

Study on the Risk-Return Mathematical Model Based on LSTM Time-Series Model and Monte Carlo Simulations

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Abstract—As the returns and risks of stocks are uncertain, decisions are particularly important when investing. For transactions in risky assets, it is difficult for investors to make investment decisions at prices known in the past. Both gold and bitcoin stocks in the U. S. market are high-risk products. Given the initial funding, it is a problem that our team requires to solve. In this paper, we establish the stock time series prediction model and the portfolio strategy model, find the equilibrium point between risk and return, and then obtain the best investment strategy. For problem 1: first, the price of the next trading day is predicted through the LSTM time-series model, and a preliminary qualitative judgment is made by comparing the relationship between the forecast price and the current price. Then, through simulation and the Sharp ratio of Monte Carlo simulation, the best investment ratio is obtained when considering risks and returns to solve this problem. For problem 2: by comparing the cumulative yield and dollar value obtained from the best portfolio, minimum risk portfolio and equal divided weight combination, the strategy obtained in this paper is the best, and proved from two aspects of theory and practice. For problem 3: Based on the reality and development trend of transaction expenses, this paper obtains the sensitivity analysis of transaction expenses to yield and investment holding shares by using derivatives. According to the analysis, the built model is relatively stable and is suitable for most transaction expenses to change. Also, we note that transaction frequency affects how sensitive a commission is to earnings. For question 4: we give the market trader suggestions from the developed model and the optimal strategy. It can be summarized as: following our best daily investment strategy, \$1,000 will be added after a 5-year investment and trade period to \$74,521.3; proving that this optimal investment strategy takes full consideration of risks and benefits. Reduce the risk through investment portfolio and ensuring the maximum return at this risk level; transaction expenses have less impact on investment decisions under certain transaction frequency. The model established in this paper fully considers the risk and return balance relationship. The LSTM time-series prediction model has the advantages of long-term memory, and the Monte Carlo simulation has an iterative function, which can save the tedious mathematical calculus and derivation process. However, the application of this model is limited due to the existence of assumed conditions. Moreover, we extended the model to predict further data than this five-year period to give market investors more favorable decisions.

Keywords—The Lstm Time-series Model, Monte Carlo Simulations, Sharp Ratio, Portfolio Strategy

I. INTRODUCTION

A. Problem Restatement

For stock investors, whether they are risk-averse or risk-averse, there is a common goal of maximizing total returns. There are two investment options available, Gold and Bitcoin; Two figures are now known, both of which are past daily prices. In order to design a model to determine whether traders should choose to hold for sale or buy shares, so that the initial \$1,000 will maximize the total return after a five-year trading period, this article will:

- Use the price data to date to build a model, giving the optimal strategy for each day, that is, how much gold and bitcoin to hold. And the initial 1000 yuan at the end of the investment period is drawn.
- Describes why this model provides an optimal strategy
- The degree to which the trading price affects the strategy is the sensitivity, and the path to achieving this effect.
- Communicate the implementation of optimal strategies, the establishment of models, and subsequent improvements and prospects.

B. Literature Review

"Compared with gold, bitcoin has a huge space for development in the financial sector due to its features of higher yield and higher volatility." Investors generally use bitcoin to complement gold for hedging. In this paper on the dynamic cointegration of gold and bitcoin, the authors used the half-parameter MIDAS quantile regression model for analysis and found that the gold market and the bitcoin market risk change trend opposite.^①

"Applying the risk-value var as a risk metric to a multi-stage portfolio model to find the optimal solution, considering both transaction costs and investment ratios in the constraints, and obtaining the optimal decision. The solution, considering both transaction costs and investment ratios in the constraints and obtaining the optimal decision."^②

In Gu Maochun's article "Theoretical Analysis of Reducing Portfolio Risk Reduction", it is believed that in the portfolio of Chinese medicine, try to choose a portfolio with a negative correlation coefficient.^③

"Many analysts believe that after the issuance of Bitcoin ETFs, funds in the traditional market will flock to the bitcoin market. However, some cautious people believe that having a compliant fund entry channel does not mean that investors will necessarily enter. Investors can invest in bitcoin, and whether to invest depends on the investor's risk appetite. Because Bitcoin is a high-risk asset. U.S. regulators are more cautious about reviewing Bitcoin ETFs. ^④

C. Our Model

First of all, the question can only make the best choice for each day based on the price data up to the day. The best decision on stock selection is determined not only by past data but also by future data, so we chose LSTM-based stock time series to predict the price of gold and Bitcoin every day during the five-year investment period. The choice of optimal investment decisions is based on our forecasted data.

Secondly, in the selection of the optimal combination, we used python to conduct a portfolio analysis of gold and bitcoin stocks, and the maximization of investment returns is regarded as investors' reasonable allocation of existing wealth in investable risk assets to maximize returns under risk or maximize cumulative returns. On the basis of calculating the daily return of the stock, three weight allocation schemes are used to calculate the investment income under different combinations. They are, given a portfolio underweight, an equally weighted portfolio, and a market-weighted portfolio. Correlation analysis of the portfolio was carried out. Including covariance matrix, correlation matrix, etc.

Thirdly, we weighed the risk-reward factors together. Using the Monte Catho simulation model, an effective boundary of a finite combination of points was determined, which was chosen by all rational investors. The strategy of investment risk minimization and investment optimal combination was carried out, and the optimal weight of the two holdings was determined by the Sharpe ratio. The comparison of these decisions proves that the model we have built can provide the best decision for the problem.

Finally, we constructed a mathematical model of the degree of influence of transaction fees on yield, the share of holdings, and income—i.e., sensitivity—to analyze the impact of trading prices on optimal investment decisions. It also includes an analysis of trends in transaction cost trends.

II. ASSUMPTIONS AND NOTATION

A. Assumptions

To simplify the problem, we made the following assumptions:

Asm.1 Short selling is non-existent in trading markets. Investors cannot borrow money to trade investments.

Asm.2 Prices follow the trend. In order to eliminate the interference of extreme events and behaviors and ensure the validity of prediction results, this hypothesis is made when LSTM model is used for time series price prediction.

Asm.3 The cash is invested entirely. Firstly, the problem is simplified from determining the weight of cash, gold and bitcoin in the portfolio to determining the weight of the latter two. Secondly, holding cash will lose the time value of money, there is potential loss. Finally, the 5-year investment and trade period is a long-term investment

activity. Through the decision of the model, the return must be positive, and the return will be higher if all the capital is invested.

Asm.4 The risk-free yield is Zero. The rate of return obtained through this model is much higher than the risk-free rate of return. The volatility of bitcoin returns is large, and Sharpe ratio can be ignored due to its small dimension. In addition, the standard of risk-free rate of return given in the market is different, so it is difficult to obtain a certain value to measure.

B. Notion

The symbols used in this article are described in Table I

TABLE I NOTION

P_{normal}	Data after normalization of actual price
P_{max} P_{min}	Represent the maximum value and minimum value of the actual price respectively
R_p	Expected rate of return
R_f	Risk free rate of return (set as 0 in this paper)
σ_p	Standard deviation of excess return
P_1, P_3	Represents the actual price of gold and bitcoin on day n-1 respectively
P_2, P_4	Represents the actual price of gold and bitcoin on the nth day respectively
P_5, P_6	Represents the predicted prices of gold and bitcoin respectively
γ_{gold} , $\gamma_{bitcoin}$	Represents the daily yield of gold and bitcoin on day n respectively
α_{gold} , $\alpha_{bitcoin}$	Represents the transaction costs of gold and bitcoin respectively
γ_p	Daily yield of portfolio
w_{gold} , $w_{bitcoin}$	Represents the purchase weight of gold and bitcoin respectively (where)
r_{total}	Cumulative yield of portfolio
Q_{gold} , $Q_{bitcoin}$	Represents the holding shares of gold and bitcoin respectively

III. ANALYSIS AND MODELING

A. Stock correlation analysis

We know that when making stock investment, try to choose the way of portfolio investment, which can disperse the risk and reduce the loss to the greatest extent. When choosing a portfolio, we should also try to choose stocks with low correlation coefficient, so as to avoid the probability of loss at the same time.

1) Correlation matrix

The correlation coefficient is the ratio of the product of the covariance and standard deviation of the investment returns of the two stocks. The correlation analysis inputs obtained with Python are visualized as shown in Table II and

Figure 1:

TABLE II CORRELATION MATRIX

	Bvalue	Gvalue
Bvalue	1.000000	0.053378
Gvalue	0.053378	1.000000

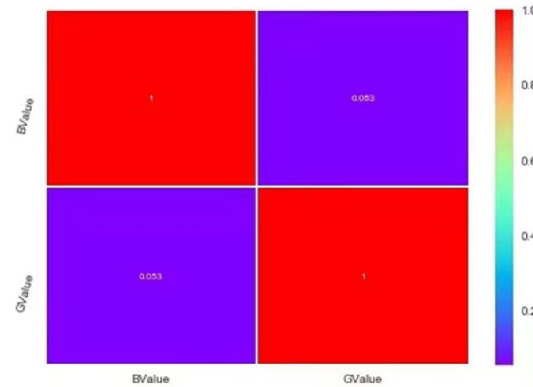


Figure 1 Heat map

According to the data in the figure, the correlation coefficient between gold and bitcoin is 0.053378. It can be seen that gold and bitcoin are basically irrelevant. Therefore, the following analysis and modeling can be carried out.

B. Analysis and modeling

The problem to be solved in this paper is: the initial \$1000, after a five-year trading period, through holding different proportions of cash, gold and bitcoin, in order to achieve the best trading strategy every day. The trading commissions of gold and bitcoin are 1% and 2% respectively, and gold can only be traded on open days. When determining the best daily strategy, only the data up to that day can be used. Considering the above problems, we establish a stock time prediction model based on LSTM and an optimal

portfolio strategy model based on Monte Carlo simulation and sharp ratio.

1) Model 1: stock time prediction model based on LSTM

According to the design information, the best trade strategy of each day can only be based on the data up to that day. Take day n as an example: according to the knowledge of "learning to buy low and sell high", the purchase, holding or sale of shares on day n can be determined by comparing the price relationship between day n and day $n + 1$. Since the price data on day $n + 1$ cannot be obtained directly, it needs to be predicted. This problem is solved by constructing a stock time prediction model based on LSTM. The specific modeling and solution ideas are shown in Figure 2:

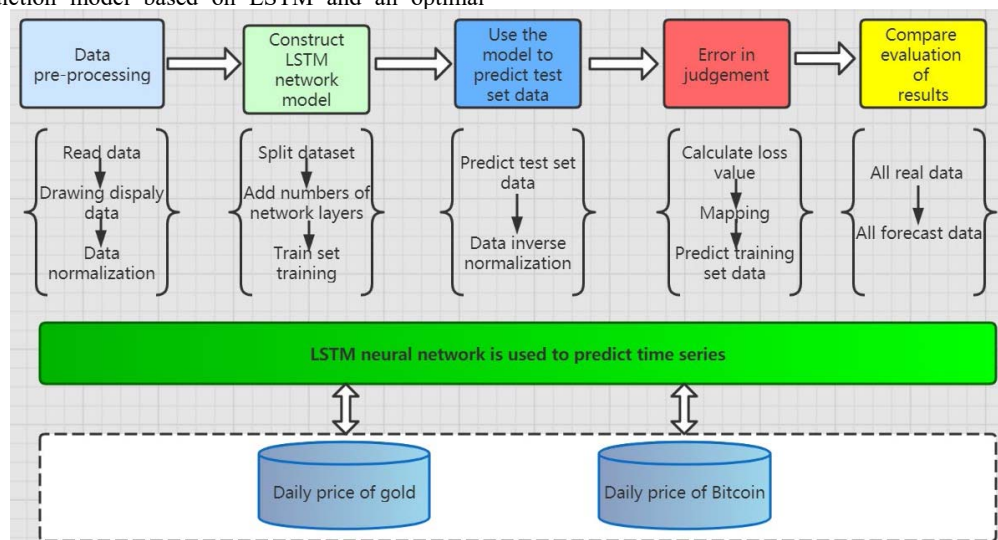


Figure 2 The specific modeling and solution ideas

- Data preprocessing

Read all price data and carry out visual output, read data data through algorithm statements and output series, and

complete visualization through pandas Reading and outputting bitcoin prices and gold prices are shown in Figure 3 and Figure 4:

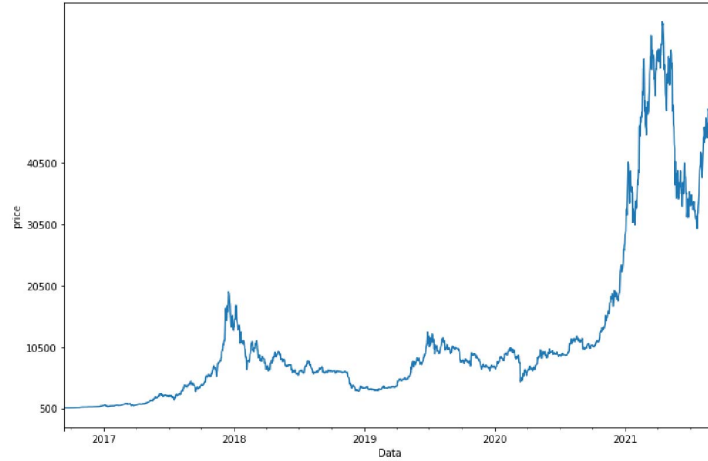


Figure 3 Reading and output of bitcoin price

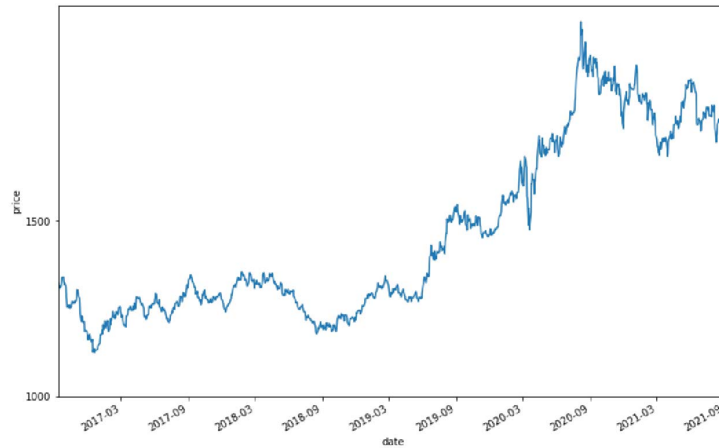


Figure 4 Reading and visual output of gold price data

As can be seen from the above chart: in the long run, the prices of gold and bitcoin are on the rise.

The normalization processing of data is provided by annex lbma-gold csv and BCHAIN-MKPRU. According to the CSV data, the prices of gold and bitcoin are in different dimensions, that is, there are strange sample data. Singular sample data is easy to increase the training time. In order to avoid this interference, normalization processing is carried out.

The linear function normalization method is used to complete the above processing, and the formula is:

$$P_{norm} = \frac{P - P_{min}}{P_{max} - P_{min}} \quad (1)$$

Of which: P_{norm} Between 0 and 1 is the price data after normalization.

P_{min} , P_{max} Represents the minimum and maximum values of the original price data respectively.

- Constructing LSTM neural network model

Principle: LSTM neural network is an improved cyclic neural network, which can solve the problem that RNN can not deal with long-distance dependence. It is also widely used in time series prediction. LSTM used in this paper refers to long-term and short-term memory artificial neural network. The long-term and short-term memory network is a time cycle neural network, that is, a cycle in time. Each cycle will use the results of the last calculation, so as to achieve the effect of learning long-term dependent information. The general operations are: ① generate model training set ② determine and adjust the network model structure ③ predict the results.

The specific creation process is as follows:

Split dataset: Take the first 80% of the five-year data as the training set and the last 20% as the prediction set

Adding network layers to build LSTM neural network: The input dimension of the first layer is 1 and the output dimension is 50; The input dimension of the second layer is 50 and the output dimension is 100; The input dimension of the third layer is 100 and the output dimension is 200

Train the training set: Start training the model with a step size of 32 and a number of iterations of 50

- Use the model to predict the test set data and judge the error

Prediction test set: The trained model is used to predict the test set.

Inverse normalization processing: The predicted data

are de normalized.

Judgment error : Calculate the loss between the predicted value of the last 20% and the real value and draw graphs, The visual output is shown in Figure 5 and Figure 6:

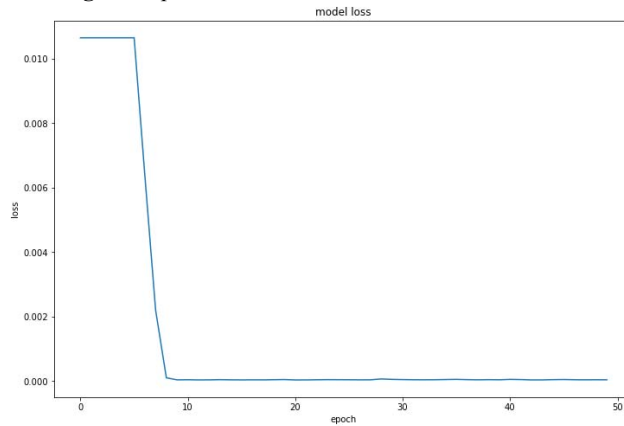


Figure 5 Bitcoin loss value

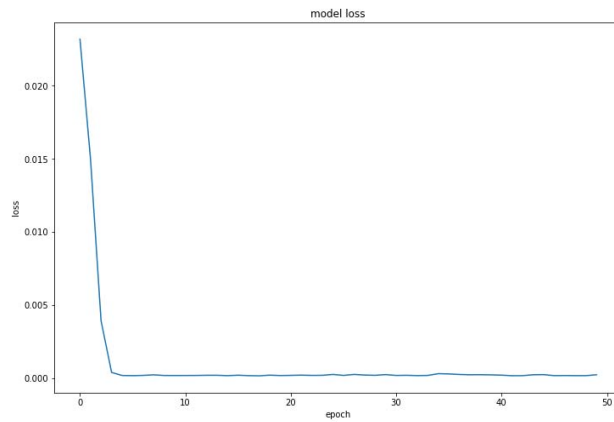


Figure 6 Gold loss value

It can be seen from the figure that the loss calculated from the predicted and real values of gold and bitcoin decreases with the increase of time dimension, and finally approaches 0.

- Evaluation of model results

Evaluate the prediction model : Compare the forecast data with the real price data, and the comparison charts are output as Figure 7 and Figure 8:



Figure 7 Comparison between predicted price and real price of bitcoin

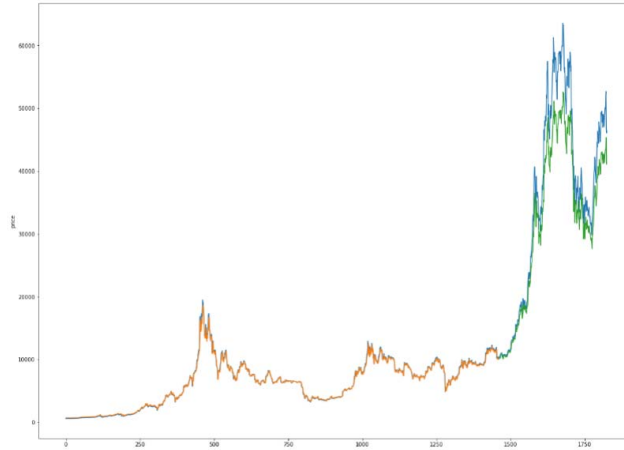


Figure 8 Comparison between predicted price and real price of gold

Among them, green represents the predicted price and blue represents the actual price.

Conclusion

The forecast price and the actual price have the same trend of change in five years, with roughly the same rise and fall.

There is an error between the predicted price and the actual price in some peaks.

Except for the peak value, the two are basically consistent.

To sum up, the prediction price obtained by the stock prediction model based on LSTM is reliable.

2) Model 2: optimal portfolio strategy model based on Monte Carlo simulation and sharp ratio

After the predicted price is obtained from model 1, it can be determined whether to buy, hold or sell the stock, which

is a qualitative decision. However, it is a quantitative decision to give the best strategy, that is, to determine the specific holding share of gold or bitcoin. To solve this decision, determine the purchase proportion of the two. In this paper, the proportion corresponding to the best combination is determined by Monte Carlo simulation of Markowitz model and Sharpe ratio, that is, the best purchase proportion. Since the transaction is a five-year period, which is a continuous investment process, and the daily rate of return varies, we introduce the cumulative rate of return. The cumulative yield of the combination can be obtained by the sum of the product of gold and bitcoin and their respective purchase proportion. Through initial funds $\times (1 + \text{cumulative rate of return of the portfolio})$ you can get the total value of the current investment. Multiply the total value by their respective purchase proportion and divide it by the price including transaction costs to obtain the holding share. The modeling process and solution are shown in Figure 9:

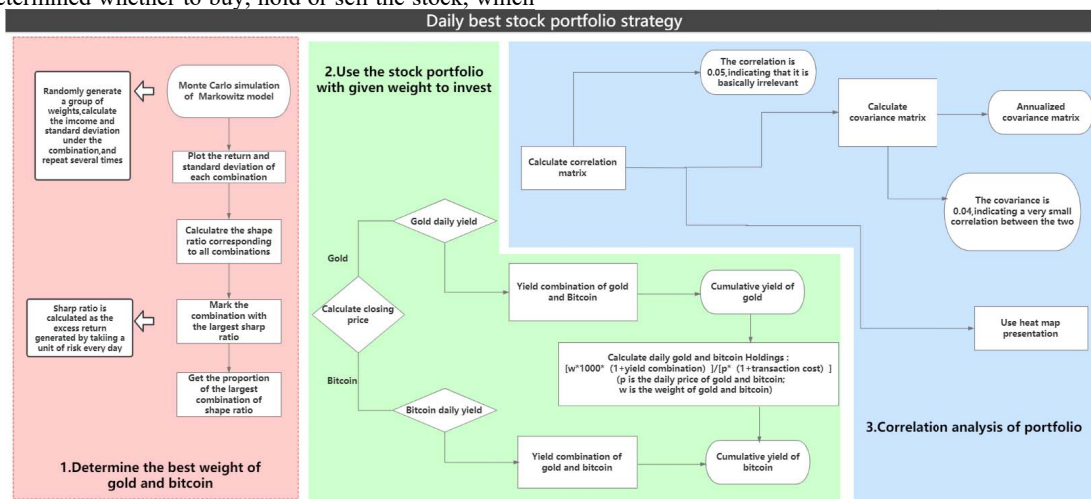


Figure 9 The modeling process and solution

- Determine the weight of gold and bitcoin in the best portfolio

Selection of indicators Monte Carlo simulation reflects the relationship between the return and standard deviation of different combinations. Therefore, we select the rate of return

and standard deviation (the standard deviation of the rate of return) as indicators.

a) The daily rate of return is calculated by using the stock returns statement algorithm to obtain the daily rate of return of stocks.

b) The standard deviation is calculated by using the calculated daily rate of return and obtained through StdevP formula in Excel.

Data preprocessing:As gold is a discontinuous transaction, it can only be traded on an open day. Therefore, on non trading days, the share of gold in the account remains unchanged. During data preprocessing, the yield of 2 non trading days is supplemented with 0.

Get the effective boundary of the portfolio:Through Monte Carlo simulation, Markowitz model can randomly generate a group of weights, calculate the standard deviation of the income under the combination, repeat the iterative process, and draw the scatter diagram of the income and standard deviation of each combination. The effective boundary of the portfolio is shown in Figure 10:

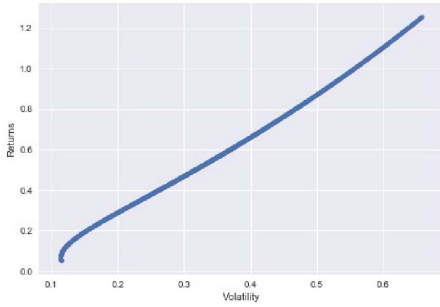


Figure 10 Effective boundary graph by Monte Carlo simulation

According to Monte Carlo portfolio theory, what rational investors pursue is to maximize the return under the same risk (standard deviation) or minimize the risk under the same return. This paper comprehensively considers the benefits and risks, and then introduces the "sharp ratio" to solve it.

Determine the optimal combination weight through sharp ratio:

Sharp ratio means that the excess return generated by each unit of total risk can help us find a balance between return and risk. The formula can be expressed as:

$$\text{Sharp ratio} = \frac{R_p - R_f}{\sigma_p} \quad (2)$$

In the above formula, is the expected rate of return, is the risk-free interest rate, and is the standard deviation of excess return.

Among them, the numerator represents the difference between the expected rate of return and the risk-free interest rate, that is, the excess rate of return.

The denominator represents the risk of obtaining excess return. The larger the denominator, the higher the risk.

a) Calculate the sharp ratio corresponding to the combination of the above Monte Carlo simulation, and draw it as the third variable in the income risk scatter diagram. Here, the visual clue of color is used to characterize the sharp ratio. The effective boundary plot for introducing the Sharpe ratio is shown in Figure 11:

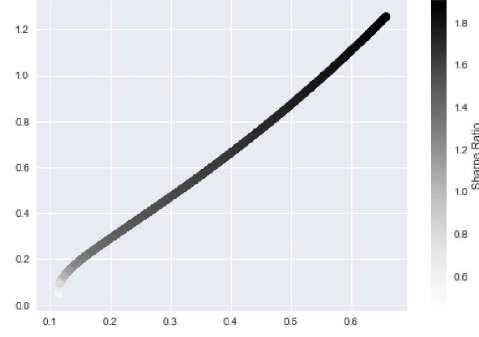


Figure 11 Effective boundary graph by introducing sharp ratio

b) Then find the combination with the largest sharp ratio and draw it in the income risk scatter chart.

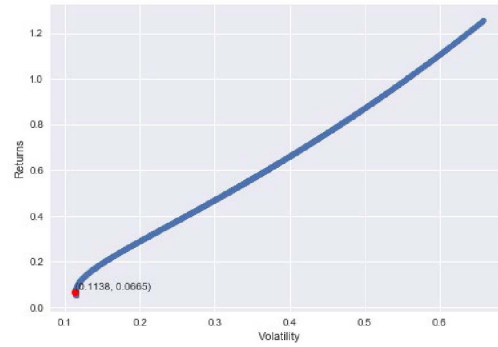


Figure 12 Determine the effective boundary graph of the maximum Sharpe ratio

c) Get the weight of the largest combination of sharp ratios.

$$[9.99612320e-01 \ 3.87679757e-04]$$

$$w_{gold} = 0.000387679757$$

$$w_{bitcoin} = 0.99612320$$

Next, use the weight to make the best combination decision.

a) Get the closing price of two stocks.

b) Calculate the daily rate of return of gold and bitcoin within five years. The formula is as follows:

Setting: the actual price of gold on day n-1 is P_1 , The actual price on day n is P_2 .

The actual price of bitcoin on day n-1 is P_3 , The actual price on day n is P_4 .

The daily yield of gold on day n is:

$$\gamma_{gold} = \frac{P_2(1 + \alpha_{gold}) - P_1(1 + \alpha_{gold})}{P_1(1 + \alpha_{gold})} = \frac{P_2 - P_1}{P_1} \quad (3)$$

The daily yield of bitcoin on day n is:

$$\gamma_{bitcoin} = \frac{P_4(1 + \alpha_{bitcoin}) - P_3(1 + \alpha_{bitcoin})}{P_3(1 + \alpha_{bitcoin})} = \frac{P_4 - P_3}{P_3} \quad (4)$$

c) Calculate the daily yield of the best combination of gold and bitcoin.

According to the portfolio theory, the rate of return of the portfolio is the weighted average of the rate of return of each

project constituting the portfolio.

The daily yield of the best combination on day n can be expressed as:

$$\gamma_p = w_{gold} \gamma_{gold} + w_{bitcoin} \gamma_{bitcoin} = w_{gold} \frac{P_2 - P_1}{P_1} + w_{bitcoin} \frac{P_4 - P_3}{P_3} \quad (5)$$

d) Visual output of daily return of portfolio is shown in Figure 13:

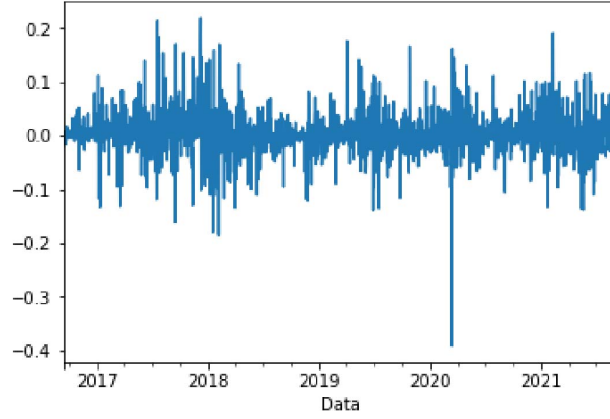


Figure 13 Daily return of portfolio

e) Calculate the cumulative daily rate of return of the portfolio and carry out visual output.

The cumulative rate of return up to day n is the sum of the daily rates of return of the best combination in the previous n days, which is expressed by the formula:

$$\gamma_{total} = \sum_{i=1}^n r_p = \sum_{i=1}^n w_{gold} \frac{P_2 - P_1}{P_1} + w_{bitcoin} \frac{P_4 - P_3}{P_3} \quad (6)$$

The cumulative return graph of the portfolio is drawn by using Python language algorithm. The visual output is shown

in Figure 14:

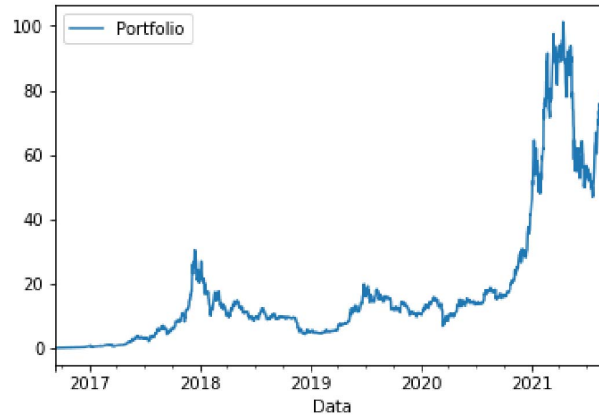


Figure 14 The cumulative return graph

As can be seen from the figure:

The cumulative rate of return of the best portfolio shows an upward trend as a whole, which shows that the daily rate

of return of the best portfolio is positive or positive after the positive and negative offsets of the daily rate of return.

In a relatively long period of time, the slope of the cumulative rate of return curve is gradually increasing, indicating that the increase range of the cumulative rate of return is gradually increasing.

According to the above contents, it can be preliminarily verified that the optimal portfolio weight determined based on Monte Carlo simulation and sharp ratio is convincing.

f) Determine the best daily investment strategy, that is,

$$\gamma_{total} = \sum_{i=1}^n r_p = \sum_{i=1}^n w_{gold} \frac{P_2 - P_1}{P_1} + w_{bitcoin} \frac{P_4 - P_3}{P_3} \quad (9)$$

Through the above formula, the holding shares of gold and bitcoin in the daily best portfolio can be obtained by using Excel.

g) The model is used to calculate the value of the initial

$$V_{1825} = 1000 \times (1 + r_{total1825}) = 1000 \times (1 + 73.52131614) = 74521.3dollars$$

C. Proof of optimal trading strategy model

The optimal trading portfolio strategy is based on Monte Carlo simulation and sharp ratio to calculate the purchase weight of them. The weight determined by the model in this paper is the optimal purchase ratio. In order to prove this model (i.e. this weight is the best purchase proportion), it is proved by comparing the cumulative rate of return obtained by the minimum risk combination and the average weight with the value of the US dollar.

the holding shares of gold and bitcoin.

Holding shares of gold and bitcoin on day n:

$$Q_{gold} = \frac{1000 \times (1 + \gamma_{total}) \times w_{gold}}{p_5 (1 + \alpha_{gold})} \quad (7)$$

$$Q_{bitcoin} = \frac{1000 \times (1 + \gamma_{total}) \times w_{bitcoin}}{p_6 (1 + \alpha_{bitcoin})} \quad (8)$$

\$1000 investment on September 10, 2021.

According to the data given in the annex, September 10, 2021 is the 1825th day of the investment period

1) Invest in the minimum risk portfolio with a cumulative yield and usd value

The Minimum Risk Portfolio strategy is the combination that selects the smallest risk and the highest return at that risk level.

a) Find the least risky portfolio algorithmically and plot it in a graph of yield versus standard deviation. The visual output is shown in Figure 15:

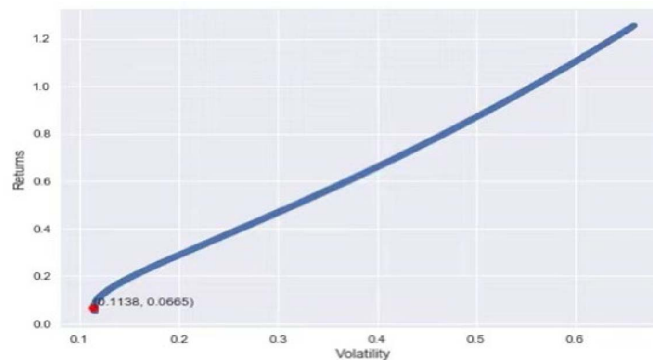


Figure 15 The efficient boundary graph for the minimum portfolio

b) Obtain the weight of the lowest risk combination and output this weight.

That is, the purchase weights of gold and bitcoin are 0.02077114 and 0.97922886 respectively.

c) Calculate the cumulative rate of return of the minimum risk portfolio.

This calculation process is the same as "making the best investment decision through given weight" in model d. Based on the daily rate of return data obtained from the above process, it can be obtained by changing the values of and in the formula of combined daily rate of return and cumulative

rate of return.

2) Cumulative rate of return of portfolio obtained by weight sharing

Determine the and in the above formula of daily rate of return and cumulative rate of return as 0.5 and calculate them.

3) Cumulative Return comparison and visual output of three trading portfolios

Summarize the cumulative income of the three trading portfolios into the same table, and compare and output by drawing a line chart. The visual output is shown in Figure 16:

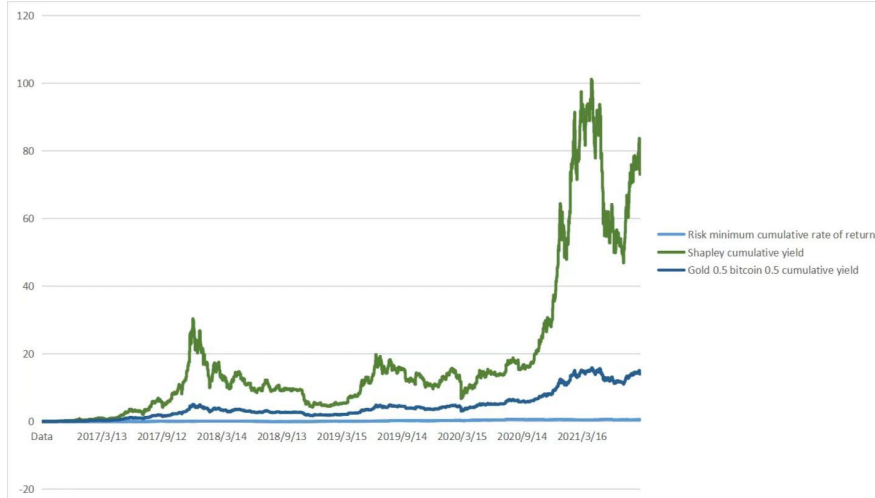


Figure 16 Visual output of cumulative returns for the portfolio

As can be seen from the figure:

The cumulative return of portfolio based on Monte Carlo simulation and sharp ratio is the largest, and the cumulative return of portfolio with equal weight of gold and bitcoin is the smallest;

Compared with the portfolio with equal weight, the portfolio determined by the model fully considers the diversification of risk;

Compared with the minimum risk portfolio, the portfolio equilibrium determined by the model considers the risk and return, that is, it can reduce the investment risk and obtain the highest return as much as possible;

From a quantitative point of view, the peak value of the cumulative return of the portfolio determined by the model is more than five times that of the minimum risk portfolio. The cumulative rate of return of the investment portfolio with equal weight is almost 0. This investment portfolio almost loses the time value of funds, which is an invalid investment.

4) Certification overview

a) Theoretical perspective: the optimal portfolio model in this paper is an iterative process based on Monte Carlo simulation and sharp ratio. The optimal weight is given by randomly generating the array weight, iterating repeatedly and comprehensively considering the risk and return.

b) Practical perspective: from the comparison of the

$$\gamma_{total} = \sum_{i=1}^n r_p = \sum_{i=1}^n w_{gold} \frac{P_2 - P_1}{P_1} + w_{bitcoin} \frac{P_4 - P_3}{P_3} \quad (10)$$

It can be seen from the above formula that the parameter "transaction cost" is not included in the expression of cumulative rate of return, so the transaction cost will not affect the output of cumulative rate of return.

above three portfolios, it can be seen that the investment strategy given by the model not only reduces the risk to the greatest extent, but also ensures the return to the greatest extent, which is the best investment portfolio strategy.

IV. SENSIVITY ANALYSIS

A. The sensitivity of the transaction costs to strategy

In the solution of the previous problem, we obtain and prove the optimal strategy. Next, sensitivity analysis is carried out to prove the reliability of the model. Sensitivity analysis is a method to study and analyze the sensitivity of the state or output change of a system (or model) to the change of system parameters or surrounding conditions. In practice, transaction costs will change. Whether the change of transaction costs will affect the applicability of our model is something we need to verify.

Sensitivity actually refers to whether changing the parameter of transaction cost in the model will change the output of the model. In our model, the output includes cumulative income and the best share of gold bitcoin. We analyze them one by one.

1) Impact of changes in transaction costs on cumulative rate of return

According to the formula of cumulative rate of return derived in model 1:

Relationship between different transaction costs and cumulative portfolio yield on the same day (keeping the price unchanged) (take September 12, 2016 as an example). The cumulative portfolio yield is shown in Figure 17:

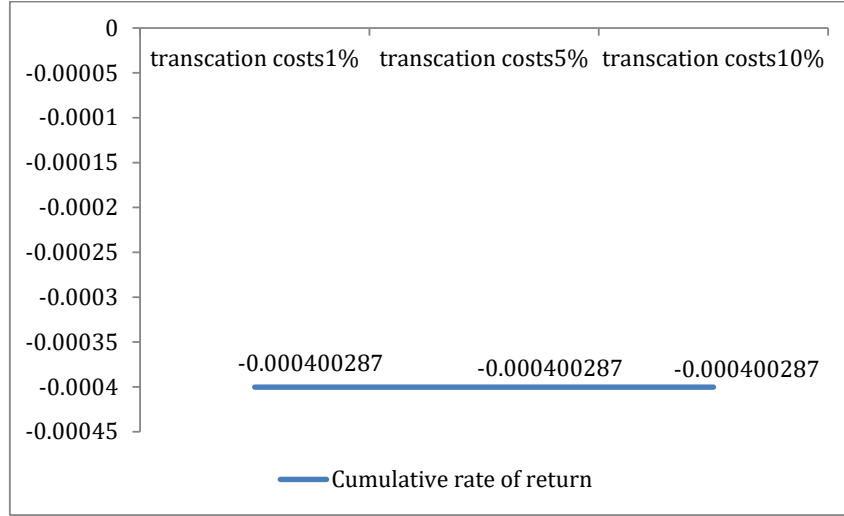


Figure 17 Cumulative portfolio yield

It can be seen from this example that transaction costs will not affect the cumulative rate of return.

2) Impact of changes in transaction costs on gold and bitcoin holding the best share

Relationship between different transaction costs and holding shares on the same day (keeping the price unchanged) (take September 12, 2016 as an example). The holding shares is shown in Figure 18:

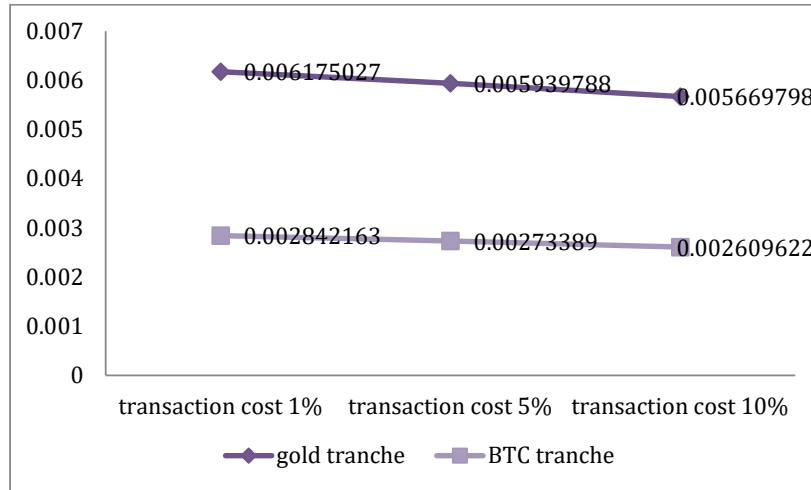


Figure 18 Holding shares

As can be seen from the figure, the change of transaction cost rate will affect the best holding share of gold and bitcoin. However, for quantitative analysis, we conducted sensitivity analysis. Taking the transaction costs of gold and bitcoin as unknown parameters, the relationship between them and the best holding share is calculated.

Share of gold held:

$$Q_{gold} = \frac{1000 \times (1 + \gamma_{total}) \times w_{gold}}{p_5 (1 + \alpha_{gold})} \quad (11)$$

Share of bitcoin held:

$$Q_{bitcoin} = \frac{1000 \times (1 + \gamma_{total}) \times w_{bitcoin}}{p_6 (1 + \alpha_{bitcoin})} \quad (12)$$

While affecting the gold share, it will also affect the bitcoin share. Take the share of gold held as an example:

$$\text{Order } \Delta \alpha_{bitcoin} \rightarrow 0,$$

$$\text{Sensitivity: } S(P_g, \alpha_{gold}) = \frac{\Delta P_g / P_g}{\Delta \alpha_{gold} / \alpha_{gold}} = \frac{dQ_{gold}}{d\alpha_{gold}} * \frac{\alpha_{gold}}{Q_{gold}} \quad (13)$$

$$= \frac{1000(R_{total} + 1) * W_{gold}}{P_5} * \frac{-1}{(1 + \alpha_{gold})^2} * \frac{\alpha_{gold}}{Q_{gold}}$$

$$\begin{aligned}
&= \frac{1000(Rtotal+1)*Wgold}{P5} * \frac{-1}{(1+agold)^2} * \frac{agold*P5}{1000*(Rtotal+1)*Wgold} \frac{(1+agold)^2}{agold*P5} \\
&= \frac{1000(\sum Wgold * \frac{P4-P3}{P3} + Wbitcoin * \frac{P2-P1}{P1} + 1) * Wgold}{P5} * \frac{-1}{(1+agold)^2} * \frac{agold*P5}{1000*(Rtotal+1)*Wgold} \\
&= (\sum Wgold * \frac{P4-P3}{P3} + Wbitcoin * \frac{P2-P1}{P1} + 1) * \frac{-1}{(1+agold)} * \frac{agold}{(Rtotal+1)}
\end{aligned}$$

Taking the data on September 12, 2016 as an example, when calculating the change of transaction cost, the change of gold share:

(when changing from 5% to 10%):

$$\frac{\Delta Qgold / Pgold}{\Delta agold / agold} = \frac{(0.002609622 - 0.00273389) / 0.002609622}{5\% / 10\%} = -0.0009524 \quad (14)$$

Take the data on September 12, 2016 as an example to calculate the change of bitcoin share when the transaction cost changes:

(when changing from 5% to 10%):

$$\frac{\Delta Qbitcoin / Pbitcoin}{\Delta abitcoin / abitcoin} = \frac{(0.005669798 - 0.005939788) / 0.005669798}{5\% / 10\%} = -0.0009524 \quad (15)$$

Transaction cost α Every time gold changes by 1 unit, the holding share of gold (bitcoin) will change by -0.0009524 units. That is, for every 1% increase in transaction costs, the holding share of gold (bitcoin) will decrease by 0.0009524%.

This change is very small, which shows that the model is stable and reliable. This model is applicable to all transaction cost rates.

Why is the transaction cost negatively related to the holding share of gold (bitcoin)? Transaction cost is the most direct cost for investors to participate in the transaction. The level of transaction costs not only affects the business income of securities companies, but also affects the interests of investors. For investors:

$$P = R - C \quad (16)$$

In the case of constant income, the increase of cost will inevitably lead to the decrease of profit. That is, investors have less return on investment.

So, for investors, is the transaction cost the less the better? In fact, the low transaction cost will make the securities companies unable to improve the intermediary service level, and eventually damage the interests of investors.

3) On the sensitivity of the model from the development trend of transaction costs

First of all, from the perspective of major countries in the world, the liberalization of transaction rate has been the main trend of development. On May 1, 1975, the United States took the lead in abolishing the fixed transaction rate system of securities trading and implementing the negotiated transaction rate system; In the later period, many countries

have introduced the transaction rate system, and great changes have taken place in the world.

Secondly, the sharp decline in transaction rates is a global trend. After the reform of the transaction rate system in the United States in 1975, the transaction cost of American investors decreased significantly, and the transaction commission per share decreased from an average of 26 cents in 1975 to 11.9 cents in 1980, and to an average of 5 cents per share in 1997 (due to the sharp decline of commission caused by online transactions). After the reform of the commission system in the UK in 1986, the average commission rate was reduced from 0.7% to 0.28%, of which the average commission rate of individual investors was reduced from 1.07% to 0.28%, and the average commission rate of institutional investors was reduced from 0.42% to 0.21%.

We have proved in 4.3.2 that the level of commission is not sensitive to the holding share of gold bitcoin, so the model is also applicable under the general trend of substantial decline in commission rate.

Finally, many countries and regions use different transaction volumes to determine different commission rates. The larger the transaction volume, the lower the cost. The Taiwan Stock Exchange has adopted a transaction rate determination mechanism of "graded and cumulative commission system, specifying the upper limit of commission rate and free negotiation between securities companies and customers", that is, when the daily transaction volume of each household is NT \$15 million, the transaction rate of NT \$10 million transaction volume shall not be higher than 1.425 ‰ (minimum NT \$20), However, the maximum transaction rate for the NT \$5 million transaction volume of 10-15 million is 1.325 ‰. When the transaction volume is large and the transaction cost is small, the cost is lower and the rate of return is higher; Conversely, when the transaction volume is small and the transaction cost is relatively high, the cost is higher and the rate of return is lower.

This system, in which the transaction volume is inversely proportional to the transaction cost, will have a certain impact on the investment decision-making. Under the same income, investors prefer large transaction volume.

V. MODEL EVALUATION AND PROSPECT

A. Model evaluation

1) Advantages of the model

- The daily optimal portfolio strategy based on the income model directly explains the daily investment choice of investors from the economic level, and the result is reasonable, intuitive and practical.
- The time series model based on LSTM neural network has the function of long-term memory, which solves the problems of gradient disappearance and gradient explosion in long series training.
- The advantage of Monte Carlo simulation is simple and fast, which saves the complicated mathematical derivation and calculation process. By using computer technology and repeated experiments, the optimal weight of gold and bitcoin is obtained.
- Sharp ratio is an index that can comprehensively consider both return and risk at the same time. It can

very well help investors pursue the maximum return under the fixed risk they can bear.

2) Disadvantages of the model

- The daily optimal portfolio established based on the income model is a decision made under the assumption that \$1000 is used for investment. It does not specifically consider whether the funds in hand should be used for investment, which will produce certain errors under special circumstances.
- LSTM neural network can not operate in parallel, and the gradient will disappear when the sequence length exceeds a certain limit.
- The disadvantage of Monte Carlo simulation is that it usually needs more tests and the convergence speed is slow.
- The disadvantage of sharp ratio is that it needs to compare long-term historical data, which has no great reference significance for short-term data.

3) Improvement of model

- On the basis of this model, consider the circumstances under which the initial \$1000 does not need to be fully invested.
- The Monte Carlo simulation in this paper is based on the assumption that the risk-free rate of return is 0. Whether gold and bitcoin have a risk-free rate of return is 0 needs further research.

B. Generalization of model

Using LSTM time series model and stock portfolio model, we can predict the daily price trend and best holding share of gold and bitcoin in the future. The following Table III shows the predicted daily price of gold and the results of the best holding share from September 13, 2021 to September 23, 2021:

TABLE III FUTURE DAILY PRICE TREND AND BEST HOLDING SHARE

date	Price (USD / oz)	Best share (ounces)
9/13/2021	1831.7264	0.0156116
9/14/2021	1830.253162	0.015624166
9/15/2021	1840.56583	0.015536624
9/16/2021	1830.253162	0.015624166
9/17/2021	1799.806235	0.015888477
9/23/2021	1799.315155	0.015780832

VI. POLICY MODEL INTERCHANGE

To: Market traders

From: Team

Subject: the choice to buy, hold, or sell assets

Date: February 21, 2022

Dear Sir:

Bitcoin is an encrypted digital currency, a digital currency in the form of p2p. In order to maximize total returns, market traders frequently buy and sell volatile assets. The price of volatile assets fluctuates, taking into account the

price and risk of the stock. The returns a trader receives are usually affected by costs and benefits. Therefore, for the "risk trading" of volatile assets, it is very difficult to judge whether to buy, hold or sell through the known trading price in the past, so as to make the best investment decision every day, such as gold and bitcoin stocks that exist in the US market. Both are high-risk products, how to make the best trade decisions based on known prices, under different conditions of commissions, and under conditions of the two trade plans. This is an urgent problem for our team.

On this basis, our team used the provided data for pre-processing, built an LSTM neural network model for time series forecasting, used the model to predict the price of gold and bitcoin after the day when the best decision needs to be made, and came to the conclusion that the forecast set is basically the same as the real set; We then considered the risks and returns in balance and determined the best stock portfolio strategy for each day, i.e. how much bitcoin and gold should be held separately in the best decision. The optimal proportion of investment was obtained using the Sharpe ratio, and finally the portfolio correlation analysis was done. While diversifying some of the risks, it also gets a higher yield.

Results:

Through the modeling and analysis of the data, we obtained the following results:

i) You invest according to the daily best investment strategy we give, \$1000 after a 5-year investment trading period, you will hold \$74521.3. The dollar value of the gold and bitcoins you buy every day is weighted by 0.000387679757 and 0.9961232, respectively. Through the cumulative yield calculated by our model, you can get a picture of the current-day value of the initial investment of \$1,000, further determine the dollar value of gold and bitcoin, and finally determine the holding share of their respective stocks.

ii) Through our proof, you can know that the investment strategy we give is fully considered for risk and return. We reduce your investment risk through portfolios while also guaranteeing you maximum returns at this level of risk. If you are not willing to take on the loss of risk, you are a risk-averse investor, and you can still choose our investment strategy in the case of your long-term investment. Compared to the least risk portfolio, the cumulative returns of the two are broadly consistent over 1 year. However, in the long run, the cumulative yield of the investment strategy we give is much higher than that of the minimum risk portfolio strategy, up to more than 5 times. According to the consensus of investing "high risk and high return", if you like risk, you should not consider a minimal risk portfolio. However, we do not recommend that you take too high a risk to invest, as investing is a long-term exercise and you need to consider both risk and return. Therefore, you can trust that our investment strategy is right for you.

iii) In a day's transaction (i.e. in the case of a fixed transaction price), the impact of transaction fees on yield is very weak, but transaction fees will affect the share of gold and Bitcoin transactions to a certain extent, but the impact is not large, so when the commission is no longer 1%, 2%, you can also continue to use our model with confidence. In addition, the increase in commission will reduce the share of gold and Bitcoin, which is because it exists as the cost of the

transaction, and under certain conditions of income, too high commission will reduce your rate of return; Of course, we also do not recommend too low commissions, so that there will be a decrease in the quality of the broker's service, thus harming your interests. Therefore, you are advised to choose a moderate commission, even if the impact on your earnings is not very large, but it still has a negative impact.

iv) In addition, according to the current system, the exchange with the larger the transaction volume, the less commission it needs to pay, so it is recommended that you give priority to the large trading volume under the same conditions, of course, this needs to be weighed in terms of the amount of commission and the price.

v) Do not trade frequently, the higher the trading frequency, the greater the impact of trading commission on earnings.

vi) Finally, we would like to say that both Bitcoin and gold are high-risk investment products. If you are a conservative investor, it is recommended that you do not spend too much money on investing in these two products, of course, according to the mathematical model we have established, you can also try to invest, because our model in long-term investment, comprehensively consider the risk and return, diversify a part of the risk at the same time, to ensure a higher yield. In addition to the daily best investment decisions for the past five years from September 10, 2016 to September 10, 2021, we provide you with the price and best

decisions for the future two stocks in the outlook of the model.

The above suggestions can help you make the optimal investment in the two stocks of gold and bitcoin, and the portfolio investment solution provided for you will be very good at reducing risk and obtaining higher returns, thereby improving your rate of return. We sincerely hope that our research will benefit you and help you in your investment decisions.

Sincerely,

Team

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