observation_operator

December 4, 2020

1 The Observation Operator for Stellar Population-Kinematic modelling

1.1 Example Usage

[1]:
```
cd ../
```

```
/Users/pjethwa/Astro/Science/uq4pk
```

[2]:
```
# get the operator

%load_ext autoreload
%autoreload 2

import uq4pk_src

from uq4pk_src.observation_operator import ObservationOperator
G = ObservationOperator(max_order_hermite=4)

# get a distribution funtion
from uq4pk_src.distribution_function import RandomGMM_DistributionFunction
df = RandomGMM_DistributionFunction(modgrid=G.ssps)

# define LOSVD parameters
V, sigma, h0, h1, h2, h3, h4 = 30., 130., 1., 0., 0., -0.05, 0.1
Theta_v = [V, sigma, h0, h1, h2, h3, h4]

# evaluate the operator
ybar = G.evaluate(df.F, Theta_v)
```

```
Distribution functions should have shape (12, 53)
```

1.2 Requirements

Before running this code you will need to install

- astropy

as well as a number of standard Python packages,

- numpy
- scipy
- matplotlib
- os

1.3 Installation

To import and run this code, you will need to put it somewhere accessible to your Python installation. If this is unfamiliar you you, I recommend the following simple solution. Set an environment variable `PYTHONPATH` to any convenient location, e.g.

```
setenv PYTHONPATH = '/Users/your_name/Library/MyPythonLibrary'
```

then create this directory, and copy the entire directory `uq4pk_src` (i.e. the source code) to that location. The following import statement should then be successful,

```
[3]: import uq4pk_src
```

1.4 Evaluating the Observation Operator

The first step is to initialise the observation operator object,

```
[4]: from uq4pk_src.observation_operator import ObservationOperator
     G = ObservationOperator(max_order_hermite=4)
```

```
Distribution functions should have shape (12, 53)
```

This initialises various properties of the SSP models and the LOSVD. One value we must provide is the maximum order of Hermite polynomial that we will use in the expansion: 4 is sufficient for now. We are also told the shape of the distributiuon function that which is consistent with the grid of SPP models.

To evaluate this, let's generate a (random, unrealsitic) input distributiuon function, namely a Gaussian mixture,

```
[5]: from uq4pk_src.distribution_function import RandomGMM_DistributionFunction
     df = RandomGMM_DistributionFunction(modgrid=G.ssps)
     print('Distribution function stored at df.F')
     print(f'This has shape {df.F.shape}')
```

```
Distribution function stored at df.F
This has shape (12, 53)
```

Let's set parameters Theta_v of the LOSVD

```
[6]: V = 30.
     sigma = 100.
     h0, h1, h2, h3, h4 = 1., 0., 0., -0.05, 0.1
```

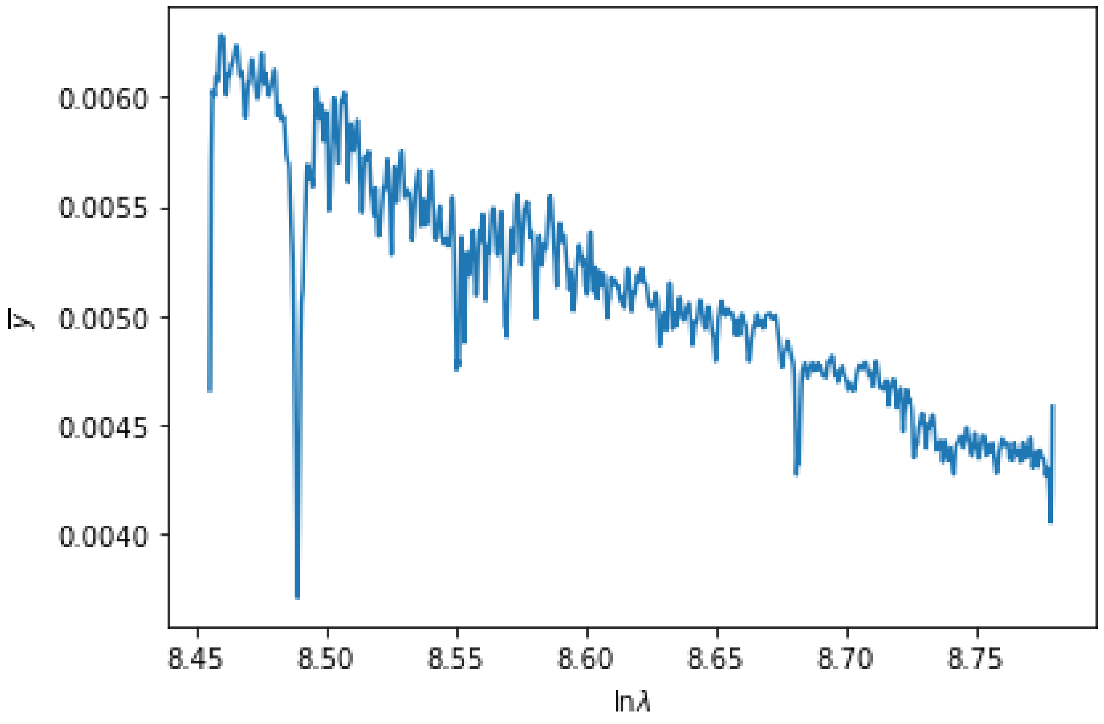
```
Theta_v = [V, sigma, h0, h1, h2, h3, h4]
```

And then to evaluate the operator,

[7]:
```
ybar = G.evaluate(df.F, Theta_v)
```

To look at the generated result,

[8]:
```
import matplotlib.pyplot as plt
plt.plot(G.ssps.w, ybar)
plt.gca().set_xlabel('$\ln \lambda$')
plt.gca().set_ylabel('$\overline{y}$')
plt.tight_layout()
```



1.5 Error: wrong definition of the GH-LOSVD

I made an error in the definition of the GH-LOSVD. The normal density should be “un-whitened”, i.e.

$$L(v;\theta_v) = \mathcal{N}(v;V,\sigma^2)\left[\sum_{m=0}^{M} h_m H_m(\hat{v})\right]$$

$$\hat{v} = \frac{v - V}{\sigma}$$

The incorrect "whitened" version differs by a factor of $1/\sigma$. All calculations in the code use the correct version.

1.6 Evaluating the Convolution

Equation (1.3) of the document "Reformulation of the model using curvelets" is a convolutuion between the composite spectrum S and the LOSVD L. We evaluate it using Fourier transforms. Here are some notes about how we evaluate this convolution.

1.6.1 Logarithmic Resampling for Convolution

See equations (2-7) of the STECKMAP paper to see the transformations applied in order to convert (1.3) into a standard convolution. One necessary transformation is $w = \ln \lambda$ i.e. going to the logarithm of wavelength. We therefore logarithmically re-sample the SSP model wavelengths. The resampled values are stored in

`G.ssps.w`

1.6.2 Pre-calculation of the Fourier Transforms of the SSP models

The Fourier transforms of the SSP models are pre-calculated when we create the `ObservationOperator` object, i.e. line 30 of `observation_operator.py`. These are stored in the attribute `G.ssps.F_tilde_s`. To get the Fourier transform of the composite spectrum S, we must evaluate the integral of FT's of the template spectra over the distribtuion funtion - i.e. the Fourier transformed version of equation (1.2) of "Reformulation of the model using curvelets". We approcimate this integral by the sum evaluated on line 101 of `observation_operator.py`.

1.6.3 Analytic Fourier Transforms of GH-LOSVDs

We use analytic expressions to evaluate FTs of the GH-LOSVDs. These can be found in Sections 2.3 and 4.4 of the PPXF paper.

1.7 Partial derivatives wrt Θ_v

A method to evaluate the partial derivatives of the observation operator,

[59]:
```
ddTheta_V_ybar = G.partial_derivative_wrt_Theta_v(df.F, Theta_v)
```

where - `ddTheta_V_ybar[:,0]` is partial derivative of signal wrt V - `ddTheta_V_ybar[:,1]` is partial derivative of signal wrt sigma - `ddTheta_V_ybar[:,2:]` are partial derivatives of signal wrt h_0,…, h_M