Code ▼

Statistical Analysis Handout for the Market Segmentation Lecture

- Cluster Analysis
- · Reading and outputing data
- · Hierarchical Clustering Analysis
 - The four-cluster solution
 - Three-Cluster Solution
 - Number of Clusters
 - Targeting the Clusters/segments
 - Demographics
 - Choice
- K-Means
 - · Three Cluster Solution obtained using K-Means
 - Find the optimal number of clusters
 - Results Comparison
 - Demographics
 - Choice
 - Hierarchical Clustering
- Latent Class Analysis
 - · Find the optimal model
 - Results Comparison
 - Demographics
 - Choice
 - Hierarchical Clustering
 - K-Means Clustering

This handout is designed to help you replicate the statistical analyses that were covered in the Market Segmentation lecture. You should have this handout handy when you work on the (Market Segmentation) programming assignment where you will be asked to apply the learnings from the lecture on a different dataset.

Cluster Analysis

Cluster Analysis refers to a class of techniques used to classify individuals into groups such that:

- Individuals within a group should be as similar as possible
- · Individuals belonging to different groups should be as dissimilar as possible

This handout shows three different cluster analysis techniques:

- 1. Hierarchical clustering
- 2. K-Means
- 3. Latent Class Analysis

In order to run these statistical methods, you need to install these R packages:

- NbClust
- mclust
- gmodels

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```
set.seed(1990)
library(NbClust)
library(mclust)
library(gmodels)
```

Reading and outputing data

The dataset includes data from 73 students (24 MBAs and 49 undergrads). These students were asked to allocate 100 points across six automobile attributes (Trendy, Styling, Reliability, Sportiness, Performance, and Comfort) in a way that reflects their importance in the purchase decision of which car to buy. We use this dataset to answer the following questions:

- 1. Are there different benefit segments among this student population?
- 2. How many segments?
- 3. How are they different in their constant-sum allocation?
- 4. How can we transform this information into actionable levers from a managerial standpoint?

Let us start by reading the data. Remember to start by setting the appropriate directory using the following code

```
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```

```
setwd("your_directory")
```

We can now read the raw data:

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```
seg_data <- read.csv(file = "SegmentationData.csv",row.names=1)
head(seg_data)</pre>
```

Hierarchical Clustering Analysis

Hierarchical Clustering Analysis is one of the most popular technique used for market segmentation. It is a numerical procedure which attempts to separate a set of observations into clusters from the bottom-up by joining single individuals sequentially until we obtain one large cluster. Hence, this technique doesn't require the pre-specification of the number of clusters, which can be assessed through the "dendogram" (a tree-like representation of the data).

More specifically, the algorithm works as follow:

- 1. Each respondent is initially assigned to his or her own cluster
- 2. Identify the distance between each cluster (intially between pairs of respondents)
- 3. The two closest clusters are combined into one
- 4. Repeat steps 2 and 3 until there is one unique cluster containing all the observations
- 5. Represent the clusters in a dendogram

A key aspect of hierarchical clustering consists in choosing how to compute the distance between two clusters. Is it equal to the maximal distance between two points from each of these clusters? Or the minimal distance? What about the distance between two points? In this handout, we will use Ward's criterion which aims to minimize the total variance within-cluster. To do so, we use the R function hclust. We start by standardizing the data so that every variable is on the same scale. We then compute the euclidean distance between observations.

```
Hide
```

```
std_seg_data <- scale(seg_data[,c("Trendy", "Styling", "Reliability", "Sportiness", "Performance", "Comfort"
)])
dist <- dist(std_seg_data, method = "euclidean")
as.matrix(dist)[1:5,1:5]</pre>
```

```
1 2 3 4 5

1 0.000000 3.730216 2.802191 1.775616 2.746615

2 3.730216 0.000000 4.218662 3.017462 2.984534

3 2.802191 4.218662 0.000000 1.974683 3.331082

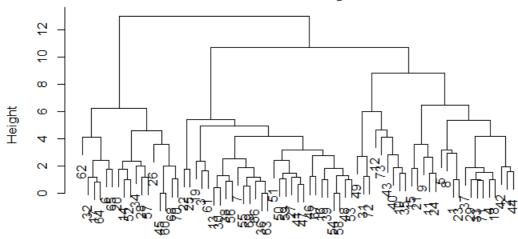
4 1.775616 3.017462 1.974683 0.000000 2.924141

5 2.746615 2.984534 3.331082 2.924141 0.000000
```

We now use the function hclust() to apply hierarchical clustering on our data. We use the Ward criterion which aims to minimize the withincluster variance. \ We obtain the dendogram below which can help us decide the number of clusters to retain. This number seems to be either 3 or 4. \ Note: It is important to set the seed to a specific value. This way you would always get the same labeling of the clusters. Otherwise, cluster 1 in one analysis may correspond to cluster 3 in another.

```
set.seed(1990)
clust <- hclust(dist, method = "ward.D2")
plot(clust)</pre>
```

Cluster Dendrogram



dist hclust (*, "ward.D2")

The four-cluster solution

We start by 4 clusters as we see below:

set.seed(1990)
clust <- hclust(dist, method = "ward.D2")
plot(clust)
h_cluster <- cutree(clust, 4)
rect.hclust(clust, k=4, border="red")</pre>

dist hclust (*, "ward.D2")

Let us now look at some description of this clustering. The table below informs us with the number of individuals in each cluster:

```
h_cluster
1 2 3 4
18 29 17 9
```

The table below reports the profiles of the four clusters (i.e., the clustering variables means by cluster). Looking at this table, we can describe the clusters as follows:

- 1. Cluster 1 values reliability and Performance
- 2. Cluster 2 values Sportiness and Comfort
- 3. Cluster 3 values Trendiness and Style
- 4. Cluster 4 values Style and Sportiness

Hence, it seems that Cluster 4 is a combination of Clusters 2 and 3. This suggests that 3 clusters may be better at capturing the heterogeneity of the subjects in this dataset.

Hide

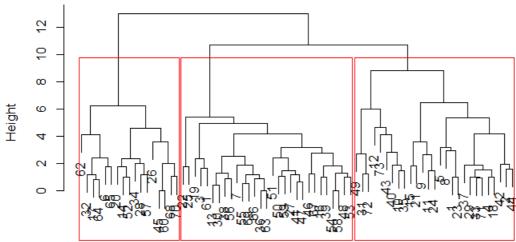
```
hclust_summary <- aggregate(std_seg_data[,c("Trendy", "Styling", "Reliability", "Sportiness", "Performance",
"Comfort")],by=list(h_cluster),FUN=mean)
hclust_summary</pre>
```

Three-Cluster Solution

Hide

```
plot(clust)
h_cluster <- cutree(clust, 3)
rect.hclust(clust, k=3, border="red")</pre>
```





dist hclust (*, "ward.D2")

Hide

```
table(h_cluster)
```

```
h_cluster
1 2 3
27 29 17
```

Hide

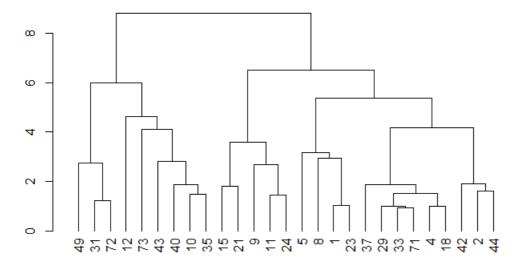
```
hclust_summary <- aggregate(std_seg_data[,c("Trendy", "Styling", "Reliability", "Sportiness", "Performance",
"Comfort")],by=list(h_cluster),FUN=mean)
hclust_summary</pre>
```

This solution seems to have clusters of similar sizes. In addition, we can easily caracterize each of them. The first cluster cares about Performance and Reliability while Cluster 2 values Comfort and Sportiness. Finally, the third cluster cares about the appearance. Below, we rename those clusters according to their characteristics.

We can also focus on a given cluster by using the following code. Here the first one on the left:

Hide

```
plot(cut(as.dendrogram(clust), h=9)$lower[[3]])
```



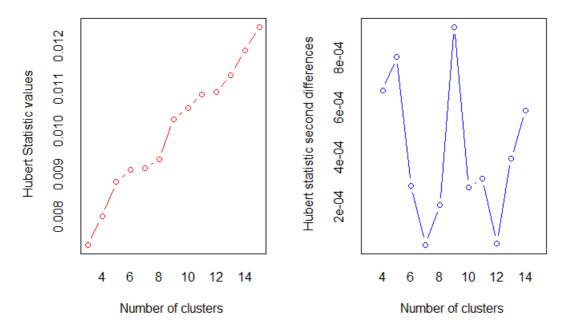
Number of Clusters

As seen above, one can use the dendogram to decide on the appropriate number of clusters. The function NbClust examines all the indexes/criteria used to determine the optimal number of clusters and outputs the optimal number based on the majority rule. Note that since it's a constant-sum allocation, we must use only 5 variables to avoid collinearity issues.

```
set.seed(1990)
NbClust(data=std_seg_data[,1:5], min.nc=3, max.nc=15, index="all", method="ward.D2")
```

***: The Hubert index is a graphical method of determining the number of clusters.

In the plot of Hubert index, we seek a significant knee that corresponds to a significant increase of the value of the measure i.e the significant peak in Hubert index second differences plot.



*** : The D index is a graphical method of determining the number of clusters.

In the plot of D index, we seek a significant knee (the significant peak in Dindex second differences plot) that corresponds to a significant increase of the value of the measure.

- * Among all indices:
- $\,{}^{\star}$ 7 proposed 3 as the best number of clusters
- \star 2 proposed 4 as the best number of clusters
- * 2 proposed 5 as the best number of clusters
- * 2 proposed 6 as the best number of clusters
- * 4 proposed 9 as the best number of clusters
- \star 3 proposed 12 as the best number of clusters
- \star 1 proposed 13 as the best number of clusters
- * 2 proposed 15 as the best number of clusters

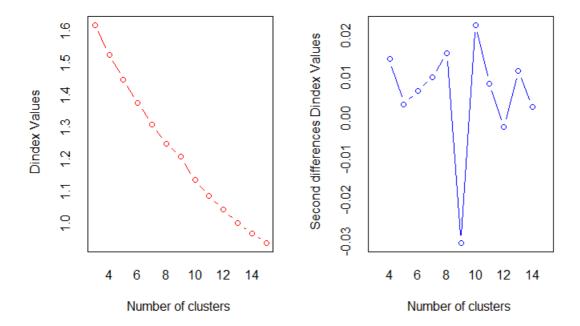
**** Conclusion ****

* According to the majority rule, the best number of clusters is 3

```
$All.index
                                                                                CH Hartigan
                                                                                                                                                                         CCC
                                                                                                                                                                                                                Scott Marriot
                                                                                                                                                                                                                                                                                                                    TrCovW TraceW Friedman Rubin Cindex
ouette Duda Pseudot2 Beale
 3\quad 4.2356\ 19.1708 \qquad 9.1557\ -3.8247\ 125.3102\ 546888028\ 3415.0147\ 232.5975 \qquad 5.4783\ 1.5477\ 0.4026\ 1.6869
0.1987 0.6935 6.6287 1.2967
4 0.3189 17.2540 8.3534 -5.0953 159.1836 611300402 2593.8512 205.6938
                                                                                                                                                                                                                                                                                                                                                                                                                            7.0207 1.7502 0.3825 1.5521
0.2111 0.7443
                                                                                7.2126 1.0261
5 0.5079 16.3550 8.9849 -6.0343 202.7795 525669059 2134.0463 183.4809
                                                                                                                                                                                                                                                                                                                                                                                                                           8.8405 1.9621 0.3560 1.5191
0.2206 0.5410
                                                                                 6.7863 2.3600
  6 \quad 1.2239 \quad 16.3655 \qquad 7.4554 \quad -5.4291 \quad 240.3721 \quad 452300734 \quad 1641.4307 \quad 162.0669 \quad 10.4134 \quad 2.2213 \quad 0.4427 \quad 1.3403 \quad 10.4134 
  .2335 0.3851 11.1763 4.3725
7 \quad 0.9987 \ 16.1533 \quad 6.8924 \ -5.0795 \ 272.0101 \ 399115542 \ 1309.7081 \ 145.8389 \quad 11.7806 \ 2.4685 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 1.1950 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.4429 \ 0.
 .2414 0.6398 6.7563 1.6266
8\quad 0.8637\ 16.0296 \qquad 7.0554\ -4.7518\ 305.5157\ 329419867\ 1063.7346\ 132.0490 \quad 13.4009\ 2.7263\ 0.4268\ 1.1583
 .2445 0.7768 8.9079 0.8713
9 \quad 1.0446 \quad 16.1775 \qquad 6.6456 \quad -4.2941 \quad 344.2663 \quad 245198975 \quad 894.1088 \quad 119.1193 \quad 15.5956 \quad 3.0222 \quad 0.3921 \quad 1.1251 \quad 1.1
 .2550 0.7288 10.7909 1.1258
10\ 0.9691\ 16.3520 \qquad 6.6364\ -3.8667\ 376.7922\ 193878168 \quad 720.2895\ 107.9138 \quad 17.7061\ 3.3360\ 0.3630\ 1.2087
 .2093 0.4678 4.5515 2.8491
11 1.2274 16.6619 5.7269 -3.3850 405.2744 158805977 566.4595 97.6295 19.4921 3.6874 0.4062 1.1481
 .2253 0.3736 5.0300 3.9357
12 1.4727 16.7917 4.4283 -3.0689 430.4837 133803651 464.2880 89.3741 21.0350 4.0280 0.4749 1.0742
 .2312 0.7289 7.8114 1.1113
13\ \ 0.9527\ \ 16.6021 \qquad 4.4291\ \ -3.0135\ \ 462.2226\ \ 101664547 \quad 418.1911 \quad 83.3250 \quad 23.7505\ \ 4.3204\ \ 0.4593\ \ 1.1319
 .2094 0.5973 3.3715 1.7587
14 1.0413 16.5171 4.2289 -2.9133 490.8040 79707962 370.8162 77.5969 25.9989 4.6394 0.4458 1.1454
```

```
2.5155 150.0010 75707502 570.0102 77.0505 25.5505 1.0551 0.1150 1.1151
.2167 0.5413 5.9328 2.3211
15 0.9571 16.4550 4.2829 -2.8227 517.1570 63774583 329.0855 72.4071 28.2284 4.9719 0.4303 1.1209
.2158 1.9784 -0.9891 -1.0319
 Ratkowsky Ball Ptbiserial Frey McClain Dunn Hubert SDindex Dindex SDbw
   5
   6
7
   8
   0.2811 16.5061
                0.5535 0.0382 1.6962 0.2200 0.0094 1.7668 1.2513 0.4271
9
   0.2725 13.2355
                 0.5828 0.9510 1.7837 0.2200 0.0103 1.7508 1.2091 0.4453
10
   0.2644 10.7914
                0.5028 0.0019 2.7486 0.1690 0.0106 2.1167 1.1366 0.4186
                0.5089 0.0231 2.7520 0.1935 0.0109 2.1415 1.0868 0.3715
11
   0.2573 8.8754
   0.2502 7.4478
                0.5125 1.7152 2.7587 0.2300 0.0109 2.0783 1.0454 0.3555
12
   0.2429 6.4096
                0.4248 0.0688 4.2473 0.2300 0.0113 2.4720 1.0019 0.3382
1.3
                0.4268 0.1162 4.3133 0.2300 0.0119 2.5598 0.9698 0.3191
  0.2365 5.5426
14
                0.4257 0.0279 4.4504 0.2300 0.0125 2.6614 0.9406 0.3038
15
  0.2305 4.8271
$All.CriticalValues
 CritValue_Duda CritValue_PseudoT2 Fvalue_Beale
       0.4234 20.4305 0.2745
3
                   22.1752
                             0.4062
4
       0.4864
5
       0.2868
                   19.8896
                             0.0573
6
       0.2552
                   20.4342
                             0.0034
7
                   19.7832
       0.3776
                             0.1668
                             0.5019
8
       0.5502
                   25.3445
                            0.3493
      0.5399
9
                   24.7090
      0.1164
                   30.3727
                            0.0422
1.0
                            0.0177
      0.0442
11
                   64.8953
12
      0.4864
                   22.1752
                            0.3589
13
      0.1725
                   23.9899
                            0.1581
      0.2552
                   20.4342
                            0.0638
14
15
      -0.0536
                   -39.3104
                             1.0000
SBest.nc
                  CH Hartigan
                             CCC Scott Marriot TrCovW TraceW Friedman Rubin Cindex
             KL
DB Silhouette Duda PseudoT2
                                          9 4.0000 6.0000 13.0000 12.0000 5.000 12
Number clusters 3.0000 3.0000
                     6.0000 15.0000 5.0000
.0000
    9.000 3.0000 3.0000
Value Index 4.2356 19.1708 1.5295 -2.8227 43.5959 32900086 821.1635 5.1859 2.7154 -0.0482 0.356 1.
    0.255 0.6935 6.6287
0742
          Beale Ratkowsky Ball PtBiserial Frey McClain Dunn Hubert SDindex Dindex
0.5828 NA 1.236 0.23
                                                   0 1.7508
Value Index 1.2967 0.3432 26.1091
$Best.partition
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 3
7 38 39 40 41 42 43 44 45 46 47 48
2 1 2 2 1 2 1 2 1 3 2 2 2
49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73
```

▶



Targeting the Clusters/segments

We can now study our demographics and choice data in light of these cluster assignments using the funcion CrossTable:

Demographics

```
Cell Contents
N / Row Total |
      N / Col Total |
Total Observations in Table: 73
        | h_cluster
seg_data$MBA | Perf. | Comfort | Appearance | Row Total |
-----|
     MBA | 14 | 6 | 4 | 24 |
| 0.583 | 0.250 | 0.167 | 0.329 |
| 0.519 | 0.207 | 0.235 |
-----|
Undergrad | 13 | 23 | 13 | 49 | 0.265 | 0.469 | 0.265 | 0.671 | 0.481 | 0.793 | 0.765 |
-----|
Column Total | 27 | 29 | 17 | 73 | 0.370 | 0.397 | 0.233 |
Statistics for All Table Factors
Pearson's Chi-squared test
```

Choice

```
CrossTable(h_cluster, seg_data$Choice, prop.chisq = FALSE, prop.r = T, prop.c = T, prop.t = F, chisq = T)
```

See lecture on how to identify variables for targeting.

K-Means

We now focus on a different method called K-Means. This method, which requires us to specify in advance the number of clusters, aims to group the observations based on their similarity using an optimization procedure. Indeed, the aim is to minimize the within-cluster variation which is defined as the sum of square of the euclidean distance between each data point to the centroid of its cluster. More precisely, the algorithm works as follow:

- 1. Start by assigning each point to a cluster randomly
- 2. Compute the centroid of each cluster and the distances of each point to each centroid
- 3. Reassign each observation to the closest Centroid
- 4. Repeat Steps 2 and 3 until the within-cluster variance is minimized

Three Cluster Solution obtained using K-Means

Let us start by observing how the algorithm works on our data for 3 segments. We use the function kmeans(). Don't forget to set the seed to a specific vaule (e.g., 1990).

```
set.seed(1990)
car_Cluster3 <-kmeans(std_seg_data, 3, iter.max=100,nstart=100)
car_Cluster3
```

```
K-means clustering with 3 clusters of sizes 18, 32, 23
Cluster means:
             Styling Reliability Sportiness Performance
      Trendy
1 -0.637247817 -0.6837159 1.1781135 -1.0328905 0.7785740 0.08642535
2 -0.003271873 -0.3788069 -0.3496669 0.4977728 -0.0445069 0.53615835
3 0.503267855 1.0621176 -0.4355087 0.1157956 -0.5473961 -0.81359668
Clustering vector:
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 3
7 38 39 40 41 42 43 44 45 46 47 48
2 1 2 2 3 2 1 3 1 2 1 2 2
49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73
        3 2 2 2 2 3 2 2 2 3 3 2 3 3 3 3 3 3
Within cluster sum of squares by cluster:
[1] 81.39207 83.90060 111.49649
 (between_SS / total_SS = 35.9 %)
Available components:
[1] "cluster"
              "centers"
                           "totss"
                                        "withinss"
                                                     "tot.withinss" "betweenss"
                                                                               "size"
            "ifault"
"iter"
                                                                                         . ▶
                                                                                         Hide
```

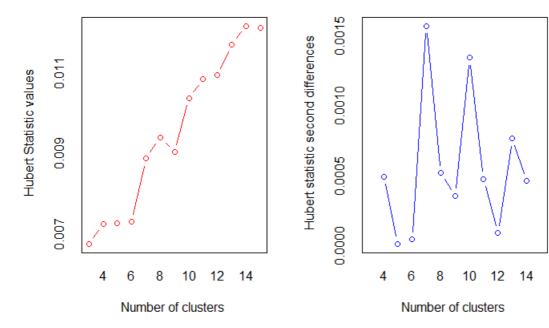
Find the optimal number of clusters

A key question when using the K-Means clustering technique consists in choosing the optimal number of segments. In order to do that, we can use the function NbClust() as in hierarchical clustering by specifying the method kmeans as below. From the output, We see that the three-cluster solution is best.

```
Set.seed(1990)
NbClust(data=std_seg_data[,1:5], min.nc=3, max.nc=15, index="all", method="kmeans")

*** : The Hubert index is a graphical method of determining the number of clusters.

In the plot of Hubert index, we seek a significant knee that corresponds to a significant increase of the value of the measure i.e the significant peak in Hubert index second differences plot.
```



*** : The D index is a graphical method of determining the number of clusters.

In the plot of D index, we seek a significant knee (the significant peak in Dindex second differences plot) that corresponds to a significant increase of the value of the measure.

```
* Among all indices:
```

- \star 7 proposed 3 as the best number of clusters
- \star 2 proposed 4 as the best number of clusters
- * 1 proposed 6 as the best number of clusters
- * 3 proposed 7 as the best number of clusters
- * 5 proposed 8 as the best number of clusters
- * 1 proposed 10 as the best number of clusters
- * 1 proposed 11 as the best number of clusters
- * 1 proposed 12 as the best number of clusters
- \star 2 proposed 15 as the best number of clusters

0.2093 1.4350 -3.6377 -0.8433

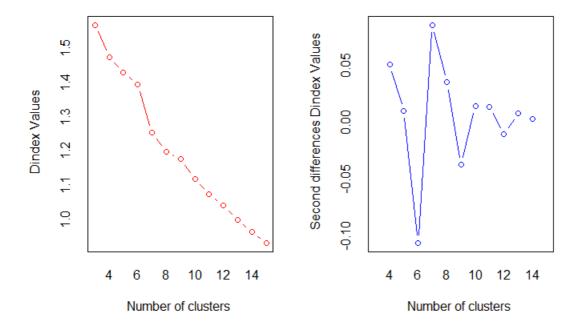
**** Conclusion ****

 * According to the majority rule, the best number of clusters is 3

```
$All.index
                                                                                             CH Hartigan
                                                                                                                                                                                          CCC
                                                                                                                                                                                                                          Scott Marriot TrCovW TraceW Friedman Rubin Cindex
                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                            DB Si
                                           KL
lhouette Duda Pseudot2 Beale
3 2.6615 23.0027 9.9985 -2.3384 161.6503 332431565 3142.6116 217.2313 6.9183 1.6572 0.4029 1.6368
                                                                                     6.8769 0.9339
0.2211 0.7619
                   2.6211 20.5604 5.9101 -3.4050 206.2171 320952678 2494.7351 190.0809
                                                                                                                                                                                                                                                                                                                                                                                                                                               8.7864 1.8939 0.3875 1.4764
0.2351 1.4900 -4.9326 -0.9006
                         0.3090 17.9546 3.5125 -5.0208 211.9276 463754305 1884.0863 175.0844
                                                                                                                                                                                                                                                                                                                                                                                                                                                9.3054 2.0562 0.3747 1.3972
0.2146 1.6044 -3.7672 -0.9825
                    0.1667 15.5757 15.2717 -5.9661 250.0332 396233308 1841.2783 166.4846 11.9435 2.1624 0.3825 1.3828
0.1678 1.0555 -0.7881 -0.1542
                 2.8226 18.2077 7.6528 -3.7082 297.2619 282403511 1117.8685 135.5809 13.1919 2.6552 0.3826 1.2398
0.2296 2.1023 -5.2433 -1.2308
8 115.6198 18.2290 3.1015 -3.2577 325.0955 251921280 850.0973 121.4935 14.7248 2.9631 0.4170 1.1432
0.2537 1.9135 -9.0706 -1.2807
                       0.0103 16.8361 6.1543 -3.8303 341.9554 253084884 780.5087 115.9604 15.5678 3.1045 0.4631 1.2340
0.1908 2.2632 -7.8141 -1.3975
10 \quad 0.7749 \ 16.8213 \quad 7.0516 \ -3.5342 \ 370.8399 \ 210349050 \quad 629.2076 \ 105.7877 \quad 17.4933 \ 3.4030 \ 0.3716 \ 1.2501 \quad 10.2501 \quad
0.2030 0.6031 2.6329 1.7167
11 5.0114 17.2605 3.1097 -2.9648 406.5994 155949665 518.0238 95.1388 19.4733 3.7839 0.4513 1.1235
0.2118 2.0940 -8.3592 -1.3081
12 \quad 0.2795 \quad 16.4907 \quad 5.3651 \quad -3.2843 \quad 446.4373 \quad 107536543 \quad 493.6384 \quad 90.5948 \quad 24.4889 \quad 3.9737 \quad 0.4432 \quad 1.1876 \quad 10.2795 \quad 
0.2075 0.9828 0.1929 0.0507
13 \quad 1.0804 \ 16.6162 \quad 5.0360 \ -3.0034 \ 473.8984 \quad 86637820 \quad 423.0730 \quad 83.2710 \quad 26.1131 \ 4.3232 \ 0.4365 \ 1.1268 \quad 4.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1.08161 \ 1
```

```
14 1.9907 16.7292 3.2865 -2.7603 487.2320 83705207 340.7177 76.8230 26.1213 4.6861 0.4219 1.1091
0.2127 1.3559 -1.3124 -0.5477
15 1.0396 16.3524 3.1017 -2.8976 510.4032 69956428 321.4077 72.7695 27.6550 4.9471 0.4171 1.0293
0.2340 9.3984 -5.3616 0.0000
                        Frey McClain Dunn Hubert SDindex Dindex SDbw
 Ratkowsky Ball Ptbiserial
3
   4
   5
   6
7
   0.2981 19.3687
                 0.4840 -0.2534 2.2978 0.1520 0.0089 1.8817 1.2548 0.4441
8
   0.2876 15.1867
                 0.5323 -5.0634 2.1224 0.2019 0.0094 1.8598 1.1989 0.4061
9
   0.2744 12.8845
                 0.3980 0.0564 3.8572 0.1324 0.0090 2.0189 1.1776 0.3576
                 0.4099 -0.3349 4.1242 0.1690 0.0104 2.2013 1.1187 0.3598
10
   0.2657 10.5788
                 0.4576 -23.5430 3.4951 0.1321 0.0109 2.1409 1.0738 0.3560
11
   0.2586 8.6490
   0.2496 7.5496
                12
   0.2430 6.4055
                0.4021 0.0993 4.7440 0.1487 0.0117 2.8269 0.9999 0.3015
13
  0.2370 5.4874 0.4017 -0.2287 4.8915 0.1309 0.0122 2.6940 0.9648 0.2900
14
  0.2305 4.8513 0.4087 0.6242 4.7589 0.1309 0.0121 2.6062 0.9323 0.2517
$All.CriticalValues
 CritValue_Duda CritValue_PseudoT2 Fvalue_Beale
               23.2312 0.4623
3
       0.4864
                              1.0000
4
       0.2552
                    43.7876
                              1.0000
5
       0.1725
                    47.9798
       0.4234
                    20.4305
6
                              1.0000
                              1.0000
7
       0.0442
                   216.3177
                              1.0000
       0.2177
8
                    68.2773
      0.1164
                   106.3046
                              1.0000
9
                              0.1675
10
      0.1725
                    19.1919
11
      0.1164
                   121.4910
                              1.0000
12
      0.3776
                   18.1346
                              0.9983
      0.2868
                    29.8345
13
                              1.0000
14
      -0.0536
                    -98.2760
                              1.0000
      -0.4373
                   -19.7211
1.5
                                NaN
$Best.nc
               KL
                    CH Hartigan
                              CCC Scott Marriot TrCovW TraceW Friedman Rubin Cind
    DB Silhouette Duda PseudoT2
Number clusters 8.0000 3.0000 6.0000 3.0000 7.0000
                                              4 7.0000 7.0000 12.0000 11.0000 10.00
00 15.0000 8.0000 3.0000 3.0000
          115.6198 23.0027 11.7591 -2.3384 47.2286 154280514 723.4098 16.8164 5.0156 -0.1911 0.371
Value Index
Beale Ratkowsky Ball PtBiserial Frey McClain Dunn Hubert SDindex Dindex
Number_clusters 3.0000 3.0000 4.0000 8.0000 2 3.0000 8.0000 0 8.0000 0 15.0000
Value Index 0.9339 0.3612 24.8902
                               0.5323 NA 1.2726 0.2019
                                                       0 1.8598
                                                                 0 0.2517
$Best.partition
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 3
7 38 39 40 41 42 43 44 45 46 47 48
3 2 3 3 3 3 2 3 2 1 2 3 3
49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73
1 3 3 1 3 3 3 3 1 3 3 1 1 1 3 3 1 3 3 1 3 1 2
```

)



Results Comparison

Looking at the cluster means above, we see that the clusters defined with the kmeans function are characterized similarly as before. Thus, we relabel them to describe them more accurately. We can now compare this clustering to the demographics and choice as well as the hierarchical clustering.

Demographics

```
Hide
```

```
Cell Contents
N / Row Total |
      N / Col Total |
Total Observations in Table: 73
       | Kmean Cluster
seg_data$MBA | Perf. KM | Comfort KM | Appearance KM |
                                          Row Total |
_____|
     MBA | 11 | 8 | 5 | 24 | 0.458 | 0.333 | 0.208 | 0.329 |
              0.611 |
                        0.250 |
                                   0.217 |
 -----|
Undergrad | 7 | 24 | 18 | 49 | 0.143 | 0.490 | 0.367 | 0.671 | 0.389 | 0.750 | 0.783 |
-----|
Column Total | 18 | 32 | 23 | 73 | 0.247 | 0.438 | 0.315 |
Statistics for All Table Factors
Pearson's Chi-squared test
Chi^2 = 8.694824 d.f. = 2 p = 0.01294026
```

Choice

```
CrossTable(Kmean_Cluster, seg_data$Choice, prop.chisq = FALSE, prop.r = T, prop.c = T, prop.t = F, chisq = T)
```

```
Cell Contents
      N / Row Total |
      N / Col Total |
Total Observations in Table: 73
        | seg data$Choice
Kmean_Cluster | BMW | Lexus | Mercedes | Row Total |
-----|-----|------|
  Perf. KM | 7 | 8 | 3 | 18 |
           0.389 | 0.444 | 0.167 | 0.247 |
    0.219 | 0.364 |
                            0.158 |
        -----|
 Comfort KM | 14 | 8 | 10 | 32 |
        0.438 | 0.250 | 0.312 | 0.438 |
        0.438 | 0.364 | 0.526 |
-----|----|-----|
Appearance KM | 11 | 6 | 6 | 23 | 0.478 | 0.261 | 0.261 | 0.315 | 0.344 | 0.273 | 0.316 |
Column Total | 32 | 22 | 19 | 73 | 0.438 | 0.301 | 0.260 |
 -----|----|-----|
Statistics for All Table Factors
Pearson's Chi-squared test
Chi^2 = 2.753465 d.f. = 4 p = 0.5998918
```

Hierarchical Clustering

```
Hide
```

Latent Class Analysis

Latent Class Analysis is a method to identify cluster membership of subjects using the observable variables that describe them. The approach consists in estimating for each individual the probability to belong to a "latent class" or cluster. In turn, each cluster is defined in terms of its "geometry" and "orientation" as cloud of points. As such, this technique belongs to the family of gaussian finite mixture models. This approach relies on a different optimization procedure that aims to maximize the likelihood (versus minimize the distances between each point). Hence, the tools to assess the optimal number of classes differ. We now perform this analysis using the package mclust. We start by determining the optimal model based on BIC using the function mclustBIC().

Find the optimal model

```
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```

```
set.seed(1990)
mclustBIC(std_seg_data[,1:5],verbose=F)
```

Hence, the optimal model is VEE with 2 segments. This means that the data can be clustered in two clusters which will both be modeled by a Normal distribution with the same covariance matrix. We obtain more details about the optimal model below:

```
set.seed(1990)
lca_clust <- Mclust(std_seg_data[,1:5], verbose = FALSE)
summary(lca_clust)</pre>
```

```
Gaussian finite mixture model fitted by EM algorithm

Mclust VEE (ellipsoidal, equal shape and orientation) model with 2 components:

log.likelihood n df BIC ICL

-431.8862 73 27 -979.6149 -990.9626

Clustering table:

1 2

47 26
```

We now interpret each cluster and rename them to describe them accurately:

Hide

Results Comparison

Let us compare this solution to the demographics and choice data as well as the hierarchical clustering and K-Means:

Demographics

Hide

Choice

Hide

Hierarchical Clustering

Hide

K-Means Clustering