

Introduction to Optimization

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Outline

- Optimization
- Decision-Making
- Basic Definitions
- Classification of Optimization Problems
- Heuristic Techniques

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Optimization

- Encountered in many areas, including everyday life
- Important step in decision making
- Aids in determining outcomes of decision-making and design processes
- Tries to arrive at the best possible decision under a given set of circumstances
- What is the *best*?
 - May be unattainable
 - May be different for different people
 - Maybe no clear definition
- So, determine some quantity to be maximized or minimized subject to some constraints.

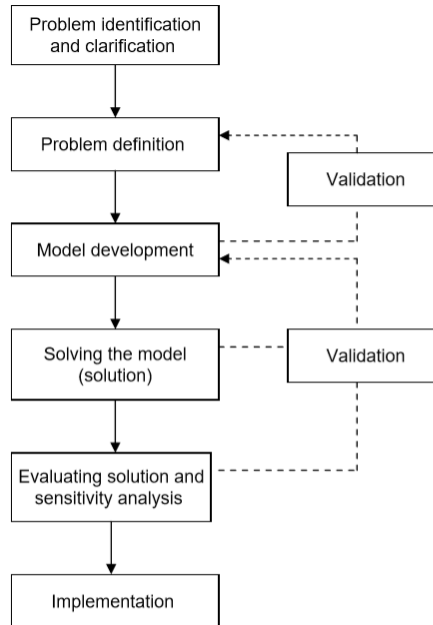
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Decision-Making

- Decision-making involves all activities necessary to identify the most optimal or preferred choice among the available alternatives.
- Basic requirements of decision-making:
 - set of goals or objectives
 - methods for evaluating alternatives
 - evaluation of consequences of each course of action
- Decision-making phases:
 - formulation of goals and objectives, enumeration of constraints, identification and evaluation of alternatives
 - selection of optimal course of action for a given set of constraints

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Steps in a Decision Making Process



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Basic Definitions -1

- A *model* is an abstraction or a mathematical representation of a problem.
- Problems that seek to minimize or maximize a mathematical function of a number of variables, subject to constraints are called *optimization problems*.
- Optimization problems: constrained x unconstrained
 - Constraints: Conditions that solutions must satisfy

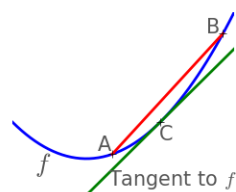
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Basic Definitions - 2

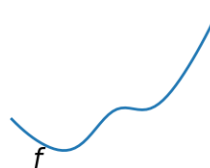
- Objective function: single objective x multiobjective
- Variables: real, integer (integer or binary), or mixed
- Constraints
 - hard x soft
 - equality x inequality
- Feasible and infeasible solutions: feasible solutions satisfy the constraints

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Convex Functions



A convex function



A nonconvex function

- In a convex function:
 - f is above all of its tangents
 - for two points A and B , $f(C)$ lies below the segment $[f(A), f(B)]$ if $A < C < B$
- Optimizing convex functions is easy

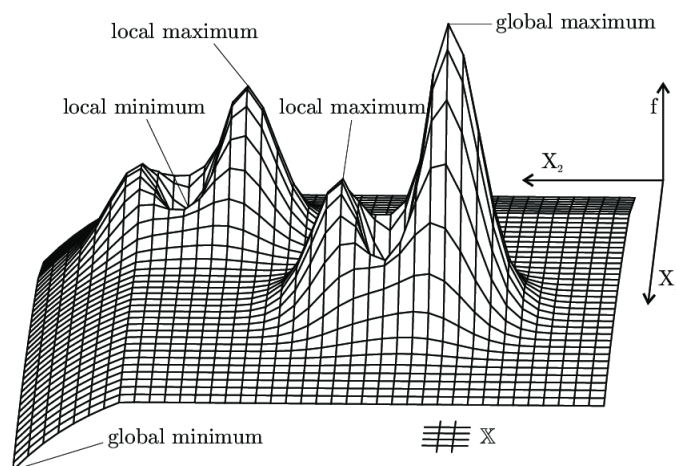
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Basic Definitions - 3

- Local and global optima:
 - local optimum: point in the search space which is better than all of its neighboring points
 - global optimum: point in the search space which is better than all of the points in the search space

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Global x Local Optima



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Basic Definitions - 4

- Problem x problem instance

e.g.

Problem: Given any set of points X, find the shortest roundtrip.

Solution: Algorithm that finds shortest roundtrips for any X.

Problem instance: Given a specific set of points P, find a shortest round trip

Solution: Shortest roundtrip for P

➡ Problems can be formalized as sets of problem instances.

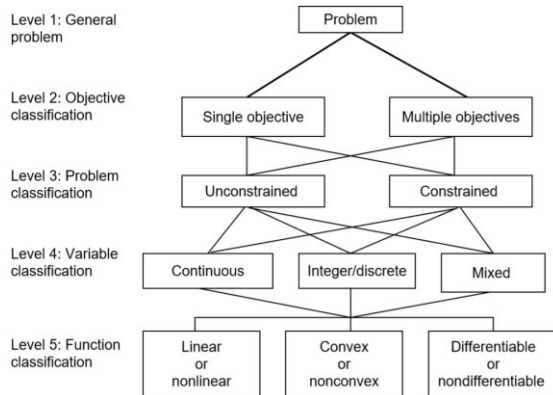
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Classification of Optimization Problems

- Classification based on
 - Type of constraints
 - Nature of the equations involved
 - Deterministic nature of the variables
 - Permissible value of the design variables
 - Separability of the functions
 - Number of objective functions

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Classification



Note:

- *Discrete optimization* deals with optimization problems with integer or discrete variables.
- Such problems are called *combinatorial problems*.
- In combinatorial problems, the solution is an object from a finite or infinite set—typically an integer, set, permutation, or graph.
- Combinatorial problems involve finding a grouping, ordering, assignment of a discrete, finite set of objects that satisfy given conditions.

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Solution Approaches

- Classical techniques
- Modern heuristics

OR

- Deterministic techniques
- Stochastic techniques

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Classical Optimization Techniques

- Linear programming
- Integer programming
- Goal programming
- Nonlinear programming
- Multiobjective models

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Heuristic Techniques

Heuristic: a rule-of-thumb approach that does not guarantee convergence and optimality, but usually works well and produces solutions of acceptable quality.

Heuristics are usually problem class dependent.

- Hill-climbing techniques
- Simulated annealing
- Tabu search
- Evolutionary algorithms
- Swarm intelligence
- ...

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