

Yusuf Bhavn

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
 (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
 (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck. *not an `Arbitrary` or `Testable`*
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

`foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m`

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

$(\text{Int} \rightarrow [\text{Int}]) \rightarrow [\text{Int}] \rightarrow [\text{Int}]$

$\text{show} :: a \rightarrow \text{String}$

$\text{foldMap} (\text{a} \rightarrow \text{String})$

$\rightarrow [\text{Char}] \subset \text{String}$

$\rightarrow \text{String}$

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\boxed{\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}}$

(d) $\lambda x l \rightarrow x : (l ++ l ++ [x])$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >= putStrLn`

$\boxed{\text{IO ()}}$

(f) `putStrLn "42" >= putStrLn "43"`

$\boxed{\text{ill-typed}}$

(g) `(,) "42"`

$a \rightarrow (\text{String}, a)$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x \neq y]$

$\boxed{\text{Eq } a \Rightarrow [a] \rightarrow [(a, a)]}$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

- (a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$$\lambda x \rightarrow x + 1$$

$(+1)$

- (b) $\text{Bool} \rightarrow [\text{Bool}]$

pure

- (c) $a \rightarrow \text{Maybe } b$

const Nothing

- (d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$$\lambda f \quad a \rightarrow \text{map} (\text{uncurry } f) \$ \text{zip } a b$$

- (e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

- (f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$$\lambda f \quad a \rightarrow \text{map } f \ a$$

- (g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$$\lambda a \quad f \rightarrow f (\text{fromSift } a) \gg= (\text{pure } \cdot (a,))$$

- (h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$$\lambda a \rightarrow \text{filter } (\text{eq } a)$$

- (i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

pure . foldMap show

- (j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

flip uncurry

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int, Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

Gives every pair of distinct integers in the list
 (in both orders)

$$\begin{aligned} \text{foo } l &= \{ \} \\ \text{foo } [1, 2, 3] &= [(1, 2), (1, 3), \\ &\quad (2, 1), (2, 3), \\ &\quad (3, 1), (3, 2)] \end{aligned}$$

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

It appends at the
 end of a list

$$\text{foo } 3 \ [] = [3]$$

$$\text{foo } 4 \ [1, 2, 3] = [1, 2, 3, 4]$$

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

$$\text{foo } [] = \{ \}$$

$$\begin{aligned} \text{foo } [\text{Nothing}] &= \{ \} \\ \text{foo } [\text{Just } 2, \text{Nothing}, \text{Just } 4] &= [2, 4] \end{aligned}$$

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer:

Gives the maximum of the 1st arg
 and a list of the same length
 as the first arg full of
 the second arg

$$\begin{aligned} &+ \text{non-empty 1st arg} \\ \text{foo } [1, 2, 3, 4] &= (4, [4, 4, 4, 4]) \end{aligned}$$

$$\begin{aligned} \text{foo } [4, 5, 6] &= (6, [4, 5, 6]) \end{aligned}$$

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Gives the tail of a list

Ans 53 = [1, 2, 3]

Ans [1, 2, 3]

= [1, 2, 3],

[2, 3],

[3],

□ []

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:
`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

$\text{weave} :: [\alpha] \rightarrow [\alpha] \rightarrow [\alpha]$
 $\text{weave } a b = \text{concat} . \text{ map } (\lambda (a, b) \rightarrow [a, b]) . \text{zip } a \ b$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:
`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$\text{toMax } l = \text{map } (\text{const } m) \ l$
where $m = \text{maximum } l$

Bonus

$\text{toMax } (x : xs) = \text{snd} \ $ \text{go } x \ xs$
where

$\text{go } e (x : xs) = (m, m : 1)$

$\text{where } (m, 1) = \text{go } (m, e) \ x$
where $m = \max e x$

$\text{go } e [] = (e, (e))$

Typeclass Definitions

```
class Semigroup a where
  (++) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (=>), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<>*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (">>>") :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

CMSC 488B: Midterm Exam (Spring 2022)

Matvey Stepanov
UID: 116684101

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
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 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
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- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
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 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap ([] : [Int]) [1..10] => m`

(i) The expression will fail to typecheck.

(ii) The monoid in this call is `Int`.

(iii) The monoid in this call is `[Int]`.

(iv) The monoid in this call is `String`.

(v) The expression is equivalent to the identity function.

(vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

(i) The expression will fail to typecheck.

(ii) The monoid in this call is `Int`.

(iii) The monoid in this call is `[Int]`.

(iv) The monoid in this call is `String`.

(v) The expression is equivalent to the identity function.

(vi) The foldable in this call is `[]` - the instance for lists.

`show :: a -> String`

`[char]`

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

$\text{IO String} \rightarrow \text{IO ()}$

(f) $\text{putStrLn "42"} >= \text{putStrLn "43"}$

ill-typed \rightarrow *should use* \gg

(g) $(,) "42"$

$a \rightarrow (\text{String}, a)$

(h) $\text{reverse} . \text{foldMap return}$

'Foldable t \Rightarrow $t a \rightarrow [a]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a,a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

ill-typed

(k) $\text{filterM} (\text{const [True, False]})$

$[a] \rightarrow [[a]]$

const [True, False]
 $\hookrightarrow a \rightarrow [\text{Bool}]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda b \rightarrow [b]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f l_1 l_2 \rightarrow f \text{map} (\lambda . \text{uncurry } f) (\text{zip } l_1 l_2)$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$\lambda f a b \rightarrow \text{liftM2 } f a b$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f v l \rightarrow [f a | a \in l, v(f a) == \text{True}]$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda m f \rightarrow$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda v l \rightarrow \text{filter } (== v) l$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\rightarrow \lambda l \rightarrow \text{foldMap show } l$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda p f \rightarrow \text{uncurry } f p$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Constructs all combinations (pairs) of distinct elements from given list.

Ex: foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

`foo [2,2] = []`

Answer: appends x to end of given list l

Ex: foo 4 [1,2,3] = [1,2,3,4]

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Creates new list containing non-Nothing values from original list. (Any Nothing values are removed and Maybe values are extracted).

Ex: foo [Maybe 4, Maybe 3, Nothing, Maybe 2]

= [4, 3, 2]

foo [Nothing, Nothing] = []

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer:

Returns a pair: first element is the max of original list OR the default m passed in if original is empty.

Second element is a list of same size as original containing only the default m repeated # of times.

length of original list

Ex: foo [] 5 = (5, [])

foo [10] 5 = (10, [5])

foo [1,2,3,4] 20 = (4, [20,20,20,20])

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

$$\text{weave } [] [] = [] \quad \text{weave} :: [\alpha] \rightarrow [\alpha] \rightarrow [\alpha]$$

$$\text{weave } (x:xs)(y:ys) = x:y:(\text{weave } xs ys)$$

$$\text{weave } \dots = \text{undefined}$$

↳ should never reach this case if lists are
of equal lengths.

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
maxl :: [Int] → (Int, Int)
maxl [] = undefined -- shouldn't happen
maxl (x:xs) = maxlhelp x xs where
    maxlhelp :: Int → [Int] → (Int, Int)
    maxlhelp m [] = (m, 1)
    maxlhelp m (h:t) =
        let (m', l') = maxlhelp (max m h) t in
            (m', l'+1)
```

Traverses given list only once to find max and length.
Then uses replicate
Hope this counts :)

```
toMax :: [Int] → [Int]
toMax l = let (max, len) = (maxl l) in
    replicate len max
```

Typeclass Definitions

```
class Semigroup a where
  (<*>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (=<), (=>), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Hammaad Khan
116640077

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

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 - (i) True
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- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
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 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Show "123456"
Show :: String -> String
a = String

String = [Char]

show "123456" = "\123456\"

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x \rightarrow x : x ++ x ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

IO ()

(f) `putStrLn 42 >>= putStrLn 43`

ill-typed

(g) `(,) "42"`

$a \rightarrow (\text{String}, a)$

(h) ~~`reverse . foldMap return`~~

$\text{Foldable } t \Rightarrow t a \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

reverse . foldMap

Γ

$(\text{const } [\text{True}, \text{False}])$

$= (\lambda x \rightarrow [\text{True}, \text{False}])$

filterM

$(a \rightarrow M \text{ Bool}) \rightarrow [a] \rightarrow M [a]$

$M = []$

$\text{foldable } t \Rightarrow (a \rightarrow M) \rightarrow t a \rightarrow M$
 $\text{Monoid } M \Rightarrow$

return: $a \rightarrow M$

$M = [a]$ by reverse

$(a \rightarrow [a]) \rightarrow t a \rightarrow [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$.

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda b \rightarrow [b, \& b]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Just } []$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f \ xs \ ys \rightarrow (\text{not } (f \ 10 \ 'a')) : (\text{ZipWith } f \ xs \ ys)$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f1 \ f2 \ (x:xs) = \text{if } (\text{not } (f2 \ (f1 \ x))) \text{ then } [f1 \ x] \text{ else } [f1 \ x]$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a,b)$

undefined

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x \ xs \rightarrow \text{if } x == x \text{ then } (x:xs) \text{ else } xs$

(i) $\text{Show } a \Rightarrow a \rightarrow \text{IO String}$

$\lambda (x:xs) \rightarrow \text{putStrLn } (\text{show } x) \gg \text{getLine}$

(j) $(a,b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (x,y) \ f \rightarrow f \ x \ y$

Map Show xs

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: All pairs of elements from l that are not equal

`foo [] = []`
`foo [1, 2, 3] = [(1, 2), (1, 3), (2, 1), (2, 3), (3, 1), (3, 2)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: Appends x to the end of l

`foo [] [] = []`

`foo 5 [1, 2, 3, 4, 5] = [1, 2, 3, 4, 5]`

`x: [4, 5, 2, 1, 7]`
`(x, 4, 5, 2, 1, 7)`
`[1, 2, 3, 4, 5]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Filters out Nothing from list of Maybe, unwraps Just

`foo [] -> []`

`foo [Nothing, Nothing] -> []`

`foo [Just 1, Nothing, Just 3, Nothing] -> [1, 3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

*Answer: Repeats m n times where n is the length of the list
 Also gives the maximum value of the list, or m if the list is empty*

`foo [] 1 = (1, [])`

`foo [1, 2, 4] 5 = (4, [5, 5, 5])`

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

$\text{foo} : [a] \rightarrow [[a]]$

foo gives all tails of the given list

$\text{foo } [1, 2, 3] \rightarrow [[1, 2, 3], [2, 3], [3]]$

$\text{foo } [] \rightarrow []$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave :: [a] → [a] → [a]`

`weave [] [] = []`

`weave (x:xs) (y:ys) = x:y:(weave xs ys)`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax :: [Int] → [Int]`

`toMax [] = []`

`toMax xs = replicate (length xs) (maximum xs)`

w/ bonus:
`lengthAndMax :: [Int] → (Int, Int)`

`lengthAndMax (x:xs) = (l+1, max x x')` where
`(l, x') = lengthAndMax xs`

`lengthAndMax [x] = (1, x)`

`toMax xs = replicate l m` where `(l, m) = lengthAndMax xs`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<>*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (">>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Yishan Zhao

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
(ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
(ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
(ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$a \rightarrow b \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

ill typed

(e) `getLine >=> putStrLn`

`IO ()`

(f) `putStrLn 42 >=> putStrLn 43`

ill typed

(g) `(,) "42"`

$a \rightarrow (a, \text{String})$

(h) `reverse . foldMap return`

ill typed

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$[a] \rightarrow [(a,a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda b \rightarrow [b \& \text{True}]$

(c) $a \rightarrow \text{Maybe } b$

undefined

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

ifA2

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

ifM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

if c a → if c fa then if as else if a

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

\ma f → do a ← ma

liftM (a,) (f a)

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

\a (x:xs) = if a == x then (x:xs) else (x:xs)

(i) $\text{Show } a \Rightarrow a \rightarrow \text{IO String}$

(putStrLn . show) >> getLine

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (a, b) f \rightarrow f a b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: all possible pairs of elements, given that elements can't pair with themselves

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: append `x` to end of the list

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
 `Nothing -> rs`
 `Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: filter out the 'Nothing's from the list

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
where `(m', xs') = foo xs m`

Answer: produces a tuple where the first is the

largest element in the list, and the second is a history of the maxima during iterations

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

power set of a list

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

$$\begin{aligned} \text{weave } (x : xs) \quad (y : ys) &= \quad x : y : (\text{weave } xs \ ys) \\ \text{weave } [] \quad [] &= \quad [] \end{aligned}$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\begin{aligned} \text{toMax } l &= \text{replicate } (\text{length } l) \quad (\text{findMax } l) \\ \text{where } \quad \text{findMax } [] &= \text{Int. MinValue} \\ \text{findMax } [x] &= x \\ \text{findMax } (x : y : xs) &= \quad \text{if } x \geq y \text{ then} \\ &\quad \text{findMax } x \ xs \\ &\quad \text{else} \\ &\quad \text{findMax } y \ xs \end{aligned}$$

Typeclass Definitions

```
class Semigroup a where
  (++) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (⊛) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (⊛=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Philip Wang

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
 (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
 (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
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 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$a \rightarrow b \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow b \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

$a \gg= \lambda x$

(f) $\text{putStrLn "42"} >= \text{putStrLn "43"}$

ill-typed

(g) $(,) "42"$

ill-typed

(h) $\text{reverse} . \text{foldMap return}$

$[m a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$([a] \rightarrow [(a, a)])$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$('a', \text{True})$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$(\text{Monad m}) \Rightarrow m [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow [x]$

(c) $a \rightarrow \text{Maybe } b$
undefined

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

Zip with

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g h \rightarrow$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

mapM

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$f = \lambda x \rightarrow \lambda y \rightarrow$

$f u (x:y) = \text{if } u == x \text{ then } ys \text{ else } xs$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$f s = \text{let } xs = \text{null } ++ s \text{ in } \text{getLine}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (a, b) f \rightarrow f a b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer: Gets all pairs (x,y) such that $x \neq y$

`foo [] = []`

(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)` `foo [1,2] = [(1,2), (2,1)]`

Answer: Inverts x to end of 1

`foo 1 [2,3] = [2,3,1]`

`foo 2 [1,2] = [2,1,2]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: `bar` returns a list whose elements are those such that `f` applied to them is not `Nothing`.

`foo [Just 1, Just 2] = [1, 2]`

`foo [] just the identity function.`

`foo [Just 1, Nothing] = [1]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer:

`foo` returns a tuple whose

`foo [] 1 = (1, [])`

first element is the max of the

`foo [2] 3 = (2, [3])`

given list and the second

element

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave [] [] = []`
`weave (x:xs) (y:ys) = [x,y] ++ weave xs ys`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

* Not sure if `max` is defined
in prelude but if not *

`toMax lst =`
`let m = max lst in`
`map ($\lambda x \rightarrow m$) lst`

`max (x:xs) = foldr (\x y \rightarrow if y > x then y else x) x`

Bonus! not sure if this counts but could do something like

Typeclass Definitions

```
class Semigroup a where
  (<*>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  ((<)), ((<=)), ((>)), ((>=)) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  ((<*>)) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  ((>>=)) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

(P6)

Zachary
Gurvitz

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
(ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
(ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

$$m a \rightarrow (a \rightarrow m b) \rightarrow m b$$

\nearrow \nwarrow

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

ill-typed

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

$\text{String} \rightarrow \text{IO ()}$

(f) `putStrLn 42 >>= putStrLn 43`

ill-typed

(g) `(,) "42"`

$a \rightarrow (a, [\text{Char}])$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$[a] \rightarrow [[a, a]]$

(j) `let f x = x in (f 'a', f True)`

ill-typed

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

$$\left(\begin{array}{l} \text{filterM} :: (a \rightarrow m \text{Bool}) \rightarrow [a] \rightarrow m [a] \\ \text{const} [\text{True}, \text{False}] :: a \rightarrow [\text{Bool}] \end{array} \right)$$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$
 (-1)

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda b \rightarrow [b]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

ZipWith

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda x y z \rightarrow \text{filter } y (\text{map } x z)$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda x y \rightarrow \text{Just } c \mid y \rightarrow \text{arbitrary} \quad | \text{Nothing } y \rightarrow \text{undefined}$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x y \rightarrow \text{filter } (x == y) y$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda x \rightarrow \text{return } (\text{concatMap } \text{show } x)$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (x, y) z \rightarrow z \times y$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer:

returns all ordered pairs with unequal terms in a list

`foo [] = []`
`foo [1,2] = [(1,2), (2,1)]`
(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)`

Answer:
put an element at the end of a list

`foo 'a' "def" = "defa"`
`foo 1 [3,4] = [3,1,4,1]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar _ [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

make a list of values inside maybes in a list. (Cat maybes)

`foo [Just 1, Nothing] = [1]` `foo [Just "ab", Just "cd"] = ["ab", "cd"]`
`foo [Nothing, Nothing] = []`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer:

returns a tuple of the max of a list, and a number repeated the length of the list times. If the list is empty, returns the number as the "max," and the repeat is zero times

`foo [1,2] 3 = (max 2 1, 3:[3]) = (2, [3,3])`

`foo [2] 3 = (2, [3])`



`foo [] 20 = (20, [])`

`foo [2,4,1] 3 = (max 4 2, 3:[3]) = (4, [3,3,3])`

`foo [4,1] 3 = (max 4 1, 3:[3]) = (4, [3,3])`

`foo [] 3 = (1, [3])`

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ?? forall a -> [a] -> [[a]]
foo = dropWhileM (const [True, False])
```

Answer:

give a list of lists which are suffixes of the original list.

$$\begin{aligned} \text{foo } [1, 2, 3] &= [[1, 2, 3], [2, 3], [3], []] \\ \text{foo } [] &= [] \end{aligned}$$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave [] [] = []`
`weave (x:xs) (y:ys) = x:y:weave xs ys`
`weave _ _ = undefined`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax [] = []`
`toMax x:xs = rep (m', n') where`
~~`(m', n') = f x (max m x)`~~
`f :: a → [a] → Int → (a, Int)`
`f m [] = (m, 1)`
`f m (x:xs) = f (max m x) (n+1)`
`rep v 0 = []`
`rep v c = v:repeat v (c-1)`
 $\rightarrow (\text{rep is replicate with reversed inputs})$

Typeclass Definitions

```
class Semigroup a where
  (++) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (⊛) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (⊛⊛) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Jacob Grünzburg

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
- (i) True
 (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
- (i) True
 (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

ill-typed

(d) $\lambda x 1 \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

$\text{IO String} \rightarrow (\text{String} \rightarrow \text{IO ()}) \rightarrow \text{IO ()}$

(f) $\text{putStrLn} 42 >= \text{putStrLn} 43$

ill-typed

(g) $(,) "42"$

$\text{String} \rightarrow (\text{Char}, \text{Char})$

(h) $\text{reverse} . \text{foldMap} \text{return}$

ill-typed

(i) $\lambda l \rightarrow [(x, y) \mid x <- 1, y <- 1, x /= y]$

$\exists a a \Rightarrow [a] \rightarrow [(a, a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(\text{Char} \rightarrow a) \rightarrow (\text{Bool} \rightarrow b) \rightarrow (a, b)$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

ill-typed

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow \text{if } x \text{ then } [x] \text{ else } [\neg x]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f x y \rightarrow \text{foldr}(\text{foldr}(\lambda x' y' \rightarrow f x' y') x) y$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g xs \rightarrow \text{foldr}(\lambda x acc \rightarrow \text{if } g(f x) \text{ then } (f x) : acc \text{ else acc}) xs$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda x f \rightarrow \text{if isJust } x \text{ then arbitrary}$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x \mid \text{if } (x =_a d x) \text{ then } d \text{ else } x$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda xs \rightarrow \text{getLine}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda f g \rightarrow \text{let } f = (x, y) \text{ in } g x y$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Creates a list of tuples of all unequal elements in a list

$\text{foo}[1, 1, 2, 3] = [(1,2), (1,3), (2,3)]$

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: appends x to the end of l

$\text{foo } 4, [1,2,3] = [1, 2, 3, 4]$

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
 `let rs = bar f xs in`
 `case f x of`
 `Nothing -> rs`
 `Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Composes a list of elements that aren't Nothing

$\text{foo } [\text{Just } 1, \text{Nothing}, \text{Nothing}] = [1]$

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
 `where (m', xs') = foo xs m`

*Answer: Returns a tuple of the maximum element of the given argument
and the list and a list of maximum
elements at each part of the list*

$\text{foo } [1, 2, 3] 4 = (4, [4, 4, 4])$

$\text{foo } [1, 2, 3] 1 = (3, [1, 2, 3])$

$\text{foo } [4, 5, 6] 5 = (6, [5, 5, 6])$

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

$$\begin{aligned} \text{weave } [] &= [] \\ \text{weave } (x:xs), (y:ys) &= x:y:(\text{weave } xs ys) \end{aligned}$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\begin{aligned} \text{maxList } (x:xs) &= \text{case foo } x \text{ } xs \text{ of} \\ &\quad (a,b) \rightarrow \text{Just } a \\ &\quad _ \rightarrow \text{Nothing} \\ \text{maxList } [] &= \text{Nothing} \end{aligned}$$

`toMax = foldr (\let a = maxList in a) []`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  ( <*> ) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  ( >>= ) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Aaron Ortwein

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
- (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
- (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
- (iii) Both of the above.
- (iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
- (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
- (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
- (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
- (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) -> (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

IO ()

(f) `putStrLn "42" >>= putStrLn "43"`

ill-typed

(g) `(,) "42"`

$b \rightarrow (\text{String}, b)$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$[a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow \text{if } x \text{ then } [x] \text{ else } [x]$

(c) $a \rightarrow \text{Maybe } b$

id

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f i c \rightarrow f (\text{head } i) (\text{head } c)$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g l \rightarrow \text{map } (\lambda x \rightarrow \text{if } g(f x) \text{ then } f x \text{ else } f x) l$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda m g \rightarrow$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x l \rightarrow \text{if } x == x \text{ then } x : l \text{ else } l$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

flip uncurry

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer: Creates pairs out of every permutation of two distinct elements in a list

`foo [1,2] = [(1,2), (2,1)]`

`foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]`

(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)`

Answer: Moves the head of a non-empty list to be its last element

`foo [1] = [1]`

`foo [1,2,3,4,5] = [2,3,4,5,1]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`

`bar [] = []`

`bar f (x:xs) =`

`let rs = bar f xs in`

`case f x of`

`Nothing -> rs`

`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`

`foo = bar id`

Answer: Extracts the values from a list of monadic Maybe values

`foo [Nothing] = []`

`foo [Just 1, Just 2, Nothing, Just 3] = [1, 2, 3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer: Returns a pair consisting of the maximum element of the list (or a default value m for empty lists) and a list of the same length where all elements are m

`foo [] 5 = (5, [])`

`foo [1] 5 = (1, [5])`

`foo [1,2,3,4,5] 5 = (5, [5, 5, 5, 5, 5])`

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave :: [a] → [a] → [a]
weave [] [] = []
weave (x:xs) (y:ys) = x : y : weave xs ys
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
toMax :: [Int] → [Int]
toMax [] = []
toMax [x] = [x]
toMax (x:xs) = let (m, _) = foo (x:xs) x in snd (foo (x:xs) m)
```

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  ( <*> ) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  ( >>= ) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Hiteesh Nukalapati

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

(Eq a, Show a) $\Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x 1 \rightarrow x : 1 ++ 1 ++ [x]$

[Add 1] \Rightarrow ill-typed

(e) `getLine >>= putStrLn`

$IO()$

(f) `putStrLn "42" >>= putStrLn "43"`

$IO()$

(g) `(,) "42"`

$a \rightarrow (\text{String}, a)$

(h) `reverse . foldMap return`

(Monoid m, Foldable t) $\Rightarrow t a \rightarrow t(m a)$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

(Eq a) $\Rightarrow [a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[Bool] \rightarrow m[\text{Bool}]$

$a \rightarrow m a$

$(a \rightarrow m \text{Bool}) \Rightarrow [a] \rightarrow m[a]$

$a \rightarrow b \rightarrow a'$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) `Int -> Int`

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) `Bool -> [Bool]`

$\lambda n \rightarrow$ if $n == \text{True}$ then $[n]$ else $[\text{False}]$.

(c) `a -> Maybe b`

(d) `(Int -> Char -> Bool) -> [Int] -> [Char] -> [Bool]`

$\lambda f il cl \rightarrow$ if $f(\text{head } il) (\text{head } cl) == \text{True}$ then $[\text{True}]$ else $[\text{False}]$.

(e) `(a -> b -> c) -> IO a -> IO b -> IO c`

(f) `(a -> b) -> (b -> Bool) -> [a] -> [b]`

(g) `Maybe a -> (a -> Gen b) -> Gen (a,b)`

(h) `Eq a => a -> [a] -> [a]`

$\lambda x l \rightarrow$ if $x == (\text{head } l)$ then $.l$ else $x : l$

(i) `Show a => [a] -> IO String`

(j) `(a,b) -> (a -> b -> c) -> c`

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int] [(Int, Int)]`.
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer:
takes an int list, creates Pairs with unequal elements from the same list.
~~foo [1,0] = [(1,0), (0,1)]~~

(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)`

Answer:

~~foo 3 [1, 2]~~

Adds an element to the front of a reversed list & then reverses this list.

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

~~(1, 2, 3)~~
~~(2, 1, 3)~~
~~(3, 2, 1)~~

(e) `foo :: [Int] -> Int -> [Int] -> (Int, [Int])`.
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

~~foo [2] 3 = (2, [3])~~
~~(2, [1, 3])~~

Answer:

~~foo [1, 2] 3 = (2, [1, 3])~~
~~= (2, [1, 3])~~

This function returns a pair, where the first element
is the maximum of the current list (list that is passed to it).
& the second element (of the pair) is a list with the rest of the elements
and 2nd parameter formed to the list.

(f) *Bonus!*

```
drcpWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
drcpWhileM _ []      = return []
drcpWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foc :: ???
foc = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

$$\cancel{\text{weave } l \& r} \quad \text{weave } l \& r = \text{reverse}(\text{weaveHelper } l \& r)$$

$$\begin{aligned} \text{weaveHelper } l \& r &:: (\alpha) \rightarrow (\alpha) \rightarrow (\alpha) \\ \text{weaveHelper } [] \& r &= r \\ \text{weaveHelper } (h_1 : t_1) \& (h_2 : t_2) &= h_1 : h_2 : \text{weaveHelper } t_1 \& t_2 \end{aligned}$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\begin{aligned} \text{toMax } l &= \text{let } \text{len} = \text{length } l \text{ in} \\ &\quad \text{let } (\text{max}, \text{temp}) = \text{foo } l \text{ in} \\ &\quad \text{listBuild max len} \end{aligned}$$

$$\begin{aligned} \text{listBuild ele size} &:: \text{Int} \rightarrow \text{Int} \rightarrow [\text{Int}] \\ \text{listBuild - } 0 &= [] \\ \text{listBuild ele size} &= \text{ele} : (\text{listBuild ele } (\text{size} - 1)) \end{aligned}$$

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Crd a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

BAHAA HARAZ

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
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- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
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 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\text{Eq } a \Rightarrow a * \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow a$

(e) `getLine >>= putStrLn`

$IO ()$

(f) `putStrLn 42 >>= putStrLn 43`

~~ill-typed~~

(g) `(,) "42"`

$a \rightarrow (a, \text{String})$

(h) `reverse . foldMap return`

~~Moved~~ $m \Rightarrow [a] \rightarrow [m a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

~~ill-typed~~ $ill \text{ typed}$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

- (a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$

(+1)

- (b) $\text{Bool} \rightarrow [\text{Bool}]$

$f b = [b \quad \{; \text{True}\}]$

- (c) $a \rightarrow \text{Maybe } b$

$f a = \text{Just}$

- (d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda p \text{ with}$

- (e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$\lambda f \text{ with}$

- (f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

~~$f g \lambda l \text{st} = \text{map } g (\text{filter } h \text{ l} \text{st})$~~

~~$f g \lambda l \text{st} = \text{filter } h (\text{map } g \text{ l} \text{st})$~~

- (g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda \gg =$

- (h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$f a \text{ l} \text{st} = \text{filter } (\lambda a \text{ == }) \text{ l} \text{st}$

- (i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

return

- (j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$f (a, b) g = g \circ b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

Examples for q6:

$\text{foo } [1, 1, 2] = [1, 2]$
 $\text{foo } [1, 2, 3] = [1, 2], (2, 3), (1, 3)$

(b) `foo :: [Int] -> [[Int, Int]]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: takes a list and creates a new list of all combinations of elements in the original list, so long as they aren't equal.

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: appends the first argument to the reversed second argument

$\text{foo } 3 [1, 2] = [3, 2, 1]$

$\text{foo } 3 [] = [3]$

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Takes a list of maybe's, removes all Nothing's and returns a list of all values inside Just's.

$\text{foo } [\text{Just } 2, \text{Nothing}] = [2]$

$\text{foo } [\text{Just } 2, \text{Just } 3] = [2, 3]$

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer: Takes a list of ints and an int and returns tuple of form (max value of list, list of second value repeated (length list) times)
 If list is empty, return (value, [])

$\text{foo } [] 3 = (3, [])$

$\text{foo } [1, 2] 3 = (2, [3, 3])$

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)
```

```
foo :: ??  
foo = dropWhileM (const [True, False])
```

Answer: Drops all elements of a list.

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave [] [] = []`
`weave (x:xs) (y:ys) = x:y:(weave xs ys)`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax [] = []`
`toMax lst = let len = length lst in let m = maximum lst in replicate len m`
BONUS version: `toMax [] = []`

~~toMax lst =~~
`let aux l = foldl (\acc x → if x > (fst acc) then (x, (snd acc) + 1) else (fst acc, (snd acc) + 1)) (0, 0) in let (m, len) = aux lst in replicate len m`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Mitchell Fanger

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
 (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
 (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

() () ()

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

Eq a, Show a $\Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >= putStrLn`

IO String \rightarrow IO()

(f) `putStrLn 42 >>= putStrLn 43`

Ill typed

(g) `(,) "42"`

$a \rightarrow (\text{String}, a)$

(h) `reverse, foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

Eq a $\Rightarrow [a] \rightarrow [[a, a]]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) `Int -> Int`

Example answers:

`\x -> x + 1`

`(+1)`

(b) `Bool -> [Bool]`

`\x -> [x, not x, x || x]`

(c) `a -> Maybe b`

`\x -> Nothing`

(d) `(Int -> Char -> Bool) -> [Int] -> [Char] -> [Bool]`

`\f(xs) (ys) -> replicate x (y + c == z)`

(e) `(a -> b -> c) -> IO a -> IO b -> IO c`

(f) `(a -> b) -> (b -> Bool) -> [a] -> [b]`

`\g xs -> [y | y <- f xs, not (g y)]`

(g) `Maybe a -> (a -> Gen b) -> Gen (a,b)`

(h) `Eq a => a -> [a] -> [a]`

`\x ys -> map (\y -> if x==y then x else y) ys`

(i) `Show a => [a] -> IO String`

`\(x:xs) -> putStrLn (show x)`

(j) `(a,b) -> (a -> b -> c) -> c`

`\(x,y) f -> f x y`

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

`foo [] -> []`
`foo [1,2] -> [(1,2), (2,1)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: adds x to the end of list l

`foo 5 [] = [5]`

`foo 5 [1,2,3,4] = [1,2,3,4,5]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`

`foo = bar id`

Answer: takes in a list of Maybes and removes the maybe, returning the elements

`foo [] = []`

`foo [Just 5, Just 4, Nothing] = [5, 4]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer: finds the max of a list and returns it along with it removed from the list

`foo [] s = (5, [])`

`foo [1] s = (1, [5])`

`foo [1,2,7] s = (7`

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave :: [a] → [a] → [a]  
weave [] [] = []  
weave (x:xs) (y:ys) = x : (y : (weave xs ys)))
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
toMax :: Ord a ⇒ [a] → [a]
toMax [] = []
toMax (x:xs) = case findMax x xs of
  (m, l) → replicate l m
  _ → []
```

```
findMax :: Ord a ⇒ a → [a] → (a, Int)
findMax m [] = (m, 0)
findMax m (x:xs) l
  | x > m = findMax x xs (l+1)
  | otherwise = findMax m xs (l+1)
```

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (">>>") :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Elvin Liu

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the `identity` function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the `identity` function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >>= \text{putStrLn}$

IO ()

(f) $\text{putStrLn "42"} >>= \text{putStrLn "43"}$

ill-typed

(g) $(,) "42"$

$a \rightarrow (\text{String}, a)$

(h) $\text{reverse} . \text{foldMap return}$

$[a] \rightarrow [a]$

$\text{reverse} \quad a \rightarrow [a] \quad + a \rightarrow []$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a, a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(\text{Char}, \text{Bool})$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$[a] \rightarrow [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$

(+1)

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow [x \wedge x]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f m z$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$\lambda f m z$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g l \rightarrow f \text{ filter } g \text{ (map } f \text{ l)}$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda x f \rightsquigarrow \text{mapM } f \text{ (mapToList } x \text{)} \geq (\lambda y \rightarrow$

$\text{either } (\text{head } (\text{mapToList } x)), y \text{)} \rightarrow ?lmao$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x l \rightarrow \text{filter } (\lambda z \Rightarrow x) l$

(i) Show $a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda x \rightsquigarrow \text{putStrLn } x \gg \text{getLine}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (x, y) f \rightsquigarrow f x y$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Several pairs of unequal elements

`foo [1,0,0,1] = [(1,0), (0,1)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: Append x to end of l

`foo 4 [1,2,3] = [1,2,3,4]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: removes Nones from list of Maysbes and retrieves elements in Justs

`foo [Just 1, Nothing, Just 2, Just 3] = [1,2,3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

1:3, 5 m', xs' = (3, [5])

$$(3, ([5]))$$

Answer: tuple of max of first 1:3:2, 8 → (3, 4:4:4)

list and list of m of length 3:2, 8 → (3, 4:0)

of first list

$$2, 8 \rightarrow (2, [0])$$

`foo (1,2) 5 = (3, [5,5])`

`foo [1,3,2] 2 = (3, [3,2,2])`

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave :: [a] → [a] → [a]  
weave [] [] = []  
weave (x:xs) (y:ys) = x:y:(weave xs ys)
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
toMax [] = []  
toMax x = replicate (length x) (maximum x)
```

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Chris Jose

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
 (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
 (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

`foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m`

For each of the following `foldMap` calls, select all options that are true:

(a -> m) ta. Foldable -> list.

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

$$>> = m\ a \rightarrow (a \rightarrow m\ b) + m\ b.$$

$\text{IO}\ \text{String} \rightarrow \text{St}$

Question 2 (20 points) $\text{IO}\ \text{String} \rightarrow (\text{String} \rightarrow \text{IO})$

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x\ y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x\ y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$a \rightarrow b \rightarrow \text{String}$

(d) $\lambda x\ l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >>= \text{putStrLn}$

$\text{IO}\ \text{String} \rightarrow (\text{String} \rightarrow \text{IO}()) \rightarrow \text{IO}()$

(f) $\text{putStrLn}\ 42 >>= \text{putStrLn}\ 43$

$\text{IO}\ \text{String} \rightarrow (\text{String} \rightarrow \text{IO}\ \text{String}) \rightarrow \text{IO}\ \text{String}$

(g) $(,) "42"$

(h) $\text{reverse} \cdot \text{foldMap} \text{return}$

Monad $\Rightarrow [a] \rightarrow [m\ a]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- 1, y <- 1, x /= y]$

$[a] \rightarrow [(a,a)]$

(j) $\text{let } f\ x = x \text{ in } (f\ 'a', f\ \text{True})$

ill typed

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$

$(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$f x = (x \text{ || } \text{False}) : []$

(c) $a \rightarrow \text{Maybe } a$

$\lambda x \rightarrow \text{pure } x \gg \text{Maybe}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$f m1 m2 \rightarrow (f ((\text{head } m1) + 1) (\text{head } m2) : [a]) : []$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$\text{lift } m2$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g \text{ lst} \rightarrow g(f(\text{head lst})) : []$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\text{Maybe } x \gg = \text{Gen } b$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x m1 \rightarrow \text{intersperse } x m1$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda x \rightarrow (\text{head } x) ++ \text{getLine}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda \text{tup } f \rightarrow f (\text{fst } \text{tup}) (\text{snd } \text{tup})$

$\lambda f m1 m2 \rightarrow [f(x + 1) | y : [a] | x \in m1, y \in m2]$

$\text{IO } \{1,2,3\}$
 $\text{IO } [3,2,1]$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

Return a list of tuples of ints of pairs of 1, where both elements of each tuple are different. $\text{foo } [1,2,3] = [(1,2), (1,3), (2,3)]$.

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

Returns the original list along with element x appended to the end of the list.
 $\text{foo } 10 [1,2,3] = [1,2,3,10]$; $\text{foo } 10 [] = [10]$.

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
 Nothing -> rs
 Just r -> r:rs

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
where $(m', xs') = \text{foo } xs\ m$

$\text{foo } [3,2,1] 6.$

Answer:

Returns a $(\text{Int}, [\text{Int}])$, where the 1st element of tuple is the max element of input list and 2nd element is a list of the 2nd arg with the same length as input list.

$\text{foo } [3,2,1] 6 = (3, [6,6,6])$

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

weave [] [] = []
weave [] b = b.
weave a [] = a.
weave m@(x:xs) n@(y:ys) =
 x:y:weave xs ys.

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

toMax lst = let max = (fst \$ foo lst) in
 snd \$ foo lst max.

>> get Given b.

Typeclass Definitions

```

class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b
    a -> ma →

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a

```

$3:[z, 1] \quad 6 = (\max^m 3, 6:xs)$
 $\text{foo}[z, 1] \quad 6.$
 $(\max^m z, 6:6:xs')$
 $\text{foo}[1] \quad 6. = (1, [6])$

Samuel Howard 115960176

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

IO ()

(f) `putStrLn 42 >>= putStrLn 43`

ill-typed

(g) `(,) "42"`

$a \rightarrow (\text{String}, a)$

(h) `reverse . foldMap return`

$\text{Foldable } f \Rightarrow f a \rightarrow [a]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- 1, y <- 1, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a,a)]$

(j) `let f x = x in (f 'a', f True)`

$(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$\text{Monad } m \Rightarrow [a] \rightarrow m [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$.

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow \text{if } x \text{ then } [x] \text{ else } [x, x]$

(c) $a \rightarrow \text{Maybe } b$

Undefined

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f \lambda i \lambda c \rightarrow \text{map } (\lambda (a, b) \rightarrow f a b) (\text{zip } i \ c)$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

~~$\lambda f \lambda a \lambda b \rightarrow$~~ $\rightarrow \lambda f a b \rightarrow \text{do}$
 $a \leftarrow a$
 $b \leftarrow b$
 $\text{putStrLn } (f)$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f_a \lambda f_b \lambda a \rightarrow \text{filter } (\lambda a \rightarrow f_b (f_a a)) \ a$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

~~$\lambda m f \rightarrow$~~ $\rightarrow \lambda m f \rightarrow \text{do}$
 $a \leftarrow m$
 $(a, f a)$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda a \lambda a \rightarrow \text{filter } (\lambda b \rightarrow a == b) \ a$

(i) $\text{Show } a \Rightarrow a \rightarrow \text{IO String}$

$\lambda L \rightarrow \text{map } \text{putStrLn } L$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (a, b) f \rightarrow (f a b)$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

Returns all pairs of ints in a list that are not equal
`foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

Appends X to the end of l

`foo l [7,8,9] = [7,8,9,1]`

`foo 'a' [] = ['a']`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar _ [] = []`

`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

Removes all 'Nothings' from a list of 'Maysbes'

`foo [Nothing, Maybe 1, Maybe 2, Nothing] = [Maybe 1, Maybe 2]`

`foo [Nothing] = []`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer:

Returns a pair of the max element of a list and a list with m repeated

`foo [3,2,1] 2 = (3, [2,2,2])`
`foo [2,1] 3 = (2, [3,3])`
`foo [1] 3 = (1, [3])`
`foo [] 3 = (3, [])`

First arg

Second argument

n times where n is the length of the first argument.
OR returns second arg paired with empty list if argument list is empty

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the `same` length.

`weave :: [a] → [a] → [a]`

`weave [] [] = []`

`weave (x:xs) (y:ys) = x:y:(weave xs ys)`

-- for invalid cases:

`weave _ _ = [] -- Not possible with assumption`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax :: [Int] → [Int]`

`toMax [] = [] -- Not possible with assumption`

`toMax (x:xs) = a`

where `(_, a) = foo (x:xs) b`

where `(b, _) = foo (x:xs) 0`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  ((<), (≤), (≥), (>) :: a -> a -> Bool)
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (⊛) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (⊛=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Vyoma Jani

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 (i) True *laziness? Is OCaml lazy?*
 (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 (i) True
 (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

$\text{foldMap} :: (\text{Monoid } m, \text{ Foldable } t) \Rightarrow (a \rightarrow m) \rightarrow t \rightarrow [m] \rightarrow [Int]$

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

$[] \rightarrow m$

$(a \rightarrow m) \rightarrow (t \rightarrow m)$
 $(a \rightarrow [char]) \rightarrow [char] \rightarrow [char]$
Monad `[char]`
Foldable `[]`

$a = a \text{ or } \text{char}?$

(can combine strings)

`show :: a → String`
`[char]`

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or "ill-typed" if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then } \text{show } x \text{ else } \text{show } (x, y)$

$\text{show} :: a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow (x : l) ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn} \quad (>>) :: m a \rightarrow (a \rightarrow m b) \rightarrow m b$

$!0 ()$

(f) $\text{putStrLn} 42 >= \text{putStrLn} 43$

Ill-Typed

(g) $(,) "42"$

$a \rightarrow (\text{String}, a)$

(h) $\text{reverse} . \text{foldMap} \text{return}$

Foldable $t \Rightarrow t a \rightarrow [a]$

$\text{reverse} (\text{foldMap} \text{return})$

$[a] \rightarrow [a]$

$[a]$

foldMap return

$(a \rightarrow m) \rightarrow [a] \rightarrow m$

$a \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) | x <- l, y <- l, x /= y]$

Eq a \Rightarrow [a] \rightarrow [(a, a)]

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

(Char, Bool)

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$[a] \rightarrow [[a]]$

$\text{filterM} :: \text{Monad } m \Rightarrow$

$(a \rightarrow m \text{ Bool}) \rightarrow ([a] \rightarrow m [[a]])$

$\text{const} [\text{True}, \text{False}]$

$b \rightarrow [\text{Bool}]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

- (a) $\text{Int} \rightarrow \text{Int}$
Example answers:
 $\lambda x \rightarrow x + 1$
 $(+1)$
- (b) $\text{Bool} \rightarrow [\text{Bool}]$
 $\lambda x \rightarrow \text{if } x \text{ then } [x] \text{ else } [\text{true}]$
- (c) $a \rightarrow \text{Maybe } b$
 $\lambda x \rightarrow \text{Nothing}$
- (d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$
 liftM2
- (e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$
 liftM2
- (f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$
 $\lambda f1 \rightarrow \lambda f2 \rightarrow \lambda xs \rightarrow [f1 a | a \leftarrow xs, f2 (f1 a)]$
- (g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\text{func } \text{Nothing} = \text{undefined}$ $\text{func } (\text{Just } a) = \text{do } b \leftarrow f a$ $\text{return } (a, b)$
--
- (h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$
 $\text{func } a [] \triangleq []$
 $\text{func } a (h:t) \triangleq \text{if } a == h \text{ then } [a] \text{ else } [h]$
- (i) $\text{Show } a \Rightarrow a \rightarrow \text{IO String}$
 $\text{func } [] \triangleq \text{gerline}$
 $\text{func } (h:t) \triangleq \text{do } \text{show } h$
 gerline
- (j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$
 $\lambda (a, b) \rightarrow \lambda f \rightarrow f a b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [[(Int, Int)]]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Gets all possible pairs of elements in the list except those where both elements are equal

`foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,3)]` `foo [] = []`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: Appends `x` to the end of the input list `l`

`foo 4 [1,2,3] = [1,2,3,4]`
`foo 3 [] = [3]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`

`foo = bar id`

Answer: Extracts the elements out of the "Just -"s in the list into their own list ignoring Nolthings

`foo [Just 3, Nothing, Just 4] = [3,4]`
`foo [Nothing] = []`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])` All will be m:m:m...
`foo (x : xs) m = (max m' x, m' : xs')`
where `(m', xs') = foo xs m`

Answer: Returns a 2-Tuple where the first element is the maximum element of the input list, or the input Int if the list is empty, and the second element is a list of the input Ints that is of the same length as the original input list

`foo [] 3 = (3, [])`

`foo [3] 4 = (3, [4])`

`foo [2, 3, 4] 5 = (4, [5, 5, 5])`

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

Weave $[] [] = []$
Weave $(h1:t1) (h2:t2) = h1 : h2 : (\text{weave } t1 \ t2)$

- (b) Implement a function `toMax`, that given a ~~non-empty~~ list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

↙ what if we misunderstood
how (4e) foo function
operates?

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

unpassing
case →

`toMax [] = []`

`toMax (lst@(_:_t)) = let mmax = getMax lst in`

`length1 (lst mmax)`

where

`length1 [] _ = []`

`length1 (h:t) mmax = mmax : (length1 t)`

`getMax [] mmax = mmax`

`getMax (h:t) mmax = if h > mmax then getMax t h
else getMax t mmax`

`toMax [] = []`
↓
`toMax lst = let max = maximum lst in`
`replicate (length lst) max`

?
partial?

Typeclass Definitions

```
class Semigroup a where
  (++)  :: a -> a -> a

class Semigroup m => Monoid m where
  mempty  :: m

class Show a where
  show   :: a -> String

class Eq a where
  (==)  :: a -> a -> Bool
  (/=)  :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min      :: a -> a -> a

class Functor f where
  fmap   :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure   :: a -> f a
  (⊛)   :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (⊛)   :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink    :: a -> [a]

class Num a where
  (+), (-), (*)      :: a -> a -> a
  negate             :: a -> a
  abs                :: a -> a
  signum             :: a -> a
  fromInteger        :: Integer -> a
```

Yuvraj Nayak

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 (i) True
 (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 (i) True
 (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$\text{Eq } a, \text{show } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >=> putStrLn`

$\text{String} \rightarrow \text{IO String}$

(f) `putStrLn 42 >=> putStrLn 43`

ill-typed

(g) `(.) "42"`

$a \rightarrow (\text{String}, a)$

(h) `reverse . foldMap return`

ill-typed

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$[a] \rightarrow [[a]]$

(j) `let f x = x in (f 'a', f True)`

$(\text{char}, \text{Bool})$

(k) `filterM (const [True, False])`

Monad $m \Rightarrow [a] \rightarrow m [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$$\lambda x \rightarrow x + 1$$

(+1)

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$$\lambda b \rightarrow [b, b]$$

(c) $a \rightarrow \text{Maybe } b$

$$\lambda x \rightarrow \text{Nothing}$$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow \text{Bool}$

$$\lambda f \text{ list } c \text{ list } \rightarrow [f i c \mid i \in \text{list}, c \in \text{clist}, f i c]$$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$$\text{fmap}$$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$$\lambda f g \text{ list } \rightarrow [f e \mid e \in \text{list}, g(f e)]$$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$$\lambda m f \rightarrow$$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$$\lambda x \text{ list } \rightarrow \text{if list} == [] \text{ then } [] \text{ else if } x == \text{head list} \text{ then } x : \text{list} \text{ else list}$$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$$\lambda (x, y) f \rightarrow f x y$$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [[Int]]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Finds all pairs of elements in l s.t. no pairs w/ same element

`foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: adds x to end of list

`foo 5 [1,2,3,4] = [1,2,3,4,5]`

`foo 5 [] = [5]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar _ [] = []`

`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Takes all Justs in a list of maybes and returns a list of elements

`foo [Just 1, Just 2, Nothing, Just 3] = [1,2,3]` *inside Just*

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

*Answer: finds the max element of a list and return it in a tuple
 with a list of ints:*

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave :: [a] → [a] → [a]
weave [] [] = []
weave (x:xs) (y:ys) = x:y:(weave xs ys)
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
toMax :: [Int] → [Int]
toMax [] =
```

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Maverick Kieu

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
- (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
- (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
- (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) -> (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap ([])` [1..10]

- (i) The expression will fail to typecheck. —
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`. —
- (v) The expression is equivalent to the `identity` function.
- (vi) The foldable in this call is `[]` - the `instance` for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the `identity` function.
- (vi) The foldable in this call is `[]` - the `instance` for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

ill-typed

(d) $\lambda x l \rightarrow x : (l ++ (l ++ [x]))$

$a \rightarrow [b] \rightarrow [b]$

(e) $\text{getLine} >= \text{putStrLn}$
 $(\text{IO String}) \rightarrow (\text{String} \rightarrow \text{IO ()}) \rightarrow \text{IO ()}$

(f) $\text{putStrLn "42"} >= \text{putStrLn "43"}$

ill-typed

(g) $(\lambda a b \rightarrow (a, b)) \rightarrow [\text{Char}] \rightarrow [\text{Char}] \rightarrow ([\text{Char}], [\text{Char}])$

(h) $\text{reverse} . \text{foldMap} \text{return}$
 $[\text{a}] \rightarrow [\text{m a}]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$[\text{Int}] \rightarrow [\text{Int}]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

ill-typed

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$a \rightarrow [\text{Bool}]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

- (a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

- (b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow [\text{True} \& \& x]$

- (c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Just } x$

- (d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\text{foo } a \ b = \text{if } (\text{head } a) > 0 \text{ then } [\text{head } b == 'a'] \text{ else } [\text{head } b == 'b']$

- (e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$\lambda _ \rightarrow \text{undefined}$

- (f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\text{foo } f \ g \ \text{lst} \rightarrow \text{let } b = \text{head } (\text{map } f \ \text{lst}) \text{ in if } (f \ b) \text{ then } [b] \text{ else } [b]$

- (g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda _ \rightarrow \text{undefined}$

- (h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\text{foo } a \ b = \text{if } a > \text{head } b \text{ then } [a] \text{ else } [a]$

- (i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

- (j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\text{foo } (a, b) \ f = f \ a \ b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:
Returns a list of numbers if it is not equal to itself.

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

$[1,3,2,1]$

Answer:
Appends to the end of list with extra steps

$f 1 [] = [1]$

$f 1 [1,2,3] = [1,2,3,1]$

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`

`bar f (x:xs) =`

`let rs = bar f xs in`
`case f x of`

`Nothing -> rs`

`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
where $(m', xs') = \text{foo } xs\ m$

max 2 1

$\text{foo } [2] 3 = (2, [3])$

$\text{foo } [] 3 = (3, [])$

$\text{foo } [1] 3 = (1, [3])$

$\text{foo } [1,2] 3 = (2, [3,3])$

$m' = 2$

$xs' = [3]$

(c) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave :: [Int] → [Int] → [Int]  
weave [] [] → []  
weave (x:xs) (y:ys) = x:y:(weave xs ys)
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
toMaxHelper :: Int → [Int] → Int  
toMaxHelper [] = 0  
toMaxHelper a (x:xs) = if a < x then toMaxHelper x xs else toMaxHelper a x  
toMax (x:xs) = fmap (λ y → maxHelper x xs) (x:xs)
```

Typeclass Definitions

```
class Semigroup a where
  (<*>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  ( <*> ) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  ( >>= ) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Segev Elazar Mittelman

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
 (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
 (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$(Eq a, Show a) \Rightarrow a \rightarrow a \rightarrow String$

(d) $\lambda x \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

$IO()$

(f) $\text{putStrLn} 42 >= \text{putStrLn} 43$

ill-typed, putStrLn 43 not a function type

(g) $(,) "42"$

$a \rightarrow (String, a)$

(h) $\text{reverse} \cdot \text{foldMap} \text{return}$

ill-typed, return doesn't return a monoid, it returns a monad

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$(Eq a) \Rightarrow [a] \rightarrow [(a, a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(Char, Bool)$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda b \rightarrow [(b \& b)]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda a \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

ZipWith

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f as \rightarrow \text{filter } p (fmap f as)$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda a \rightarrow \text{undefined} \quad (\text{can't handle the Nothing case})$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda a as \rightarrow \text{if } a == a \text{ then } (a : as) \text{ else } as$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda as \rightarrow \text{return } (\text{foldMap show as})$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

flip uncurry

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: calculates list of all pairs of nonequal elements of the input list of ints

`foo [] = []`, `foo [1,2] = [(1,2), (2,1)]`

`foo [1] = []`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: appends the element x to the end of the list l.

`foo 1 [] = [1]`

`foo 1 [2] = [2,1]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`

`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: foo takes a list of Maybe a values, and returns a list of all the values in Just constructors in the input list, removing the Nones.

`foo [] = []`, `foo [Nothing] = []`, `foo [Just 1, Just 2, Nothing, Just 3] = [1, 2, 3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer: returns a pair of the maximum element in the list and a list repeating m a number of times equal to the length of the input list

`foo [1,2,3] 0 = (3, [0,0,0])`

`foo [2,3] 0 = (3, [0,0])`

`foo [3] 0 = (3, [0])`

`foo [5,5,5] 1 = (5, [1,1,1])`

`foo [] 1 = (1, [])`

(f) *Eonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ [] = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)
```

foo :: ?? $[a] \rightarrow [[a]]$
foo = dropWhileM (const [True, False])

Answer: returns the list of all tails of a list including the original list

$$\text{foo } [] = [[]]$$

$$\text{foo } [1, 2, 3] = [[1, 2, 3], [2, 3], [3], []]$$

$$\text{foo } "bel" = ["bel", "el", "l", ""]$$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave :: [a] → [a] → [a]`
`weave [] [] = []`
`weave (a:as) (b:bs) = a:b:weave as bs`
`weave _ _ = error "lists diff size"`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax :: [Int] → [Int]`
`toMax [] = []`
`toMax (x:xs) = let (m, l) = foo (x:xs) x`
`in fmap (const m) as`

Bonus: `toMax [] = []`
`toMax (x:xs) = let (m, l) = toMax' x 1 xs in replicate l m`
where
`toMax' m l [] = (m, l)`
`toMax' m l (a:as) = if a > m = toMax' a (l+1) as`
otherwise = `toMax' m (l+1) as`

`replicate n m | n ≤ 0 = []`
otherwise = `m : replicate (n-1) m`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>*) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Azhdaha Farzad

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
- (i) True
(ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
(ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
- (i) True
(ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
- (i) The program will fail to typecheck.
(ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

Ⓐ (a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Ⓑ (b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$\text{Eq } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >>= \text{putStrLn}$

$\text{IO } u$

(f) $\text{putStrLn "42"} >>= \text{putStrLn "43"}$

ill-typed

(g) $(,) "42"$

$a \rightarrow (a, \text{String})$

(h) $\text{reverse}, \text{foldMap}, \text{return}$

$\text{Monoid } m \Rightarrow t \rightarrow m a$

for monad

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$(\text{Ord } a, \text{Eq } a) \Rightarrow [a] \rightarrow [(a, a)]$

(j) $\text{let } f x = x \text{ in } (f \text{ } a', f \text{ True})$

ill typed

(k) $\text{filterM} (\text{const } [\text{True}, \text{False}])$

~~$a \rightarrow [\text{Bool}] \rightarrow t \rightarrow [[a]]$~~
 $[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda a \rightarrow \text{if } a \text{ then } (\text{replicate } a \text{ 4}) \text{ else } [a]$

~~(c)~~ $a \rightarrow \text{Maybe } b$

$\lambda a \rightarrow \text{Just } 4$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

zipWith

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g \rightarrow \text{map } f$

~~(g)~~ $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\text{fun ma f = do a \leftarrow ma}$

$b \leftarrow (f a)$

$\text{return } (a, b)$

~~(h)~~ $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda a \text{ lst} \rightarrow \text{filter } (==a) \text{ lst}$

~~(i)~~ $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$\lambda \text{ lst} \rightarrow \text{do map show lst}$

getLine

~~(j)~~ $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda x y \rightarrow \text{uncurry } y \ x$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

Creates list of tuples of elements of l . It is the cartesian product without tuples in the form (a, a)
`foo [1,2,3] = [(1,2), (1,3), (2,1), (2,3), (3,1), (3,2)]`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

Appends x to end of list

(d) `foo 4 [1,2,3] = [1,2,3,4]`
`bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`
`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

Returns a list of the Just values of the parameter.

(e) `foo [Just 1, Just 2, Just 4, Nothing] = [1, 2, 4]`
`foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`
`6 [7] 6 7`

Answer:

returns a tuple with the first element as the biggest element in the first argument (list) and as the second element of the tuple as a list where each element is the second argument and has a length that is equal to the length of the first argument
`foo [3,6] 7 = (6, [7,7])`

`foo [3,4,5] 7 = (5, [7,5,7])`

`foo [6] 7 = (6, [7])`

```
(f` Bonus!
  dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
  dropWhileM _ []      = return []
  dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

  foo :: ???
  foo = dropWhileM (const [True, False])
```

Answer:

$\text{foo} : [a] \rightarrow [[a]]$

$\text{foo } \{1, 2, 3\} =$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function weave that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

$\text{weave } [1, 2, 3] \ [4, 5, 6] = [1, 4, 2, 5, 3, 6]$

You can assume that the lists have the same length ✓

$\text{weave} :: [a] \rightarrow [a] \rightarrow [a]$

$\text{weave } [] \ [] = []$

$\text{weave } (x:xs) \ (y:ys) = x:y: \text{weave } xs \ ys$

- (b) Implement a function toMax, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

$\text{toMax } [1, 4, 2, 5, 3] = [5, 5, 5, 5, 5]$

You can use the foo function of problem (4e) if it helps.

BONUS: Implement toMax so that it only traverses a list once!

$\text{toMax} :: [\text{Int}] \rightarrow [\text{Int}]$

~~toMax lst~~

$\text{toMax lst} = \text{replicate}(\text{maximum lst}) (\text{length lst})$

~~maximum = foldr (\x y) max (x:xs)~~

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<>>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Susan
wen

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
- (i) True
 (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
(ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
- (i) True
(ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"` \equiv `foldMap show [1..6]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$(\text{show } a, \text{Eq } a) \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

IO ()

(f) $\text{putStrLn} 42 >= \text{putStrLn} 43$

ill-typed

(g) $(,) "42"$

$a \rightarrow (\text{String}, a)$

(h) $\text{reverse} . \text{foldMap} \text{return}$

$[a] \rightarrow [\text{Maybe } a]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow \text{List } [(a,a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(\text{Char}, \text{Bool})$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

ill-typed

$[a] \rightarrow [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow (x \&& \text{True}) : []$

(c) $a \rightarrow \text{Maybe } b$

~~foo~~ $x = \text{undefined}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$\lambda f x y \rightarrow [f x' y' \mid x' \leftarrow x, y' \leftarrow y]$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

(e) $\lambda f x y \rightarrow \text{do}$
 $x' \leftarrow x$
 $y' \leftarrow y$
 $\text{return } f x' y'$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g xs \rightarrow \text{filter } g (\text{map } f xs)$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$\lambda x f \rightarrow \text{case } x \text{ of } \text{Nothing} \rightarrow \text{Nothing}$
 $\text{Just } y \rightarrow \text{do}$
 $z \leftarrow fy$
 $\text{return } (y, z)$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x ls \rightarrow \text{filter } (= x) ls$

(i) Show $a \Rightarrow [a] \rightarrow \text{IO String}$

~~%R/X1/X6/~~ $\text{foo} = \text{getLine}$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (x, y) f \rightarrow f x y$

$\lambda f g xs \rightarrow \text{filter } g (\text{map } f xs)$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [[Int]]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: create list of all pairings of non-equal numbers in list l.
`foo [] = []`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

add x to end of list l
`foo [] = []`

`foo o [1,2,3] = [1,2,3,0]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`

let rs = bar f xs in
 case f x of
 Nothing -> rs
 Just r -> r:rs

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: remove all Noltinys and keep values in Just's

`foo [Nothing, Just 1] = [1]`

`foo [] = []` `foo [Just 1, Just 2] = [1,2]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
 where `(m', xs') = foo xs m`

`foo [3] 0 = (3, [0])`

`foo [2,3] 0 = (3, [0,0])`

`foo [1,2,3] 0 = (3, [0,0,0])`

Answer:

return make a tuple where first is the largest number in the list^{plus m} and second is

a list of m's length list of (x:xs)

`foo [] 0 = (0, [])` `foo [2,3] 4 = (3, [4,4])`

(f) Bonus!

```
dropWhileM :: (Monad m) -> (a -> m Bool) -> [a] -> m [a]
dropWhileM _ [] = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ???
foo = dropWhileM (const [True, False])
```

Answer:

removes first False
generates list with False

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave [] [] = []
weave (x:xs) (y:ys) = x: y: (weave xs ys)
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

```
+ toMax (x:xs) = foldr (\y a → if y > a then y else a) x xs
+ in      snd (foo (x:xs)) max
```



```
+ toMax ls@(x:xs) = snd (foo ls)
+ where m = foldr (\y a → if y > a then y
+                   else a) x ls
```

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
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  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
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class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Siddharth Taneja

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True
- (ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
- (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
- (iii) Both of the above.
- (iv) None of the above.

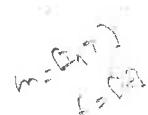
- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
- (ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
- (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
- (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
- (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts



The standard library defines `foldMap` with the following type:

`foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m`

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$\text{Eq } a, \text{Show } a \Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

$\text{IO String} \rightarrow \text{IO ()}$

(f) $\text{putStrLn} 42 >= \text{putStrLn} 43$

ill-typed

(g) $(,) "42"$

$a \rightarrow (\text{String}, a)$

(h) $\text{reverse} . \text{foldMap return}$

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x \neq y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a,a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(\text{Char}, \text{Bool})$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

$m = [] \quad a = a$

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$
 $(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow [x \& \& \text{True}]$

(c) $a \rightarrow \text{Maybe } b$

$\lambda x \rightarrow \text{Nothing}$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

zipWith

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftA2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$\lambda f g \rightarrow \text{map } f$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a,b)$
 $\lambda (\text{Just } x) f \rightarrow \text{do } y \leftarrow f x ; \text{return } (x,y)$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$\lambda x xs \rightarrow [y \mid y \leftarrow xs, x \neq y]$

(i) $\text{Show } a \Rightarrow a \rightarrow \text{IO String}$

$\lambda xs \rightarrow \text{do } x \leftarrow xs ; \text{putStrLn } (\text{show } x); \text{getLine}$

(j) $(a,b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$\lambda (x,y) f \rightarrow f x y$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <= 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int] [(Int, Int)]`
`foo l = [(x,y) | x <= 1, y <= 1, x /= y]`

Answer: Computes all pairs of unequal elements in a list
`foo [1] = []`

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: Puts `x` at the end of the given list `l`
`foo 1 [2,3,4] = [2, 3, 4, 1]`

`foo 7 [] = [7]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`

`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Takes a list of `Maybe a`'s and removes all the `Nothing`'s and the `Just`'s
`foo [Just 1, Nothing, Just 2] = [Just 1, Just 2]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`

`foo [x] m = (x, [m])`

`foo (x:xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer: `foo` returns the the maximum of the integer list, along with a list with each element replaced by `m`. If the list is empty then we return `m` as the "default" max value.
`foo [1,2,4,5] 3 = (5, [3,3,3,3])`

`foo [1,2,3] 7 = (3, [7,7,7])`

`foo [] 4 = (4, [])`

$m = \{ \}$

(i) Bonus!

$\text{dropWhileM} :: (\text{Monad } m) \Rightarrow (a \rightarrow m \text{ Bool}) \rightarrow [a] \rightarrow m [a]$

$\text{dropWhileM } [] = \text{return } [] \rightsquigarrow \{ \}$

$\text{dropWhileM } p (x:xs) = \text{do}$

$q \leftarrow p x$
 $\text{if } q \text{ then dropWhileM } p xs \text{ else return } (x:xs)$

$\text{foo} :: ?? [a] \rightarrow [a]$

$\text{foo} = \text{dropWhileM } (\text{const } [\text{True}, \text{False}])$

Answer: $\text{foo takes a list } l$

and returns all suffix sets

of l in decreasing size order

$\text{foo } [1, 2, 3] = [[1, 2, 3], [2, 3], [3], []]$

$\text{f. } "abcd" = ["abcd", "bcd", "cd", "d", []]$

$\text{foo } [] = [[]]$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave :: [a] → [a] → [a]`

$$\text{weave } (x : xs) \ (y : ys) = x : (y : (\text{weave } xs \ ys))$$

$$\text{weave} \ _{-} \ _{-} = []$$

↑
If the lists are of unequal length then we return [].

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax :: [Int] → [Int]`

$$\text{toMax } l = \text{replicate } (\text{length } l) \ (\text{maximum } l)$$

Bonus:

`dfold :: (Int → (Int → Int) → (Int → Int)) → (Int → Int) → [Int] → [Int]`
`dfold p fcc acc () = []`
`dfold p fcc acc (x : xs) = do`
 `(f', l') ← dfold p (p x fcc) acc xs`
 `(p' x f', (f' x) : l')`

$$\text{toMax } l = \text{snd } (\text{dfold } (\lambda x f \rightarrow \lambda y \rightarrow \max x (f y)) \ \text{id} \ [] \ l)$$

Typeclass Definitions

```
class Semigroup a where
  (++) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (=<), (=>), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Reid Hustley

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$(Eq, Show) \Rightarrow a \rightarrow a \rightarrow String$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

IO ()

(f) `putStrLn 42 >>= putStrLn 43`

ill-typed

(g) `(,) "42"`

$a \rightarrow (String, a)$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$Eq a \Rightarrow [a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(Char, Bool)$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

- (a) $\text{Int} \rightarrow \text{Int}$
Example answers:
 $\lambda x \rightarrow x + 1$
(+1)
- (b) $\text{Bool} \rightarrow [\text{Bool}]$
 $\lambda x \rightarrow [x]$
- (c) $a \rightarrow \text{Maybe } b$
 const Nothing
- (d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$
 $\lambda f x y \rightarrow f \$ x <*> y$
- (e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$
 $\lambda f x y \rightarrow f \$ x <*> y$
- (f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [o]$
 $\lambda f p x \rightarrow \text{filter } p (\text{map } f x)$
- (g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$
 $\lambda m f \rightarrow \text{return undefined}$
- (h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$
 $\lambda x \rightarrow \text{filter } (x ==)$
- (i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$
 $\lambda xs \rightarrow \text{return } (\text{foldMap show } xs)$
- (j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$
 $\lambda (a, b) f \rightarrow \text{flip uncurry } f a b$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer:

Constructs every pair of distinct Ints that can be made using the elements of a passed-in list

(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)`

Answer:

Inserts an element at the end of a list

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

Takes a list of Maybes and returns a list of the values contained in the Just elements in the same order

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
where `(m', xs') = foo xs m`

Answer:

Returns the max value amongst a list of Ints, or a passed-in Int if the list is empty,

as well as a list that's the same length as the passed-in list and only contains copies of the passed-in Int

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)
foo :: ?? [a] ~ [[a]]
foo = dropWhileM (const [True, False])
```

Answer:

Takes a list and returns every slice of that list
(consequently)

which ends with the last element, as well as the empty list.

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`weave :: [a] → [a] → [a]`
`weave [] _ = []`
`weave _ [] = []`
`weave (x:xs) (y:ys) = x:y:weave xs ys`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

~~`toMax :: [Int] → [Int]`~~
`toMax l = replicate (length l) max`
where `(max, _) = foo l 0`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Claude
Zou

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
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- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
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 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

$(Eq a) \Rightarrow a \rightarrow a \rightarrow String$

(d) $\lambda x \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

$IO()$

(f) `putStrLn 42 >>= putStrLn 43`

ill typed

(g) `(,) "42"`

$a \rightarrow (String, a)$

(h) `reverse . foldMap return`

ill typed

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$(Eq a) \Rightarrow [a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

$(Char, Bool)$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$$\lambda x \rightarrow x + 1$$

$(+1)$

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$$\lambda x \rightarrow [\text{not } x]$$

(c) $a \rightarrow \text{Maybe } b$

$$\lambda x \rightarrow \text{Nothing}$$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$$\lambda f \text{zipWith } f$$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

$$\lambda f \text{liftM2 } f$$

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$$\lambda f1 f2 L \rightarrow \text{filter } (\lambda x \rightarrow f2 x) (map f1 L)$$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$$\lambda x f \rightarrow \text{do } a \leftarrow x, b \leftarrow f a, \text{return } (a, b)$$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$$\lambda x L \rightarrow \text{filter } (\lambda y \rightarrow y == x) L$$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$$\lambda x \rightarrow \text{pure } (\text{Show } (\text{head } x))$$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$$\lambda f (a, b) f1 = f1 a b$$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: tuples of all pairs in a list with all other non-equal pairs in the list.

(c) `foo :: a -> [a] -> [a]` eg. $[1, 0, 1] \rightarrow [(1,0), (0,1)]$
`foo x l = reverse (x : reverse l)`

Answer: append to end of list

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
 `let rs = bar f xs in`
 `case f x of`
 `Nothing -> rs`
 `Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: removes all `Nothing`s from a list and "unwraps" the `Justs`.

eg $[\text{Just } 1, \text{Nothing}] \rightarrow [1]$

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
 where $(m', xs') = \text{foo } xs\ m$

Answer: returns a tuple of the max element of the list and a list of the same size with the second argument as the only element

$[1, 3, 5] 4 \rightarrow (5, [4, 4, 4])$

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a],  
dropWhileM _ [] = return []  
dropWhileM p (x:xs) = do  
    q <- p x  
    if q then dropWhileM p xs else return (x:xs)
```

```
foo :: ??  
foo = dropWhileM (const [True, False])
```

Answer:

$[a] \rightarrow [[a]]$

return the powerset of a list

e.g. $[1, 0] \rightarrow [[], [1], [0], [1, 0]]$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

$$\text{weave } [] \quad [] = []$$

$$\text{weave } (x:xs) \quad (y:ys) = \quad x: (y: (\text{weave } xs \ ys))$$

$$\text{weave } _ \quad _ = \text{undefined}$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\text{toMax } [] = []$$

$$\text{toMax } L = \text{replicate } (\text{length } L) \quad (\text{maximum } L)$$

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

GUIDO AMBASE

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 (i) True
 (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 (iii) Both of the above.
 (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler. (Assuming type checker ≠ compiler)
 (i) True
 (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

$a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x \rightarrow x : 1 ++ 1 ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >=> putStrLn`

IO ()

(f) `putStrLn 42 >=> putStrLn 43`

ill-typed

(g) `(,) "42"`

~~String~~ $a \rightarrow \text{String}, a$

(h) `reverse . foldMap return`

~~foldMap~~ $\text{FOLDABLE } C, \text{MONAD } m \Rightarrow t a \rightarrow m$

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

$\text{Eq } a \Rightarrow [a] \rightarrow [(a,a)]$

(j) `let f x = x in (f 'a', f True)`

~~MAP~~ $(\text{Char}, \text{Bool})$

(k) `filterM (const [True, False])`

$\text{Monad } m \Rightarrow [a] \rightarrow m[a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) `Int -> Int`
Example answers.
 $\lambda x \rightarrow x + 1$
(+1)

(b) `Bool -> [Bool]`
 $\lambda b \rightarrow \text{if } b \text{ then } [b] \text{ else } [b]$

(c) `a -> Maybe b`
 $\lambda a \rightarrow \text{Nothing}$

(d) `(Int -> Char -> Bool) -> [Int] -> [Char] -> [Bool]`
 ~~$\lambda f g h \rightarrow \text{if } f(\text{head } h) (\text{head } h) \text{ then } \text{True} \text{ else False}$~~
IFTM2

(e) `(a -> b -> c) -> IO a -> IO b -> IO c`
IFTM2

(f) `(a -> b) -> (b -> Bool) -> [a] -> [b]`
 ~~$\lambda f g \lambda a \rightarrow \text{let } b = f(\text{head } a) \text{ in if } g b \text{ then } [b] \text{ else } [b]$~~

(g) `Maybe a -> (a -> Gen b) -> Gen (a,b)`
 $\lambda ma f \rightarrow \text{arbitrary}((\text{fromJust } ma), F(\text{fromJust } ma))$

(h) `Eq a => a -> [a] -> [a]`
 $\lambda a \lambda a \rightarrow \text{if } a == \text{head } a \text{ then } [a] \text{ else } [a]$

(i) `Show a => [a] -> IO String`
 $\lambda a \rightarrow \text{return } (\text{Show } (\text{head } a))$

(j) `(a,b) -> (a -> b -> c) -> c`
 $\lambda (a,b) f \rightarrow f a b$

~~QUESTION 4~~

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int, Int] [(Int, Int)]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: Returns a list with all pairs of integers that are not equal

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: Appends an element to the end of the list.

`foo [] = []`

`foo 4 [1,2,3] = [1,2,3,4]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in -- result`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Takes a list of maybes and returns a list with all values that are not Nothing.

`foo [] = []`

~~foo~~ `foo [Just 1, Just 2, Just 3] = [1, 2, 3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer: Returns a tuple with the maximum element of the list ~~current~~ first

and a list with the maximum between the current head and the parameter m.

`foo [1,4,2] 3 = (4, [3,4,3])`

`foo [] 1 = (1, [])`

`foo [2] 1 = (2, [1])`

~~QUESTION 4 ANSWER~~ `foo [1,2] 3 = (2, [3,3])`

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ?? [a] -> m [a]
foo = dropWhileM (const [True, False])
```

Answer: This will just return the empty list, wrapped in a
~~maybe~~

~~dropWhileM~~

foo [] = []

foo [1,2,3] = []

foo ["hope", "in", "right"] = []

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

```
weave [] [] = []
weave [x] [y] = [x,y]
weave (xs:xs') ys = [x,y] ++ (weave xs' ys)
```

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

~~toMax~~
-- Should only traverse once.
toMax lst =
let max = ~~maximum~~ lst in
~~repeat~~ take (length lst) (repeat max)

-- Without bonus.
toMax lst =
let max = ~~maximum~~ lst in
~~repeat~~ (map (_ -> max) lst)

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Grayson Wolff

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
- (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
- (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
- (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

Eq a, Show a $\Rightarrow a \rightarrow a \rightarrow String$

(d) $\lambda x \ 1 \rightarrow x : 1 ++ 1 ++ [x]$

$[a] \rightarrow a \rightarrow [a]$

(e) `getLine >= putStrLn`

$| O ()$

(f) `putStrLn 42 >= putStrLn 43`

ill typed

(g) `(,) "42"`

$a \rightarrow ([Char], a)$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$[c] \rightarrow [[a, a]]$

(j) `let f x = x in (f `a`, f True)`

$([Char], Bool)$

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) `Int -> Int`
Example answers:
`\x -> x + 1`
`(+1)`

(b) `Bool -> [Bool]`

`return`

(c) `a -> Maybe b`

`undefined`

(d) `(Int -> Char -> Bool) -> [Int] -> [Char] -> [Bool]`

`liftM2`

(e) `(a -> b -> c) -> IO a -> IO b -> IO c`

`liftM2`

(f) `(a -> b) -> (b -> Bool) -> [a] -> [b]`

`if g |> filter g $ map f |`

(g) `Maybe a -> (a -> Gen b) -> Gen (a,b)`

`undefined`

(h) `Eq a => a -> [a] -> [a]`

`\x |> filter (== x) |`

(i) `Show a => [a] -> IO String`

`return $ foldMap show`

(j) `(a,b) -> (a -> b -> c) -> c`

`\t |> g -> uncurry g +`

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

returns all pairs of elements that differ from each other

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

adds x to end of list

`foo 3 [1,2] = [1,2,3]`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`

`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`

`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

removes all `Nothing`s and "un-Just's" the remaining elems

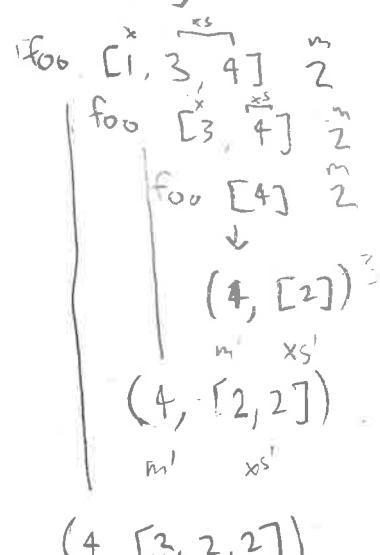
`foo [Just 1, Just 2, Nothing, Just 3] = [1,2,3]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer:

Extracts the max element,
replaces each element w/ m
and returns the tuple
containing the

(max elem, new list)



(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ??[a] → [[a]]
foo = dropWhileM (const [True, False])
```

Answer:

equivalent to tails (The "true" branch drops the element and continues, the "false" branch stops the computation)

foo [1,2,3] =

$[[1,2,3], [2,3], [3], []]$

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6],`

You can assume that the lists have the same length.

Weave $x \times y = \text{concatMap} (\text{uncurry} (\text{:})) \$ \text{zip } x \times y$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$\text{toMax } x = \text{map} (\text{const } \$ \text{maximum } x) \times$

BONUS

$\text{toMax } (x:xs) = \text{snd } \$ f \times (x:xs) \text{ where}$

$f @ [] = (a, [])$

$f @ (y:ys) = (q_r, q_f:qs) \text{ where}$

$(q_r, qs) = f (\max a y) \times ys$

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (≤), (≥), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (⊛) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (⊛=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Garrett Hill

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.

- (i) True *Lazy*
(ii) False

- (b) The constraint `Semigroup a => Monoid a` implies that:

- (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
(iii) Both of the above.
(iv) None of the above.

- (c) Typeclass laws are enforced by the Haskell compiler.

- (i) True
(ii) False

- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:

- (i) The program will fail to typecheck.
 (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
(iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
(iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:
 $a \rightarrow a$

(b) $\lambda x y \rightarrow (x,y)$

$a \rightarrow b \rightarrow (a,b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x,y)$

~~Eq a~~ $\Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >>= \text{putStrLn}$

~~IoC~~ ~~Monoid~~ ~~22~~

(f) $\text{putStrLn} 42 >> \text{putStrLn} 43$

: ill typed

(g) $(,) "42"$

ill typed

(h) $\text{reverse} . \text{foldMap return}$

~~Foldable~~ $t, \text{Monoid } m \Rightarrow t a \rightarrow [ma]$

(i) $\lambda l \rightarrow [(x,y) \mid x <- l, y <- l, x /= y]$

~~Eq a~~ $\Rightarrow (\text{Foldable } t, \text{Eq } a) \Rightarrow t a \rightarrow [a,a]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

$(\text{Char}, \text{Bool})$

(k) $\text{filterM} (\text{const} [\text{True}, \text{False}])$

~~Eq - Char~~

~~Foldable~~ $t \Rightarrow t a \rightarrow t a$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer: ~~Catenate x/y for every x, y pair in l then return the list of all (x, y) pairs~~ $\text{foo}([1,2,3]) = [(1,1), (1,2), (1,3), (2,1), (2,2), (2,3), (3,1), (3,2), (3,3)]$

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer: ~~reverses~~ appends x to l

`foo CJ = CAJ`

`foo 2 [1,3,4] = [1,3,4,2]`

$\text{foo CJ} = CJ$
 $\text{foo } [C,1,2] = [C,1,2]$
 $\text{foo } [2,1] = [2,1]$
 $\text{foo } [2,1,2] = [2,1,2]$

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`

`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: ~~Prunes~~ Prunes a list of maybes to only the values stored in

`foo [] = []`
`foo [Nothing] = []`

"Just"s

`foo [(Just 1), Nothing] = [1]`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

Answer: ~~replaces the highest value in the list with m~~

Replaces all values lower than m with m

`foo [0,1,2,3] 2 = [2,2,2,3]`

`foo [3,4,5,6] 2 = [3,4,5,6]`

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ □ = return []
dropWhileM p (x:xs) = do
    q <- p x [True, False] always!
    if q then dropWhileM p xs else return (x:xs)

foo :: ??[a] -> m[a]
foo = dropWhileM (const [True, False])
```

Answer: Returns the list of all possible tails of a given list

foo [] = [[]]

foo [1,2] = [[1,2], [2], []]

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

$$\text{weave } (x:xs) (y:ys) = x:y:(\text{weave } xs ys)$$
$$\text{weave } [] [] = []$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\text{toMax } [] = []$$
$$\text{toMax } t = \text{foo } (\underset{\text{maximum}}{\text{max } t})$$

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (<=), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>=) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

Talha Mahib

CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
 - (iv) None of the above.
- (c) Typeclass laws are enforced by the Haskell compiler.
 - (i) True
 - (ii) False
- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
 - (ii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `Int` and `Char` to generate inputs and test `foo`.
 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

ill-typed

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) `getLine >>= putStrLn`

Applicative IO => Monad IO

(f) `putStrLn 42 >>= putStrLn 43`

ill-typed

(g) `(,) "42"`

$(\text{Char} \rightarrow \text{Char} \rightarrow (\text{Char}, \text{Char})) \rightarrow \text{String}$

(h) `reverse . foldMap return`

$[a] \rightarrow [a]$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

$[a] \rightarrow [(a, a)]$

(j) `let f x = x in (f 'a', f True)`

ill-typed

(k) `filterM (const [True, False])`

$[a] \rightarrow [[a]]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as `[]`, `Nothing`, or `undefined`) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$\lambda x \rightarrow x + 1$

(+1)

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$\lambda x \rightarrow \text{if } x \text{ then } [\text{True}] \text{ else } [\text{False}]$

(c) $a \rightarrow \text{Maybe } b$

Monad maybe $\Rightarrow (\gg=)$

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

liftM2 (a b \rightarrow True) list1 list2

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftM2 & & & undefined

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

(\& & & list \rightarrow map & list)

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

(\& & & \rightarrow undefined)

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

(\& & (a:t) \rightarrow if h==a then a:tail else tail)

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

(\& & \rightarrow getLine)

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

(\& & & \rightarrow uncurry & t)

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo l = [x * x | x <- l, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.
`foo [] = []`
`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo l = [(x,y) | x <- l, y <- l, x /= y]`

Answer:

Makes a list of tuples where no tuple has duplicates

(c) `foo :: a -> [a] -> [a]`
`foo x l = reverse (x : reverse l)`

Answer:

Reverses l, then prepends x, then reverses again

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer:

Filters out elements that result to Nothing when passed into f

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m' : xs')`
`where (m', xs') = foo xs m`

Answer:

Returns a tuple where the first element is the max of the list, and the second element is a list of m

(f) *Bonus!*

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)

foo :: ?? [a] -> [a]
foo = dropWhileM (const [True, False])
```

Answer:

Powered by 

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:
- ```
weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]
```

You can assume that the lists have the same length.

$$\begin{aligned} \text{weave } [] &= [] \\ \text{weave } (h:t) &(h':t') = h : (h' : (\text{weave } t t')) \end{aligned}$$

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:
- ```
toMax [1,4,2,5,3] = [5,5,5,5,5]
```

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

$$\text{toMax list} = \text{let } m = \text{maximum list} \text{ in } \text{map } (\lambda h \rightarrow m) \text{ list}$$

Bonus:

$$\text{toMax Aux} :: [a] \rightarrow [a] \rightarrow [a]$$

$$\text{toMaxAux} [] t = [] ++ t$$

$$\begin{aligned} \text{toMaxAux} (h:t) &(h':t') = \text{let } m = \text{if } h > h' \text{ then } [h] \text{ else } [h'] \text{ in} \\ &(\text{toMaxAux } t t') ++ m \end{aligned}$$

Typeclass Definitions

```
class Semigroup a where
  (++) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (=<), (>=), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (<*>) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```

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CMSC 488B: Midterm Exam (Spring 2022)

Question 1 (20 points)

Part 1 - 8pts

For each of the following questions, select the appropriate response.

- (a) There exist OCaml programs that don't terminate, whose Haskell equivalents do terminate.
 - (i) True
 - (ii) False
- (b) The constraint `Semigroup a => Monoid a` implies that:
 - (i) Every type that has a `Semigroup` instance, also has a `Monoid` instance.
 - (ii) Every type that has a `Monoid` instance, also has a `Semigroup` instance.
 - (iii) Both of the above.
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- (c) Typeclass laws are enforced by the Haskell compiler.
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- (d) Given a function `foo :: Int -> Char -> Bool`, consider the function call `quickCheck foo`. Select what will happen if that was inside a `main` function:
 - (i) The program will fail to typecheck.
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 - (iii) After typeclass resolution, the resulting program will use the `Arbitrary` instance for `()` as default to generate inputs and test `foo`.
 - (iv) Haskell will require user-provided generators for integers and characters in order to run the tests

Part 2 - 12pts

The standard library defines `foldMap` with the following type:

```
foldMap :: (Monoid m, Foldable t) => (a -> m) -> t a -> m
```

For each of the following `foldMap` calls, select all options that are true:

(a) `foldMap (:) [] [1..10]`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

(b) `foldMap show "123456"`

- (i) The expression will fail to typecheck.
- (ii) The monoid in this call is `Int`.
- (iii) The monoid in this call is `[Int]`.
- (iv) The monoid in this call is `String`.
- (v) The expression is equivalent to the identity function.
- (vi) The foldable in this call is `[]` - the instance for lists.

Question 2 (20 points)

For each of the Haskell expressions below, write their (most general) Haskell type or “ill-typed” if it contains a type error. The type signatures of all functions below are provided in the appendix at the end.

(a) $\lambda x \rightarrow x$

Example answer:

$a \rightarrow a$

(b) $\lambda x y \rightarrow (x, y)$

$a \rightarrow b \rightarrow (a, b)$

(c) $\lambda x y \rightarrow \text{if } x == y \text{ then show } x \text{ else show } (x, y)$

Show a $\Rightarrow a \rightarrow a \rightarrow \text{String}$

(d) $\lambda x l \rightarrow x : l ++ l ++ [x]$

$a \rightarrow [a] \rightarrow [a]$

(e) $\text{getLine} >= \text{putStrLn}$

String \rightarrow IO String \rightarrow IO ()

(f) $\text{putStrLn "42"} >= \text{putStrLn "43"}$

IO ()

(g) $(,) "42"$

$a \rightarrow (a, \text{String})$

(h) $\text{reverse} . \text{foldMap return}$

Monad m $\Rightarrow [a] \rightarrow m a$

(i) $\lambda l \rightarrow [(x, y) \mid x <- l, y <- l, x /= y]$

Ord a $\Rightarrow a \rightarrow [(a, a)]$

(j) $\text{let } f x = x \text{ in } (f 'a', f \text{ True})$

(Char, Bool)

(k) $\text{filterM} (\text{const [True, False]})$

Monad m $\Rightarrow [a] \rightarrow m [a]$

Question 3 (20 points)

For each of the types below, write a Haskell expression that has that type. Don't write trivial expressions (such as [], Nothing, or undefined) unless there is no other option. You can use any function from the appendix, do syntax, list comprehensions, or any valid Haskell.

(a) $\text{Int} \rightarrow \text{Int}$

Example answers:

$$\lambda x \rightarrow x + 1$$

(+1)

(b) $\text{Bool} \rightarrow [\text{Bool}]$

$$\lambda x \rightarrow \text{replicate } 5 \ x$$

(c) $a \rightarrow \text{Maybe } b$

Just

(d) $(\text{Int} \rightarrow \text{Char} \rightarrow \text{Bool}) \rightarrow [\text{Int}] \rightarrow [\text{Char}] \rightarrow [\text{Bool}]$

$$\lambda x \ y \rightarrow x > 0 \ \& \ y == 'a'$$

(e) $(a \rightarrow b \rightarrow c) \rightarrow \text{IO } a \rightarrow \text{IO } b \rightarrow \text{IO } c$

liftA2

(f) $(a \rightarrow b) \rightarrow (b \rightarrow \text{Bool}) \rightarrow [a] \rightarrow [b]$

$$\lambda f \ g \ l \rightarrow \text{foldr } (g \cdot f) \ [] \ l$$

(g) $\text{Maybe } a \rightarrow (a \rightarrow \text{Gen } b) \rightarrow \text{Gen } (a, b)$

$$\lambda x \ f \rightarrow \text{do. } \begin{cases} y \in x \\ z \in f \end{cases} \ \text{return } (y, z)$$

(h) $\text{Eq } a \Rightarrow a \rightarrow [a] \rightarrow [a]$

$$\lambda x \ l \rightarrow \text{case } l \ \text{of} \ \begin{cases} [] \rightarrow [] \\ (h : r) \rightarrow \text{if } x == h \ \text{then } r \ \text{else } [x] \end{cases}$$

(i) $\text{Show } a \Rightarrow [a] \rightarrow \text{IO String}$

$$\lambda l \rightarrow \text{case } l \ \text{of} \ \begin{cases} [] \rightarrow \text{do getLine} \\ (h : t) \rightarrow \text{do putStrLn } h \ \text{getLine} \end{cases}$$

(j) $(a, b) \rightarrow (a \rightarrow b \rightarrow c) \rightarrow c$

$$\lambda (x, y) \ f \rightarrow f \ x \ y$$

Question 4 (20 points + 10pt bonus!)

For each of the following functions, write down a short description of what `foo` does, and some output examples.

(a) `foo :: [Int] -> [Int]`
`foo 1 = [x * x | x <- 1, x > 0]`

Example answer:

Calculates the squares of all positive numbers in a list.

`foo [] = []`

`foo [1,0,2,-1] = [1, 4]`

(b) `foo :: [Int] -> [Int]`
`foo 1 = [(x,y) | x <- 1, y <- 1, x /= y]`

Answer: Constructs a list of all pairs of non-equal integers in a list.

`foo [1,2] = [(1,2), (2,1)]`

(c) `foo :: a -> [a] -> [a]`
`foo x 1 = reverse (x : reverse 1)`

Answer: Adds an element to the end of a list

`foo 3 [1, 2] = [1, 2, 3]`
`foo 'c' "abc" = "abc"`

(d) `bar :: (a -> Maybe b) -> [a] -> [b]`
`bar [] = []`
`bar f (x:xs) =`
`let rs = bar f xs in`
`case f x of`
`Nothing -> rs`
`Just r -> r:rs`

`foo :: [Maybe a] -> [a]`
`foo = bar id`

Answer: Removes the maybe monad from a list.

`foo [Nothing] = []`
`foo [Maybe 2, Nothing, Maybe 4] = [2,4]`
`foo [] = []`

(e) `foo :: [Int] -> Int -> (Int, [Int])`
`foo [] m = (m, [])`
`foo [x] m = (x, [m])`
`foo (x : xs) m = (max m' x, m : xs')`
`where (m', xs') = foo xs m`

*Answer: On non-empty list: replace all elements of the list with m and return (largest element in list, list of ms.)
On empty list: return (m, [])*

`foo [] 2 = (2, [])`
`foo [2,4,3] 5 = (4, [5,5,5])`

(f) Bonus!

```
dropWhileM :: (Monad m) => (a -> m Bool) -> [a] -> m [a]
dropWhileM _ []      = return []
dropWhileM p (x:xs) = do
    q <- p x
    if q then dropWhileM p xs else return (x:xs)
```

```
foo :: ??  
foo = dropWhileM (const [True, False])
```

Answer: Drops elements from the start of the list, each with a 50% chance, and stops when an element is not dropped.

Example:
↳ [1,2,3] could be const [1,2,3]
or const [3]
but not const [2]

Question 5 (20 points + 10pt bonus!)

Implement the following Haskell functions:

- (a) Implement a function `weave` that given two lists with elements of the same type, returns a list with elements alternating between the two lists. For example:

`weave [1,2,3] [4,5,6] = [1,4,2,5,3,6]`

You can assume that the lists have the same length.

`Weave [] [] = []`

`weave (x:xs) (y:ys) = x : y : (weave xs ys)`

`weave _ _ = error "lists not same length"`

- (b) Implement a function `toMax`, that given a non-empty list of integers, returns a list of the same length, where each element has been replaced by the maximum element of the list. For example:

`toMax [1,4,2,5,3] = [5,5,5,5,5]`

You can use the `foo` function of problem (4e) if it helps.

BONUS: Implement `toMax` so that it only traverses a list once!

`toMax xs = replicate (length xs) (maximum xs)`

Typeclass Definitions

```
class Semigroup a where
  (<>) :: a -> a -> a

class Semigroup m => Monoid m where
  mempty :: m

class Show a where
  show :: a -> String

class Eq a where
  (==) :: a -> a -> Bool
  (/=) :: a -> a -> Bool

class Eq a => Ord a where
  compare :: a -> a -> Ordering
  (<), (=<), (=>), (>) :: a -> a -> Bool
  max, min :: a -> a -> a

class Functor f where
  fmap :: (a -> b) -> f a -> f b

class Functor f => Applicative f where
  pure :: a -> f a
  (=>*) :: f (a -> b) -> f a -> f b

class Applicative m => Monad m where
  return :: a -> m a
  (=>>) :: m a -> (a -> m b) -> m b

class Foldable t where
  foldMap :: Monoid m => (a -> m) -> t a -> m

class Arbitrary a where
  arbitrary :: Gen a
  shrink :: a -> [a]

class Num a where
  (+), (-), (*) :: a -> a -> a
  negate :: a -> a
  abs :: a -> a
  signum :: a -> a
  fromInteger :: Integer -> a
```