7-3 FIRST-ORDER LOW-PASS BUTTERWORTH FILTER

Figure 7–2 shows a first-order low-pass Butterworth filter that uses an *RC* network for filtering. Note that the op-amp is used in the noninverting configuration; hence it does not load down the *RC* network. Resistors R_1 and R_F determine the gain of the filter.

According to the voltage-divider rule, the voltage at the noninverting terminal (across capacitor C) is

$$v_1 = \frac{-jX_C}{R - jX_C} v_{in} \tag{7–1a}$$

where

$$j = \sqrt{-1} \quad \text{and} \quad -jX_c = \frac{1}{j2\pi fC}$$

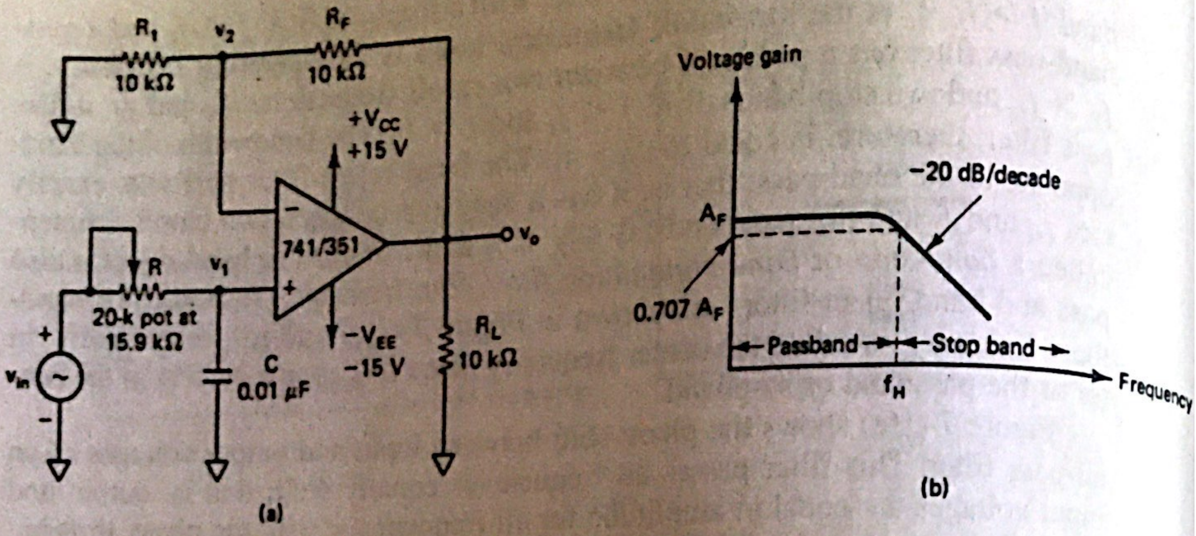


FIGURE 7-2 First-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

Simplifying Equation (7–1a), we get

$$v_1 = \frac{v_{in}}{1 + j2\pi fRC}$$

and the output voltage

$$v_o = \left(1 + \frac{R_F}{R_1}\right) v_1$$

That is,

$$v_o = \left(1 + \frac{R_F}{R_1}\right) \frac{v_{in}}{1 + j2\pi fRC}$$

or

$$\frac{v_o}{v_{in}} = \frac{A_F}{1 + j(f/f_H)} \tag{7–1b}$$

where $\dfrac{v_o}{v_{in}}$ = gain of the filter as a function of frequency

$A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal

$f_H = \dfrac{1}{2\pi RC}$ = high cutoff frequency of the filter

The gain magnitude and phase angle equations of the low-pass filter can be obtained by converting Equation (7–1b) into its equivalent polar form, as follows:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}} \tag{7–2a}$$

$$\phi = -\tan^{-1}\left(\frac{f}{f_H}\right) \tag{7–2b}$$

where ϕ is the phase angle in degrees.

The operation of the low-pass filter can be verified from the gain magnitude equation, (7–2a):

1. At very low frequencies, that is, $f < f_H$,

$$\left|\frac{v_o}{v_{in}}\right| \cong A_F$$

2. At $f = f_H$,

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{2}} = 0.707A_F$$

3. At $f > f_H$,

$$\left|\frac{v_o}{v_{in}}\right| < A_F$$

Thus the low-pass filter has a constant gain A_F from 0 Hz to the high cutoff frequency f_H. At f_H the gain is $0.707A_F$, and after f_H it decreases at a constant rate with an increase in frequency [see Figure 7–2(b)]. That is, when the frequency is increased tenfold (one decade), the voltage gain is divided by 10. In other words, the gain decreases 20 dB (= 20 log 10) each time the frequency is increased by 10. Hence the rate at which the gain rolls off after f_H is 20 dB/decade or 6 dB/octave, where octave signifies a twofold increase in frequency. The frequency $f = f_H$ is called the *cutoff frequency* because the gain of the filter at this frequency is down by 3 dB (= 20 log 0.707) from 0 Hz. Other equivalent terms for cutoff frequency are *−3 dB frequency, break frequency,* or *corner frequency.*

7-3-1 Filter Design

A low-pass filter can be designed by implementing the following steps:

1. Choose a value of high cutoff frequency f_H.
2. Select a value of C less than or equal to 1 μF. Mylar or tantalum capacitors are recommended for better performance.

3. Calculate the value of R using

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, select values of R_1 and R_F dependent on the desired passband gain A_F using

$$A_F = 1 + \frac{R_F}{R_1}$$

7–3–2 Frequency Scaling

Once a filter is designed, there may sometimes be a need to change its cutoff frequency. The procedure used to convert an original cutoff frequency f_H to a new cutoff frequency f'_H is called *frequency scaling*. Frequency scaling is accomplished as follows. To change a high cutoff frequency, multiply R or C, but not both, by the ratio of the original cutoff frequency to the new cutoff frequency. In filter design the needed values of R and C are often not standard. Besides, a variable capacitor C is not commonly used. Therefore, choose a standard value of capacitor, and then calculate the value of resistor for a desired cutoff frequency. This is because for a nonstandard value of resistor a potentiometer can be used (see Examples 7–1 and 7–2).

EXAMPLE 7–1

Design a low-pass filter at a cutoff frequency of 1 kHz with a passband gain of 2.

SOLUTION

Follow the preceding design steps.

1. $f_H = 1$ kHz.
2. Let $C = 0.01\ \mu$F.
3. Then $R = 1/(2\pi)(10^3)(10^{-8}) = 15.9$ kΩ. (Use a 20-kΩ potentiometer.)
4. Since the passband gain is 2, R_1 and R_F must be equal. Therefore, let $R_1 = R_F = 10$ kΩ. The complete circuit with component values is shown in Figure 7–2(a).

EXAMPLE 7–2

Using the frequency scaling technique, convert the 1-kHz cutoff frequency of the low-pass filter of Example 7–1 to a cutoff frequency of 1.6 kHz.

SOLUTION

To change a cutoff frequency from 1 kHz to 1.6 kHz, we multiply the 15.9-kΩ resistor by

$$\frac{\text{original cutoff frequency}}{\text{new cutoff frequency}} = \frac{1 \text{ kHz}}{1.6 \text{ kHz}} = 0.625$$

Therefore, new resistor $R = (15.9 \text{ k}\Omega)(0.625) = 9.94 \text{ k}\Omega$. However, 9.94 kΩ is not a standard value. Therefore, use $R = 10 \text{ k}\Omega$ potentiometer and adjust it to 9.94 kΩ. Thus the new cutoff frequency is

$$f_H = \frac{1}{(2\pi)(0.01\ \mu\text{F})(9.94 \text{ k}\Omega)} = 1.6 \text{ kHz}$$

EXAMPLE 7-3

Plot the frequency response of the low-pass filter of Example 7-1.

SOLUTION

To plot the frequency response, we have to use Equation (7-2a). The data of Table 7-1 are, therefore, obtained by substituting various values for f in this equation. Equation (7-2a) will be repeated here for convenience:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^2}}$$

where $A_F = 2$ and $f_H = 1$ kHz. The data of Table 7-1 are plotted as shown in Figure 7-3.

TABLE 7-1 Frequency Response Data for Example 7-3.

Input frequency, f (Hz)	*Gain magnitude,* $\lvert v_o/v_{in}\rvert$	*Magnitude (dB) =* $20 \log\lvert v_o/v_{in}\rvert$
10	2	6.02
100	1.99	5.98
200	1.96	5.85
700	1.64	4.29
1,000	1.41	3.01
3,000	0.63	−3.98
7,000	0.28	−10.97
10,000	0.20	−14.02
30,000	0.07	−23.53
100,000	0.02	−33.98

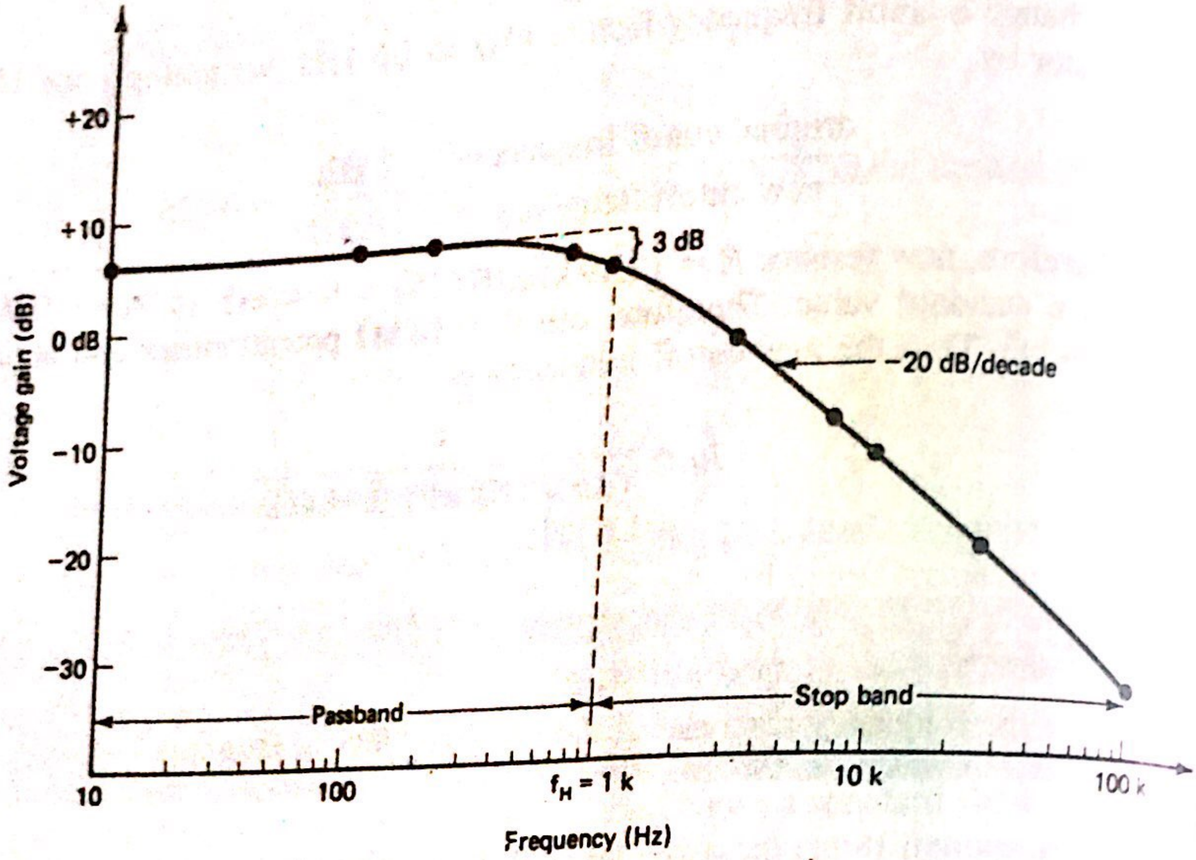


FIGURE 7–3 Frequency response for Example 7–3.

7–4 SECOND-ORDER LOW-PASS BUTTERWORTH FILTER

A stop-band response having a 40-dB/decade roll-off is obtained with the second-order low-pass filter. A first-order low-pass filter can be converted into a second-order type simply by using an additional *RC* network, as shown in Figure 7–4.

Second-order filters are important because higher-order filters can be designed using them. The gain of the second-order filter is set by R_1 and R_F, while the high cutoff frequency f_H is determined by R_2, C_2, R_3, and C_3, as follows:

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}} \tag{7–3}$$

For the derivation of f_H, refer to Appendix C.

Furthermore, for a second-order low-pass Butterworth response, the voltage gain magnitude equation is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F}{\sqrt{1 + (f/f_H)^4}} \tag{7–4}$$

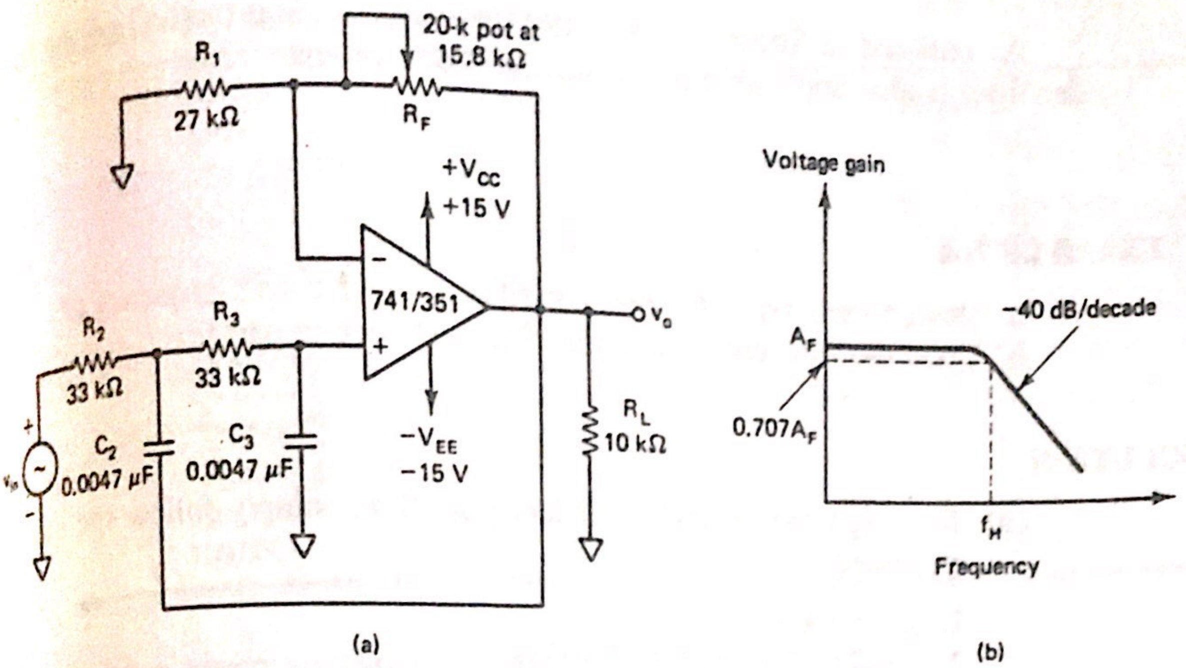


FIGURE 7–4 Second-order low-pass Butterworth filter. (a) Circuit. (b) Frequency response.

where $A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$$f_H = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}} = \text{high cutoff frequency (Hz)}$$

7-4-1 Filter Design

Except for having twice the roll-off rate in the stopband, the frequency response of the second-order low-pass filter is identical to that of the first-order type. Therefore, the design steps of the second-order filter are identical to those of the first-order filter, as follows:

1. Choose a value for the high cutoff frequency f_H.
2. To simplify the design calculations, set $R_2 = R_3 = R$ and $C_2 = C_3 = C$. Then choose a value of $C \leq 1\ \mu F$.
3. Calculate the value of R using Equation (7–3):

$$R = \frac{1}{2\pi f_H C}$$

4. Finally, because of the equal resistor ($R_2 = R_3$) and capacitor ($C_2 = C_3$) values, the passband voltage gain $A_F = (1 + R_F/R_1)$ of the second-order low-pass filter has to be equal to 1.586. That is, $R_F = 0.586R_1$. This gain is necessary to guarantee Butterworth response. Hence choose a value of $R_1 \leq 100\ k\Omega$ and calculate the value of R_F.

TABLE … **Data for Example 7–4.**

Frequency, f (Hz)	Gain magnitude, $\lvert v_o/v_{in}\rvert$	Magnitude (dB) = $20 \log \lvert v_o/v_{in}\rvert$
10	1.59	4.01
100	1.59	4.01
200	1.58	4.00
700	1.42	3.07
1,000	1.12	1.00
3,000	0.18	−15.13
7,000	0.03	−29.80
10,000	0.02	−35.99
30,000	1.76×10^{-3}	−55.08
100,000	1.59×10^{-4}	−75.99

5 FIRST-ORDER HIGH-PASS BUTTERWORTH FILTER

High-pass filters are often formed simply by interchanging frequency-determining resistors and capacitors in low-pass filters. That is, a first-order high-pass filter is formed from a first-order low-pass type by interchanging components R and

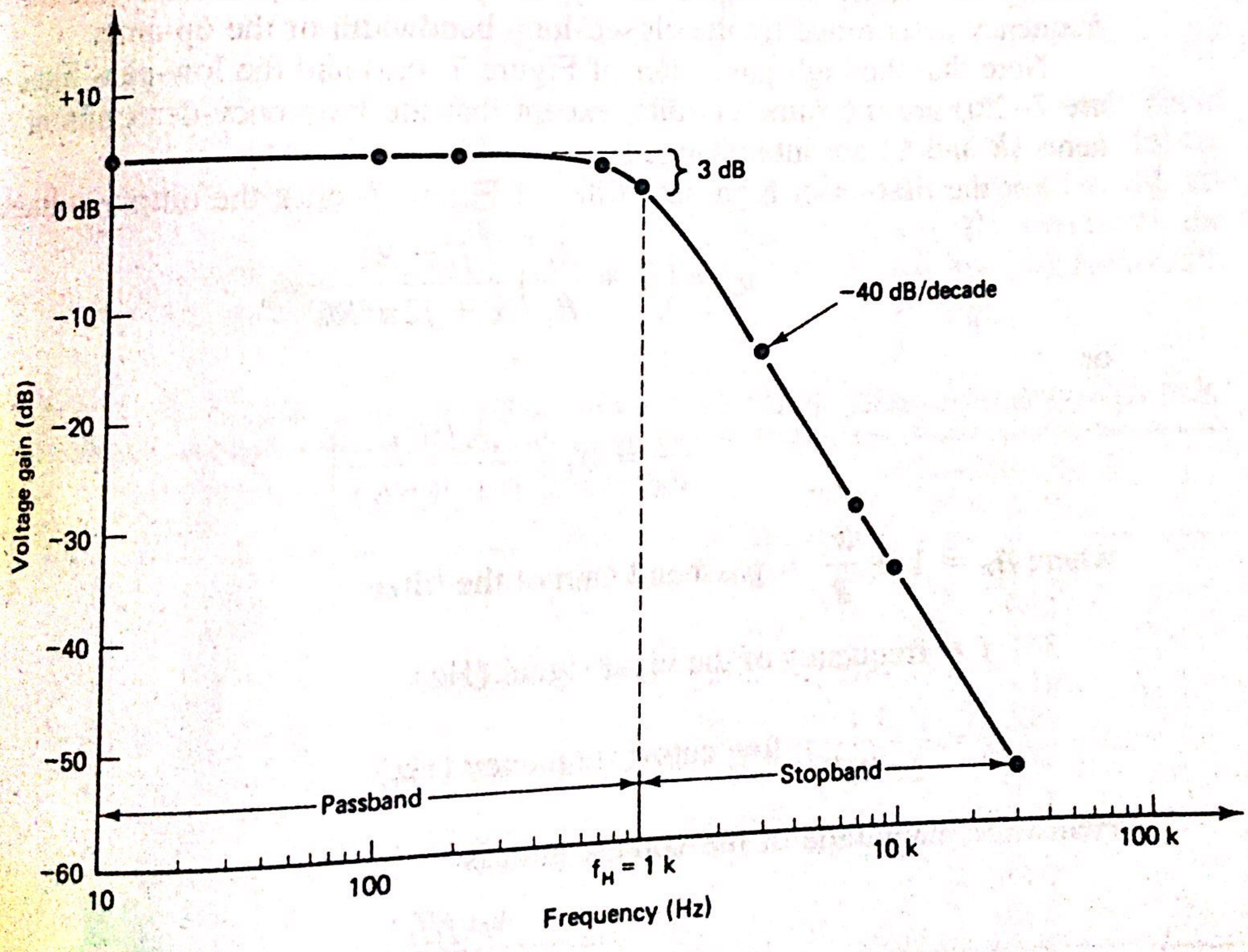


FIGURE 7–5 Frequency response for Example 7–4.

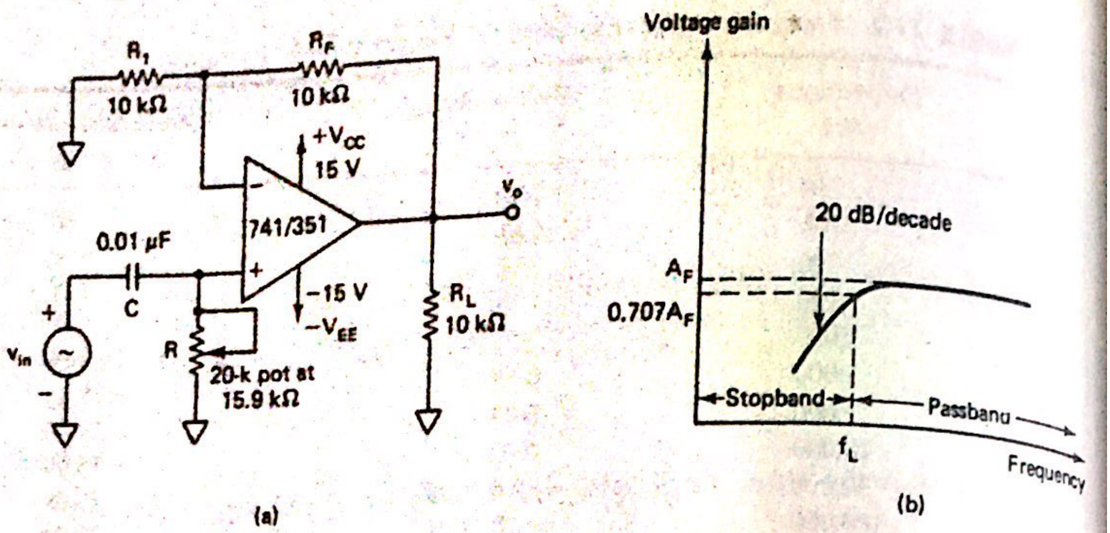


FIGURE 7–6 (a) First-order high-pass Butterworth filter. (b) Its frequency response.

C. Similarly, a second-order high-pass filter is obtained from a second-order low-pass filter if R and C are interchanged, and so on. Figure 7–6 shows a first-order high-pass Butterworth filter with a low cutoff frequency of f_L. This is the frequency at which the magnitude of the gain is 0.707 times its passband value. Obviously, all frequencies higher than f_L are passband frequencies, with the highest frequency determined by the closed-loop bandwidth of the op-amp.

Note that the high-pass filter of Figure 7–6(a) and the low-pass filter of Figure 7–2(a) are the same circuits, except that the frequency-determining components (R and C) are interchanged.

For the first-order high-pass filter of Figure 7–6(a), the output voltage is

$$v_o = \left(1 + \frac{R_F}{R_1}\right)\frac{j2\pi fRC}{1 + j2\pi fRC}\, v_{in}$$

or

$$\frac{v_o}{v_{in}} = A_F\left[\frac{j(f/f_L)}{1 + j(f/f_L)}\right] \tag{7–5}$$

where $A_F = 1 + \dfrac{R_F}{R_1}$ = passband gain of the filter

f = frequency of the input signal (Hz)

$f_L = \dfrac{1}{2\pi RC}$ = low cutoff frequency (Hz)

Hence the magnitude of the voltage gain is

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}} \tag{7–6}$$

Since high-pass filters are formed from low-pass filters simply by interchanging R's and C's, the design and frequency scaling procedures of the low-pass filters are also applicable to the high-pass filters (see Sections 7–3–1 and 7–3–2).

EXAMPLE 7–5

a. Design a high-pass filter at a cutoff frequency of 1 kHz with a passband gain of 2.
b. Plot the frequency response of the filter in part (a).

SOLUTION

a. Use the same values of R and C that were determined for the low-pass filter of Example 7–1, since $f_L = f_H = 1$ kHz. That is, $C = 0.01\ \mu F$ and $R = 15.9\ k\Omega$. Similarly, use $R_1 = R_F = 10\ k\Omega$, since $A_F = 2$.

b. The data for the frequency response plot can be obtained by substituting for the input frequency f values from 100 Hz to 100 kHz in Equation (7–6). These data are included in Table 7–3. Equation (7–6) is repeated here for convenience:

$$\left|\frac{v_o}{v_{in}}\right| = \frac{A_F(f/f_L)}{\sqrt{1 + (f/f_L)^2}}$$

where $A_F = 2$ and $f_L = 1$ kHz. The frequency response data of Table 7–3 are plotted in Figure 7–7. In the stopband (from 100 Hz to 1 kHz) the gain increases at the rate of 20 dB/decade. However, in the passband (after $f = f_L = 1$ kHz) the gain remains constant at 6.02 dB. Moreover, the upper-frequency limit of the passband is set by the closed-loop bandwidth of the op-amp.

TABLE 7–3 Frequency Response Data for the First-Order High-Pass Filter of Example 7–5.

Frequency, f (*Hz*)	*Gain magnitude,* $\lvert v_o/v_{in}\rvert$	*Magnitude (dB)* = 20 $\log\lvert v_o/v_{in}\rvert$
100	0.20	−14.02
200	0.39	−8.13
400	0.74	−2.58
700	1.15	1.19
1,000	1.41	3.01
3,000	1.90	5.56
7,000	1.98	5.93
10,000	1.99	5.98
30,000	2	6.02
100,000	2	6.02

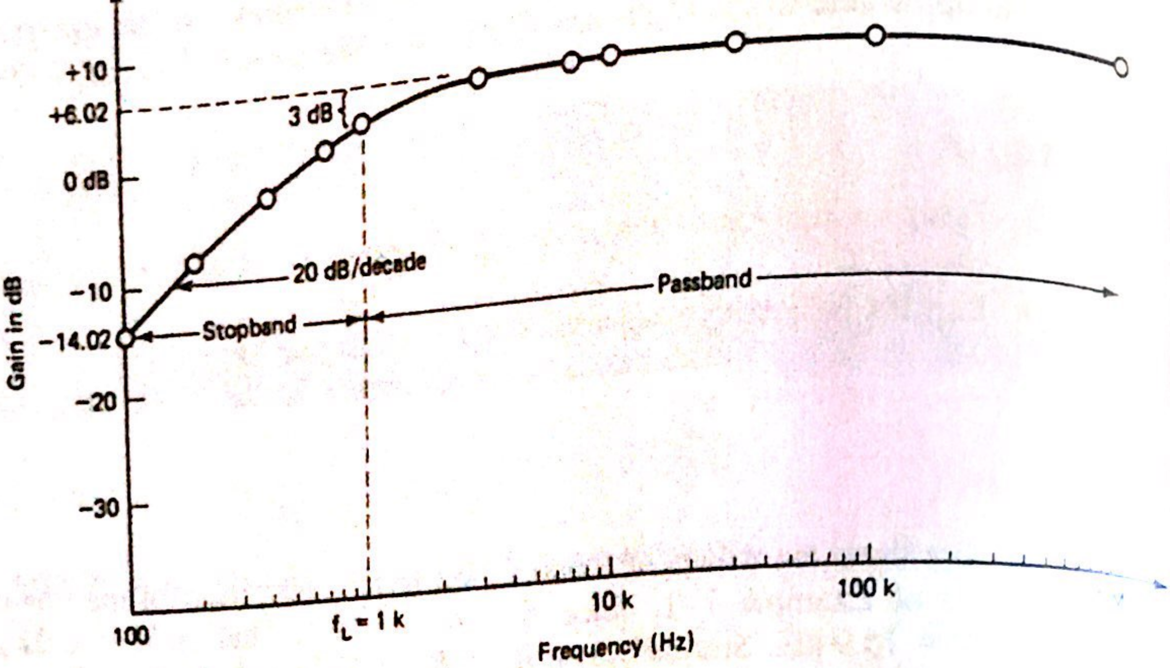


FIGURE 7–7 Frequency response for Example 7–5.

7–6 SECOND-ORDER HIGH-PASS BUTTERWORTH FILTER

As in the case of the first-order filter, a second-order high-pass filter can be formed from a second-order low-pass filter simply by interchanging the frequency-determining resistors and capacitors. Figure 7–8(a) shows the second-order high-pass filter.

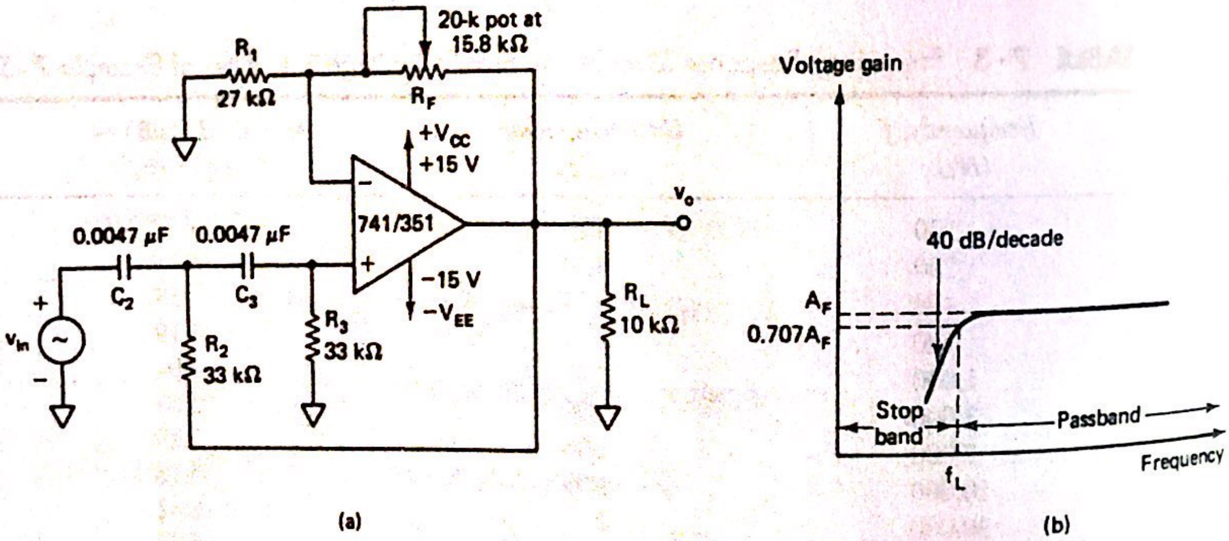


FIGURE 7–8 (a) Second-order high-pass Butterworth filter. (b) Its frequency response.

The voltage gain magnitude equation of the second-order high-pass filter is as follows:

$$\left|\frac{v_o}{v_{\text{in}}}\right| = \frac{A_F}{\sqrt{1 + (f_L/f)^4}} \tag{7-7}$$

where A_F = 1.586 = passband gain for the second-order Butterworth response
f = frequency of the input signal (Hz)
f_L = low cutoff frequency (Hz)

Since second-order low-pass and high-pass filters are the same circuits except that the positions of resistors and capacitors are interchanged, the design and frequency scaling procedures for the high-pass filter are the same as those for the low-pass filter.

EXAMPLE 7-6

a. Determine the low cutoff frequency f_L of the filter shown in Figure 7-8(a).
b. Draw the frequency response plot of the filter.

SOLUTION

a.

$$f_L = \frac{1}{2\pi\sqrt{R_2R_3C_2C_3}}$$

$$= \frac{1}{2\pi\sqrt{(33\text{ k}\Omega)^2(0.0047\ \mu\text{F})^2}} \cong 1\text{ kHz}$$

b. The frequency response data in Table 7-4 are obtained from the voltage gain magnitude equation, (7-7), which is repeated here for convenience:

$$\left|\frac{v_o}{v_{\text{in}}}\right| = \frac{A_F}{\sqrt{1 + (f_L/f)^4}}$$

where A_F = 1.586 and f_L = 1 kHz. The resulting frequency response plot is shown in Figure 7-9.

7-7 HIGHER-ORDER FILTERS

From the preceding discussions of filters we can conclude that in the stopband the gain of the filter changes at the rate of 20 dB/decade for first-order filters and at 40 dB/decade for second-order filters. This means that, as the order of the filter is increased, the actual stopband response of the filter approaches its ideal stopband characteristic.

TABLE 7–4 Frequency Response Data for Second-Order High-Pass Filter of Example 7–

Input frequency, f(Hz)	Gain magnitude, $\lvert v_o/v_{in} \rvert$	Magnitude (dB) = 20 log$\lvert v_o/v_{in} \rvert$
100	0.01586	−35.99
200	0.0634	−23.96
700	0.6979	−3.124
1,000	1.1215	0.9960
3,000	1.5763	3.953
7,000	1.5857	4.004
10,000	1.5859	4.006
30,000	1.5860	4.006
100,000	1.5860	4.006

Higher-order filters, such as third, fourth, fifth, and so on, are formed simply by using the first- and second-order filters. For example, a third-order low-pass filter is formed by connecting in series or cascading first- and second-order low-pass filters; a fourth-order low-pass filter is composed of two cascaded second-order low-pass sections, and so on. Although there is no limit to the order of the filter that can be formed, as the order of the filter increases, so does its size. Also, its accuracy declines, in that the difference between the actual stopband response and the theoretical stopband response increases with an increase in the order of the filter. Figure 7–10 shows third- and fourth-order low-pass Butterworth filters. Note that in the third-order filter the voltage gain of the first-order section is *one*, and that of the second-order section is *two*. On the other hand, in the fourth-

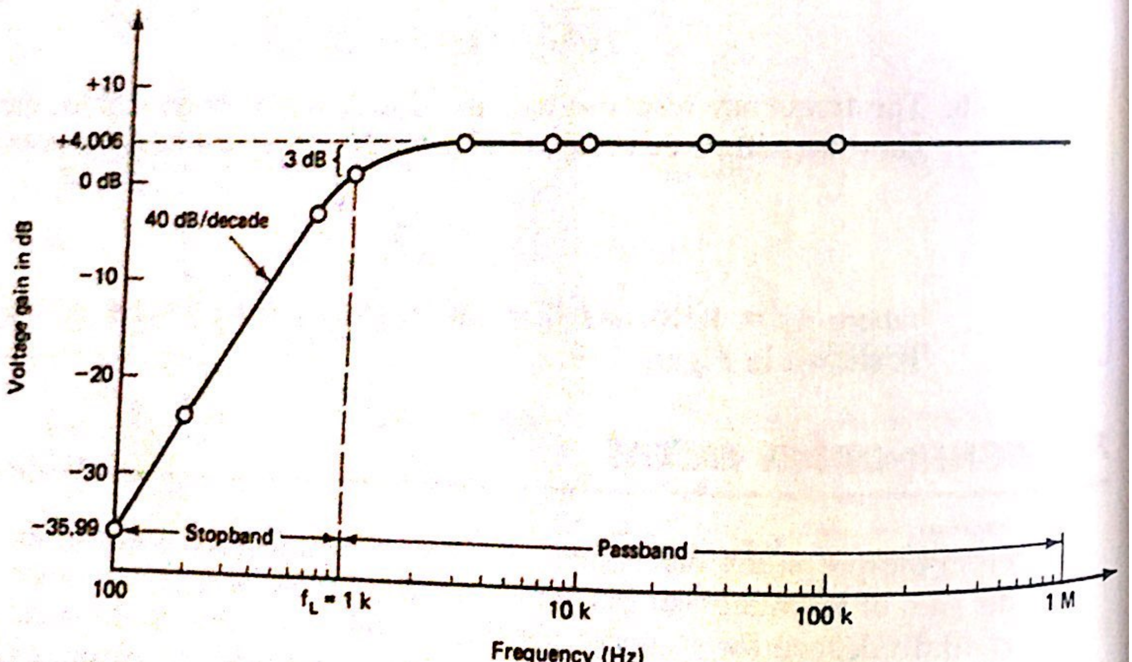


FIGURE 7–9 Frequency response for Example 7–6.

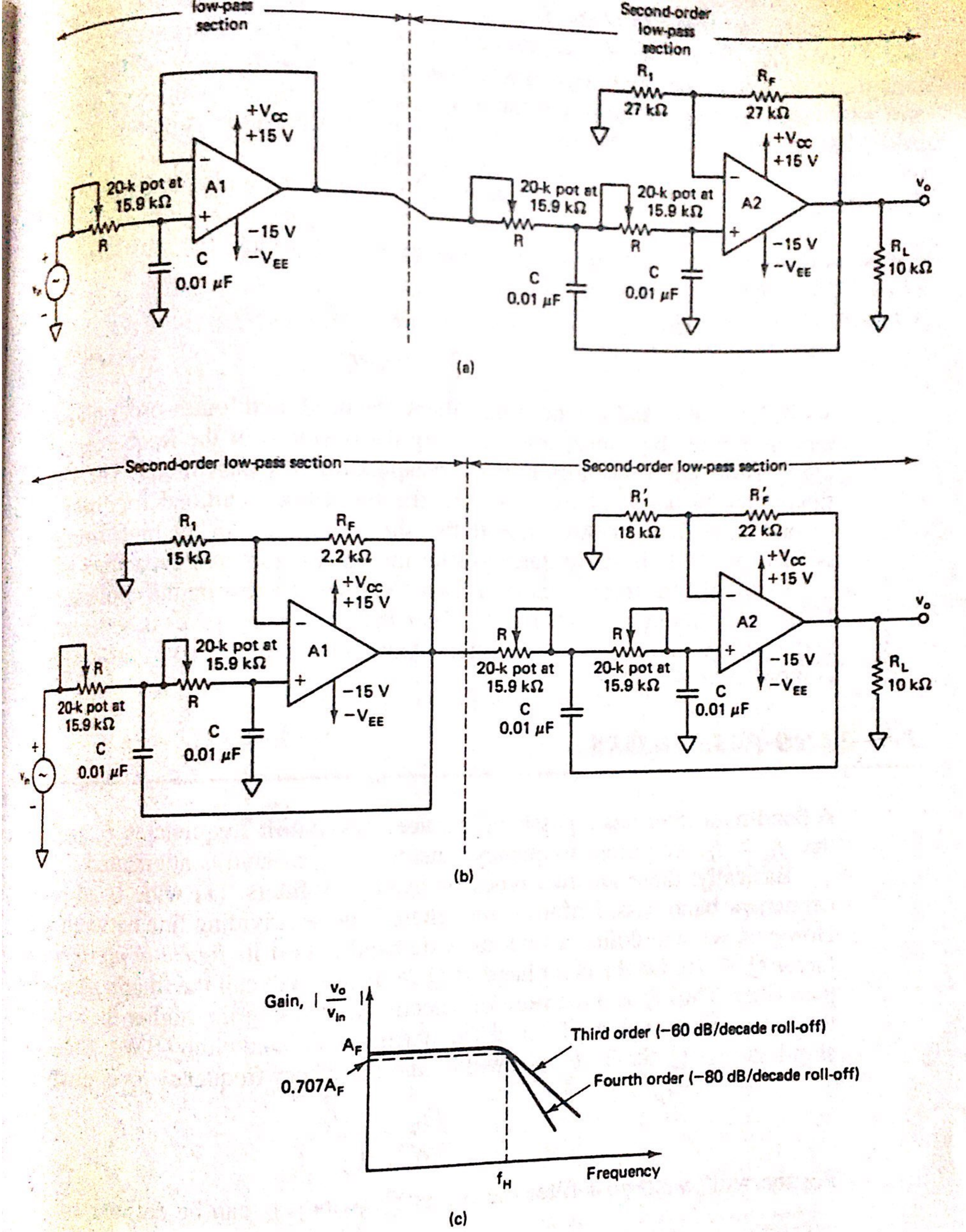


FIGURE 7–10 (a) Third-order and (b) fourth-order low-pass Butterworth filters. (c) Their frequency responses. A_1 and A_2 dual op-amp: 1458/353.

order filter the gain of the first section is *1.152*, while that of the second section is *2.235*. These gain values are necessary to guarantee Butterworth response and must remain the same regardless of the filter's cutoff frequency. Furthermore, the overall gain of the filter is equal to the product of the individual voltage gains of the filter sections. Thus the overall gain of the third-order filters is 2.0, and that of the fourth order filters is $(1.152)(2.235) = 2.57$.

Since the frequency-determining resistors are equal and the frequency-determining capacitors are also equal, the high cutoff frequencies of the third- and fourth-order low-pass filters in Figure 7–10(a) and (b) must also be equal. That is,

$$f_H = \frac{1}{2\pi RC} \tag{7–8}$$

As with the first- and second-order filters, the third- and fourth-order high-pass filters are formed by simply interchanging the positions of the frequency-determining resistors and capacitors in the corresponding low-pass filters. The high-order filters can be designed by following the procedures outlined for the first- and second-order filters. However, note that the overall gain of the higher-order filters is *fixed* because all the frequency-determining resistors and capacitors are equal.

Generally, the minimum-order filter required depends on the application specifications. Although a higher-order filter than necessary gives a better stop-band response, the higher-order type filter is more complex, occupies more space, and is more expensive.

7–8 BAND-PASS FILTERS

A band-pass filter has a passband between two cutoff frequencies f_H and f_L such that $f_H > f_L$. Any input frequency outside this passband is attenuated.

Basically, there are two types of band-pass filters: (1) wide band pass, and (2) narrow band pass. Unfortunately, there is no set dividing line between the two. However, we will define a filter as wide band pass if its *figure of merit* or *quality factor* $Q < 10$. On the other hand, if $Q > 10$, we will call the filter a narrow band-pass filter. Thus Q is a measure of selectivity, meaning the higher the value of Q, the more selective is the filter or the narrower its bandwidth (BW). The relationship between Q, the 3-dB bandwidth, and the center frequency f_C is given by

$$Q = \frac{f_C}{\text{BW}} = \frac{f_C}{f_H - f_L} \tag{7–9a}$$

For the wide band-pass filter the center frequency f_C can be defined as

$$f_C = \sqrt{f_H f_L} \tag{7–9b}$$

where f_H = high cutoff frequency (Hz)

f_L = low cutoff frequency of the wide band-pass filter (Hz)

In a narrow band-pass filter, the output voltage peaks at the center frequency.

7-8-1 Wide Band-Pass Filter

A wide band-pass filter can be formed by simply cascading high-pass and low-pass sections and is generally the choice for simplicity of design and performance. To obtain a ± 20 dB/decade band-pass, first-order high-pass and first-order low-pass sections are cascaded; for a ± 40-dB/decade band-pass filter, second-order high-pass and second-order low-pass sections are connected in series, and so on. In other words, the order of the band-pass filter depends on the order of the high-pass and low-pass filter sections.

Figure 7–11 shows the ± 20-dB/decade wide band-pass filter, which is composed of first-order high-pass and first-order low-pass filters. To realize a band-pass response, however, f_H must be larger than f_L, as illustrated in Example 7–7.

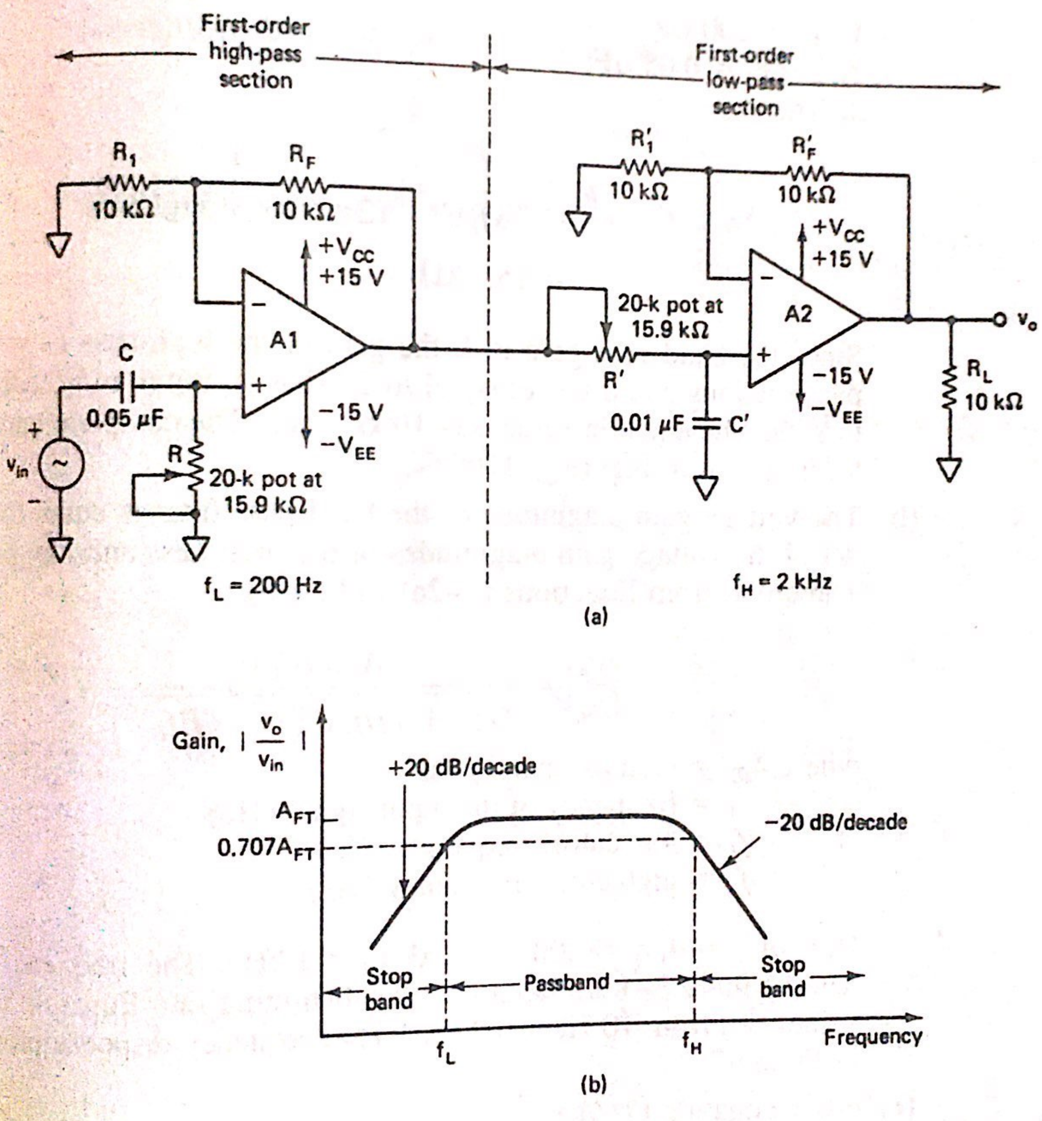


FIGURE 7–11 (a) ± 20 dB/decade-wide band-pass filter. (b) Its frequency response. A_1 and A_2 dual op-amp: 1458/353.

7-8-2 Narrow Band-Pass Filter

The narrow band-pass filter using multiple feedback is shown in Figure 7–13. As shown in this figure, the filter uses only one op-amp. Compared to all the filters discussed so far, this filter is unique in the following respects:

1. It has two feedback paths, hence the name *multiple-feedback filter*.
2. The op-amp is used in the *inverting* mode.

Generally, the narrow band-pass filter is designed for specific values of center frequency f_C and Q or f_C and bandwidth [see Equation (7–9a)]. The circuit components are determined from the following relationships.

To simply the design calculations, choose $C_1 = C_2 = C$.

$$R_1 = \frac{Q}{2\pi f_C C A_F} \tag{7-11}$$

$$R_2 = \frac{Q}{2\pi f_C C(2Q^2 - A_F)} \tag{7-12}$$

$$R_3 = \frac{Q}{\pi f_C C} \tag{7-13}$$

where A_F is the gain at f_C, given by

$$A_F = \frac{R_3}{2R_1} \tag{7-14a}$$

The gain A_F, however, must satisfy the condition

$$A_F < 2\,Q^2 \tag{7-14b}$$

Another advantage of the multiple feedback filter of Figure 7–13 is that its center frequency f_C can be changed to a new frequency f'_C without changing the gain or bandwidth. This is accomplished simply by changing R_2 to R'_2 so that

$$R'_2 = R_2\left(\frac{f_C}{f'_C}\right)^2 \tag{7-15}$$

(see Example 7–8).

EXAMPLE 7–8

a. Design the bandpass filter shown in Figure 7–13(a) so that f_C = 1 kHz, $Q = 3$, and $A_F = 10$.
b. Change the center frequency to 1.5 kHz, keeping A_F and the bandwidth constant.

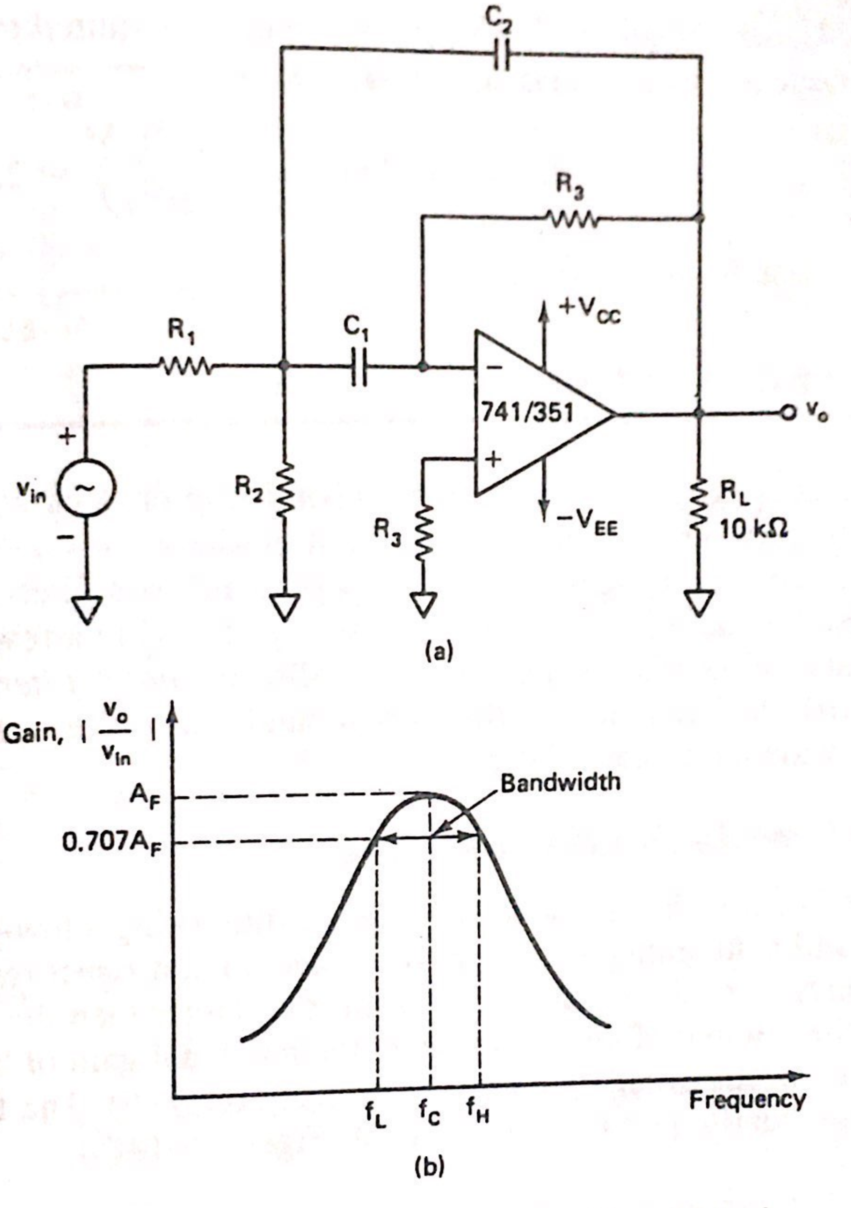


FIGURE 7–13 (a) Multiple-feedback narrow band-pass filter. (b) Its frequency response.

b. Using Equation (7–15), the value of R_2' required to change the center frequency from 1 kHz to 1.5 kHz is

$$R_2' = (5.97\ \text{k}\Omega)\left(\frac{1(10^3)}{1.5(10^3)}\right)^2 = 2.65\ \text{k}\Omega$$

(Use $R_2' = 2.7\ \text{k}\Omega$.)

7-9 BAND-REJECT FILTERS

The band-reject filter is also called a ***band-stop*** or ***band-elimination*** filter. In this filter, frequencies are attenuated in the stopband while they are passed outside this band, as shown in Figure 7–1(d). As with band-pass filters, the band-reject filters can also be classified as (1) wide band-reject or (2) narrow band-reject. The narrow band-reject filter is uncommonly called the ***notch filter***. Because of its higher Q (>10), the bandwidth of the narrow band-reject filter is much smaller than that of the wide band-reject filter.

7-9-1 Wide Band-Reject Filter

Figure 7–14(a) shows a wide band-reject filter using a low-pass filter, a high-pass filter, and a summing amplifier. To realize a band-reject response, the low cutoff frequency f_L of the high-pass filter must be larger than the high cutoff frequency f_H of the low-pass filter. In addition, the passband gain of both the high-pass and low-pass sections must be equal (see Example 7–9). The frequency response of the wide band-reject filter is shown in Figure 7–14(b).

EXAMPLE 7-9

Design a wide band-reject filter having f_H = 200 Hz and f_L = 1 kHz.

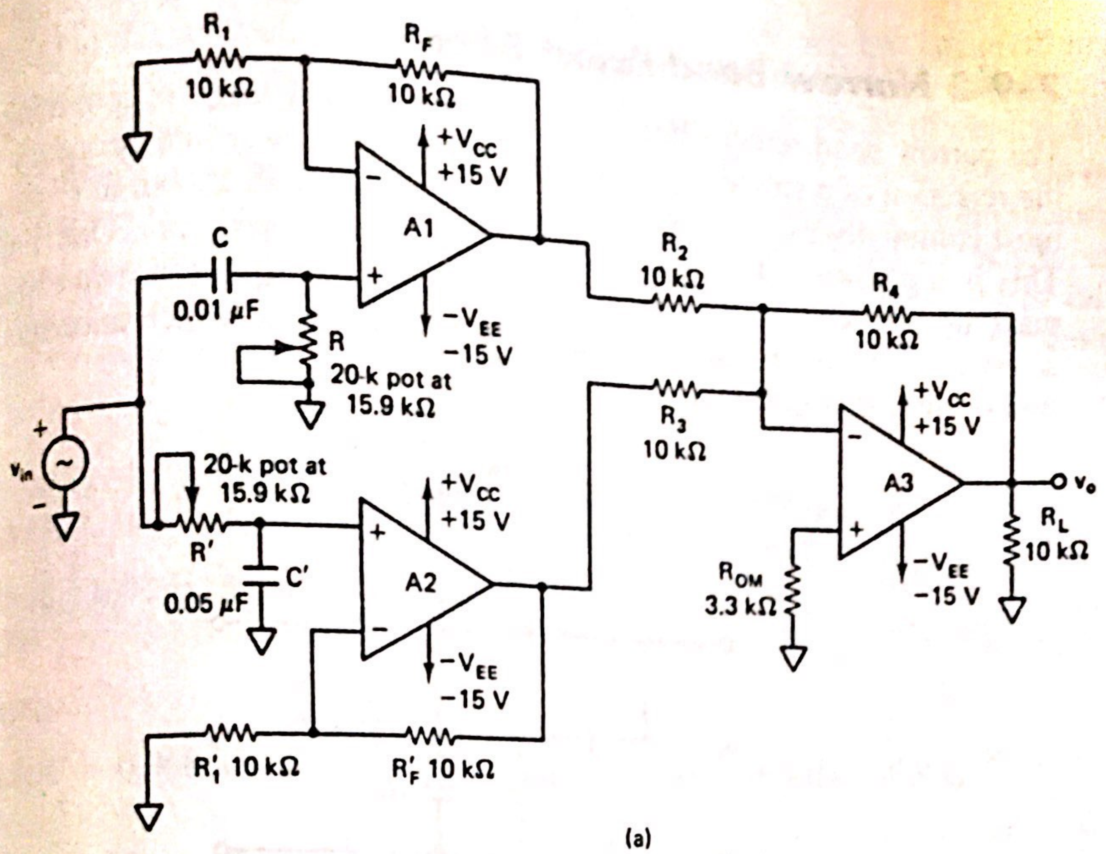
SOLUTION

In Example 7–7, a wide band-pass filter was designed with f_L = 200 Hz and f_H = 1 kHz. In this example these band frequencies are interchanged, that is, f_L = 1 kHz and f_H = 200 Hz. This means that we can use the same components as in Example 7–7, but interchanged between high-pass and low-pass sections. Therefore, for the low-pass section, $R' = 15.9\ \text{k}\Omega$ and $C' = 0.05\ \mu\text{F}$, while for the high-pass section

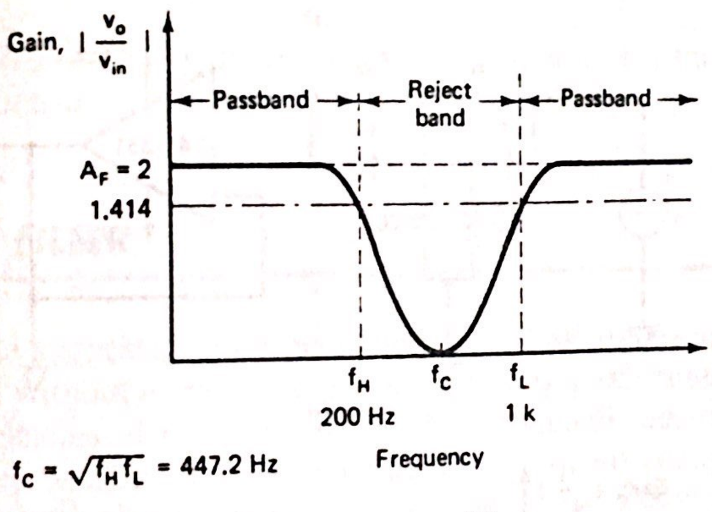
$$R = 15.9\ \text{k}\Omega \text{ and } C = 0.01\ \mu\text{F}$$

Since there is no restriction on the passband gain, use a gain of 2 for each section. Hence let

$$R_1 = R_F = R_1' = R_F' = 10\ \text{k}\Omega$$



(a)



(b)

FIGURE 7–14 Wide band-reject filter. (a) Circuit. (b) Frequency response. For A_1, A_2, and A_3 use quad op-amp μAF774/MC34004.

Furthermore, the gain of the summing amplifier is set at 1; therefore,

$$R_2 = R_3 = R_4 = 10\ \text{k}\Omega$$

Finally, the value of $R_{OM} = R_2 \| R_3 \| R_4 \cong 3.3\ \text{k}\Omega$.

The complete circuit is shown in Figure 7–14(a), and its response is shown in Figure 7–14(b). The voltage gain changes at the rate of 20 dB/decade above f_H and below f_L, with a maximum attenuation occurring at f_C.

7-9-2 Narrow Band-Reject Filter

The narrow band-reject filter, often called the *notch filter*, is commonly used for the rejection of a single frequency such as 60-Hz power line frequency hum. The most commonly used notch filter is the *twin-T* network shown in Figure 7–15(a). This is a *passive filter* composed of two T-shaped networks. One T network is made up of two resistors and a capacitor, while the other uses two capacitors and a resistor. The *notch-out* frequency is the frequency at which maximum attenuation occurs; it is given by

$$f_N = \frac{1}{2\pi RC} \tag{7-16}$$

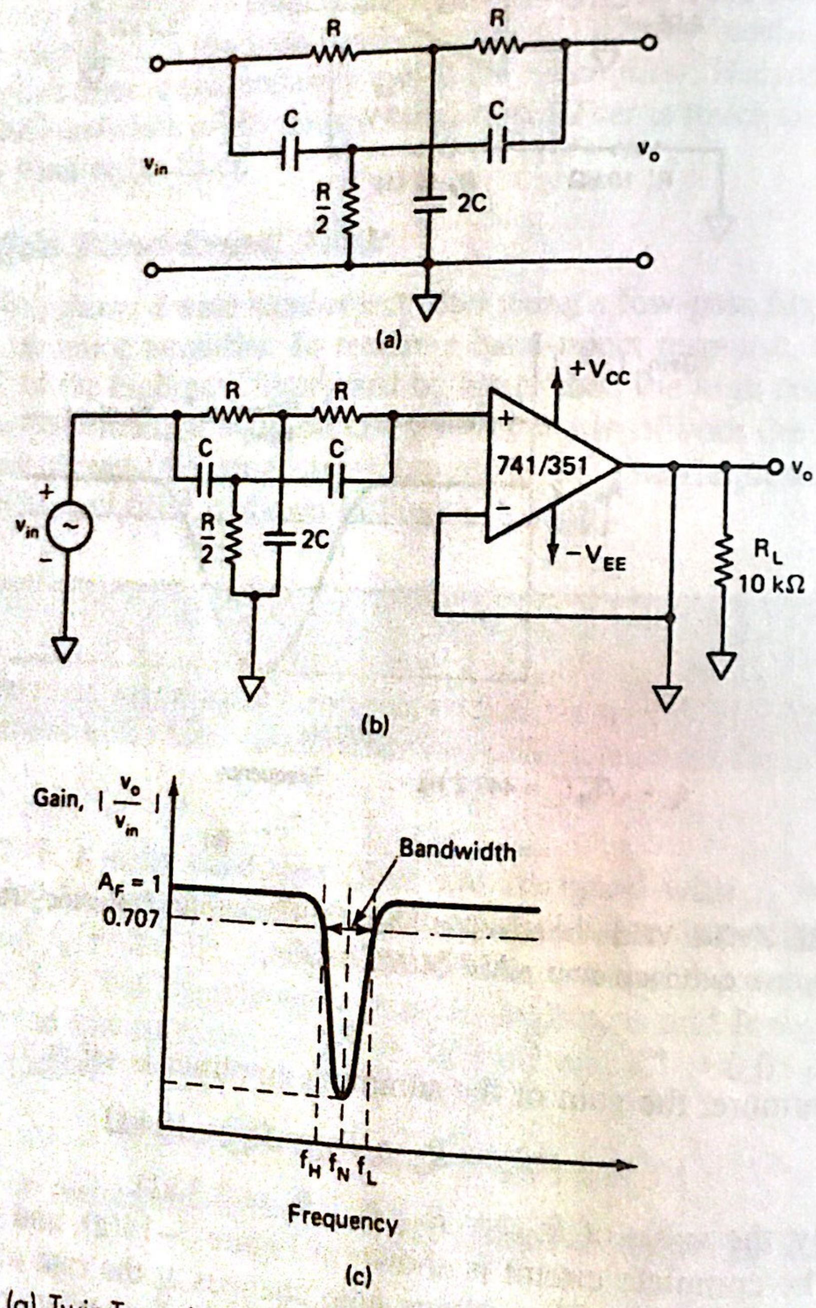


FIGURE 7–15 (a) Twin-T notch filter. (b) Active notch filter. (c) Frequency response of the active notch filter.

Unfortunately, the passive twin-T network has a relatively low figure of merit Q. The Q of the network can be increased significantly if it is used with the voltage follower as shown in Figure 7–15(b). The frequency response of the active notch filter of Figure 7–15(b) is shown in Figure 7–15(c). The most common use of notch filters is in communications and biomedical instruments for eliminating undesired frequencies. To design an active notch filter for a specific notch-out frequency f_N, choose the value of $C \leq 1\ \mu F$ and then calculate the required value of R from Equation (7–16). For the best response, the circuit components should be very close to their indicated values.

EXAMPLE 7–10

Design a 60-Hz active notch filter.