$$a(n,0) = n \quad a(n,1) = \frac{n}{n+1} \quad a(n,i+1) = \frac{a(n,i)^2}{a(n,i-1)(a(n,i)+1)}$$

$$f(n,k) = \frac{1}{n}\sum_{i=0}^{k} a(n,i) = 1 + \sum_{j=1}^{k} \frac{1}{1 + \sum_{i=1}^{j} 2^{i-1} \frac{(j+i)!}{(j-i)!\,(2\,i)!}\, n^i}$$

$$\sqrt{3} = \lim_{k\to\infty} f(1,k)$$

$$\sqrt{2} = \lim_{k\to\infty} f(2,k)$$

$$\sqrt{n\,(n+2)} = n \lim_{k\to\infty} f(n,k)$$

$$\sqrt{n} = \frac{x-1}{y} \lim_{k\to\infty} f(x-1,k)$$

where x, y is solution to Pell equation

$$x^2 - n\,y^2 = 1$$

$$\frac{x-1}{y} f(x-1,k) \equiv (0 \bmod n)$$