6-12 THE INTEGRATOR

A circuit in which the output voltage waveform is the integral of the input voltage waveform is the *integrator* or the *integration amplifier.* Such a circuit is obtained by using a basic inverting amplifier configuration if the feedback resistor R_F is replaced by a capacitor C_F [see Figure 6–23(a)].

The expression for the output voltage v_o can be obtained by writing Kirchhoff's current equation at node v_2:

$$i_1 = I_B + i_F$$

Since I_B is negligibly small,

$$i_1 \cong i_F$$

Recall that the relationship between current through and voltage across the capacitor is

$$i_c = C\frac{dv_c}{dt}$$

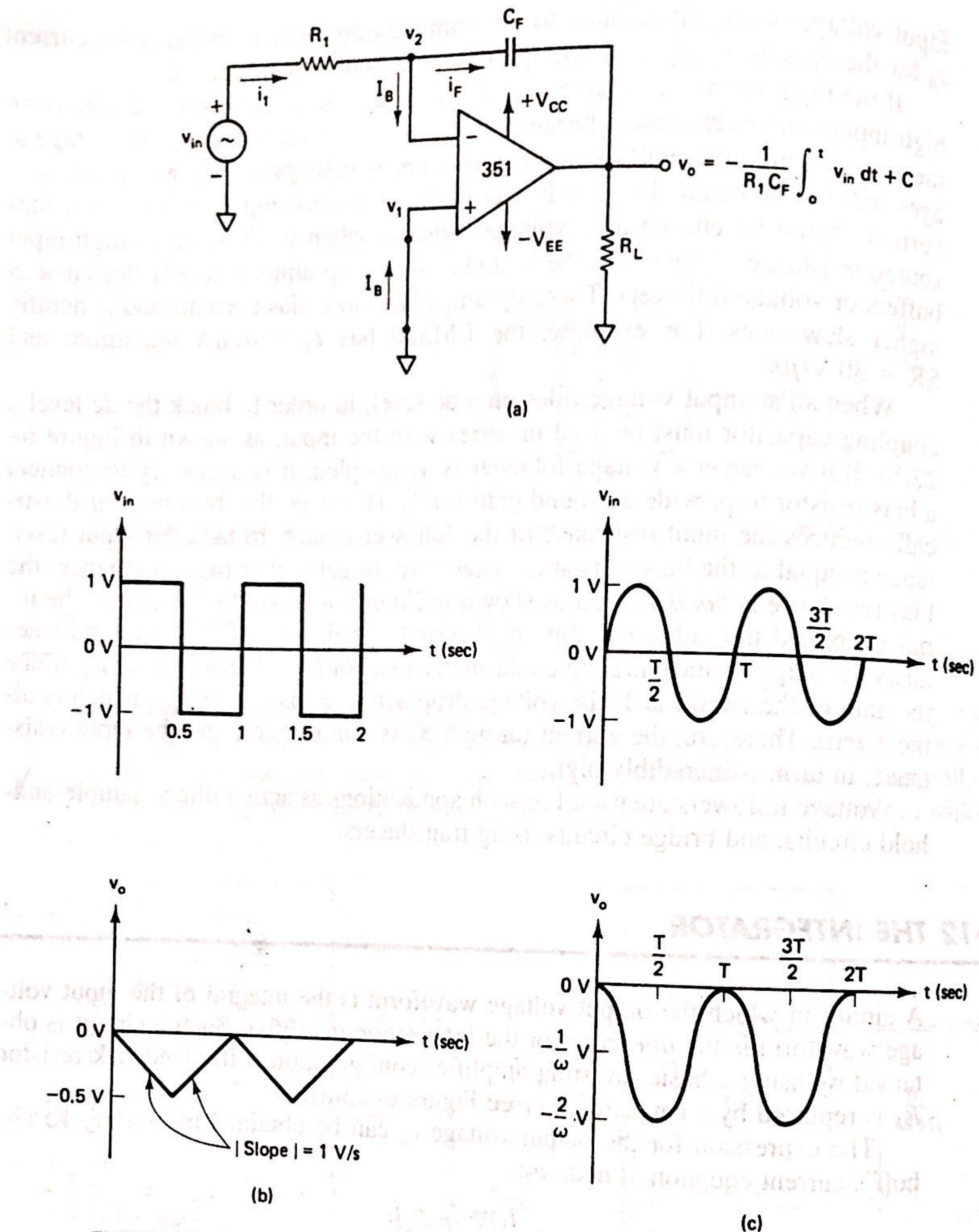

FIGURE 6–23 (a) The integrator circuit. (b) and (c) Input and ideal output waveforms using a square wave and sine wave, respectively. $R_1C_F = 1$ second and $V_{ooT} = 0$ V assumed.

Therefore,

$$\frac{v_{\text{in}} - v_2}{R_1} = C_F\left(\frac{d}{dt}\right)(v_2 - v_o)$$

However, $v_1 = v_2 \cong 0$ because A is very large. Therefore,

$$\frac{v_{\text{in}}}{R_1} = C_F \frac{d}{dt}(-v_o)$$

The output voltage can be obtained by integrating both sides with respect to time:

$$\int_0^t \frac{v_{\text{in}}}{R_1}\,dt = \int_0^t C_F \frac{d}{dt}(-v_o)\,dt$$

$$= C_F(-v_o) + v_o\big|_{t=0}$$

Therefore,

$$v_o = -\frac{1}{R_1 C_F}\int_0^t v_{\text{in}}\,dt + C \qquad \textbf{(6–23)}$$

where C is the integration constant and is proportional to the value of the output voltage v_o at time $t = 0$ seconds.

Equation (6–23) indicates that the output voltage is directly proportional to the negative integral of the input voltage and inversely proportional to the time constant R_1C_F. For example, if the input is a sine wave, the output will be a cosine wave; or if the input is a square wave, the output will be a triangular wave, as shown in Figure 6–23(c) and (b), respectively. Note that these waveforms are drawn with the assumption that $R_1C_F = 1$ second and $V_{ooT} = 0$ V; that is, $C = 0$.

When $v_{\text{in}} = 0$, the integrator of Figure 6–23(a) works as an open-loop amplifier. This is because the capacitor C_F acts as an open circuit ($X_{CF} = \infty$) to the input offset voltage V_{io}. In other words, the input offset voltage V_{io} and the part of the input current charging capacitor C_F produce the error voltage at the output of the integrator. Therefore, in the practical integrator shown in Figure 6–25, to reduce the error voltage at the output, a resistor R_F is connected across the feedback capacitor C_F. Thus, R_F limits the low-frequency gain and hence minimizes the variations in the output voltage.

The frequency response of the basic integrator is shown in Figure 6–24. In this figure, f_b is the frequency at which the gain is 0 dB and is given by

$$f_b = \frac{1}{2\pi R_1 C_F} \qquad \textbf{(6–24)}$$

For the derivation of Equation (6–24), refer to Appendix C.

Both the stability and the low-frequency roll-off problems can be corrected by the addition of a resistor R_F as shown in the *practical integrator* of Figure 6–25. The term stability refers to a constant gain as frequency of an input signal is varied over a certain range. Also, low-frequency roll off refers to the rate of decrease in gain at lower frequencies. The frequency response of the practical

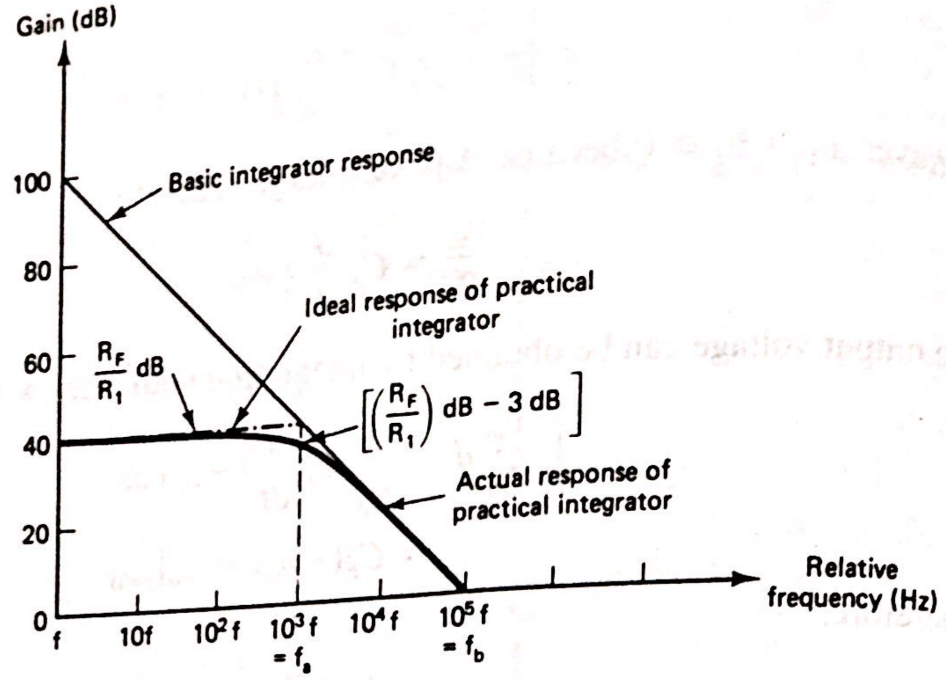

FIGURE 6–24 Frequency response of basic and practical integrators. $f_a = 1/(2\pi R_F C_F)$ and $f_b = 1/(2\pi R_1 C_F)$.

integrator is shown in Figure 6–24 by a dashed line. In this figure, f is some relative operating frequency, and for frequencies f to f_a to gain R_F/R_1 is constant. However, after f_a the gain decreases at a rate of 20 dB/decade. In other words, between f_a and f_b the circuit of Figure 6–25 acts as an integrator. The gain-limiting frequency f_a is given by

$$f_a = \frac{1}{2\pi R_F C_F} \tag{6–25}$$

Refer to Appendix C for derivation.

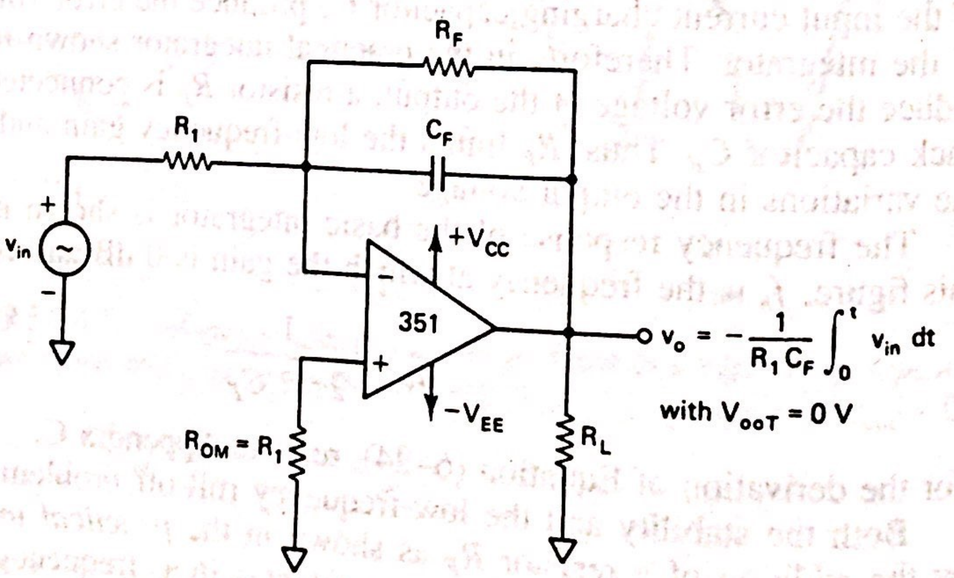

FIGURE 6–25 Practical integrator.

Generally, the value of f_a and in turn R_1C_F and R_FC_F values should be selected such that $f_a < f_b$. For example, if $f_a = f_b/10$, then $R_F = 10R_1$. In fact, the input signal will be integrated properly if the time period T of the signal is larger than or equal to R_FC_F. That is,

$$T \geq R_FC_F \tag{6–26}$$

where

$$R_FC_F = \frac{1}{2\pi f_a}$$

The integrator is most commonly used in analog computers and analog-to-digital (ADC) and signal-waveshaping circuits.

6–13 THE DIFFERENTIATOR

Figure 6–27(a) shows the *differentiator* or *differentiation amplifier.* As its name implies, the circuit performs the mathematical operation of differentiation; that is, the output waveform is the derivative of the input waveform. The differentiator may be constructed from a basic inverting amplifier if an input resistor R_1 is replaced by a capacitor C_1.

The expression for the output voltage can be obtained from Kirchhoff's current equation written at node v_2 as follows:

$$i_C = I_B + i_F$$

Since $I_B \cong 0$,

$$i_C = i_F$$

$$C_1 \frac{d}{dt}(v_{in} - v_2) = \frac{v_2 - v_o}{R_F}$$

But $v_1 = v_2 \cong 0$ V, because A is very large. Therefore,

$$C_1 \frac{dv_{in}}{dt} = -\frac{v_o}{R_F}$$

or

$$v_o = -R_F C_1 \frac{dv_{in}}{dt} \tag{6-27}$$

Thus the output v_o is equal to $R_F C_1$ times the negative instantaneous rate of change of the input voltage v_{in} with time. Since the differentiator performs the reverse of the integrator's function, a cosine wave input will produce a sine wave output, or a triangular input will produce a square wave output. However, the differentiator

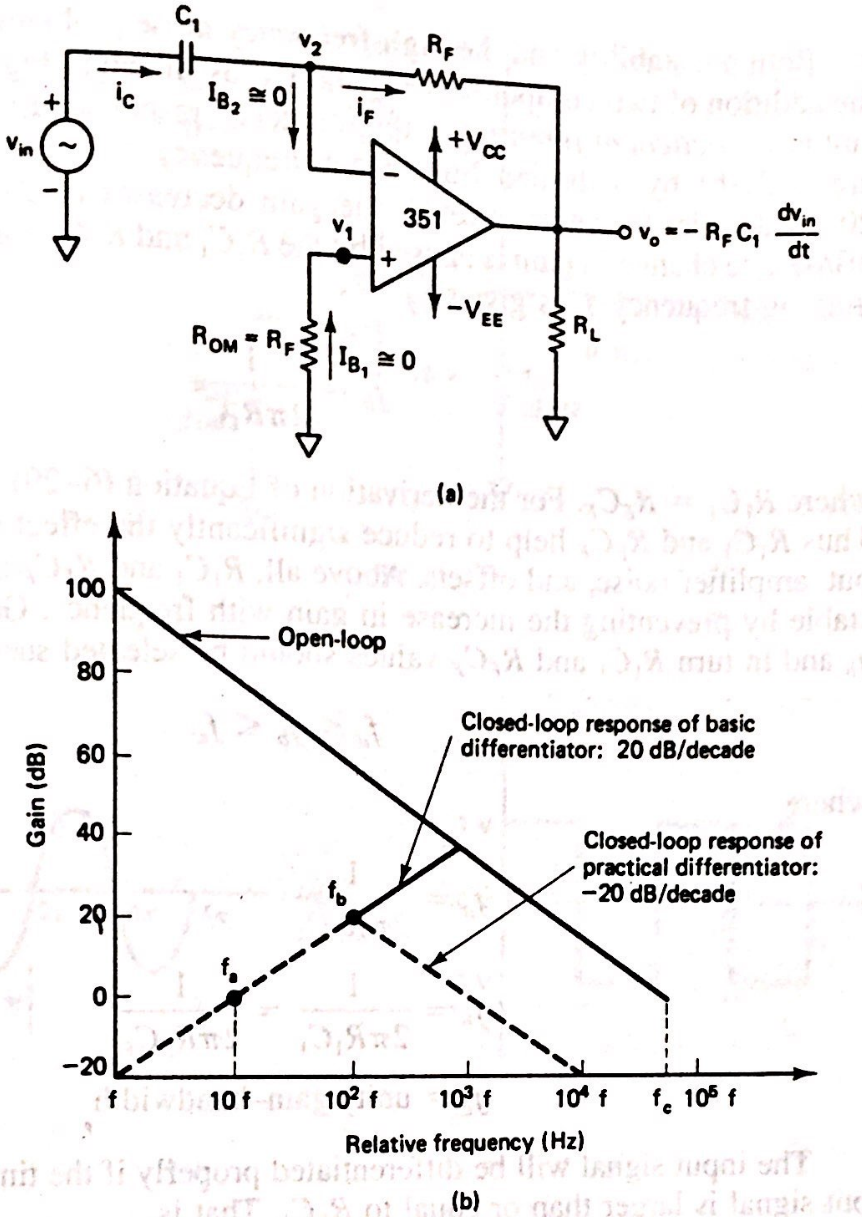

FIGURE 6–27 Basic differentiator. (a) Circuit. (b) Frequency response.

of Figure 6–27(a) will not do this because it has some practical problems. The gain of the circuit (R_F/X_{C1}) increases with increase in frequency at a rate of 20 dB/decade. This makes the circuit unstable. Also, the input impedance X_{C1} decreases with increase in frequency, which makes the circuit very susceptible to high-frequency noise. When amplified, this noise can completely override the differentiated output signal. The frequency response of the basic differentiator is shown in Figure 6–27(b). In this figure, f_a is the frequency at which the gain is 0 dB and is given by

$$f_a = \frac{1}{2\pi R_F C_1} \tag{6–28}$$

Also, f_c is the unity gain–bandwidth of the op-amp, and f is some relative operating frequency. For the derivation of Equation (6–28), refer to Appendix C.

Both the stability and the high-frequency noise problems can be corrected by the addition of two components: R_1 and C_F, as shown in Figure 6–28(a). This circuit is a *practical differentiator*, the frequency response of which is shown in Figure 6–27(b) by a dashed line. From frequency f to f_b, the gain increases at 20 dB/decade. However, after f_b the gain decreases at 20 dB/decade. This 40-dB/decade change in gain is caused by the R_1C_1 and R_FC_F combinations. The gain-limiting frequency f_b is given by

$$f_b = \frac{1}{2\pi R_1 C_1} \qquad \text{(6–29)}$$

where $R_1C_1 = R_FC_F$. For the derivation of Equation (6–29), refer to Appendix C. Thus R_1C_1 and R_FC_F help to reduce significantly the effect of high-frequency input, amplifier noise, and offsets. Above all, R_1C_1 and R_FC_F make the circuit more stable by preventing the increase in gain with frequency. Generally, the value of f_b and in turn R_1C_1 and R_FC_F values should be selected such that

$$f_a < f_b < f_c \qquad \text{(6–30)}$$

where

$$f_a = \frac{1}{2\pi R_F C_1}$$

$$f_b = \frac{1}{2\pi R_1 C_1} = \frac{1}{2\pi R_F C_F}$$

$$f_c = \text{unity gain–bandwidth}$$

The input signal will be differentiated properly if the time period T of the input signal is larger than or equal to R_FC_1. That is,

$$T \geq R_F C_1$$

Figure 6–28(b) and (c) show the sine wave and square wave inputs and resulting differentiated outputs, respectively, for the practical differentiator. A workable differentiator can be designed by implementing the following steps (see Example 6–16):

1. Select f_a equal to the highest frequency of the input signal to be differentiated. Then, assuming a value of $C_1 < 1\ \mu$F, calculate the value of R_F.
2. Choose $f_b = 20f_a$ and calculate the values of R_1 and C_F so that $R_1C_1 = R_FC_F$.

The differentiator is most commonly used in waveshaping circuits to detect high-frequency components in an input signal and also as a rate-of-change detector in FM modulators.

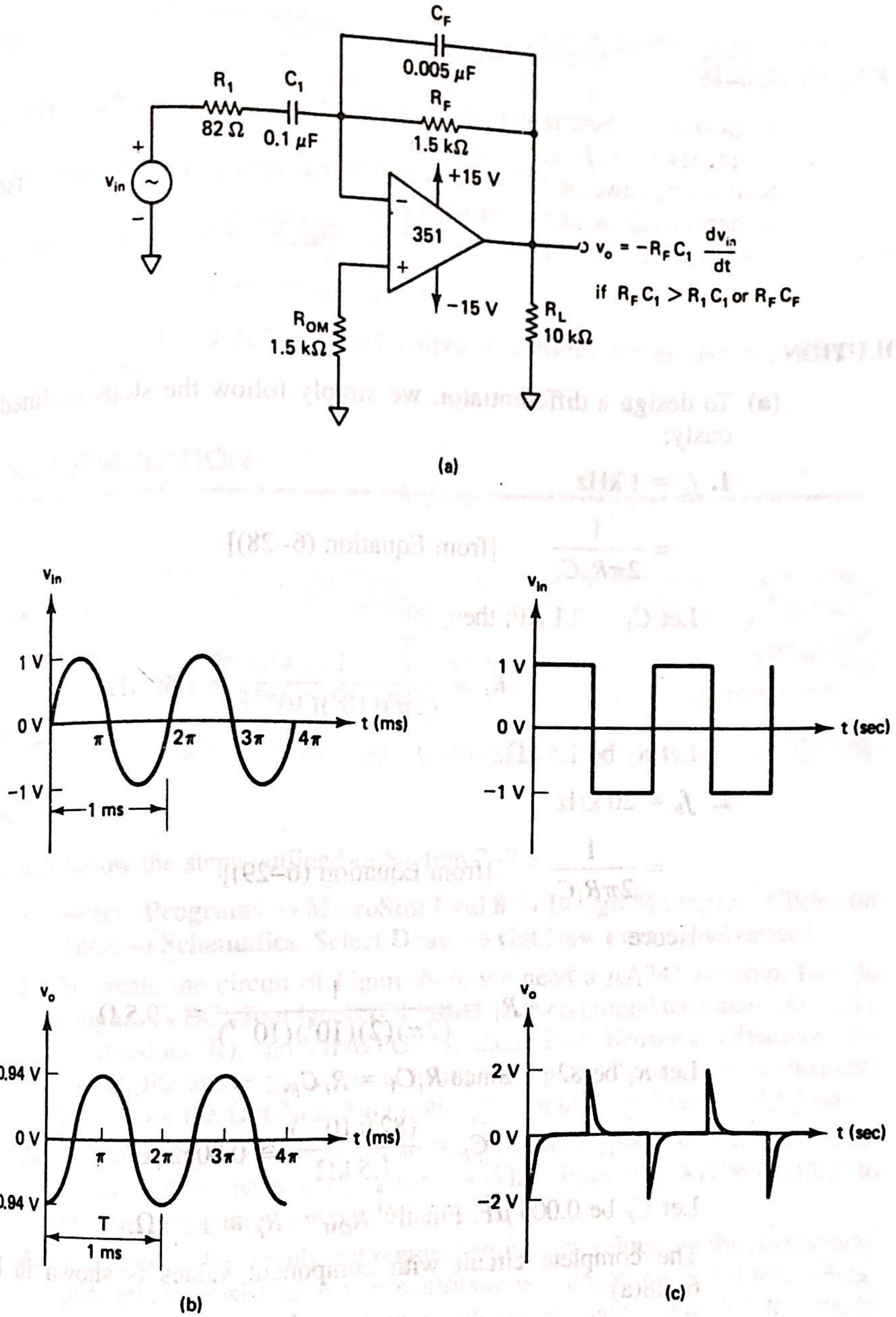

FIGURE 6–28 Practical differentiator. (a) Circuit. (b) Sine wave input and resulting cosine wave output. (c) Square wave input and resulting spike output.