Bayesian Network

- DAG
- Each node corresponds to a variable
- Acts as a filter which requires distributions to have certain conditional independence properties if they want to factorize w.r.t the graph
- Joint probability factorizes $p(\mathbf{x}) = \prod_{k=1}^{K} p(x_k|\,\mathrm{pa}_k)$
- Observed vs Latent variables
  - Latent variables allow us to build complex distributions upon simpler ones
  - Latent variables are often parameters
  - Observed variables are often data
  - Latent variables do not need to have physical interpretation
- Generative model
  - We can sample from it
  - It expresses the process by which the observed data arose
  - It encodes the causal process
- Two extremes
  - Fully-connected: no independent variables
  - No edge: joint = multiplication of marginals
  - Something in between: dropping edges to control model complexity
- Control the growth of parameter with the number of nodes
  - Alter the topology
  - Share parameters
  - Use parameterized model (e.g. Logistic vs N binary-valued)

Conditional Independence

Three types of connection

- Tail-to-tail
  - `a <--- c ---> b`
  - conditional independent if C is **observed**
- Head-to-tail
  - `a ---> c ---> b`
  - conditional independent if C is **observed**
- Head-to-head
  - `a ---> c <--- b`
  - conditional independent if C and all C's descendants are **not observed**

D-separation

Just the above types extended to set of nodes.

A path from A to B is blocked if either

- The path includes a Tail-to-tail or Head-to-tail node, and the node is conditioned on, or
- The path includes a Head-to-head node, but neither the node nor its descendants are conditioned on

If **all** paths from A to B are blocked, A and B are conditional independent given the conditioning set.

Parameters are treated as observed nodes, and have no marginal distribution, thus

- They must be tail-to-tail nodes on the paths through them
- They have no parent nodes
- They block paths through them

Markov Blanket

Markov blanket of a node contains its

- Parents
- Children
- Co-parents

And nodes in the blanket are the only ones it depends on. All other nodes are conditional independent with it.