1 Exercise 7

Given 5 women and 9 men. Let $\mathbb{P}(F)$ denote the probability that a female member is chosen. Let $\mathbb{P}(F \mid F)$ denote the probability that a female member is chosen, given that one woman was already chosen before.

We assume independent events with uniform distribution when choosing members.

1.1 $\mathbb{P}(F)$

In 5 of 14 cases, the chosen member will be female.

$$\mathbb{P}(F) = \frac{5}{14}$$

```
def prob(s):
  women, men = 5, 9
  p = '1.0'
  for x in s:
    if x.upper() == 'F':
      p += ' * ({} / {})'.format(women, men + women)
      women -= 1
    elif x.upper() == 'M':
      p += ' * ({} / {})'.format(men, men + women)
      men -= 1
  return p

# prob('FFMM')
```

1.2 $\mathbb{P}(F|F)$

As one woman is chosen, we are left with 4 out of 13 cases.

$$\mathbb{P}(F|F) = \frac{4}{13}$$

1.3 Probability that two women are chosen

$$\begin{aligned}
&\mathbb{P}(\text{FFMM}) + \mathbb{P}(\text{FMFM}) + \mathbb{P}(\text{FMMF}) + \mathbb{P}(\text{MFFM}) + \mathbb{P}(\text{MFMF}) + \mathbb{P}(\text{MMFF}) \\
&= \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{9}{12} \cdot \frac{8}{11} + \frac{5}{14} \cdot \frac{9}{13} \cdot \frac{4}{12} \cdot \frac{8}{11} + \frac{5}{14} \cdot \frac{9}{13} \cdot \frac{8}{12} \cdot \frac{4}{11} \\
&+ \frac{9}{14} \cdot \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{8}{11} + \frac{9}{14} \cdot \frac{5}{13} \cdot \frac{8}{12} \cdot \frac{4}{11} + \frac{9}{14} \cdot \frac{8}{13} \cdot \frac{5}{12} \cdot \frac{4}{11} \\
&= \frac{1440}{24024} + \frac{1440}{24024} + \frac{1440}{24024} + \frac{1440}{24024} + \frac{1440}{24024} + \frac{1440}{24024} \\
&= 6 \cdot \frac{1440}{24024} = \frac{8640}{24024} = \frac{360}{1001}
\end{aligned}$$

1.4 Probability that three women are chosen

$$
\begin{aligned}
&\mathbb{P}(\mathrm{FFFM}) + \mathbb{P}(\mathrm{FFMF}) + \mathbb{P}(\mathrm{FMFF}) + \mathbb{P}(\mathrm{MFFF}) \\
&= \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{3}{12} \cdot \frac{9}{11} + \frac{5}{14} \cdot \frac{4}{13} \cdot \frac{9}{12} \cdot \frac{3}{11} + \frac{5}{14} \cdot \frac{9}{13} \cdot \frac{4}{12} \cdot \frac{3}{11} + \frac{9}{14} \cdot \frac{5}{13} \cdot \frac{4}{12} \cdot \frac{3}{11} \\
&= \frac{540}{24024} + \frac{540}{24024} + \frac{540}{24024} + \frac{540}{24024} \\
&= 4 \cdot \frac{540}{24024} = \frac{2160}{24024} = \frac{90}{1001}
\end{aligned}
$$

1.5 Probability that two or three women are chosen

$$
\frac{360}{1001} + \frac{90}{1001} = \frac{450}{1001}
$$