

# PLUMED

## The heart of the matter

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# The equation

**Probability**

$$F(\mathbf{s}) = -k_B T \ln P(\mathbf{s})$$

**Free energy**

**Temperature**

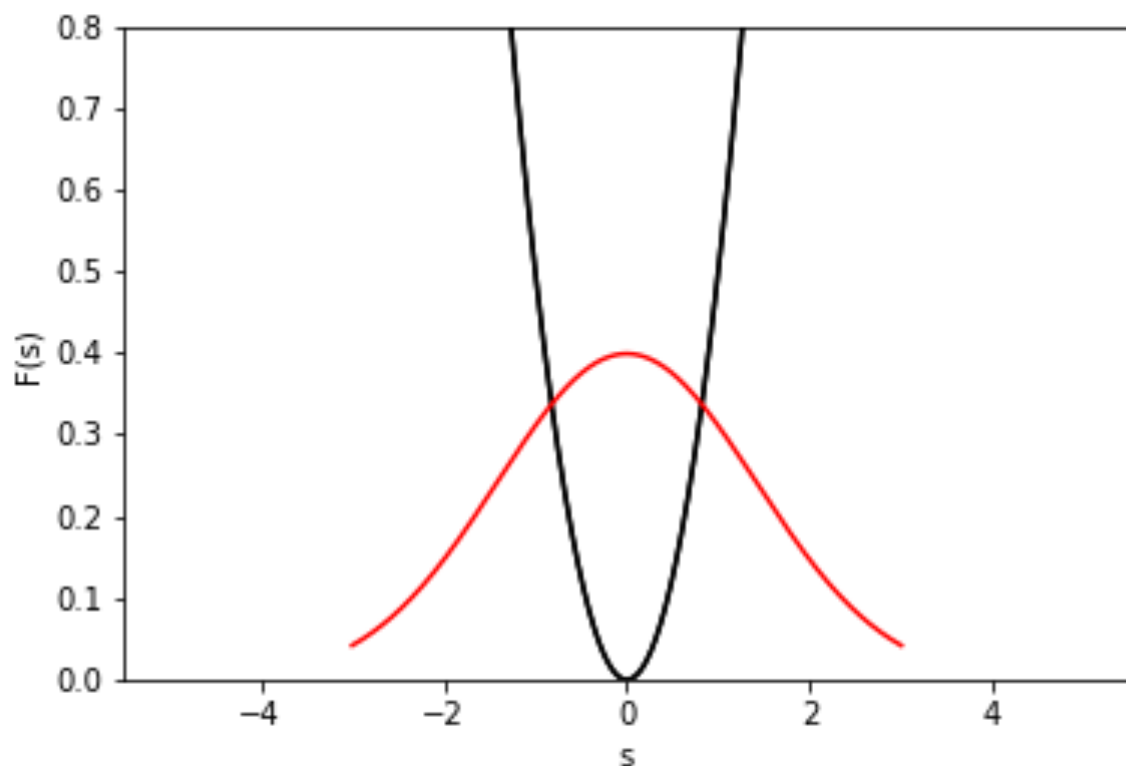
# Undergraduate (classical) statistical mechanics

$$F(\mathbf{s}) = -k_B T \ln P(\mathbf{s})$$

$$F(s) - F(a) = \cancel{F'(a)}(s - a) + \frac{F''(a)}{2!}(s - a)^2 + \dots$$

0

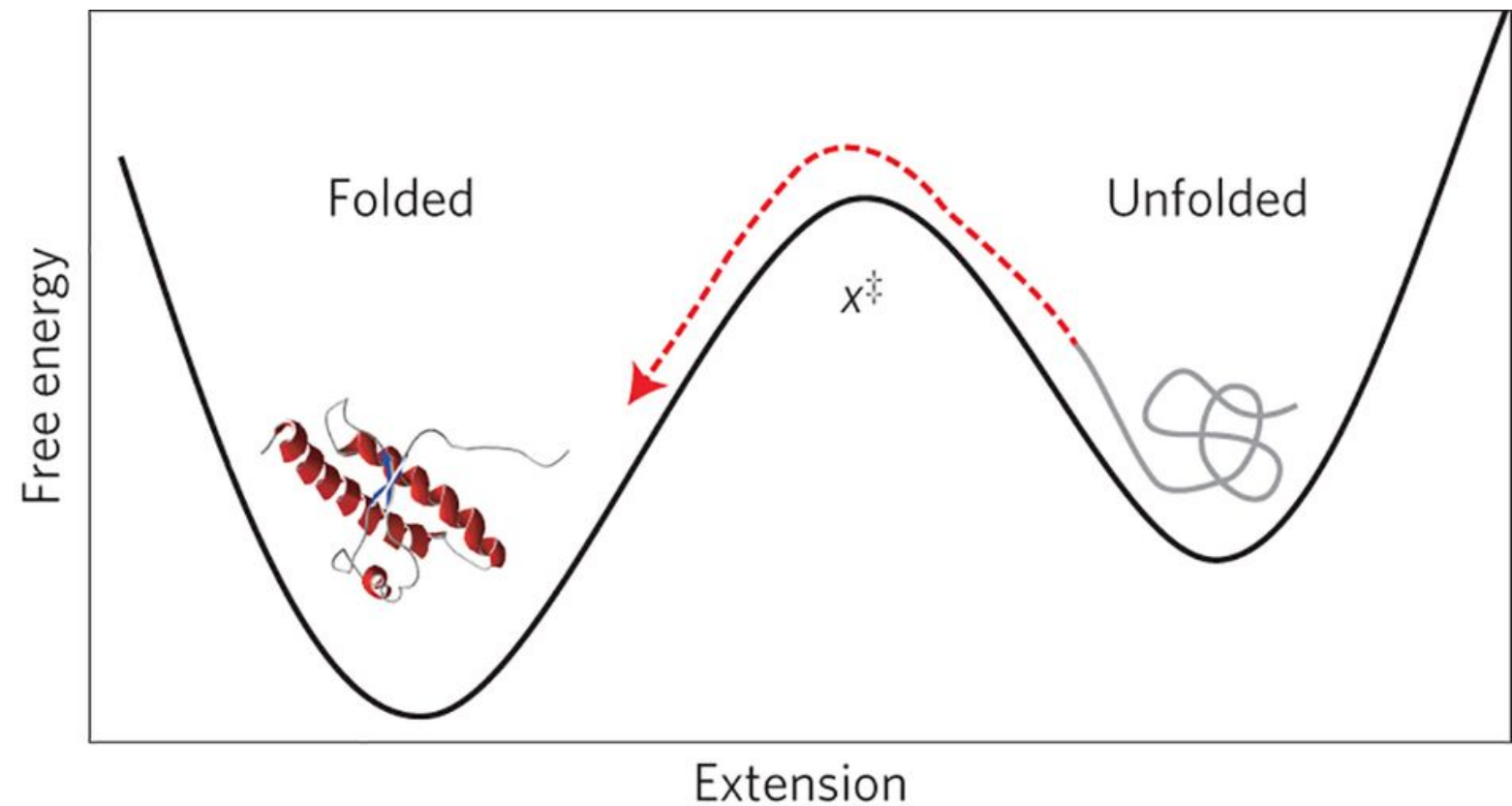
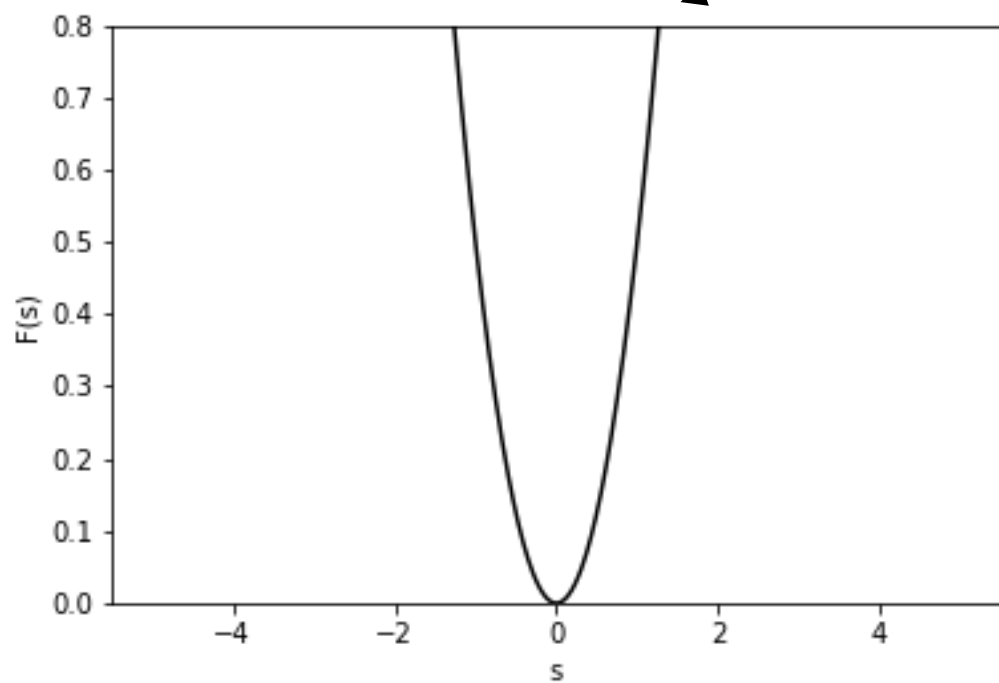
**What we use MD to investigate**



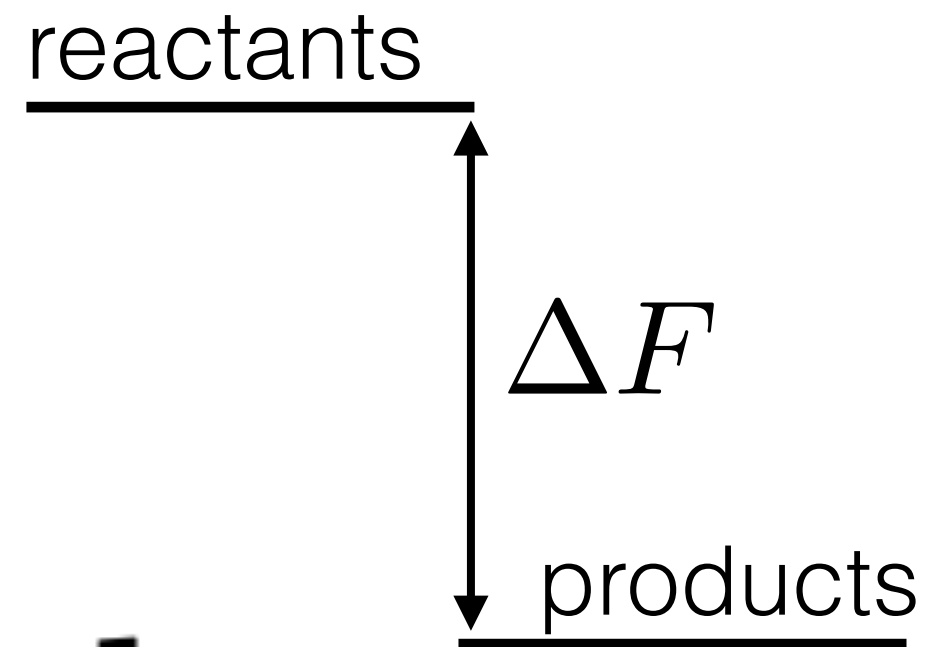
$$P(s) \propto \exp \left( -\frac{F''(a)(s - a)^2}{2k_B T} \right)$$

# Why we care about anharmonic effects

$$F(s) - F(a) = F'(a)(s - a) + \frac{F''(a)}{2!}(s - a)^2 + \dots$$



# The reaction free energy is an integral



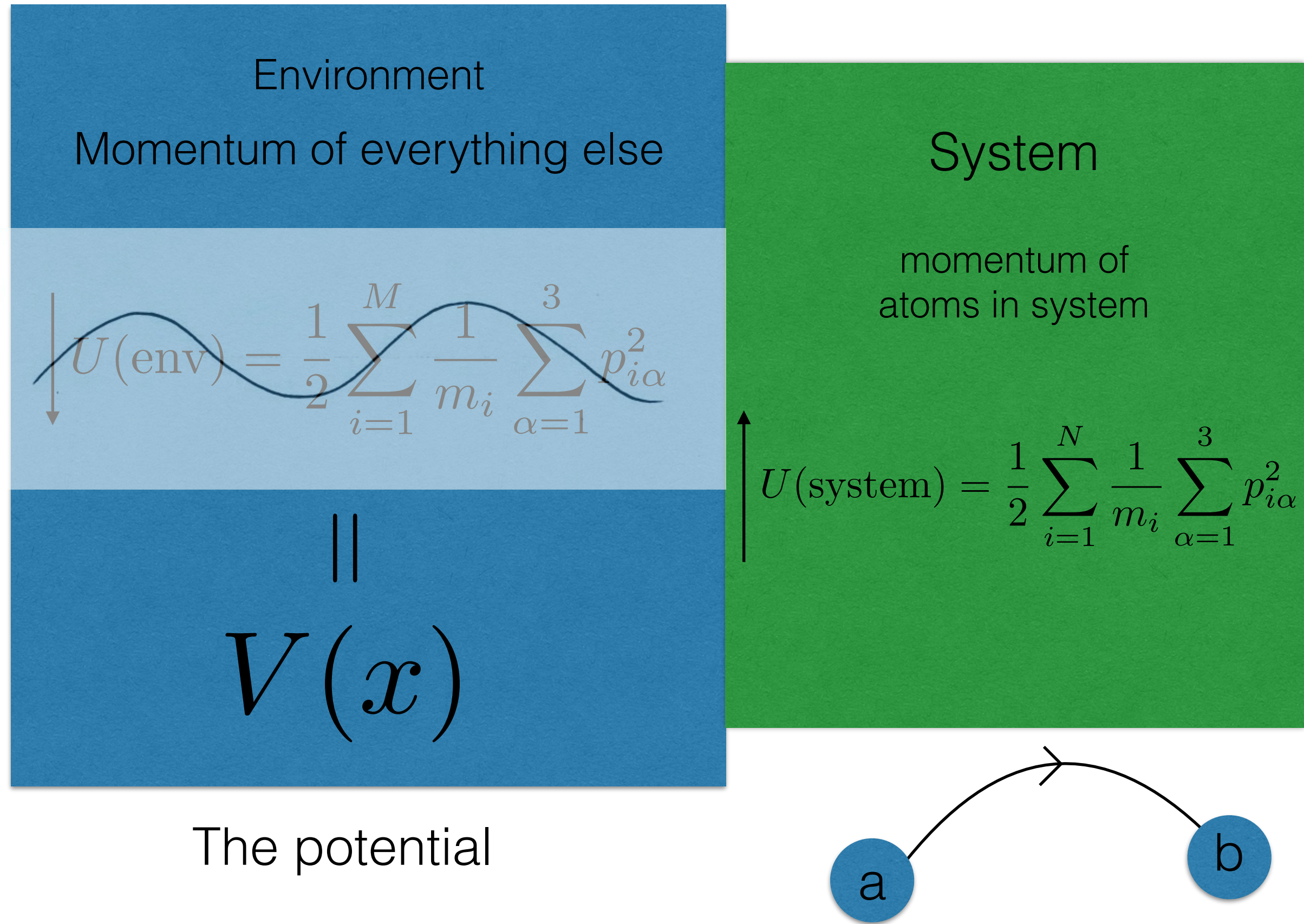
$\int_{\text{reactants}}^{\text{products}}$

$F'(x)dx$

derivative of a  
thermodynamic  
potential

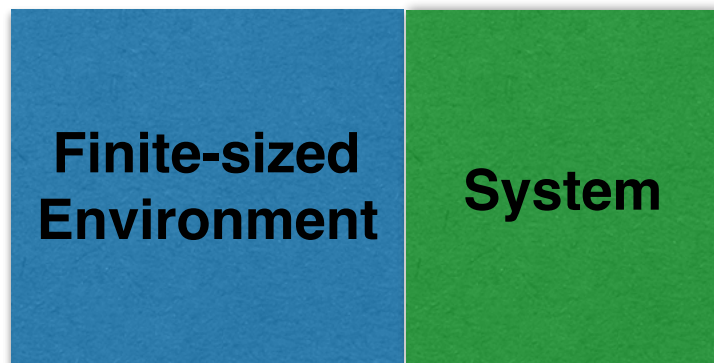
The reaction coordinate

# The free energy





# System and Environment



$$E = V(\mathbf{x}) + K(\mathbf{p})$$

environment

system

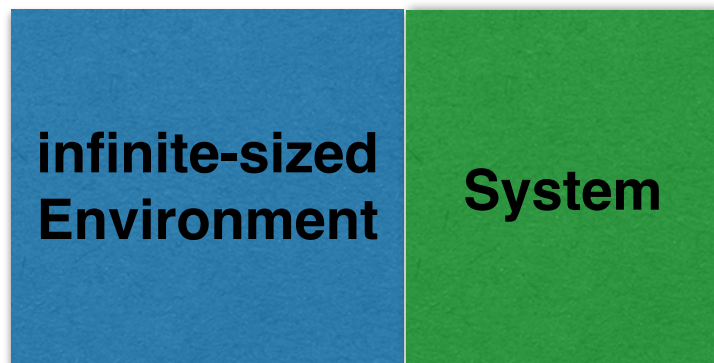
fixed

**All extensive quantities for the universe are fixed**

$$P(\xi) = \frac{\int \int \cdots \int \delta(V(\mathbf{x}, \mathbf{p}) - V) \delta(E(\mathbf{x}, \mathbf{p}) - E) \delta(\xi(\mathbf{x}) - \xi) dx_1 dx_1 \dots dx_N dp_1 dp_1 \dots dx_N}{\int \int \cdots \int \delta(V(\mathbf{x}, \mathbf{p}) - V) \delta(E(\mathbf{x}, \mathbf{p}) - E) dx_1 dx_1 \dots dx_N dp_1 dp_1 \dots dx_N}$$

**“count”      must have volume V      must have energy E      must have right cv      all micro states**  
**Total number of states with energy E and volume V**

# System and Environment



**Intensive variables  
for system are fixed**

**must be finite**

$$\langle E \rangle = \int E(\mathbf{x}, \mathbf{p}) P(\mathbf{x}, \mathbf{p}) dx_1 dx_2 \dots dx_N dp_1 dp_2 \dots dp_N$$

**lots of maths\***

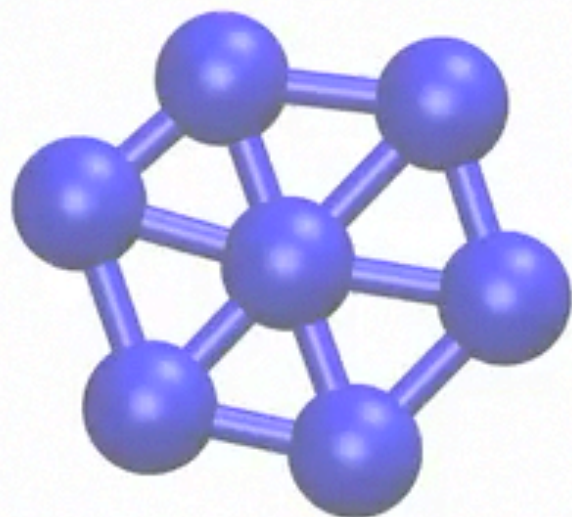
**The distribution sampled by  
molecular dynamics if we run with a  
(suitable) thermostat**

$$P(\mathbf{x}, \mathbf{p}) \propto e^{-\frac{E(\mathbf{x}, \mathbf{p})}{k_B T}}$$

$$P(\xi) = \frac{\int \int \dots \int \delta(V(\mathbf{x}) - V) \delta(\xi(\mathbf{x}) - \xi) e^{-\frac{E(\mathbf{x}, \mathbf{p})}{k_B T}} dx_1 dx_2 \dots dx_N dp_1 dp_2 \dots dp_N}{\int \int \dots \int \delta(V(\mathbf{x}) - V) e^{-\frac{E(\mathbf{x}, \mathbf{p})}{k_B T}} dx_1 dx_2 \dots dx_N dp_1 dp_2 \dots dp_N} \approx \frac{1}{T} \sum_{t=0}^T \Theta(|s(t) - s'| < \epsilon)$$

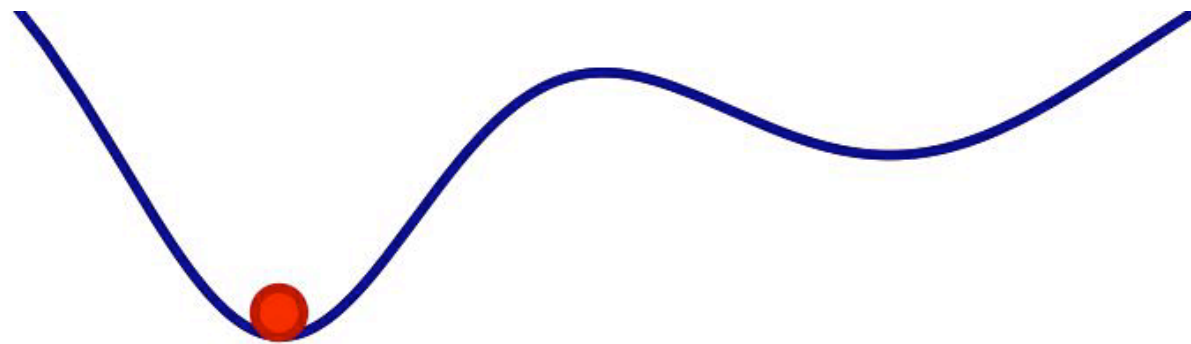
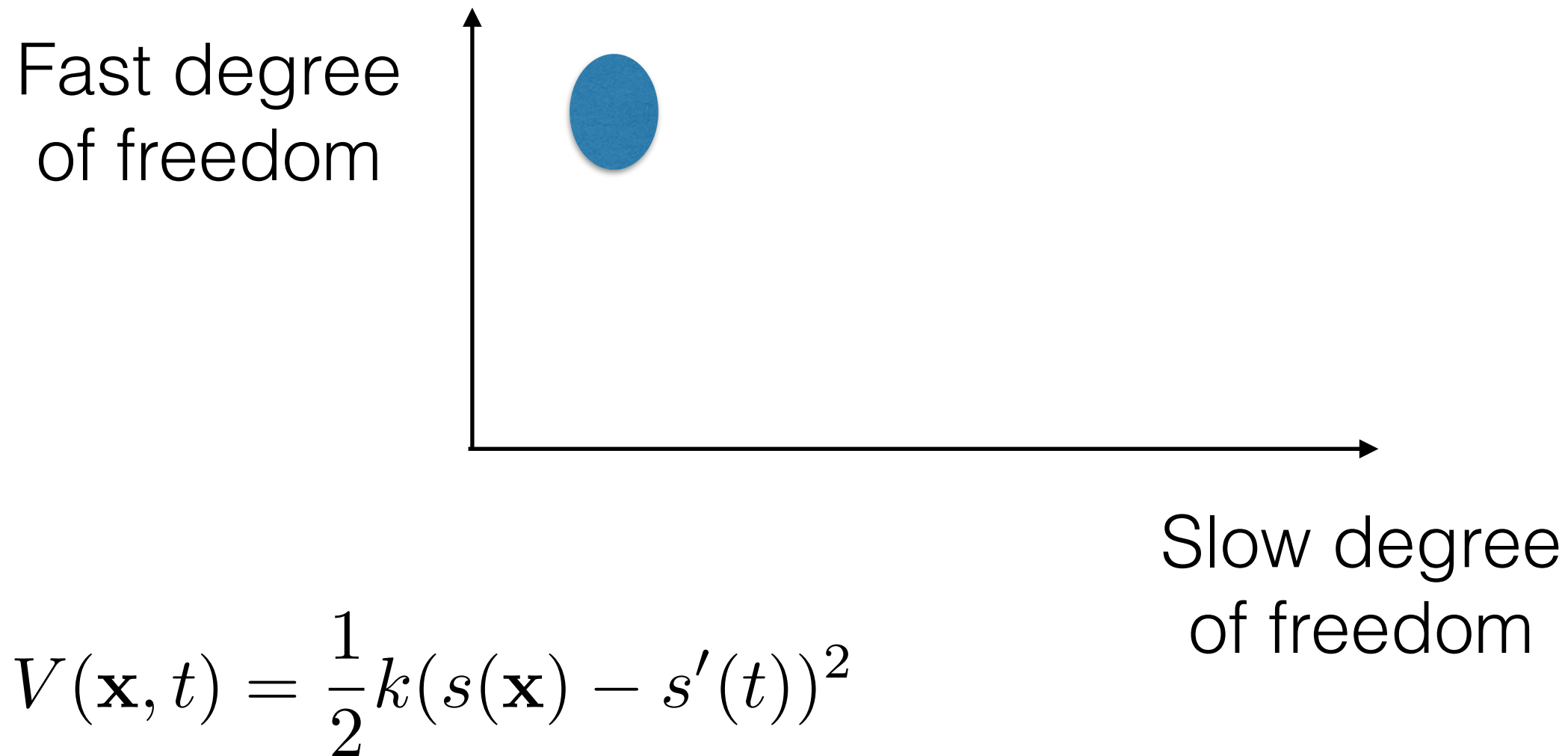
**\*See videos: <http://gtribello.github.io/mathNET/AMA40043-understand.html>  
<http://gtribello.github.io/mathNET/AMA40042-understand.html>**





$$F(\mathbf{x}) = -k_B T \log[P(\mathbf{x})]$$

# Non-ergodicity (life is a b\*#ch)



# The equation

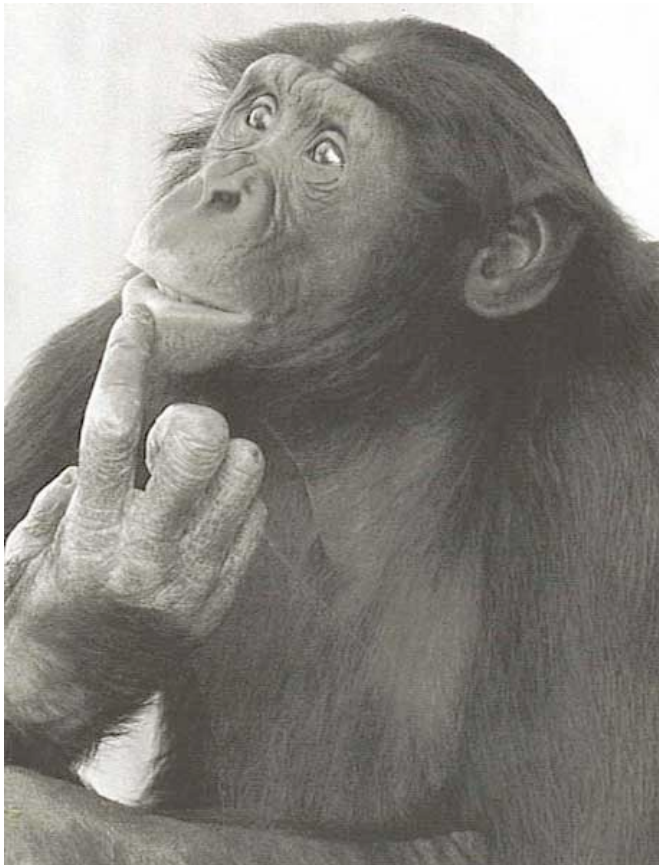
**Probability**

$$F(\mathbf{s}) = -k_B T \ln P(\mathbf{s})$$

**Free energy**

**Temperature**

# The only question left



**What collective variable  
should I use?**

$$F(\mathbf{s}) = -k_B T \ln P(\mathbf{s})$$

# More videos

**Random variables:**

**<http://gtribello.github.io/mathNET/SOR30121-understand.html>**

**Central limit theorem:**

**<http://gtribello.github.io/mathNET/SOR30121-understand.html>**

**Markov chains:**

**<http://gtribello.github.io/mathNET/SOR30123-understand.html>**

**<http://gtribello.github.io/mathNET/SOR30124-understand.html>**

**Thermodynamics:**

**<http://gtribello.github.io/mathNET/AMA40041-overview.html>**

**Statistical Mechanics:**

**<http://gtribello.github.io/mathNET/AMA40043-understand.html>**

**<http://gtribello.github.io/mathNET/AMA40042-understand.html>**