

Discrete Optimization Assignment 2

Knapsack

1 Problem Statement

In this assignment, you will design an algorithm to solve the well-known Single Knapsack Problem. You are provided with a knapsack with limited space and a collection of items with different values and weights. Your task is to maximize the value of items packed into your knapsack without exceeding its total capacity.

2 Assignment

Write an algorithm to solve the knapsack problem. The problem is mathematically formulated in the following way. Given n items to choose from, each item $i \in \{0, \dots, n-1\}$ has a value v_i and a weight w_i . The knapsack has a limited capacity K . Let x_i be a variable that is 1 if you choose to take item i and 0 if you leave item i out. Then, the knapsack problem is formalized as the following optimization problem,

Maximize:

$$\sum_{i \in \{0, \dots, n-1\}} v_i x_i$$

Subject to:

$$\begin{aligned} \sum_{i \in \{0, \dots, n-1\}} w_i x_i &\leq K \\ x_i &\in \{0, 1\} \quad (i \in \{0 \dots n-1\}) \end{aligned}$$

3 Input and Output Data Format

A knapsack input contains $n + 1$ lines. The first line contains two integers, the first is the number of items in the problem, n . The second number is the capacity of the knapsack, K . The remaining lines represent the data for each of the items. Each line, $i \in \{0, \dots, n-1\}$ contains two integers, the item's value v_i followed by its weight w_i .

Input format:

n	K
v_0	w_0
v_1	w_1
\dots	
v_{n-1}	w_{n-1}

The output contains a knapsack solution and is made up of two lines. The first line contains the objective value obj . This is the total value of the items selected to go into

the knapsack. The next line is a list of n 0/1 values, one for each of the x_i variables. This line encodes the solution.

Output Format:

obj
$x_0 \ x_1 \ x_2 \ \cdots \ x_{n-1}$

The value order in the solution output MUST match the value order of the input. Otherwise, the grader will misinterpret the output.

Examples

Input:

4	11
8	4
10	5
15	8
4	3

Output:

19
0 0 1 1

In this example, only the third and the fourth items are included in the knapsack, yielding a total value of $15+4=19$.

4 Instructions

For now, please start to work on your computer locally. For uploading to the test system, see the file “instructions.pdf”.