$$f(n,\,k) = 1 + \sum_{j=1}^{k} \frac{1}{1 + \sum_{i=1}^{j} 2^{i-1} \frac{(j+i)!}{(j-i)!\,(2\,i)!}\, n^i}$$

$$\sqrt{3} = \lim_{k\to\infty} f(1,\,k)$$

$$\sqrt{2} = \lim_{k\to\infty} f(2,\,k)$$

$$\sqrt{n\,(n+2)} = n \lim_{k\to\infty} f(n,\,k)$$

$$\sqrt{n} = \frac{x-1}{y} \lim_{k\to\infty} f(x-1,\,k)$$

where $x$, $y$ is solution to Pell equation

$$x^2 - n\,y^2 = 1$$

$$\frac{x-1}{y} f(x-1,\,k) \equiv (0 \bmod n)$$