

1 Exercise 9

1.1 Straight

This is five cards in a sequence (e.g., 4,5,6,7,8), with aces allowed to be either 1 or 13 (low or high) and with the cards allowed to be of the same suit (e.g., all hearts) or from some different suits. The number of such hands is $10 \cdot \binom{4}{1}^5$. The probability is 0.003940

1.2 3 of a kind

This hand has the pattern AAABC where A, B, and C are from distinct kinds. The number of such hands is $\binom{13}{1}\binom{4}{3}\binom{12}{2}\binom{4}{1}^2 = 54912$. After dividing by $\binom{52}{5}$, the probability is 0.021128.

1.3 2 pair

This hand has the pattern AABBC where A, B, and C are from distinct kinds. The number of such hands is $\binom{13}{2}\binom{4}{2}\binom{4}{2}\binom{11}{1}\binom{4}{1}$. After dividing by $\binom{52}{5}$, the probability is 0.047539.

1.4 Pair

This the hand with the pattern AABCD, where A, B, C and D are from the distinct "kinds" of cards: aces, twos, threes, tens, jacks, queens, and kings (there are 13 kinds, and four of each kind, in the standard 52 card deck). The number of such hands is $\binom{13}{1} \cdot \binom{4}{2} \cdot \binom{12}{3} \cdot \binom{4}{1}^3$. If all hands are equally likely, the probability of a single pair is obtained by dividing by $\binom{52}{5}$. This probability is 0.422569.

1.5 High card

We have to choose 5 distinct kinds $\binom{13}{5}$ but exclude any straights (subtract 10). We can have any pattern of suits except the 4 patterns where all 5 cards have the same suit: $4^5 - 4$. The total number of such hands is $[(\binom{13}{5} - 10) \cdot (4^5 - 4)]$. The probability is 0.501177.