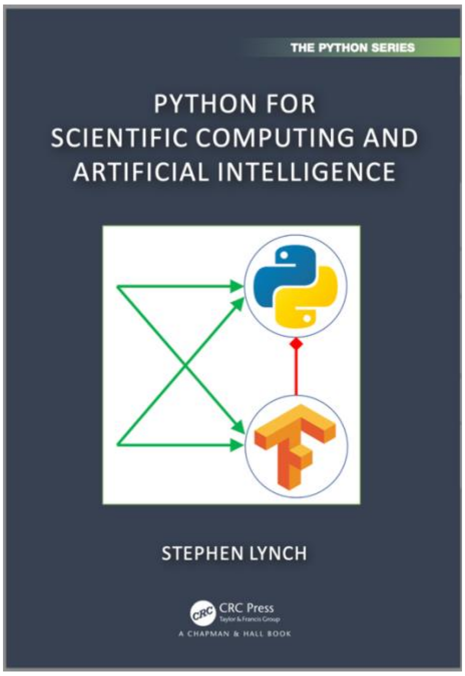
Minor Typographical Errors:

Please contact me if you notice any other typos:

s.lynch@mmu.ac.uk



See Page 49: The program is correct.

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7. Graphs and Transformations: Plot the functions $y_1 = \sin(t)$ and $y_2 = 2 + 2\sin(3t + \pi)$ on one graph. See Figure 4.3.

```
import numpy as np
import matplotlib.pyplot as plt
t = np.linspace(-2 * np.pi, 2 * np.pi, 100)
plt.plot(t, np.sin(t), t, 2 + 2 * np.sin(3 * t + np.pi))
plt.xlabel("t")
plt.ylabel("$y_1, y_2$")
plt.show()
```

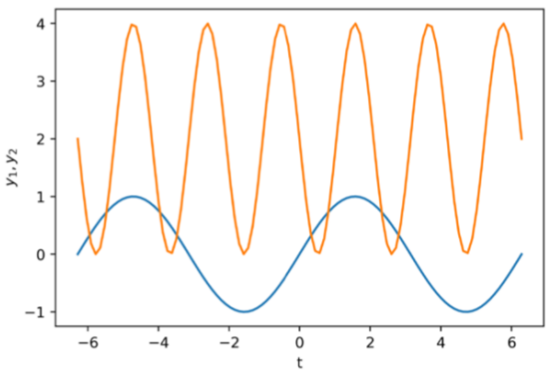

Figure 4.3 The functions $y = \sin(t)$ and $y = 2 + 2\sin(3t + \pi)$.

PLEASE TURN OVER...

See Pages 250 and 251: Program_17e.py, which plots Figure 17.7(b), is correct in the book.

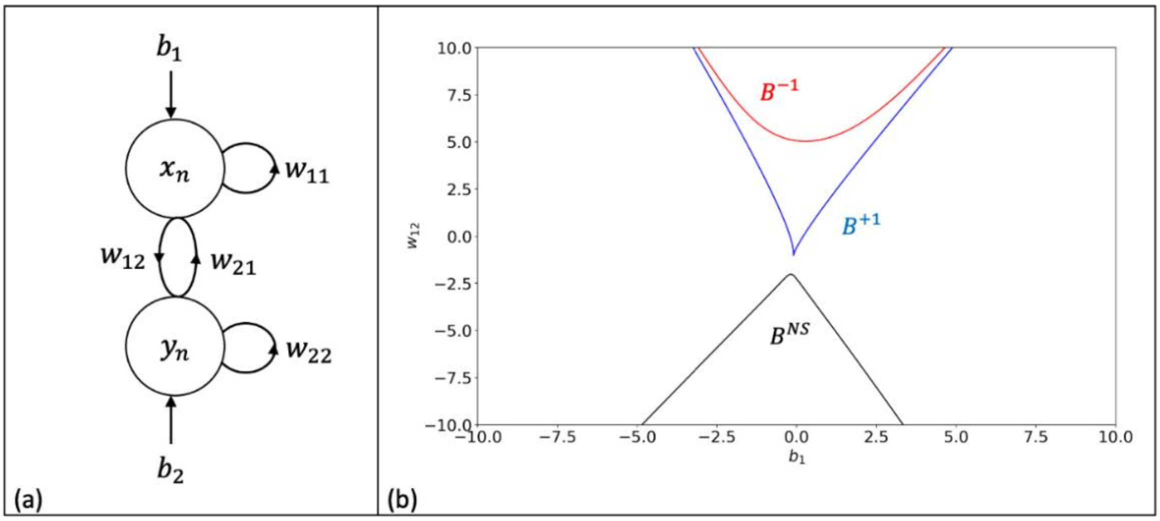

Figure 17.7 (a) Two-neuron module. (b) Stability diagram in the (b_1, w_{12}) plane when $b_2 = -1$, $w_{11} = 1.5$, $w_{21} = 5$, $\alpha = 1$, and $\beta = 0.1$. B^{+1} is the bistable boundary curve, where the system displays hysteresis, B^{-1} is the unstable boundary curve, where the system is not in steady state, and B^{NS} is the Neimark-Sacker boundary curve, where the system can show quasiperiodic behaviour.

17.4 NEURODYNAMICS

This section introduces the reader to neurodynamics and determining stability regions for dynamical systems by means of a simple example. Figures 17.7(a) and 17.7(b) show a two-neuron module and corresponding stability diagram, respectively. The discrete dynamical system that models the two-neuron module is given as:

$$x_{n+1} = b_1 + w_{11}\phi_1(x_n) + w_{12}\phi_2(y_n), \quad y_{n+1} = b_2 + w_{21}\phi_1(x_n) + w_{22}\phi_2(y_n), \quad (17.4)$$

where b_1, b_2 are biases, w_{ij} are weights, x_n, y_n are activation potentials, and the transfer functions are $\phi_1(x) = \tanh(\alpha x)$, $\phi_2(y) = \tanh(\beta y)$.

Example 17.4.1. To simplify the analysis, assume that $w_{22} = 0$ in equations (17.4). Take $b_2 = -1$, $w_{11} = 1.5$, $w_{21} = 5$, $\alpha = 1$, and $\beta = 0.1$ Use a stability analysis from dynamical systems theory to determine parameter values where the system is stable, unstable, and quasiperiodic. Use Python to obtain Figure 17.7(b).

Solution. The fixed points of period one, or steady-states, satisfy the equations $x_{n+1} = x_n = x$, say, and $y_{n+1} = y_n = y$, say. Thus,

$$b_1 = x - w_{11}\tanh(\alpha x) - w_{12}\tanh(\beta y), \quad y = b_2 + w_{21}\tanh(\alpha x). \quad (17.5)$$

Use the Jacobian matrix to determine stability conditions. Take $x_{n+1} = P$ and $y_{n+1} = Q$ in equation (17.4), then

$$J = \begin{pmatrix} \frac{\partial P}{\partial x} & \frac{\partial P}{\partial y} \\ \frac{\partial Q}{\partial x} & \frac{\partial Q}{\partial y} \end{pmatrix} = \begin{pmatrix} \alpha w_{11}\mathrm{sech}^2(\alpha x) & \beta w_{12}\mathrm{sech}^2(\beta y) \\ \alpha w_{21}\mathrm{sech}^2(\alpha x) & 0 \end{pmatrix}.$$

The stability conditions of the fixed points are determined from the eigenvalues, and the trace and determinant of the Jacobian matrix. The characteristic equation is given by $\det(J - \lambda I) = 0$, which gives:

$$\lambda^2 - \alpha w_{11} \mathrm{sech}^2(\alpha x)\lambda - \alpha\beta w_{12} w_{21} \mathrm{sech}^2(\alpha x)\mathrm{sech}^2(\beta y) = 0. \tag{17.6}$$

The fixed points undergo a fold bifurcation (indicating bistability) when $\lambda = +1$. The boundary curve is labelled B^{+1} in Figure 17.7(b). In this case, equation (17.6) gives

$$w_{12} = \frac{1 - \alpha w_{11} \mathrm{sech}^2(\alpha x)}{\alpha\beta w_{21} \mathrm{sech}^2(\alpha x)\mathrm{sech}^2(\beta y)}. \tag{17.7}$$

The fixed points undergo a flip bifurcation (indicating instability) when $\lambda = -1$. The boundary curve is labelled B^{-1} in Figure 17.7(b). In this case, equation (17.6) gives

$$w_{12} = \frac{1 + \alpha w_{11} \mathrm{sech}^2(\alpha x)}{\alpha\beta w_{21} \mathrm{sech}^2(\alpha x)\mathrm{sech}^2(\beta y)}. \tag{17.8}$$

The fixed points undergo a so-called Neimark-Sacker bifurcation (indicating quasiperiodicity) , when $\det(J) = 1$, and $|\mathrm{trace}(J)| < 2$. The boundary curve is labelled B^{NS} in Figure 17.7(b). Equation (17.6) gives

$$w_{12} = -\frac{1}{\alpha\beta w_{21} \mathrm{sech}^2(\alpha x)\mathrm{sech}^2(\beta y)}. \tag{17.9}$$

THE END