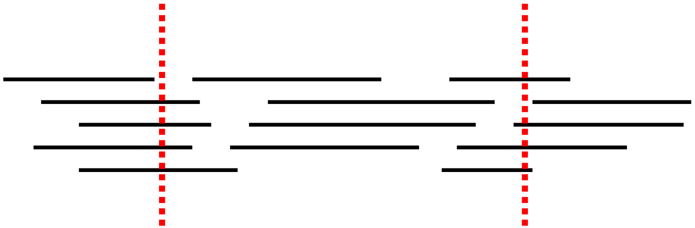
Where:

- p is the player of interest.
- o denotes the opponent.
- a_p is the action taken by player p.
- a_o is the action taken by opponent o.
- v_p is the value function of player p.

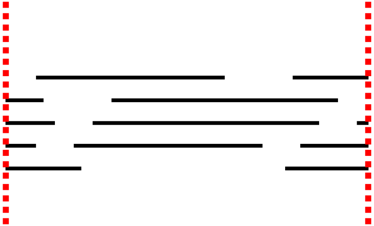
Confining the Space



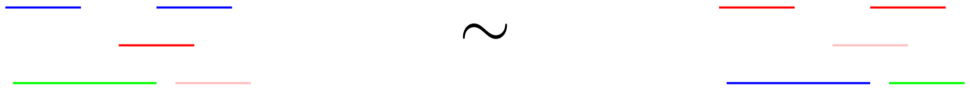
Confining the Space



Confining the Space



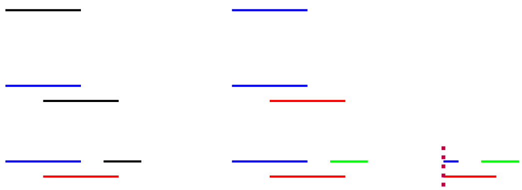
State equivalence



Problem definition

A **game** in rounds:

- The *builder* places an interval s.t. the max clicque size is not exceeded
- The colouring *algorithm* chooses a colour for that interval
- The *builder* **may** restrict the domain



$R(\omega) :=$ what is the maximum number of colours the *builder* can force the *algorithm* to use without exceeding the max clicque size ω?

State

$$E = \{1, 2, ..., \omega, \dashv\}$$
$$S = (e_1, e_2, ..., e_k),\ e_i \in E$$

Our work

$\lfloor 3\omega/2 \rfloor \leq R(\omega) \leq 2\omega - 1$

- $R(1) = 1$
- $R(2) = 3$
- $\mathbf{R(3) = 5}$
- $R(4) = 7$
- $R(5) = \ ?$
- $R(6) = \ ?$
- **...**

Bibliography

Israel R. Curbelo, Hannah R. Malko, *On the on-line coloring of unit interval graphs with proper interval representation*, 2024