1 Exercise 8

1.1 Royal Flush

10, jack, queen, king, ace in one suit

$$\sum_{s \in \{\clubsuit, \diamondsuit, \diamondsuit, \spadesuit\}} \frac{5}{52} \cdot \frac{4}{51} \cdot \frac{3}{50} \cdot \frac{2}{49} \cdot \frac{1}{48} = 4 \cdot \frac{120}{311875200} \approx 1.5390 \cdot 10^{-6}$$

Consider one suit. It is okay to pick one of five cards. Then 4 admissible cards are left. Then 3 cards. Then 2. Then 1. For four suits.

1.2 Straight Flush

5 cards in same suit with consecutive rank.

The ranks can begin with 1, 2, 3, ... up to 10. There are 4 suits.

$$\frac{4 \cdot 10}{311875200}$$

1.3 Poker / Four of a kind

4 cards of same rank.

$$\frac{1}{1} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{1}{49} = \frac{6}{124950}$$

1.4 Full House

3 cards of same rank and 2 cards of same rank.

$$\frac{1}{1} \cdot \frac{3}{51} \cdot \frac{2}{50} \cdot \frac{48}{49} \cdot \frac{3}{48} = \frac{864}{5997600} \approx 0.0001$$

The first choice is arbitrary, the second choice must be of same rank (3 of same suit are left, 51 cards left), as well as the third choice. The fourth choice excludes 4 cards of same rank (the fourth card must not be picked!), hence $52-3-1 = 48$. The fifth choice must be of same rank (there are 3 suits of same rank left).

1.5 Flush

5 cards of same suit

$$\frac{1}{1} \cdot \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{11880}{5997600} \approx 0.0020$$

Verified with Wikipedia: Poker probability

2 Whiteboard solution

The solutions do not necessarily respect the previous solutions (a Royal Flush is a Flush).

$$\mathbb{P}(\text{“Royal Flush”}) = \frac{\binom{4}{1}\binom{47}{1}}{\binom{52}{5}} \approx 1.539 \cdot 10^{-6}$$

There are 4 different Royal Flushs (because of 4 suits).

$$\Omega = \{A \subseteq \{1, \dots, 4\} \times \{2, \dots, 14\} \,|\, |A| = 5\}$$

$$\mathbb{P}(\text{“Straight Flush”}) = \frac{\binom{4}{1}\binom{9}{1}}{\binom{52}{5}} = \frac{36}{\binom{52}{5}} \approx 0.0000138$$

$$\mathbb{P}(\text{“Poker”}) = \frac{\binom{13}{1}\binom{48}{1}}{\binom{52}{5}} \approx 0.0002401$$

$$\mathbb{P}(\text{“Full House”}) = \frac{\binom{13}{1}\binom{4}{3}\binom{12}{1}\binom{4}{2}}{\binom{52}{5}} \approx 0.00144$$

$$\mathbb{P}(\text{“Flush”}) = \frac{\binom{4}{1}\binom{13}{5}}{\binom{52}{5}}$$

minus Royal Flush: $-\frac{36}{\binom{52}{5}} - \frac{4}{\binom{52}{5}} \approx 0.001965$