Discrete Optimization Assignment 1

Graph Coloring

1 Problem Statement

In this assignment, you will write a program that properly colors a graph using the fewest number of colors. Properly coloring a graph means coloring each node of the graph with a color such that all pairs of nodes joined by an edge do not have the same color. The colors will be represented by numbers starting at zero.

2 Assignment

Write an algorithm to minimize the chromatic number of a graph. The problem is mathematically formulated in the following way. Given a graph $G = (V, E)$, where V represents the set of nodes and E represents the set of edges, let c_i be a variable denoting the color of node i. Then, the graph coloring problem is formalized as the following optimization problem,

Minimize:

$$\max_{i=0,\ldots,|N|-1} c_i$$

subject to:

$$c_i \neq c_j \;\; \forall (i,j) \in E$$

3 Input and Output Data Format

The input consists of $|E|+1$ lines. The first line contains two numbers $|N|$ and $|E|$. It is followed by $|E|$ lines, where each line represents an edge (u_i, v_j), u_i and v_j being nodes.

Input format:

$\|N\|$	$\|E\|$
u_0	v_0
u_1	v_1
$\ldots$	
$u_{\|E\|-1}$	$v_{\|E\|-1}$

The output consists of two lines. The first line contains the objective value *obj* (this is the number of colors used in the coloring). The next line is a list of $|N|$ values, one for each of the c_i variables. This line encodes the solution.

Output Format:

obj
$c_0 \;\; c_1 \;\; c_2 \;\; \ldots \;\; c_{\|N\|-1}$

Examples

Input:

```
4  3
0  1
1  2
1  3
```

This means that there are 4 nodes with 3 links. Node 0 is connected to Node 1. Node 1 is connected to Node 2. Node 1 is connected to Node 3.

Output:

```
3
0 1 2 2
```

This means that 3 colors were needed. Node 0 was assigned color 0, node 1 was assigned color 1, node 2 and node 3 were assigned color 2.

4 Instructions

For now, please start to work on your computer locally. For uploading to the test system, see the file "instructions.pdf".