

# Total Coloring Conjecture for Cayley Graphs

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22 January 2020

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- Unsolved problem for 50+ years
- Techniques to solve particular cases far-reaching applications to further the subject of graph theory

# Tentative Courseplan

Coursework	May 2020
Comprehensive Exam	June 2020
Qualifying Exam	December 2020
Open Seminar -1	January 2021
Open Seminar-2	June 2021
Synopsis	September 2021
Thesis Submission	December 2021

- Linear Algebra (18MAT638)
- Graph Theory (MA804)
- Data Structures and Algorithms (18MAT671)

- Advanced Algebra
- Advanced Linear Algebra
- Applied Statistics for Research

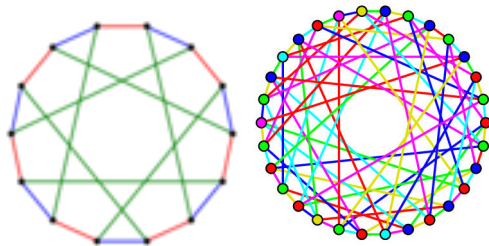
- Advanced Algorithms and Analysis
- Advanced Graph Theory
- Research Methodology

# Introduction

- Graph theory
- Possible Applications- Time tabling, scheduling, routing
- Graph coloring-A certain partitioning of vertices
- Edge coloring-A certain partitioning of edges
- Total Coloring-Simultaneous partitioning of vertices and edges
- Total Coloring Conjecture-Behzad and Vizing(1965)



# Introduction-continued



- Cayley Graphs-graphs defined on groups, high level of symmetry
- Total Coloring Conjecture for Cayley graphs-still unproven
- Partial Results-true for even powers of cycles and unitary cayley graphs

# Literature Review-Survey Sources

- Electronic Notes in Discrete Mathematics,  
<https://www.journals.elsevier.com/electronic-notes-in-discrete-mathematics/>
- Electronic Notes in Theoretical Computer Science,  
<https://www.sciencedirect.com/journal/electronic-notes-in-theoretical-computer-science>
- arXiv paper archive, <https://arxiv.org/list/math.CO>

# Literature Review-Survey Articles

- Total Colourings - A survey Geetha Jayabalan, Narayanan N, K Somasundaram, <https://arxiv.org/pdf/1910.03922> (2019)
- Total Colourings of Graphs-Yap, Hian Poh, springer-Verlag, Berlin[7](2006)

# Literature Review-Journals

- Discrete Mathematics, Elsevier, <https://www.journals.elsevier.com/discrete-mathematics/>
- Journal Of Combinatorial Theory, Elsevier, <https://www.journals.elsevier.com/journal-of-combinatorial-theory-series-a/>
- Discrete Applied Mathematics, Elsevier, <https://www.journals.elsevier.com/discrete-applied-mathematics/>
- Electronic Journal of Combinatorics, [www.combinatorics.org](http://www.combinatorics.org)
- Annals of Combinatorics, Springer, <https://www.springer.com/journal/26>

- Hung-Lin Fu, National Chiao-Tung University, Taiwan[2]
- Amanda Chetwynd, Lancaster University, UK[3]
- Celina de Mello, Brazil[1]
- De-Zheng Xie, China[6]
- Alexandr Kostochka, University of Illinois, USA[4]

- Here, we try to prove the TCC for Cayley graphs. As of now, we have submitted a paper to journal(s) ([5]) regarding total coloring of some powers of cycles and other related regular graphs like Cayley graphs on nilpotent groups, mock-threshold graphs, odd graphs and unitary cayley graphs.

- We are working on extending the method for coloring powers of cycles to circulant graphs, and further, to cayley graphs of nilpotent groups on all generating sets and also other regular graphs



# References

- [1] C. Campos and C. P. de Mello. A result on the total colouring of powers of cycles. *Discrete Applied Mathematics*, 155(5):585–597, 2007.
- [2] B.-L. Chen and H.-L. Fu. Total colorings of graphs of order  $2n$  having maximum degree  $2n-2$ . *Graphs and Combinatorics*, 8(2):119–123, 1992.
- [3] A. G. Chetwynd. Total colourings. 1990.
- [4] A. V. Kostochka. The total chromatic number of any multigraph with maximum degree five is at most seven. *Discrete Mathematics*, 162(1-3):199–214, 1996.
- [5] S. Prajnanaswaroopu, J. Geetha, K. Somasundaram, and H. Lin-Fu. On total coloring of powers of cycles and other cayley graphs. *arXiv preprint arXiv:1910.03922*, 2019.
- [6] D. Xie and W. Yang. The total chromatic number of graphs of even order and high degree. *Discrete mathematics*, 271(1-3):295–302, 2003.
- [7] H. P. Yap. *Total colourings of graphs*. Springer, 2006.

Thank You , Any Questions?