Log Normal Trading Function Calculations

First, we set up the basic functions we need throughout the notebook.

First are the CDF and inverse CDF (PPF) functions.

```
Φ[x_] := CDF[NormalDistribution[0,1], x]
Φinv[y_] := Quantile[NormalDistribution[0, 1], y]
```

Next let’s define some helper functions. These will appear often in calculations.

$$d_1[S\_,K\_,\sigma\_,\tau\_] := \frac{\text{Log}\left[\frac{S}{K}\right] + \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

$$d_2[S\_,K\_,\sigma\_,\tau\_] := \frac{\text{Log}\left[\frac{S}{K}\right] - \frac{1}{2}\sigma^2\tau}{\sigma\sqrt{\tau}}$$

Now let’s define functions that are more explicitly used for the DFMM.

These are functions used to get initial liquidity given a token amount and a price.

In[44]:=

$$L_X[x\_,S\_,K\_,\sigma\_,\tau\_] := \frac{x}{1 - \Phi[d_1[S,K,\sigma,\tau]]}$$

$$L_Y[y\_,S\_,K\_,\sigma\_,\tau\_] := \frac{y}{K\ \Phi[d_2[S,K,\sigma,\tau]]}$$

$$X[y\_,S\_,K\_,\sigma\_,\tau\_] := K\ L_Y[y,S,K,\sigma,\tau]\ (1 - \Phi[d_1[S,K,\sigma,\tau]])$$

$$Y[x\_,S\_,K\_,\sigma\_,\tau\_] := K\ L_X[x,S,K,\sigma,\tau]\ \Phi[d_2[S,K,\sigma,\tau]]$$

These are functions that are used to get prices from either a balance in X or a balance in Y.

In[48]:=
$$P_X[x\_,L\_,K\_,\sigma\_,\tau\_] := K\ \mathrm{Exp}\left[\Phi_{inv}\left[1 - \frac{x}{L}\right]\sigma - \frac{1}{2}\sigma^2\right]$$
$$P_Y[y\_,L\_,K\_,\sigma\_,\tau\_] := K\ \mathrm{Exp}\left[\Phi_{inv}\left[\frac{y}{K\ L}\right]\sigma + \frac{1}{2}\sigma^2\right]$$

Let’s initialize a pool with some constants and some liquidity.

First, let’s set the parameters for our curve, including the fee parameter γ

```
In[40]:= {K0, σ0, τ0, γ0} = {1, 1, 1, 1}; Echo[K0, "K0 = "]; Echo[σ0, "σ0 = "]; Echo[τ0, "τ
```

» K_0 = 1

» σ_0 = 1

» τ_0 = 1

» γ_0 = 1

Now, let’s set the initial liquidity by providing an amount of X and a price S.

```
In[41]:= {x0,S0} = {1, 1}; Echo[x0, "x0 = "]; Echo[x0, "S0 = "];
```

» x_0 = 1

» S_0 = 1

From this, let’s see what we will get for the initial amount of Y and L.

```
In[50]:= L0 = LX[x0,S0,K0,σ0,τ0]; Echo[N[L0, 18], "L0 = "];
         y0 = Y[x0,S0,K0,σ0,τ0]; Echo[N[L0, 18], "y0 = "];
```

» L_0 = 3.24109670456696994

» y_0 = 3.24109670456696994

Let’s check that the prices are correct after the fact.

```
In[52]:= Echo[PX[x0,L0,K0,σ0,τ0], "PX = "]; Echo[PY[y0,L0,K0,σ0,τ0], "PY = "];
```

» P_X = 1

» P_Y = 1

Just to verify that we could have done this the other way, and show the flow, let’s do that real fast.