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The Equilibrium Temperature of Planetary Bodies in Hyperbolic Trajectories

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ABSTRACT
Some comets, asteroids, and interstellar objects such as 1I/'Oumuamua move in hyperbolic trajectories and experience large variations in temperature while passing a stellar system. While it is possible to estimate their dynamic temperatures , it is not straightforward to calculate their average thermal state since they are not periodic objects. Here we derive analytic solutions for the average equilibrium temperature of planetary bodies in hyperbolic trajectories. This average temperature provides a reference standard to compare the net thermal effect of a star on transient objects. We found that 1I/'Oumuamua experienced a mild change in temperature compared to comets in similar trajectories and this probably contributed to the inactivity of any ices in its surface.

Key words: physical data and processes – comets: general – minor planets, asteroids

1 INTRODUCTION

The planetary body 1I/'Oumuamua is the first known extrasolar visitor of the Solar System (Meech et al. 2017a). It was shown to describe a hyperbolic trajectory consistent with an extrasolar origin (de la Fuente Marcos & de la Fuente Marcos 2017; Mamajek 2017) and its light curve suggest an elongated shape (up to 10:1 ratio) with an effective spherical radius of 102 ± 4 m and little to no cometary activity (Meech et al. 2017b). The source and formation mechanism of 1I/'Oumuamua is still unknown but suggestions include an ejection from a binary system, among many other alternatives (*e.g.*, Gaidos et al. 2017; Schneider 2017; Zhang 2018; Ćuk 2018; Portegies Zwart et al. 2017; Dybczyński & Królikowska 2017; Ferrin & Zuluaga 2017; Zuluaga et al. 2017; Feng & Jones 2017; Do et al. 2018).

The average equilibrium temperature of a planetary body helps to define its general thermal state between the extremes at periastron and apastron. This temperature is easy to define for bodies in elliptical orbits since they are periodic (Méndez & Rivera-Valentín 2017). However, bodies in hyperbolic trajectories could experience dramatic thermal variations, from the long interstellar environment to transient but strong stellar fluxes at periastron. This is true for 1I/'Oumuamua and many known comets and asteroids with near hyperbolic trajectories in the Solar System (NASA/JPL SSD 2018). Other planetary bodies in highly eccentric elliptical orbits could experience similar effects (*e.g.* Comet 1P/Halley).

It will be good here to expand on how the equilibrium temperature is related to the surface temperature of comets and asteroids. How bond albedo is calculated from geometric albedo, is there a general relation for comets and asteroids? What are the critical temperatures for ices sublimation? How the thermal inertia plays with the sublimation during the short exposition time near periastron? Also add interesting examples with well known asteroids or comets.

Discuss relevance of visiting rogue exoplanets, if any. (e.g. expected comet-like tails, atmospheric erosion.)

In this study we derive analytic solutions for the temporal average of orbital distance, stellar flux, and equilibrium temperature of planetary bodies in hyperbolic trajectories. These averages were derived as a function of the time within the Solar System influence (*e.g.* the heliopause ~ 122 AU, Cairns & Fuselier (2017)). We also explored other boundary conditions to calculate these averages. Our goal is to derive solutions that could be used to compare the thermal effect of a star on planetary bodies in elliptical and hyperbolic orbits. Section 2 describes averages conditions of elliptical orbits for comparison purposes. Section 3 presents our new derivations for hyperbolic trajectories. Section 4 discuss the validation of our results with numerical simulations and applications to Solar System and extrasolar planetary bodies, including I1/'Oumuamua.

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2 TEMPORAL AVERAGES FOR ELLIPTICAL ORBITS

Temporal averages for orbital distance $\langle r \rangle$, stellar flux $\langle F \rangle$, and equilibrium temperature $\langle T_{eq} \rangle$ for elliptic orbits were derived by Méndez & Rivera-Valentín (2017). They were solved by integrating in time t over the orbital period T and making the substitutions $r = a(1 - e\cos E)$, $M = E - e\sin E$, and $M = (2\pi/T)t$; where r is position, e is the orbital eccentricity, a is the semi-major axis in astronomical units, E is the eccentric anomaly, and M is the mean anomaly. These averages are

$$\langle r \rangle = a\left(1 + \frac{e^2}{2}\right), \tag{1}$$

$$\langle F \rangle = \frac{L}{a^2\sqrt{1-e^2}}, \tag{2}$$

$$\langle T_{eq} \rangle = T_o \left[\frac{(1-A)L}{\beta\epsilon a^2}\right]^{\frac{1}{4}} \frac{2\sqrt{1+e}}{\pi}\,\mathbf{E}\left(\frac{2e}{1+e}\right) \tag{3}$$

$$\approx T_o \left[\frac{(1-A)L}{\beta\epsilon a^2}\right]^{\frac{1}{4}} \left[1 - \tfrac{1}{16}e^2 - \tfrac{15}{1024}e^4 + O(e^6)\right], \tag{4}$$

where L is the stellar luminosity in solar units, A the bond albedo, ϵ the emissivity, β the redistribution factor, T_o = 278.5 K, and $\mathbf{E}$ is the complete elliptic integral of the second kind (Weisstein 2016; GSL 2016). The definition of $\mathbf{E}$ used in equation 3 is the one implemented in software packages like *Mathematica* (*i.e.*, `EllipticE`) and *Maxima* (*i.e.*, `elliptic_ce`). The *GNU Scientific Library* (GSL) uses `gsl_sf_ellint_Ecomp` which requires first the square root of the argument of E.

For elliptical orbits the average distance ($a \le \langle r \rangle < \frac{3}{2}a$) and stellar flux ($F|_{e=0} \le \langle F \rangle < \infty$) increase with eccentricity. However, the average equilibrium temperature ($T_{eq}|_{e=0} \le \langle T_{eq} \rangle < \frac{2\sqrt{2}}{\pi} T_{eq}|_{e=0}$) slowly decreases with eccentricity until converging to ~ 90% of the equilibrium temperature for circular orbits. Table 1 shows these properties calculated for some minor planetary bodies with highly eccentric orbits.

Probably derive new elliptic solutions for a time frame smaller than a period, if possible, for comparison purposes with hyperbolic solutions.

3 TEMPORAL AVERAGES FOR HYPERBOLIC TRAJECTORIES

We derived analytic solutions for hyperbolic trajectories with respect to time as similarly described in Méndez & Rivera-Valentín (2017). The period of integration T was taken from entering to leaving the stellar system where the time to periastron is the semi-period $\frac{1}{2}T$. The integrals were solved by making the substitutions $r = a(1 - e\cosh H)$, $M = e\sinh H - H$, and $M = (2M_o/T)t - M_o$; where H is the hyperbolic eccentricity and the objects move between time zero to T, corresponding to $-M_o$ to $+M_o$ in the mean anomaly and $-H_o$ to $+H_o$ in the hyperbolic anomaly. The distance, stellar flux, and equilibrium temperature averages are given with respect to the hyperbolic anomaly as

$$\langle r \rangle = -a\left[\frac{e\,(e\cosh H_o - 4)\sinh H_o + \left(e^2+2\right)H_o}{2\,(e\sinh H_o - H_o)}\right] \tag{5}$$

$$\langle F \rangle = \frac{L}{a^2\sqrt{e^2-1}}\left[\frac{2}{e\sinh H_o - H_o}\tan^{-1}\left(\frac{(e+1)\tanh\frac{1}{2}H_o}{\sqrt{e^2-1}}\right)\right] \tag{6}$$

$$\langle T_{eq} \rangle = T_o\left[\frac{L(1-A)}{\beta\epsilon a^2}\right]^{\frac{1}{4}}\left[\frac{-2i\sqrt{e-1}}{e\sinh H_o - H_o}\,\mathbf{E}\left(i\frac{H_o}{2}\,\middle|\,\frac{2e}{e-1}\right)\right] \tag{7}$$

where $\mathbf{E}$ is the incomplete elliptic integral of the second kind (Weisstein 2016; GSL 2016). As in equation 3, the definition of $\mathbf{E}$ used in equation 7 is the one implemented in *Mathematica* or *Maxima* and not in GSL. Note that equation 7 also involves operations with imaginary numbers.

Add series solution to the thermal equation.

4 DISCUSSION

We validated our analytic solutions for hyperbolic trajectories with numerical solutions. Interpretation of solutions and plots of behavior of averages as a function of e. Does eccentricity increases the temperature?

We compared the average equilibrium temperature for planetary bodies in elliptical and hyperbolic trajectories using a time frame of one year. Table for hyperbolic examples.

Implications to 1I/'Oumuamua and other objects of interest.

For those interested in aliens and interstellar travel, it is possible to calculate the stellar energy gain to recharge batteries from interstellar flybys using equation 6.

5 CONCLUSION

We derived analytic solutions for the average distance, stellar flux, and equilibrium temperature of planetary bodies in hyperbolic trajectories. We validated our derivations with numerical solutions.

We find that the easiest way to compare the thermal state between planetary bodies in elliptic and hyperbolic trajectories is under equal time frames that are close to periastron.

Our analysis on 1I/'Oumuamua shows that its average temperature of XXX K was not high enough to sublimate any ices in its surface. This low temperature and an organic crust as suggested by Fitzsimmons et al. (2017) are probably responsible to its lack of cometary activity.

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Table 1. Temporal averages of orbital distance $\langle r \rangle$, stellar flux $\langle F \rangle$, and equilibrium temperature $\langle T_{eq} \rangle$ for some minor planetary bodies with highly eccentric orbits.

	Object Description[a]			Temporal Averages[b]		
Name	a (AU)	e	A	$\langle r \rangle$ (AU)	$\langle F \rangle$	$\langle T_{eq} \rangle$ (K)
1P/Halley	17.834145	0.96714291	0.04	26.174	0.0123	69.7
2P/Encke	2.2151323	0.84832024	0.046	3.0122	0.384	174
67P/Churyumov-Gerasimenko	3.4647374	0.64058232	0.06	4.1756	0.1084	143
90377 Sedna	487.7651	0.8440912	0.32	661.53	7.84×10^{-6}	11.7
1566 Icarus	1.0779459	0.8268093	0.14	1.4464	1.53	251
101955 Bennu	1.126391	0.20374511	0.046	1.1498	0.805	259

[a] Object elements from the NASA Solar System Dynamic website https://ssd.jpl.nasa.gov/?sb_elem and albedos from REF (bond albedos, not geometric albedos).
[b] Average values calculated with equations 1, 2, and 3, respectively.

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APPENDIX A: ORBITAL ELEMENTS OF ELLIPTICAL ORBITS

This appendix includes a summary of the equations used with a figure for visualization purposes.

APPENDIX B: ORBITAL ELEMENTS OF HYPERBOLIC TRAJECTORIES

This appendix includes a summary of the equations used with a figure for visualization purposes.

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