Contents

1	Ran	nking Algorithm, v1	1
	1.1	Description of the problem]
	1.2	Rankings	2
	1.3	Greedy Algorithm	2

1 Ranking Algorithm, v1

1.1 Description of the problem

The ranking algorithm in myocardio operates on "exercises". In nonformal language an exercise has

- 1. a time when it was were last executed, which might be "never"
- 2. a list of associated muscles involved (with every muscle involved to the same degree)

To model this mathematically, assume we have a (finite) set **Muscle** of muscles. For example:

$$\mathbf{Muscle} = \{ \mathbf{biceps}, \mathbf{triceps} \} \tag{1}$$

We'd like to model a point in time as a value from \mathbb{N} , the natural numbers. But we have to deal with the special value "never". So, we define $T = \mathbb{N} \cup \{\bot\}$, where \bot stands for this special value. This makes T a set that is not a ring anymore, it's not even ordered.

Modeling all possible exercises as simply

$$E = \mathcal{P}(\mathbf{Muscle}) \tag{2}$$

where \mathcal{P} is the power set operator — doesn't work, though, because you might have two exercises which have the same muscle groups but aren't the same.

So we assume another (finite) set $\mathbf{Exercise}$ of exercise names. We then assign two functions

 $m: \mathbf{Exercise} \to \mathcal{P}(\mathbf{Muscle})$

descibing the muscles of an exercise and

 $\ell : \mathbf{Exercise} \to T$

describing the latest execution time point.

The **input** of the algorithm is the three-tuple (**Exercise**, m, l). Its **output** should be a **reordering of the exercises**, which we can model with a function $o : \mathbb{N} \to \mathbf{Exercise}$.

1.2 Rankings

We define two rankings for an exercise. One ranks by time ρ_t and one by complexity ρ_c (i.e. the number of muscles involved).

Ranking by complexity is easy: $\rho_c(e) = |m(e)|$, it's just the number of muscles involved. More muscles equals a better ranking.

Ranking by time is a bit more complicated. We want to interpolate all exercises' times by the latest/oldest one. However, we cannot interpolate across T because we have no ordering (because of the special element \bot).

To fix this, we model ρ_t as follows:

- 1. $\rho_t(e) = 0$ for all $e \in \mathbf{Exercise}$ if $\bot \in \ell(\mathbf{Exercise})$. Meaning the ranking is most if one of the exercises has never been performed.
- 2. Otherwise determine the minimum and maximum time stamps: l_{\min}, l_{\max} (which is now well-defined). Linearly interpolate between the two and rescale to [0,100].

This gives exercises that were just executed a low ranking (i.e. close to 0) and exercises that weren't performed recently higher ones (more towards 100).

The total ranking for an exercise is simply $\rho(e) = \rho_t(e) + \rho_c(e)$

1.3 Greedy Algorithm

We could now just rank all our exercises by ρ . However, two exercises that feature the same or similar muscle groups could be included back-to-back in this plan, and that's a non-goal.

In order to rank the exercises, we employ a greedy algorithm in order to construct an order o_i for exercises (where $i \in \mathbb{N}$). We assume $|\mathbf{Exercise}| \geq 2$.

- 1. Let i = 0. Take the exercise $e_i = \operatorname{argmax}_{e \in \mathbf{Exercise}} \{ \rho(e) \}$ with the highest ranking. Define $o_i = e_i$ Define the rest $R_i = \mathbf{Exercise} \setminus \{o_i\}$
- 2. If $|R_i| = \{e_{i+1}\}$, choose $o_{i+1} = e_{i+1}$ and terminate.
- 3. Define $d_i: R_i \to \mathbb{N}$ as $d_i(e) = |m(e) \cap m(o_i)|$. This is a ranking of how many muscle groups are the same as the first exercise (d for "Durchschnitt", meaning intersection in German).

4. Define $D_i: \mathbb{N} \to \mathcal{P}(R_i)$ as

$$D_i(n) = \{ e \in R_i | d_i(e) = n \}$$
 (3)

This is a ranking of how many muscle groups are the same as the first exercise.

- 5. Define $N_i = \operatorname{argmin}_{n \in \mathbb{N}} \{D_i(n) | D_i(n) \neq \emptyset\}$. Thus, $D_i(N_i)$ will be the group of exercises with the least number of muscles in common with the chosen one, o_i . This exists, because we assumed to have at least one exercise left in step 2, and even if all exercises have all muscles in common, we will have a group for them.
- 6. From $D_i(N_i)$, choose the exercise with the highest rank ρ as the next exercise o_{i+1} . Define $R_{i+1} = R_i \setminus \{o_{i+1}\}$. Go to step 2 (reindexing $i \to i+1$).

This algorithm will terminate after at most $|\mathbf{Exercise}|$ steps, since $|R_i|$ is strictly monotonically decreasing as i increases.