

## 1 Probability method

Take a die. If the outcome is 5 or 6, roll again. If the outcome is between 1 and 4, the number denotes the student's group.

If one group has reached cardinality 4, it is excluded just like 5 and 6.

**Rationale:** As the result of 5 or 6 does not influence the probability of outcomes 1 to 4, this defines a uniform discrete probability distribution.

## 2 Probability space

$$\Omega = \text{permutations}(\{1, 1, 1, 1, 2, 2, 2, 2, 3, 3, 3, 3, 4, 4, 4, 4\})$$

where the groups are implicitly defined by indices 1–4, 5–8, 9–12 and 13–16.

$$\mathcal{A} = \mathcal{P}(\Omega)$$

$$\mathbb{P}(A) = \frac{1}{4}^{|A|}$$

## 3 In a group of friends

Let Peter and Alice be Moritz' friends. Let  $\mathbb{P}[M = 1]$  be the probability that Moritz is assigned to group 1. Let  $\mathbb{P}[P = N]$  and  $\mathbb{P}[A = N]$  be correspondingly.

$$\mathbb{P}[M = 1] = \frac{1}{4}$$

$$\mathbb{P}[M = N] = \frac{1}{4}$$

$$\mathbb{P}[M = 1, P = 1] = \frac{1}{4} \cdot \frac{1}{4}$$

$$\mathbb{P}[M = N, P = N] = \sum_{N=1}^4 \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{4}$$

$$\mathbb{P}[M = 1, P = 1, A = 1] = \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1^3}{4}$$

$$\mathbb{P}[M = N, P = N, A = N] = \sum_{N=1}^4 \frac{1^3}{4} = 4 \frac{1^3}{4}$$