3) MARS model is given by

$$f(x) = \beta_0 + \sum_{m=1}^{M} \beta_m h_m(x) \quad \text{—①} \qquad h_m(x) = \begin{cases} (x-t) & \text{if } x > t \\ 0 & \text{otherwise} \end{cases}$$

where $h_m(x)$ is a piecewise linear basis functions.

Now consider the classifications and regression trees algorithm (that is CART) the formula is:

$$f(x) = \sum_{m=1}^{M} c_m I\{x \in R_m\} \quad \text{—②}$$

where I is a identity function that returns 1 if x is in subset R_m

Thus we can see that MARS to be a modification of CART algorithm with a better regression setting.

Hence by replacing the peicewise linear basis functions by step identity functions $I(x-t>0)$ & $I(x-t \leq 0)$ i.e.,

$$f(x) = \sum_{m=0}^{M} \beta_m I_m(x-t) \qquad \text{where } I_m(x-t) \begin{cases} R_1 & x > t \\ R_2 & x < t \end{cases}$$

In otherwords MARS multiplicative model h_m is replaced with interaction method I_m, a reflected step function pair.

Finally, To get a binary tree representation of, the step function should be restricted to not to split more than more than once.

By following the two steps, we can modify MARS method to behave like a decision tree.

(b) Since MARS use piece-wise linear basis functions, they are more powerful regression compared to identity step functions. Hence, MARS can express/represent better the underlying data distribution. better in comparision to binary decision trees. Hence for very high dimensional inputs which is common in real world applications, MARS method is a better regressor model.

At the same time, the process of adding a basis function to regressor model in MARS is a very computationally expensive procedure. If can be shown that for 'N' data points & 'p' predictors and m backfitting algorithm cycles,

(i) trees take $2(pN \log N)$ operations. (worst case $pN \log N + N^2 p$)

(ii) For a M-term MARS model require $NM^3 + pM^2N$ computations, if M is reasonable fraction of N then it is very expensive.

Clearly MARS methods due to its complexity can get prohibitively expensive compared to decision trees.