# BLG501E – Discrete Mathematics

2021 – 2022 Fall Term

Asst. Prof. Gökhan SEÇİNTİ

# Scope of this Course

- Introduction
  - Inductively Defined Sets
  - Peano axioms
  - Order-relations
  - Principles of mathematical induction
  - Recursive functions
- Number Theory
  - Modular arithmetic
  - Euclid algorithm
  - Greatest common divisor / least common multiple
  - Solving Congruences, Fermat and Chinese remainder theorem
  - Euler theorem

# Scope of this Course

- Counting
  - Principle of inclusion-exclusion
  - Permutations and combinations
  - Surjective functions
- Algebraic Structures
  - Groups, Rings …
  - Homomorphism and isomorphism
  - Pólya's inventory theorem
  - Group coding
  - Applications

The context of this course is compiled by Prof. Emre HARMANCI.

# Grading and Term Plan

| | | |
|---|---|---|
| 8.10.2021 | 1st Week | |
| 15.10.2021 | 2nd Week | |
| 22.10.2021 | 3rd Week | |
| 29.10.2021 | 4th Week | Republic Day |
| 5.11.2021 | 5th Week | |
| 12.11.2021 | 6th Week | Midterm 1 |
| 19.11.2021 | 7th Week | |
| 26.11.2021 | | Term Break |
| 3.12.2021 | 8th Week | |
| 10.12.2021 | 9th Week | |
| 17.12.2021 | 10th Week | |
| 24.12.2021 | 11th Week | Midterm 2 |
| 31.12.2021 | 12th Week | |
| 7.01.2022 | 13th Week | |
| 14.01.2022 | 14th Week | |

**Grading:**

- 2 x Midterms : 2 x 30%
- Final Exam: 40%

All exams will be held **face-to-face**.

**Textbooks:**

- Discrete Math. and its applications, K.H. Rosen
- Discrete and Combinational Math., R.P. Grimaldi
- Elements of Discrete Math., C.C. Liu

# Introduction

## Inductively Defined Sets

**Definitions**:
$U$: Universal Set
$E$: Set to be defined $E \subseteq U$

Induction Layout:

$$B_0 = B$$
$$B_1 = B_0 \cup \Omega(B_0)$$
$$B_2 = B_1 \cup \Omega(B_1)$$
$$\dots$$
$$B_{i+1} = B_i \cup \Omega(B_i)$$
---
$$E = \bigcup_{i>0} B_i$$

Three steps are required:
1) Basis: $B$ base set, $x \in B \implies x \in E$
Elements are explicitly defined.

2) Set of rules: $\Omega = \{f_{n_1}^1, f_{n_2}^2 \; \dots f_{n_m}^m\}$
Rules may be n-ary functions.
$\forall x_i \in E \wedge x = f_n^j(x_1, x_2 \dots x_n) \implies x \in E$

3) Closure:
Set E is the smallest subset of U, which contains B and is stable in terms of rules Ω.

$$\Omega(E) \subseteq E \iff \forall f \in \Omega, f(E) \subseteq E$$

# Introduction

## Inductively Defined Sets

**Definitions**:

$U$: Universal Set

$E$: Set to be defined $E \subseteq U$

Induction Layout:

$$B_0 = B$$
$$B_1 = B_0 \cup \Omega(B_0)$$
$$B_2 = B_1 \cup \Omega(B_1)$$
$$\ldots$$
$$B_{i+1} = B_i \cup \Omega(B_i)$$
---
$$E = \bigcup_{i>0} B_i$$

$B_{\mathrm{i}} = B_{i+1}$ then
the set is stable

$Definition\ of\ height$:
$h(x) = \min(i|x \in B_i)$

# Inductively Defined Sets

**Example 1: Even numbers**

i) $0 \in E$

ii) $n \in E \implies (n+2) \in E$

iii) There is no other elements in set E.

**Example 2: Well-formed formula set**

$A$: Atomic propositions

$S$: $\{\wedge, \vee, \neg, \rightarrow, \leftrightarrow, ), (\ \}$ operators set

$U = (A \cup S)^*$ all possible expressions

i) $B = A$

ii) $\Omega = \{f_1(x) = \neg x,\ \ f_2(x,y) = (x \wedge y),\ \ f_3(x,y) = (x \vee y),$
$f_4(x,y) = (x \rightarrow y), f_5(x,y) = (x \leftrightarrow y)\}$

iii) There is no other elements in set E.

# Peano Axioms

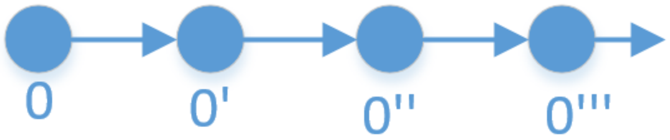
## Defining natural numbers set N

$i)\quad 0 \in N$

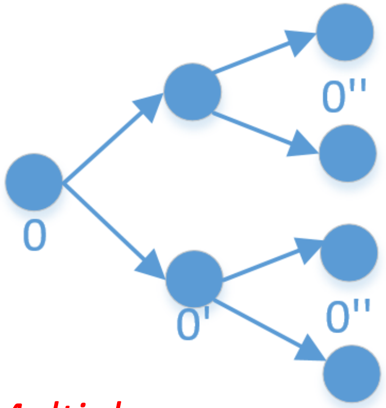
$ii)\ \forall n \in B\ \exists succ(n) \in N$

$iii)\ \forall P \subseteq N\ [0 \in P \land \forall n \in P\ (succ(n) \in P) \Rightarrow P = N]$

**Construct to be formed:**

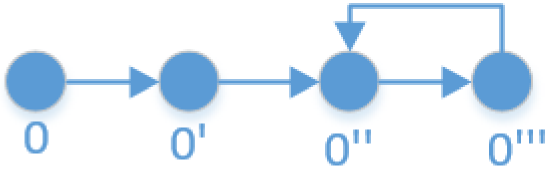


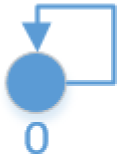

**Constructs that can be formed by the rules above:**




*Multiple succs.*

*Overlapping succs.*




*0 can never be succs.*

# Peano Axioms

**Peano Axioms:**

$i)\ \ 0 \in N$ **and** $\neg(\exists \boldsymbol{n}, \boldsymbol{succ}(\boldsymbol{n}) = \mathbf{0})$

$ii)\ \forall n, m \in B\ \exists!\, succ(n) \in N, \boldsymbol{succ}(\mathbf{n}) \wedge \boldsymbol{succ}(\boldsymbol{m}) \Rightarrow \boldsymbol{m} = \boldsymbol{n}$

$iii) \forall P \subseteq N\ [0 \in P \wedge \forall n \in P\ (succ(n) \in P) \Rightarrow P = N]$

# Additional Reads

Prerequisites:

- Chap. 1 and 2, Discrete Math. and its applications, K.H. Rosen
- https://ninova.itu.edu.tr/tr/dersler/bilgisayar-bilisim-fakultesi/142/blg-112/ekkaynaklar/

Reads:

- Appendix 1, Discrete Math. and its applications, K.H. Rosen
- https://mathworld.wolfram.com/PeanosAxioms.html