

# 1 Exercise 10

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#!/usr/bin/env python

import itertools

def experiment(n):
    distribution = [0] * (n + 1)
    for outcome in itertools.product({'L', 'R'}, repeat=2*n-1):
        bags = {'L': n, 'R': n}
        for out in outcome:
            bags[out] -= 1
            if bags[out] == 0:
                break

        print(outcome, bags)

        outcome_k = max(bags.values())
        distribution[outcome_k] += 1

    return distribution

print(experiment(5))
```

For  $n = 5$ , it gives  $k = 1$  in 140 cases,  $k = 2$  in 140 cases,  $k = 3$  in 120 cases,  $k = 4$  in 80 cases and  $k = 5$  in 32 cases.

$$\Omega = \{0, 1\}^{2n-k}$$

$$\mathcal{A} = \mathcal{P}(\Omega)$$

$$\mathbb{P}(A_k) = \frac{|A_k|}{|\Omega|}$$

0 represents a left drawing, 1 represents a right drawing.

$$\implies A_k = R_k + L_k$$

Regards  $L_k$ , there must be a 1 at its tail. Before that  $(n+1)$  ones and  $n-k$  zeros will be distributed on  $2n-k-1$  places.

$$\implies \binom{2n-k-1}{n-1}$$

For the remaining places in  $\omega$ , there are  $2^k$  possibilities.  $L_k$  and  $L_k$  have the same cardinality.

$$\implies \mathbb{P}(A_k) = \frac{|A_k|}{|\Omega|} = 2 \cdot \frac{|L_k|}{|\Omega|} = 2 \cdot \frac{\binom{2n-k-1}{n-1} \cdot 2^k}{2^{2n-1}} =$$