SEO Pricing with Marketability Restriction

A Monte Carlo Method with Stochastic Return and Volatility

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***Abstract*—In capital market, second equity offering (SEO) is the main method of refinancing for listed companies. Private placement is a representative example of SEO. And in recent years, more and more irrational phenomena have been rising with private placement in Chinese stock market because of lack of efficient pricing method. The research on stock pricing for private placement in China is of great significance.**

We consider the problem of price estimation in SEO with marketability restriction. In SEO with marketability restriction, especially in private placement, the stock price is determined by price discount and initial price. To estimate the price discount, we employ Longstaff's framework of opportunity cost and extend Longstaff's assumption. In our extended assumption, return and volatility of stock price are given by independent stochastic process. To estimate the initial price of stock in private placement, we introduce residual income method into our pricing model. Monte Carlo method is adopted to simulate the price movement in order to numerically estimate price discount in private placement. And result of empirical analysis shows that our model can effectively price the stock in private placement in China.

Keywords-private placement; marketability restriction; stochastic return and volatility; Monte Carlo method; price discount; initial price

I. INTRODUCTION

In developed foreign capital market, second equity offering (SEO) is the main method of refinancing for listed companies. In China, SEO is also an important method of refinancing for listed companies. Private placement is an representative example of SEO with marketability restriction, and in recent years, more and more researchers have been focusing on the pricing method of private placement. The fact that there is not an ideal pricing method for private placement in China means that the research on stock pricing for private placement is of great significance.

The stock price of private placement, which is a kind of SEO with marketability restriction, is based on the price discount and initial price (or inherent value) of the issuing company.

Liquidity is proved an important factor accounting for stock price discount both theoretically and empirically. The price discount is mainly determined by illiquidity especially for shares with trade or marketability restriction. Pricing of illiquid stock with marketability restriction depends on not only transaction cost or bid-ask spread but also opportunity cost. Amihud and Mendelson (1986) suggest that compared with price discount corresponding with opportunity cost, the discount relating to bid-ask spread can be neglected and the price without trade restriction can be treat as the market price in a frictionless environment [1].

Longstaff (1995) sets the price of shares outstanding as exogenous variable, and then he deduces the upper bound of the price discount. His analysis framework is a typical opportunity cost based method [2].

Empirical researches also reveal that price discount is very important and can't be ignored in private placement. Shane A. Corwin (2003) investigates the price discount of SEO from 1980 to 1998 in the United States. He finds that the average price discount of the U.S. listed companies with private placement is 2.2%. Among which the listed company in New York Stock Exchange suffer lower discount rate of private placement than those in Nasdaq [3].

On the other hand, the stock price in private placement is determined by initial price. Most researches about initial price focus on the residual income of company with the pioneering work of Ohlson (1995) [4].

Plenborg (2002) compares the residual income method with the cash flow discount method. To simplify the analysis, Plenborg releases several hypotheses and concludes that the residual income method overcomes the cash flow discount method [5].

Song ping (2006) considers that the nature of stock represents the ownership of residual income of company. In other words, it is the ownership of equity. So the company value assessment can be based on the current value of equity and consider the earning ability and developing potential in the same time [6].

The research work of stock pricing in private placement have been carried out for decades with a lot of research findings abroad, however, for Chinese stock market these findings may not be relevant. Chinese financial market is different in some aspects from these abroad and relative researches with pertinence are still needed especially in the area of stock pricing in private placement.

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II. THE PRICING MODEL FOR PRIVATE PLACEMENT IN CHINA

A. The Price Discount without Marketability Restriction

The lockup period coming up with private placement makes the issued stock illiquid for a certain time, hence the stock price has to be discounted to compensate for the illiquidity.

According to the analysis of Amihud, Mendelson and Longstaff, we suggest that the price discount equals to the opportunity cost under marketability restriction [1, 2].

Let P_t denote the current price of stock that is continuously traded in a frictionless market. We assume that the equilibrium dynamics of P_t are given by the stochastic process:

$$dP_t = \mu P_t dt + \sigma P_t dW_t \tag{1}$$

Where μ is the expected return rate of stock, σ is variance of stock price and W_t is a standard Wiener process.

Let M_t denote the time t payoff to the stock and T denote the trade restriction period. If the sale of stock could be timed optimally, where $M_t = \max(e^{r(T-t)}P_t), (0 < t < T)$. As long as the stock holder cannot sell the stock prior to time T, he cannot benefit from having perfect market timing ability. And $M_t - P_t$ can be viewed as the payoff of a liquidity swap in which V_t is swapped for M_t at time T.

Let $F(P_t, T)$ denote the present value of $M_t - P_t$, then

$$F(P,T) = e^{-rT}E[M_T] - e^{-rT}E[P_T] \tag{2}$$

According to Longstaff (1995), the closed-form solution for equation (2) is

$$F(P_t, T) =$$

$$P_t(2+\frac{\sigma^2 T}{2})N(\frac{\sqrt{\sigma^2 T}}{2}) + P_t\sqrt{\frac{\sigma^2 T}{2\pi}}\exp(-\frac{\sigma^2 T}{8}) - P_t \tag{3}$$

Where $N(\bullet)$ is the cumulative normal distribution function.

Equation (3) is suitable to estimate price discount in stock market without price limit. While in China, the highest daily price of stock listed in Shenzhen and Shanghai Stock Exchange must not be higher than 1.1 times of the previous closing price and the lowest price must not be lower than 0.9 times of the previous closing price. Here the price limits is just what Longstaff does not consider in his analysis.

B. The Price Discount with Marketability Restriction

To efficiently estimate the price discount for private placement in China, we have to develop a new method considering the price limit. The core concept of Longstaff is still relevant and we can get a numerical solution instead of the analytical solution with Monte Carlo Simulation.

Still under Longstaff's opportunity cost framework, we further assume that the price, return and volatility of stock are given by the stochastic process, respectively, which mean:

$$dP_t = \mu_t P_t dt + \sigma_t P_t dW_p \tag{4}$$

$$d\mu_t = -\alpha(\mu_t - \lambda)dt + kdW_\mu \tag{5}$$

$$d\sigma_t = -\beta(\sigma_t - \theta)_t dt + ldW_\sigma \tag{6}$$

Where α, β, λ, θ, k and l are constant, W_p, W_μ and W_σ are independent Wiener process, respectively [7, 8]. The solution of equation (5) is

$$\mu_t = \lambda + e^{-\alpha t}(\mu_0 - \lambda) + k\int_0^t e^{\alpha s} dW_\mu(s) \tag{7}$$

We can conclude that μ_t is given by normal distribution with expectation of $\overline{\mu}$ and variance of σ_μ^2:

$$\overline{\mu} = E[\mu_t] = \lambda + e^{-\alpha t}(\mu_0 - \lambda) \tag{8}$$

$$\sigma_\mu^2 = Var[\mu_t] = \frac{k^2}{2\alpha^2}(1 - e^{-2\alpha t}) \tag{9}$$

Similarly, we have the solution of equation (6) as follows:

$$\sigma_t = \theta + e^{-\beta t}(\sigma_0 - \theta) + \phi\int_0^t e^{\beta s} dW_\beta(s) \tag{10}$$

From equation (10) we can also conclude that σ_t is given by normal distribution with expectation of $\overline{\sigma}$ and variance of σ_σ^2:

$$\overline{\sigma} = E[\sigma_t] = \theta + e^{-\beta t}(\sigma_0 - \theta) \tag{11}$$

$$\sigma_\sigma^2 = Var[\sigma_t] = \frac{l^2}{2\beta^2}(1 - e^{-2\beta t}) \tag{12}$$

According to Ito's Lemma, we have

$$d(\ln P_t) = (\mu_t - \frac{\sigma_t^2}{2})dt + \sigma_t dW_p \tag{13}$$

$$\ln P_{t+\Delta t} - \ln Pt = (\mu_t - \frac{\sigma_t^2}{2})\Delta t + \sigma_t Z_t\sqrt{\Delta t} \tag{14}$$

$$P_{t+\Delta t} = P_t \bullet \exp[(\mu_t - \frac{\sigma_t^2}{2})\Delta t + \sigma_t Z_t\sqrt{\Delta t}] \tag{15}$$

Where Z_t is given by standard normal distribution.

At time $t + \Delta t$, we have

$$\mu_{t+\Delta t} = \lambda + e^{-\alpha\Delta t}(\mu_t - \lambda) + k\int_t^{t+\Delta t} e^{\alpha s} dW_\mu(s) \tag{16}$$

$$\sigma_{t+\Delta t} = \theta + e^{-\beta\Delta t}(\sigma_t - \theta) + \phi\int_t^{t+\Delta t} e^{\beta s} dW_\beta(s) \tag{17}$$

Then $\mu_{t+\Delta t} \sim N(\overline{\mu}(t+\Delta t),\ \sigma_\mu^2(t+\Delta t))$, where

$$\overline{\mu}(t+\Delta t) = E[\mu_{t+\Delta t}] = \lambda + e^{-\alpha\Delta t}(\mu_t - \lambda) \tag{18}$$

$$\sigma_\mu^2(t+\Delta t) = Var[\mu_{t+\Delta t}] = \frac{k^2}{2\alpha^2}(1 - e^{-2\alpha\Delta t}) \tag{19}$$

And $\sigma_{t+\Delta t} \sim N(\overline{\sigma}(t+\Delta t),\ \sigma_\sigma^2(t+\Delta t))$, where

$$\overline{\sigma}(t+\Delta t) = E[\sigma_{t+\Delta t}] = \theta + e^{-\beta\Delta t}(\sigma_t - \theta) \tag{20}$$

$$\sigma_\sigma^2(t+\Delta t) = Var[\sigma_{t+\Delta t}] = \frac{l^2}{2\beta^2}(1 - e^{-2\beta\Delta t}) \tag{21}$$

From $t = 0$, if P_0, μ_0 and σ_0 is determined, select Z_0 randomly from a population subject to standard normal distribution, then the price $P_{\Delta t}$ at time Δt is determined by

$$P_{\Delta t} = P_0 \bullet \exp[(\mu_0 - \frac{\sigma_0^2}{2})\Delta t + \sigma_0 Z_0 \sqrt{\Delta t}] \tag{22}$$

Also at time $2\Delta t$, the price $P_{2\Delta t}$ can be determined by $P_{\Delta t}$, with $\mu_{\Delta t}$, $\sigma_{\Delta t}$ and another $Z_{\Delta t}$ selected at random according to their distribution respectively.

Let M denote the number of trading days, P_j denote closing price of stock at day j, then $\Delta t = 1/M$. Assume

$$X_j = \frac{1}{M}(\mu_j - \frac{\sigma_j^2}{2}) + \frac{1}{\sqrt{M}}\sigma_j Z_j \tag{23}$$

Because $Z_j \sim N(0,1)$, then

$$X_j \sim (\frac{1}{M}(\mu_j - \frac{\sigma_j^2}{2}), \frac{1}{\sqrt{M}}\sigma_j) \tag{24}$$

The initial price of stock is denoted by P_0, during trade restricted period T, if the number of trading days is M, then we have a stock price movement path composed of M prices:

$$P_1 = P_0 e^{X_0},\ P_2 = P_1 e^{X_1},\ ...,\ P_m = P_{m-1} e^{X_{m-1}} \tag{25}$$

The path $P_1 = P_0 e^{X_0}, P_2 = P_1 e^{X_1}, ..., P_m = P_{m-1} e^{X_{m-1}}$ is achieved without any price limit. What is different from Longstaff's work is that the return and volatility are not assumed constant but stochastic process.

In order to reflect the real situation in China stock market, we have to make further adjustment to the simulated price movement. Let $p \ge 0$ denote the upper and lower price limit during a single trading day, in other words, in day t the closing price P_t of stock must satisfy

$$(1-p)P_{t-1} \le P_t \le (1+p)P_{t-1},\ (t \ge 1) \tag{26}$$

Then the price series should be adjusted as (27) and (28). Now, $P_1, P_2, ..., P_m$ is a relevant simulated stock price movement path through Monte Carlo method. The simulation steps are $(Z_j, \mu_j, \sigma_j) \to X_j \to P_j$

$$\begin{aligned} P_1 &= P_0 + (\mathrm{sign}(X_0)) \times \min(\left|e^{X_0} - 1\right|, p) \times P_0, \\ P_2 &= P_1 + (\mathrm{sign}(X_1)) \times \min(\left|e^{X_1} - 1\right|, p) \times P_1, \\ &\dots\dots \\ P_m &= P_{m-1} + (\mathrm{sign}(X_{m-1})) \times \min(\left|e^{X_{m-1}} - 1\right|, p) \times P_{m-1} \end{aligned} \tag{27}$$

Where

$$\mathrm{sign}(X_t) = \begin{cases} -1, & X_t < 0 \\ 1, & X_t \ge 0 \end{cases} \quad (0 < t < M) \tag{28}$$

In a certain path, the highest stock price is $\max P_t$, $(0 < t < T)$. If the risk free rate is r, the opportunity cost during the lockup period is $(\max P_t)(1+r)^{-t} - P_0$, then the price discount in private placement should be compensated by the amount of $(\max P_t)(1+r)^{-t} - P_0$.

Through a certain times of simulation, we can get the average value of $(\max P_t)(1+r)^{-t}$ which is denoted by $\overline{P}_{\max}$, and the estimated price discount D_T for private placement is

$$D_T = \overline{P}_{\max} - P_0 \tag{29}$$

And the rational price for private placement is

$$P = P_0 - D_T = P_0 - (\overline{P}_{\max} - P_0) = 2P_0 - \overline{P}_{\max} \tag{30}$$

Where P_0 is the initial price or inherent value of stock in private placement.

C. *The Initial Price of Stock*

The initial stock price reflects the inherent value of stock. And the inherent value of stock is directly related to the inherent value of the corresponding company. Then we can reasonable consider that the stock price in private placement is based on the inherent value of the issuing company.

To evaluate the inherent value of stock we employ the residual income method under the following assumptions [4]:

Assumption I: The inherent value of stock is the discounted expected dividend.

Assumption II: The accounting treatments satisfy the clean surplus relation (CSR).

Assumption III: Linear Information Dynamics (LID). Current aggregate residual income satisfies first order auto-regressive process. Aggregate residual during next period is affected by current residual income and other information.

With assumption I, II and III, we have the following equation:

$$V = \sum_{t=1}^{+\infty} \frac{d_t}{(1+r_w)^t} \tag{31}$$

$$B_t = B_{t-1} + NI_t - d_t \tag{32}$$

Where V denote inherent value of stock, d_t denote dividend at time t and r_w denote weighted average cost of capital. B_t denotes book value of company at time t, B_{t-1} denotes book value of company at time $t-1$, NI_t denote net income during time $t-1$ and t, d_t denote cash dividend paid at time t.

From (31) and (32), we have

$$V = \sum_{t=1}^{+\infty} \frac{B_{t-1} - B_t + NI_t}{(1+r_w)^t} \tag{33}$$

The value of stock equals book value of equity plus discounted future residual income, and the deduced residual value assessment model is

$$V = B_0 + \sum_{t=1}^{+\infty} \frac{NI_t - r_W \times B_{t-1}}{(1+r_W)^t} = B_0 + \sum_{t=1}^{+\infty} \frac{RE_t}{(1+r_W)^t} \quad (34)$$

Equation (34) is basic form of residual income model. RE represents the residual earnings defined by Ohlson, which is the core concept of residual income and equals net income minus weighted average cost of capital.

We suppose that, in China, refinancing through private placement improves the performance of issuing company and during the next 5 years after private placement the issuing company can achieve excess earnings above normal. Then equation (34) can be written as

$$V = B_0 + \sum_{t=2}^{6} \frac{RE_t}{(1+r_W)^t} \quad (35)$$

The inherent value of stock equals inherent value of company divided by total amount of shares outstanding. The inherent stock value V_0 with totaling N shares after private placement is

$$V_0 = \frac{V}{N} = \frac{1}{N}[B_0 + \sum_{t=2}^{6} \frac{RE_t}{(1+r_W)^t}] \quad (36)$$

Also we have concluded that the initial price or the inherent value of stock is V_0, then $P_0 = V_0$, so we have

$$D_T = \overline{P}_{\max} - V_0 \quad (37)$$

And the private placement price is

$$P = 2P_0 - \overline{P}_{\max} = \frac{2}{N}[B_0 + \sum_{t=2}^{6} \frac{RE_t}{(1+r_W)^t}] - \overline{P}_{\max} \quad (38)$$

Equation (38) is our pricing model to estimate price discount in practice.

III. EMPIRICAL ANALYSIS

We randomly select 10 listed companies that refinance though private placement in Shenzhen and Shanghai Stock Exchange to estimate the corresponding price discount and private placement price respectively. The estimation result is summarized in Table 1.

From Table 1 we can see that estimated private placement price of SHHC is most accurate with a percentage error of 4.6%. The estimated price of CWTC is least accurate with a percentage error of 11.5%. The average percentage error of sample is 7.9%.

The estimated price discount of GDRT is most accurate with a percentage error of 1.1%. The estimated price discount of CET is least accurate with a percentage error of 14.9%. The average percentage error of sample is 9.1%.

Generally speaking, few estimation of private placement price and price discount of sample stocks are relative inaccurate, however, the average result of estimation is in line with actual.

TABLE I. PRICING RESULTS FOR PRIVATE PLACEMENT

	Estimated Result (yuan)		**Real Result (yuan)**	
Stock	***private placement price***	***price discount***	***private placement price***	***price discount***
J.I.C	7.72	12.85	8.63	13.43
CWTC	7.54	6.88	6.76	7.66
Xingfa	8.02	21.39	8.57	23.55
SNA	15.63	1.94	17.82	1.72
BUCID	11.99	9.79	11.38	10.94
HLHS	9.88	12.91	10.43	11.48
SHHC	16.53	31.02	15.8	30.24
SDPG	11.24	1.12	12.05	0.99
GDRT	7.17	4.48	7.57	4.43
CET	11.5	4.09	12.95	3.56

IV. CONCLUSION

In SEO with marketability restriction, especially in private placement, one determining factor of stock price is price discount. Because there exist illiquidity caused by marketability or trading restriction, the stock price have to be discounted to compensate for the loss of marketability. Another determining factor of stock price in private placement is the initial price of stock which is based on the inherent value of the issuing company.

To estimate the price discount in private placement, we adopt Longstaff's idea of opportunity cost. We further assume the return and volatility of stock price are given by independent stochastic process respectively. Then we use residual income model to estimate the initial price of stock in private placement.

Monte Carlo simulation is adopted to estimate the price discount. Empirical analysis shows that the model we have developed is efficient in practice.

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