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# LOCALIZED OVERHEATING IN AEROBEE REGENERATIVELY COOLED THRUST CHAMBERS

BY

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## ABSTRACT

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Localized overheating, and in several cases, a "burn-through," has occurred in the convergent section of the thrust chamber nozzle of the regeneratively cooled Aerobee sounding rocket engine. Examination of recovered thrust chambers and other analysis indicates that the overheating was probably due to the cross section configuration and depth of the helix in the regenerative cooling system and associated manufacturing tolerances. Because of the cost of changing this configuration and conducting static firings to qualify the new design, it was decided instead to apply more stringent manufacturing tolerances, which should reduce the probability of burn-through.

Author

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# LOCALIZED OVERHEATING IN AEROBEE REGENERATIVELY COOLED THRUST CHAMBERS

by
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## INTRODUCTION

On August 26, 1964 the sustainer thrust chamber on NASA Aerobee 4.126 GG "burned through" at 46.6 seconds after liftoff, resulting in extreme rocket underperformance and loss of altitude. On April 24, 1965 a similar thrust chamber failure occurred on Aerobee 4.114 GG at 41.7 seconds of flight, again resulting in a drastic loss of altitude. A third burn-through was recently observed on Aerobee flight 4.112 NA (6-29-65) but with no concomitant performance degradation. In each case, the holes created by the burn-through occurred in approximately the same area of the convergent section of the thrust chamber nozzle. As a result of the severe altitude losses on two of the three flights, their experimental objectives were not fully achieved.

This report presents a pictorial review of the burn-through problem, with photographs of damaged thrust chambers and intact (but overheated) thrust chambers used on other NASA Aerobee flights. Also included are statistics for each flight discussed, general information on the Aerobee thrust chamber, and the partial results of a flow analysis conducted at the Space General Corporation, El Monte, California (Appendix A).

## SYSTEM DESCRIPTION

The liquid propulsion system used on Aerobee sounding rockets is a gas-pressure fed, bipropellant liquid-rocket engine, employing a regenerative process for cooling the thrust chamber as it is burning by flowing the fuel around the chamber nozzle prior to its combustion. Under a regulated helium pressure of approximately 500 psia, fuel and oxidizer enter the thrust chamber where they are ignited upon contact by a hypergolic reaction. Oxidizer must reach the thrust chamber before fuel to reduce the possibility of excessive fuel accumulation prior to ignition which would cause high starting pressure in the chamber, i.e., a "hard start."

Figure 1 shows the thrust chamber with arrows indicating the flow direction of the fuel through the helix area within the thrust chamber jacket. The critical area in which overheating consistently occurs is also indicated. In almost all of the pictures presented in this report, the regenerative

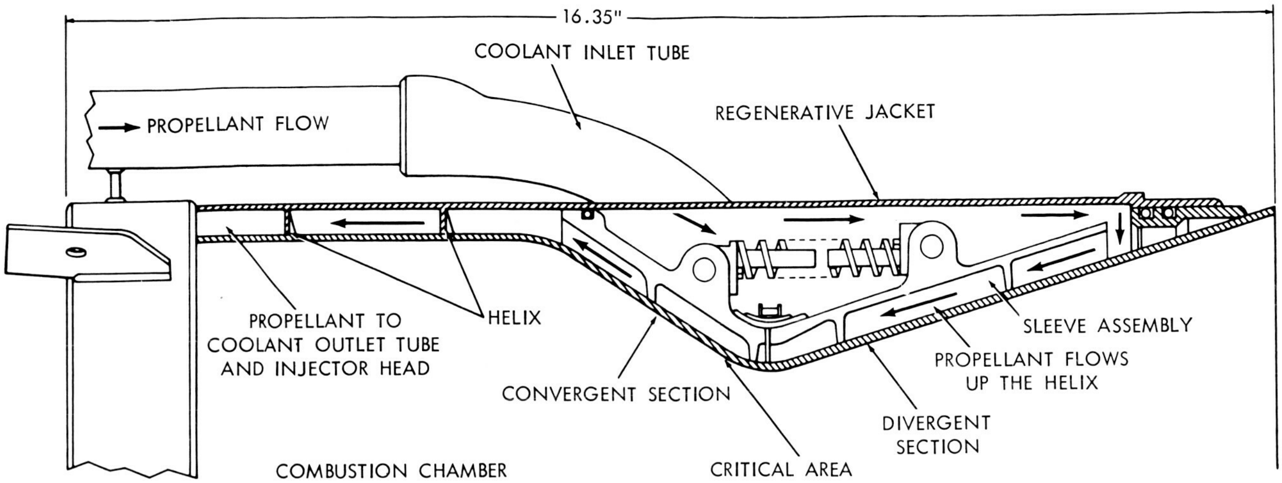



Figure 1—Longitudinal view of half section of Aerobee thrust chamber indicating propellant flow.

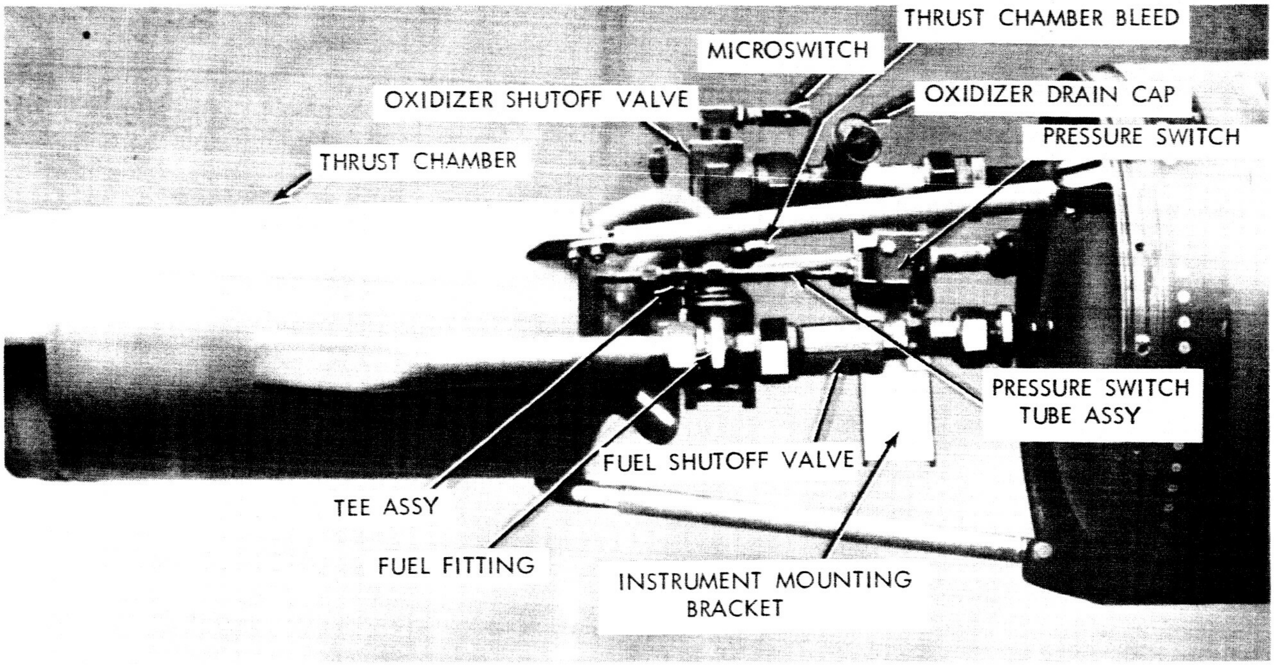
Table 1

Aerobee Thrust Chamber Performance Characteristics.

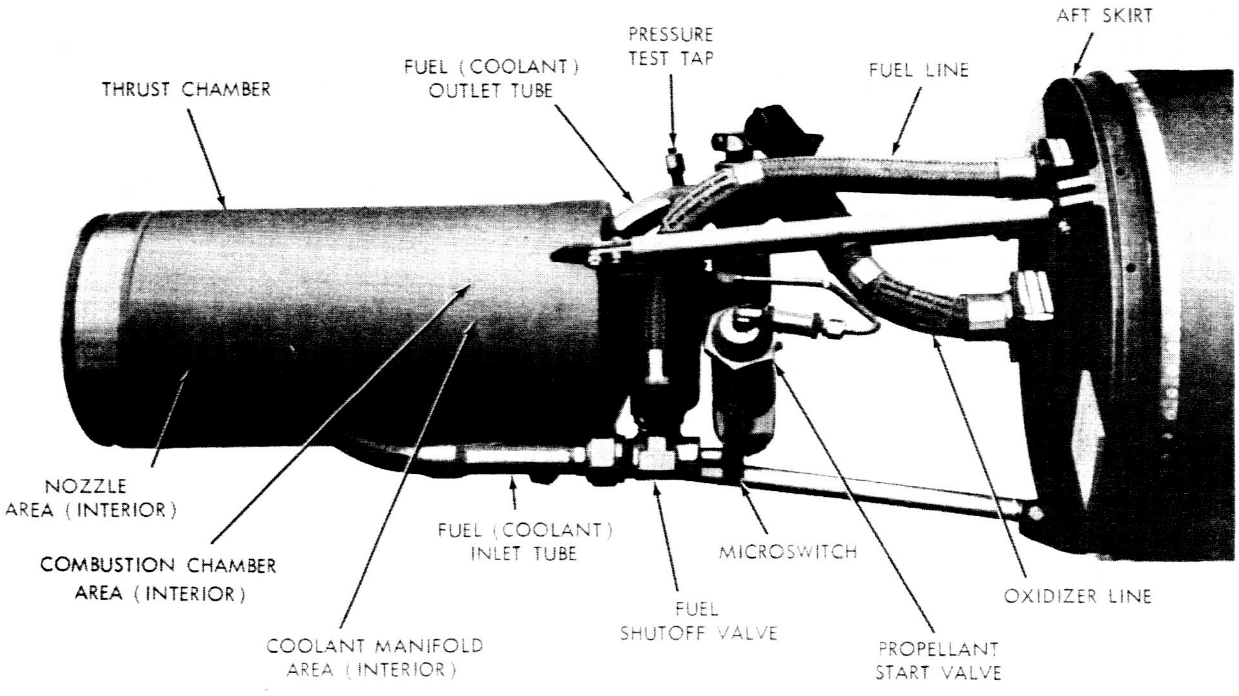
| Parameter and Unit of Measure | Aerobee 150 | Aerobee 150A |
|---|---|---|
| Nominal Thrust (lb)* | 4100 | 4100 |
| Thrust Duration (sec) | 51.3 | 50.9 |
| Specific Impulse, $I_{sp}$ (sec) | 198 | 198 |
| Total Impulse (lb/sec) | 210,330 | 208,690 |
| Thrust Coefficient* | 1.37 | 1.37 |
| Thrust Chamber Pressure (psia) | 324 | 324 |
| Exhaust Velocity (ft/sec) | 4650 | 4650 |
| Average Total Propellant Flow Rate (lb/sec) | 20.71 | 20.71 |
| Average Fuel Flow Rate (lb/sec) | 5.82 | 5.82 |
| Average Oxidizer Flow Rate (lb/sec) | 14.89 | 14.89 |
| Average Instantaneous Mixture Ratio | 2.56 | 2.56 |
| Chamber Diameter (in.) | 6.75 | 6.75 |
| Nozzle Throat Area (sq. in.) | 9.24 | 9.24 |
| Nozzle Exit Area (sq. in.) | 42.75 | 42.75 |
| Nozzle Exit Diameter (sq. in.) | 7.378 | 7.378 |
| Nozzle Area Ratio | 4.6 | 4.6 |
| Chamber-Throat Area Ratio | 3.88 | 3.88 |

*At sea level unless otherwise specified.

jacket and the magnesium sleeve assembly (often referred to as throat blocks) have been removed so that photographs could be taken of the critical area. Figures 2a and 2b show the Aerobee tail assembly including the location of propellant control fittings. Table 1 gives thrust chamber design performance parameters.




(a)




(b)

Figure 2—Tail assembly with location of propellant control fittings shown: (a) Aerobee 150; (b) Aerobee 150A.

## BURN-THROUGH FLIGHTS

### NASA Flight 4.126 GG

Ignition appeared normal on NASA 4.126 GG. The sustainer motor burned steadily until 46 seconds at which time the flame appeared to extinguish for approximately a second, re-ignite brightly for approximately a second, and then to extinguish completely. Telemetry records of the flight indicated rough chamber pressures beginning at 46 seconds with shutdown occurring at 48.7 seconds by "g" reduction timer. Statistics for this flight are given in Table 2.

Figure 3a shows an external view of the burned-through portion of convergent section of the nozzle of 4.126 GG. Most of the outer skin of the regenerative jacket and the entire assembly (Figure 1) have been removed. The fit marks where the sleeve assembly bears on the chamber assembly are clearly noticeable. Note the helical pattern and the extreme burned condition of the metal adjacent to the hole. Figure 3b shows another view of the motor taken at approximately 90° from Figure 3a. Here, the heat-affected zones on the convergent section and the aniline-furfuryl (ANFA) resin deposits covering the convergent and divergent sections are apparent as well as the burned-through area. The external photograph of the chamber in Figure 3c shows the injector, jacket, and sleeve assembly removed. The distortion on the forward end is due to the free-fall impact of the sustainer, rather than the burn-through.

(a) (b) (c)

Figure 3—External photographs of NASA 4.126 GG burned-through thrust chamber: (a) jacket partially cut off; (b) chamber turned 90°; (c) jacket, injector, sleeve assembly cut off.

Figure 4 shows the opposite half of the thrust chamber (rotated 180° from Figure 3a). This view shows some heat-affected area and burned fuel deposits, but no burn-through. The welded portion of the helix can be seen at the top of the section. The welded helix mates with the sleeve assembly at the bottom of the helix at the point as shown in the figure. Figure 5 provides an excellent close up external view of the burned-through nozzle area. Observe the errosion of metal around the burned area, the result of high velocity flow of the combustion gases.

Figures 6a and 6b are internal and external views of two of the four magnesium filler blocks which make up the sleeve assembly. Note the helix passage (in Figure 6a) which fits against the thrust chamber nozzle. It is through this passage that the ANFA flows during burning. The broken piece shown in Figure 6b is due to impact.

Table 2

Flight Statistics on NASA Aerobee 4.126 GG.

| | |
|---|---|
| Launch Site: | White Sands, New Mexico |
| Launch Date: | 22 August 1964 |
| Sustainer Serial No.: | 127-3 |
| Thrust Chamber Serial No.: | A-385 |
| Peak Altitude | |
| (a) predicted: | 118.5 st. miles |
| (b) actual: | 76.6 st. miles |
| Peak Time | |
| (a) predicted: | 233 sec |
| (b) actual: | 190 sec |
| Burnout Time: | 48.7 sec |
| Burnout Altitude: | 101,800 ft |
| Burnout Velocity: | 4,420 ft/sec |
| Coolant Jacket Pressure Drop: | 54.6 psi |
| Oxidizer Injector Pressure Drop: | 87.3 psi |
| Fuel Injector Pressure Drop: | 83.1 psi |
| Ambient Temperature at Launch: | 85°F |

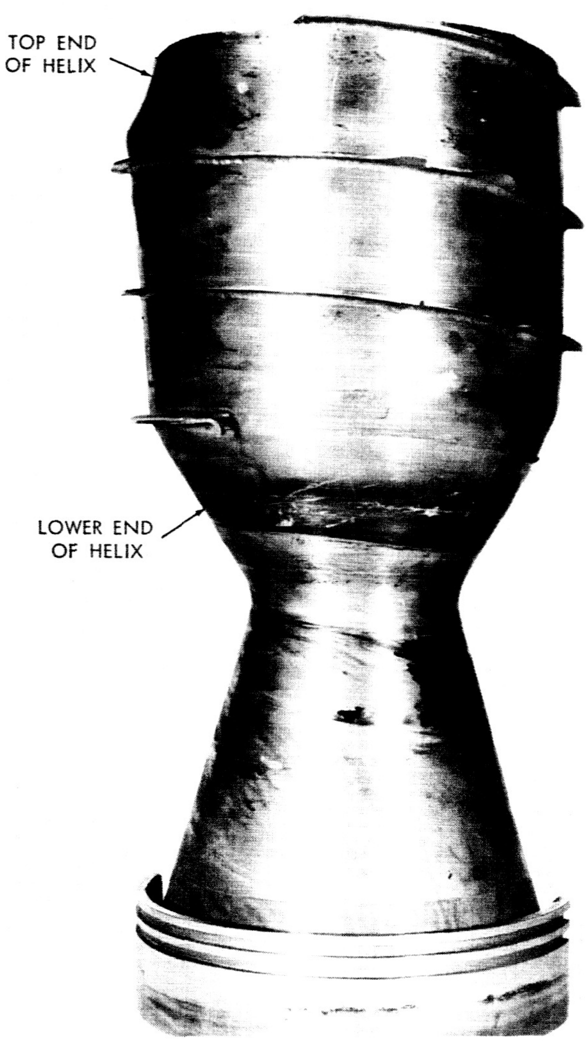



Figure 4—Thrust chamber rotated 180° from Figure 3a.

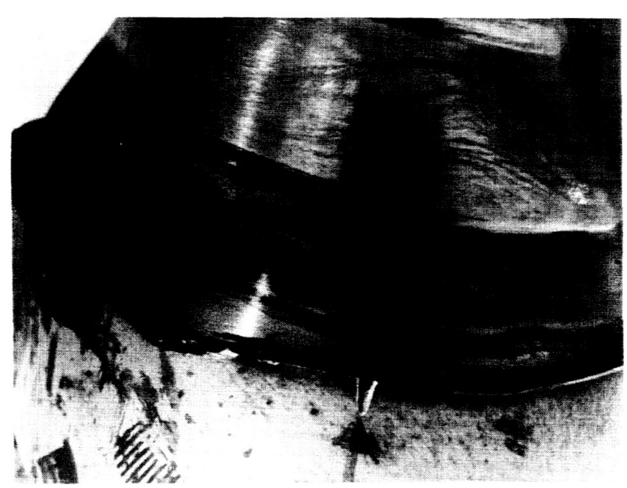
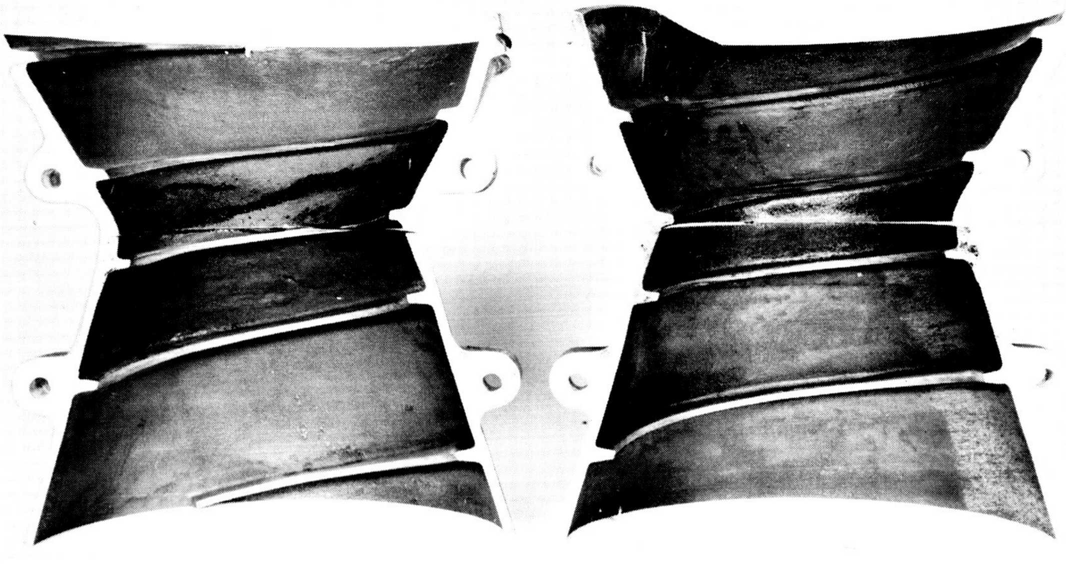
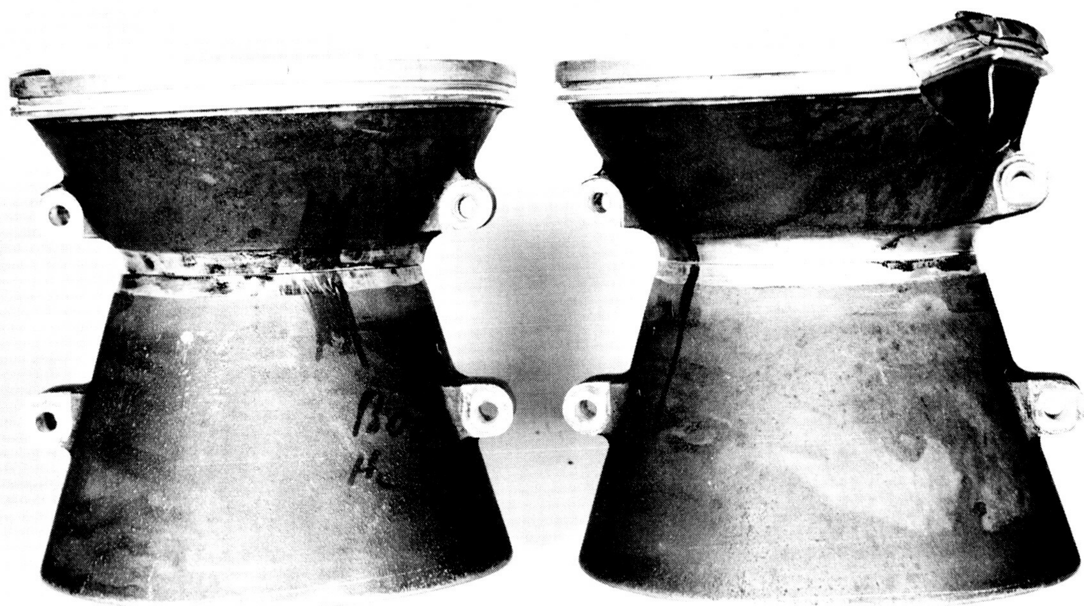


Figure 5—Close-up view of burned-through nozzle area.



(a)



(b)

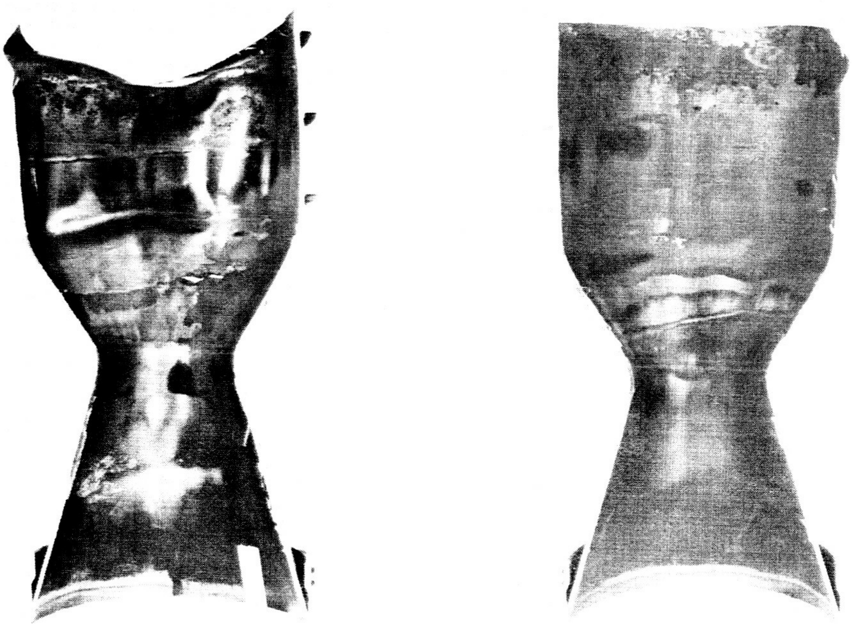
Figure 6—Magnesium filler blocks: (a) internal view; (b) external view.

Figure 7 shows the thrust chamber injector head. The outer ring of the orifices are for the oxidizer. The orifices for the fuel (much less noticeable) are inside the outer ring of the oxidizer orifices. The fuel holes are drilled in the frustrum of a cone which projects into the thrust chamber. Both sets of holes are canted to produce impinging propellant jets in the thrust chamber.

The holes were determined to have been properly drilled on this chamber. The carbonaceous material on the face of the plate (visible in the figure) is a normal product of the propellants; however, the amount in this instance is excessive, presumably the result of a fuel-rich mixture ratio under which the chamber operated for a short time after the burn-through occurred.

Figure 7—Thrust chamber injector head.

Figures 8 and 9 are internal views of the sectioned thrust chamber. Notice that the carbonaceous deposits (Figure 8b) are especially heavy on the side opposite the area where the burn-through occurred. Figure 9 is a closer view of the convergent section of nozzle area opposite the burned-through spot. Some erosion is evident on the left of the picture. In the upper



(a)

(b)

Figure 8—Thrust chamber cut in half: (a) internal view showing burned-through side; (b) internal view, opposite side.

left of the figure two segments of part of the regenerative jacket helix can be seen. These are welded to the combustion chamber.

## NASA Flight 4.114 GG

Shortly after T + 40 seconds, the propulsion system failed. No irregularities were observed prior to this time. At 41.72 seconds, chamber pressure and accelerometer outputs began to fluctuate and decreased until 44.43 seconds, when propulsion ceased. Although the sustainer was not found for two months, the cause of failure was determined from telemetered flight data to be a thrust chamber burn-through. This was confirmed when the thrust chamber was subsequently returned to the Goddard Space Flight Center for analysis. Table 3 provides flight statistics for this rocket. The pictures that follow are views of the thrust chamber burn-through.

Figure 10 shows the thrust chamber of NASA 4.114 GG with the outer skin of the regenerative jacket removed. The deposits shown in this figure are a combination of mud, rust and aniline - the results of approximately two months underwater. Figures 11a and 11b

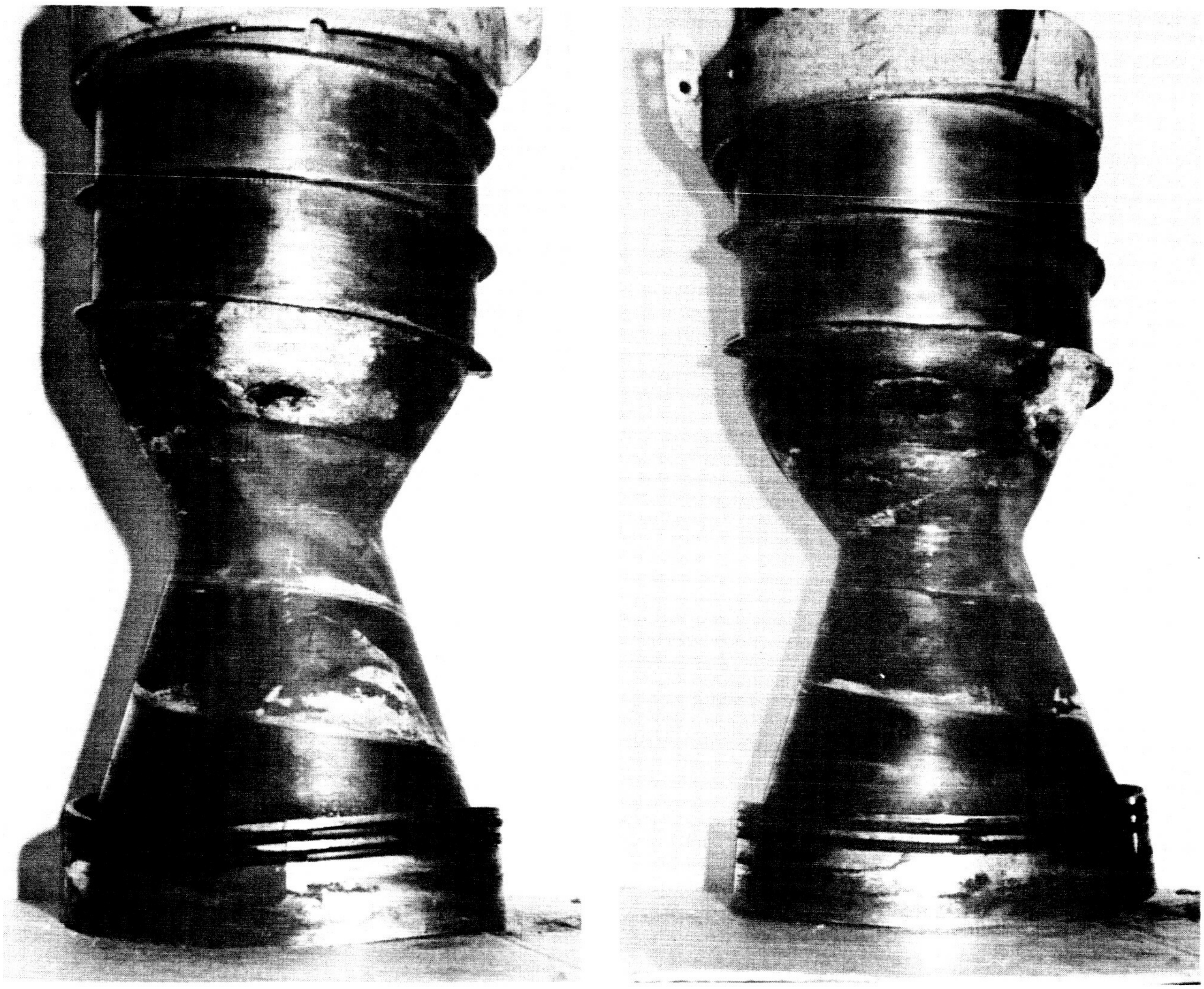
Table 3

Flight Statistics on NASA Aerobee 4.114 GG.

| | |
|---|---|
| Launch Site: | White Sands, New Mexico |
| Launch Date: | 23 April 1965 |
| Sustainer Serial No.: | 145-3 |
| Thrust Chamber Serial No.: | A-349 |
| Peak Altitude | |
| (a) predicted: | 107 st. miles |
| (b) actual: | 51 st. miles |
| Peak Time | |
| (a) predicted: | 222 sec |
| (b) actual: | 150 sec |
| Burnout Time: | 44.43 sec |
| Burnout Altitude: | 78,000 ft |
| Burnout Velocity: | 3,300 ft/sec |
| Coolant Jacket Pressure Drop: | 55.4 psi |
| Oxidizer Injector Pressure Drop: | 87.8 psi |
| Fuel Injector Pressure Drop: | 84.6 psi |
| Ambient Temperature at Launch: | 85°F |

Figure 9—Close-up, internal view of thrust chamber.

Figure 10—4.114 GG thrust chamber before cleaning, throat blocks on.



(a) (b)

Figure 11—External views of 4.114 GG thrust chamber: (a) 0° view of burned-through nozzle; (b) 90° view of burned-through nozzle.

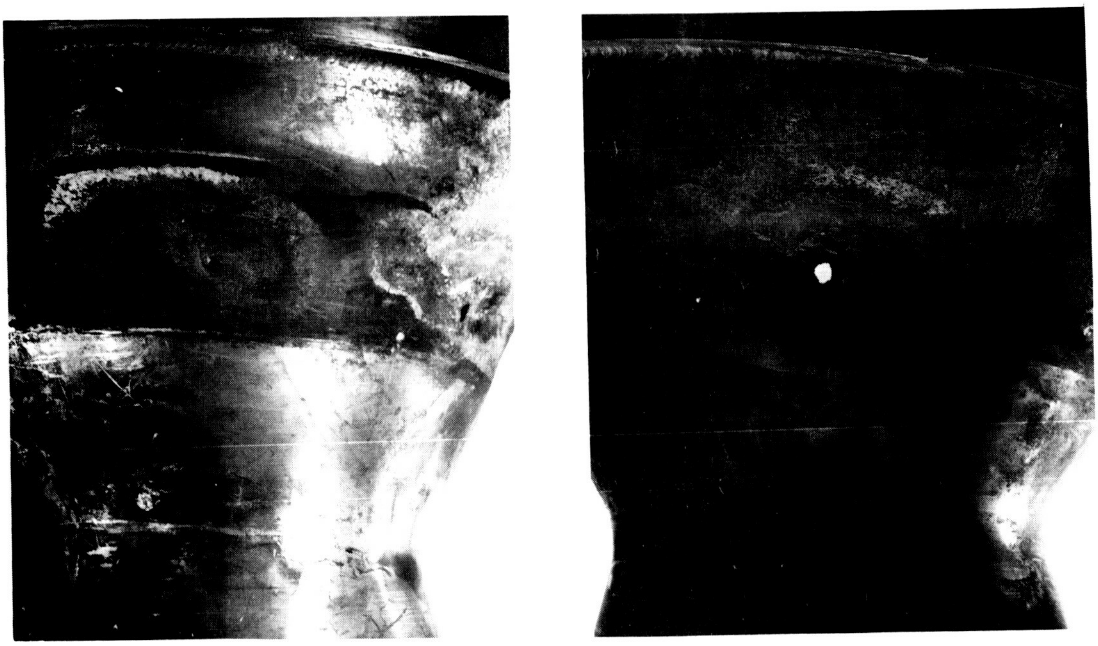
are views of the thrust chamber approximately 90° apart. Again the rust and aniline-furfuryl alcohol deposits are clearly evident. The burned-through spots are easily seen.

Figures 12 and 13 are external close-up photographs showing burned and burn-through areas.

Figure 14 presents close-up views of burned-through and erroded areas inside the thrust chamber. The inside of the combustion chamber was unusually clean of carbonaceous deposits—probably the result of oxidizer flowing through the chamber after the fuel had depleted. This chamber continued to blow down (flow propellants) for approximately fourteen seconds after the burn-through.

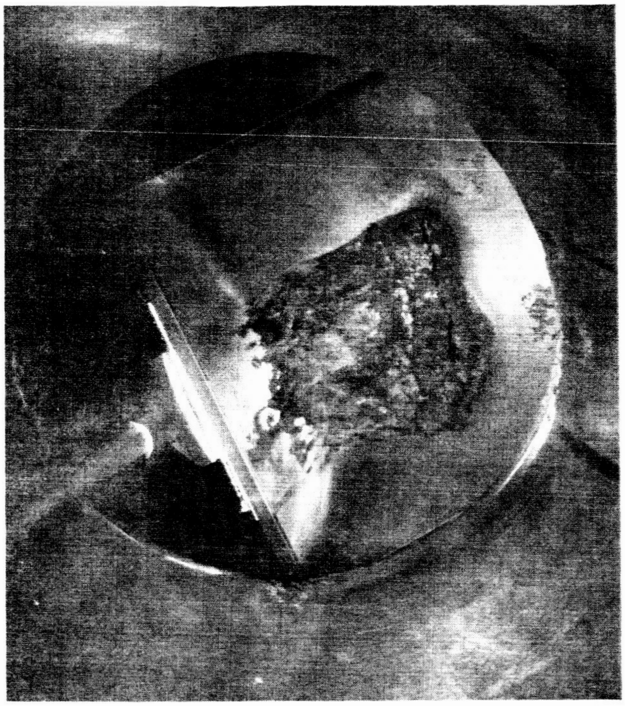
In the case of 4.126 GG, the flow continued for only approximately one second after burn-through; thus carbonaceous material was not removed.

Figure 12—Close-up external view of burned area on 4.114 GG thrust chamber.



(a) (b)

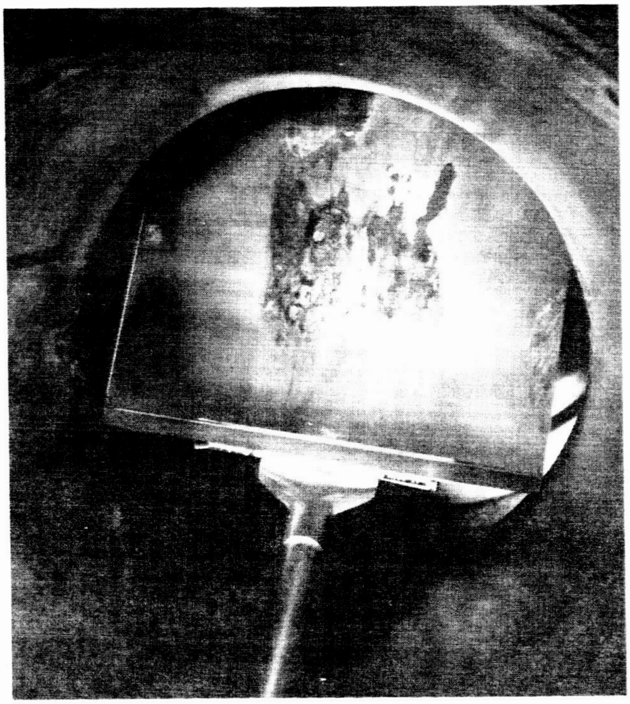
Figure 13—Close-up external views of burned-through areas on 4.114 GG thrust chamber.



(a)

(b)

(c)



(d)

Figure 14—Inside views of 4.114 GG thrust chamber.

## NASA Flight 4.112 NA

NASA 4.112 NA, in contrast to 4.126 GG and 4.114 GG, performed as predicted, reaching the planned peak altitude and useful experimental data were taken during the flight. When the sustainer motor was returned, however, it was observed that a burn-through had occurred. Table 4 provides flight statistics for this rocket.

Figure 15 shows the tail section of the recovered sustainer for flight 4.112 NA. The fin roots are observed on the periphery of the sustainer tail can (painted white). The forward end of the thrust chamber (inside) is also noticeable. The chamber was in excellent condition considering the condition of the remainder of the sustainer. Figure 16 shows an external view of the thrust chamber with the sleeve assembly still in place. The "belly band" (white teflon strip) is improperly assembled. Also, the manner in which the pieces of the sleeve assembly fit together, and in which they fit to the thrust chamber nozzle was incorrect. These manufacturing anomalies did not contribute to the burn-through.

Figure 17a through 17c are external photographs of the 4.112 NA motor with the regenerative jacket and sleeve assembly removed. The burn-through is especially noticeable in Figure 17c.

Table 4

Flight Statistics on NASA Aerobee 4.112 NA.

| | |
|---|---|
| Launch Site: | White Sands, New Mexico |
| Launch Date: | 29 June 1965 |
| Sustainer Serial No: | 147-3 |
| Thrust Chamber Serial No: | A-394 |
| Peak Altitude | |
| (a) predicted: | 121 st. miles |
| (b) actual: | 125 st. miles |
| Peak Time | |
| (a) predicted: | 235 sec |
| (b) actual: | 235 sec |
| Burnout Time: | 54.8 sec |
| Burnout Altitude: | 130,000 ft |
| Burnout Velocity: | 5,600 ft/sec |
| Coolant Jacket Pressure Drop: | 63.2 psi |
| Oxidizer Injector Pressure Drop: | 84.6 psi |
| Fuel Injector Pressure Drop: | 80.6 psi |
| Temperature (5 hr. check) | |
| fuel: | 81°F |
| oxidizer: | 78°F |

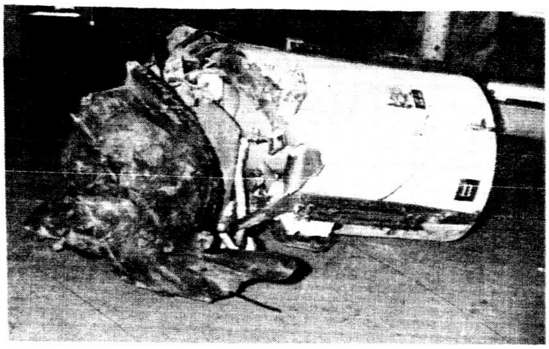


Figure 15—Flight 4.112 NA tail can damage.

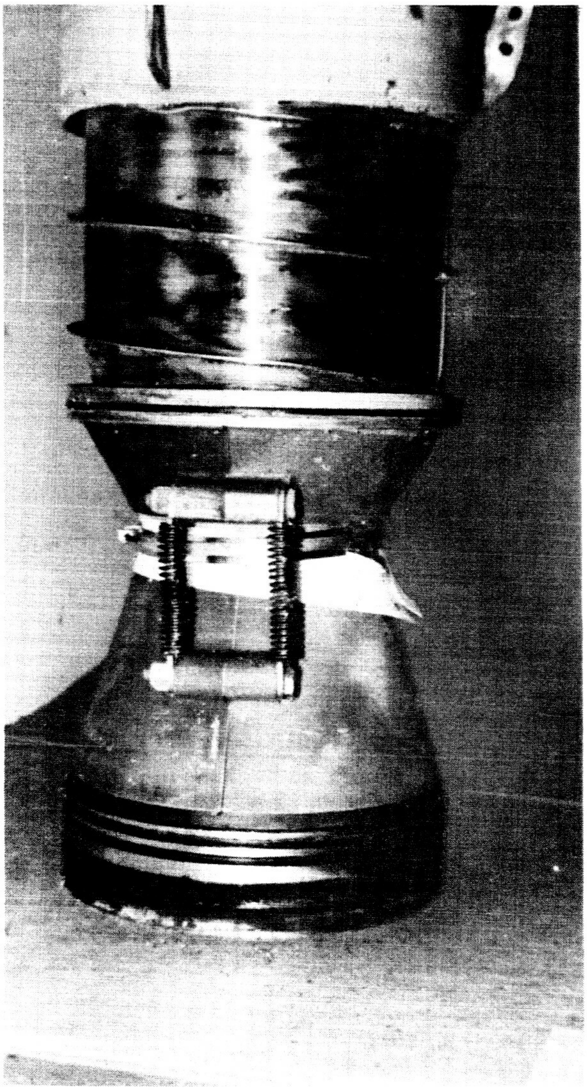


Figure 16—External view of 4.112 NA thrust chamber with throat blocks on.

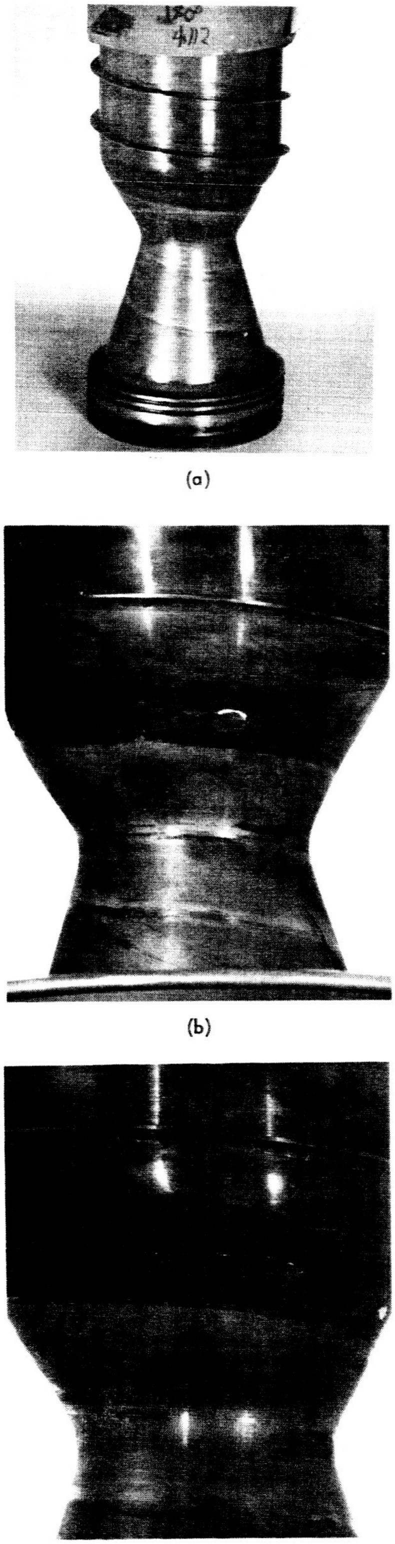



Figure 17—External views of 4.112 NA thrust chamber.

Figure 18 is an internal view of the burn-through and errosion which occurred in close proximity to each other. The carbonaceous deposits in the lower portion of the photo (in the divergent section of the nozzle) are normal.

It is possible that the burn-through on 4.112 NA was the result of fuel depletion, (i.e., if the thrust chamber uses all the fuel in combustion, there is no fuel left outside the chamber to cool it in the final second of burning.) Because of this possibility, the 4.112 NA burn-through must be considered apart from the burn-throughs on flights 4.126 GG and 4.114 GG.

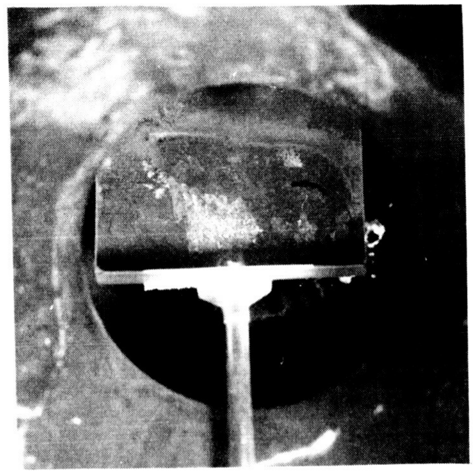
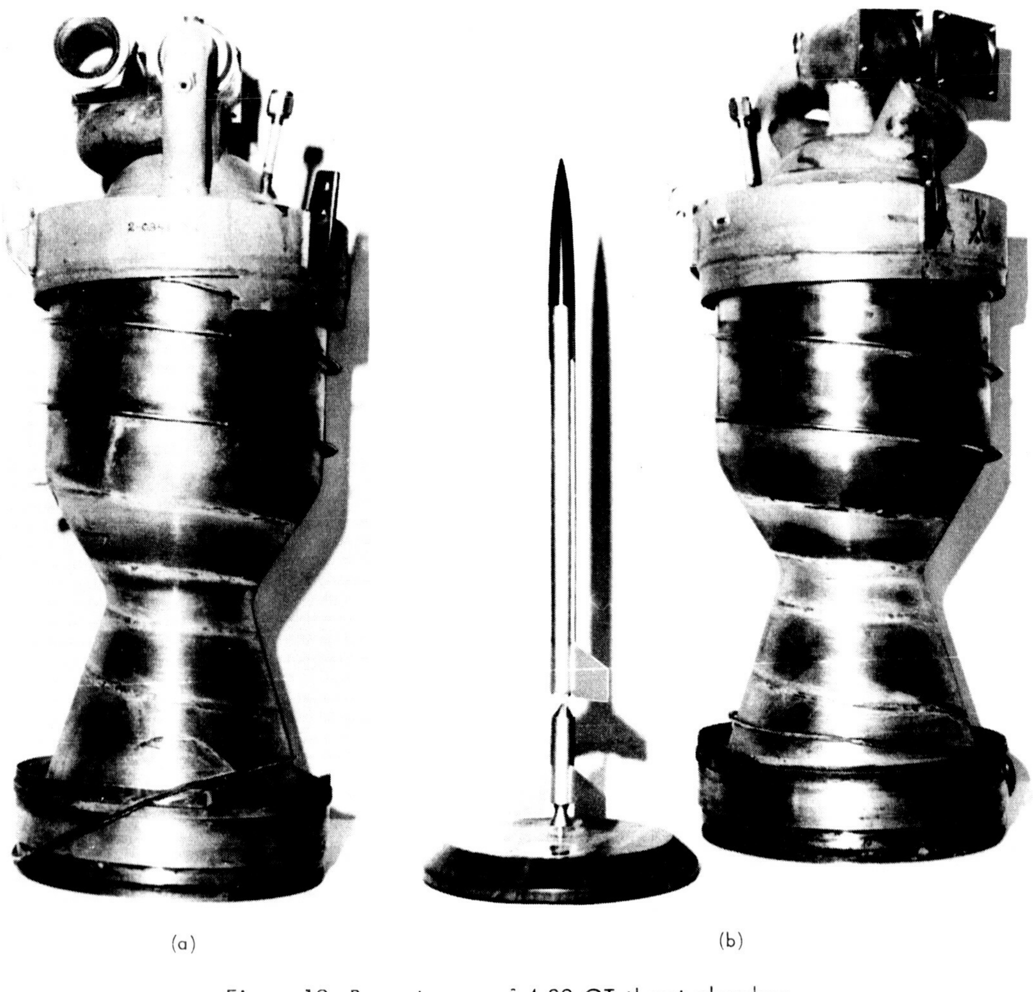


Figure 18—Internal view of 4.112 NA thrust chamber burning.

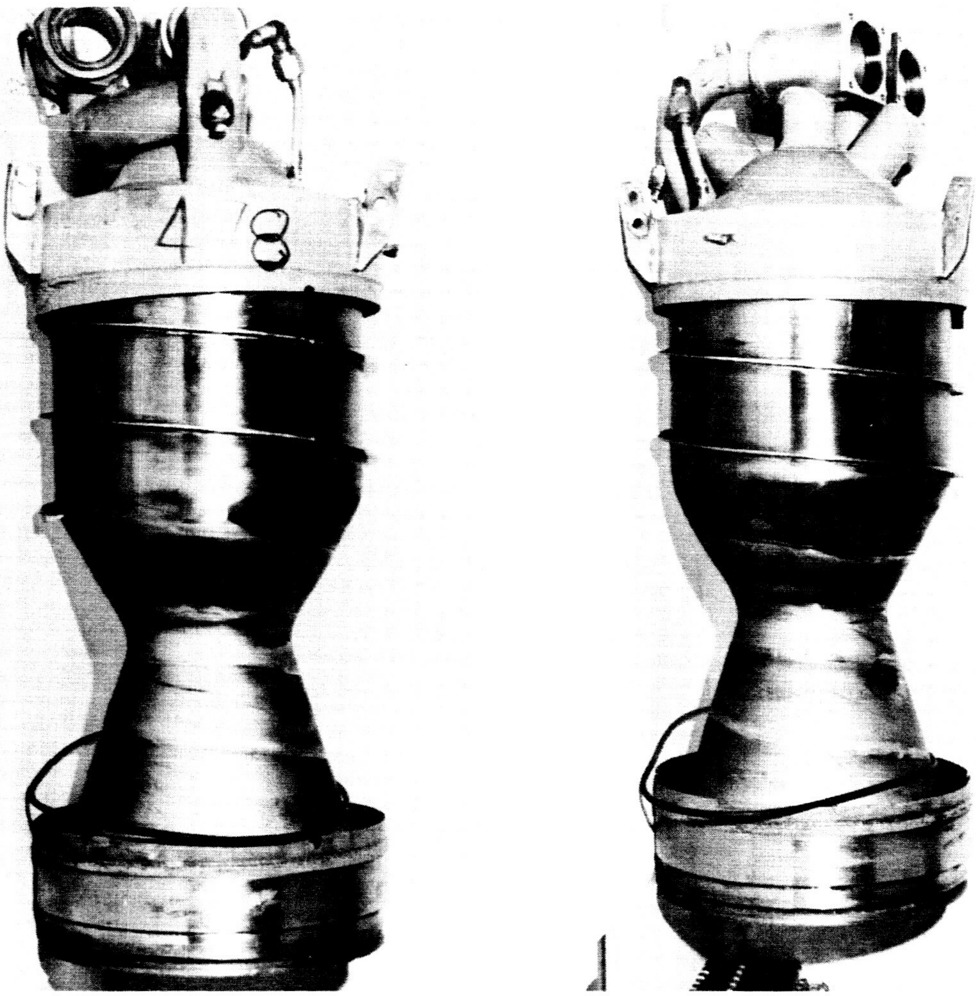
## BURNED THRUST CHAMBERS ON OTHER FLIGHTS

Figures 19-24 give a representative sample of Aerobee thrust chambers which were disassembled during this



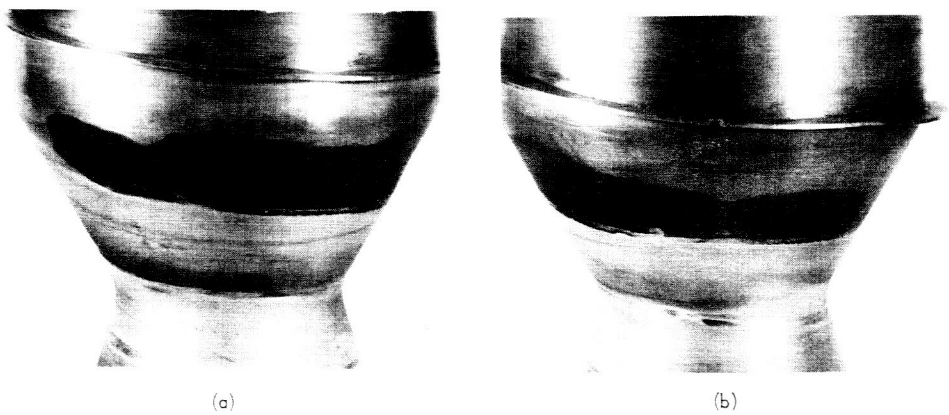
(a) (b)

Figure 19—Burned areas of 4.88 GT thrust chamber.



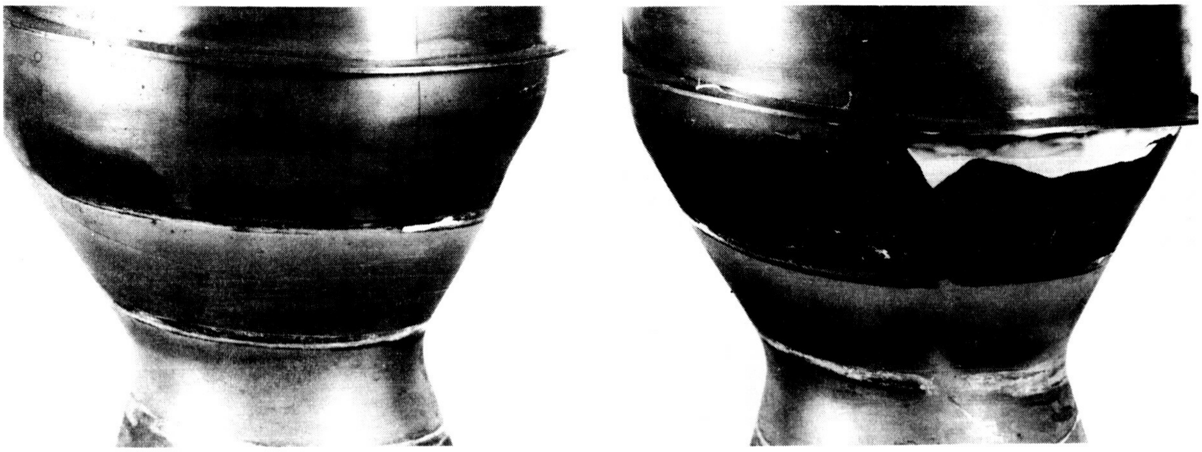
(a) (b)

Figure 20—Burned areas of 4.78 GG thrust chamber.



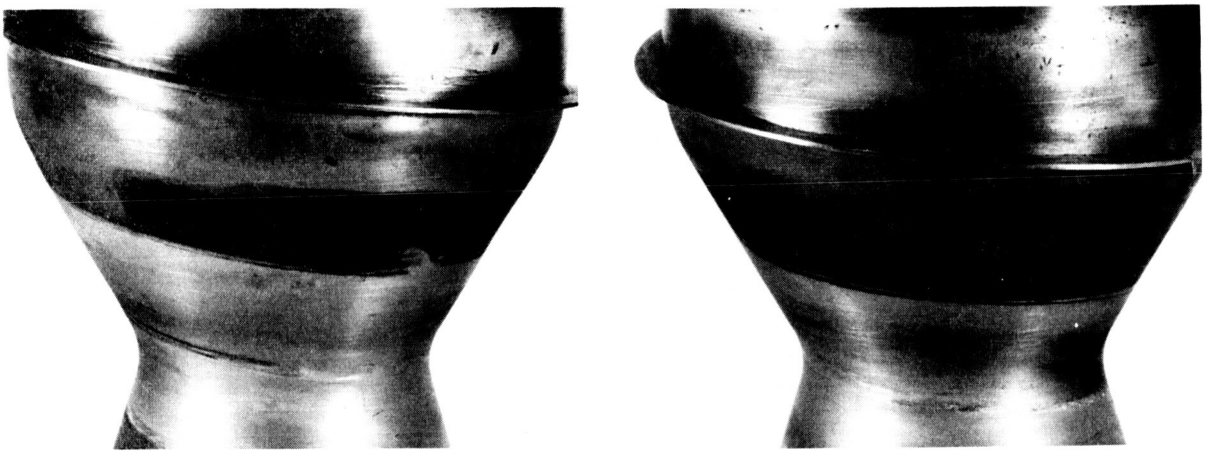
(a) (b)

Figure 21—Burned areas of 4.110 GG thrust chamber



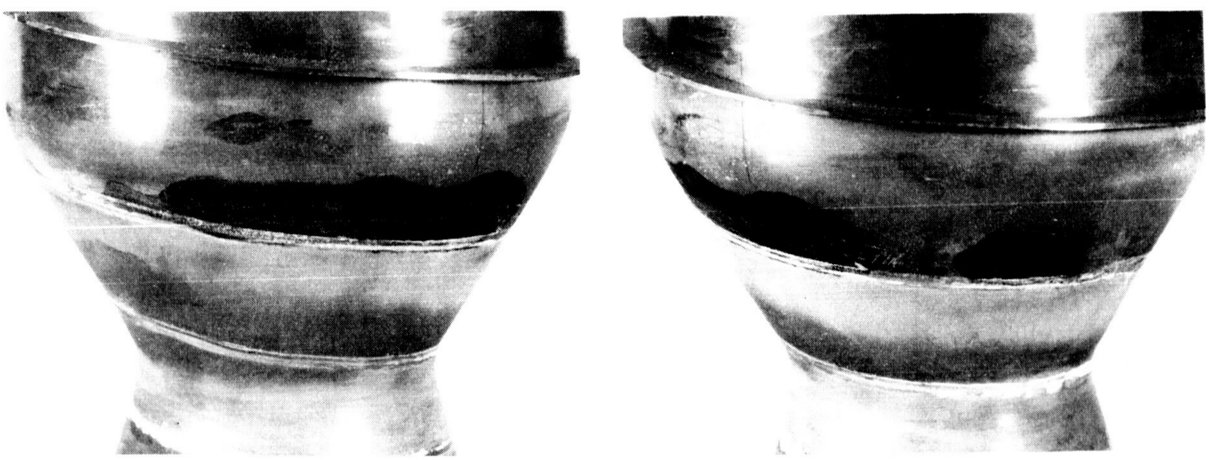
(a) (b)

Figure 22—Burned area of 4.120 CG thrust chamber.



(a) (b)

Figure 23—Burned areas of 4.123 CG thrust chamber.



(a) (b)

Figure 24—Burned areas of 4.132 GA-GI thrust chamber.

Table 5

Statistics on Successful Aerobee Flights When Thrust Chamber Overheating Occurred.

| | | | | | | |
|---|---|---|---|---|---|---|
| Flight Number | 4.78 GS | 4.88 GT | 4.110 GG | 4.120 CG | 4.123 CG | 4.132 GAGI |
| Launch Site | WSMR | WSMR | WSMR | WSMR | WSMR | WSMR |
| Launch Date | 10-1-63 | 1-28-64 | 11-14-64 | 10-2-64 | 10-27-64 | 12 16-64 |
| Sustainer Serial No. | 98-3 | 101-3 | 124-3 | 105-3 | 123-3 | AF-146-3 |
| Thrust Chamber Serial No. | A-258 | A-247 | A-323 | A-283 | A-272 | A-347 |
| Peak Altitude (est.) (st. miles) | 313 | 114 | 133.9 | 93.4 | 121.9 | 130 |
| Peak Altitude (actual) (st. miles) | 134 | 123.7 | 128.8 | 89.4 | 119.5 | 128.5 |
| Peak Time (est.) (sec) | 242 | 230 | 245 | 210.75 | 2.37 | 244 |
| Peak Time (actual) (sec) | 250 | 252.6 | 243.6 | 203.8 | 237 | 243 |
| Burnout Time (sec) | 253.7 | 52.3 | 53.4 | 51.1 | 52.4 | 54.08 |
| Burnout Altitude (ft) | 139,000 | 120,162 | 132,000 | 109,600 | 123,183 | 137,300 |
| Burnout Velocity (fps) | 5,990 | 5,459 | 5,730 | 4,776 | 5,290 | 5,742 |
| Coolant Jacket Pressure Drop (psi) | 59.4 | 53.8 | 56.4 | 66.3 | 71.0 | 64.6 |
| Oxidizer Injector Pressure Drop (psi) | 91.3 | 91.6 | 89.6 | 84.2 | 87.4 | 87.4 |
| Fuel Injector Pressure Drop (psi) | 84.6 | 83.3 | 84.1 | 82.4 | 80.2 | 78.9 |
| Launch Temperature (°F) | 60 | 55 | 63 | 78 | 69 | 35 |

investigation (4.88 GT, 4.78 GS, 4.123 CG, 4.110 GG, 4.120 GG, 4.132 GA-GI). As noted previously, although performance was satisfactory, there was localized heating in the critical region of the convergent section of the nozzle. Flight statistics for each rocket are summarized in Table 5. The extent of burning varies widely. Figure 25 shows another improperly assembled sleeve assembly on the thrust chamber motor for 4.120 CG. The gasket is protruding in the wrong direction for this anomaly to have occurred upon impact since the direction of force would be exactly opposite.

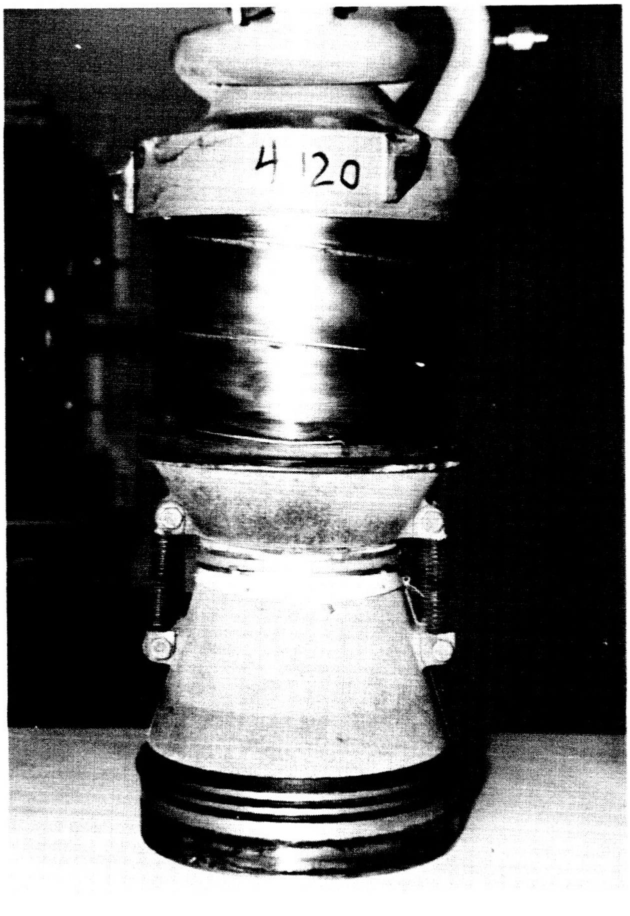



Figure 25—Thrust chamber of 4.120 CG with throat blocks on showing manufacturing discrepancy.

## CONCLUSIONS

The cause of the low performance of NASA flight 4.126 GG (August 26, 1964) was determined to be a severe burn-through in the convergent section of the thrust chamber nozzle. Subsequently, a comprehensive investigation was undertaken by Space General Corporation and Goddard Space Flight Center Sounding Rocket Vehicles Section personnel. This investigation included a disassembly of the sustainer motor, an inspection of all flight performance data, pressure data, flight hardware, and manufacturing records, and a detailed mathematical analysis of thrust chamber performance.

Following the failure of flight 4.126 GG, blown thrust chambers including ones which provided predicted rocket performance were disassembled and inspected. It is significant that even the engines which performed successfully, upon inspection, exhibited (to widely varying degrees) some effects of localized overheating in the convergent section of the nozzle.

Extensive examination of thrust chambers recovered after flight and other analysis suggested that the overheating in the nozzle convergent area was most probably due to the cross section configuration and depth of the helix regenerative cooling system in this area and associated manufacturing tolerances. In those cases where overheating was not enough to burn through, it was concluded that the primary factor was the slightly greater depth of the regenerative coolant passage, which permitted somewhat more fluid velocity and consequent increased cooling. Appendix A presents the basis for this conclusion.

Because of the costs involved in redesigning the helix configuration and conducting static firings to qualify the new design (at least $100,000), a decision was made to mitigate the problem through closer manufacturing tolerances. Quality control and inspection techniques were also improved.

Other manufacturing aspects were also carefully scrutinized through material hardness tests, dimensional checks, checking for proper injector hole sizes, chemical tests on materials, etc. All tests indicated that material quality and workmanship were not contributing factors to these failures. The chambers that were inspected were flown from WSMR and recovered after a free-fall impact. The criteria for inspecting a chamber was its condition after impact.

It is believed that, while the overheating problem may not be entirely eliminated by improving manufacturing procedures, the probability of a burn-through on future Aerobee thrust chambers has been significantly reduced.* If manufacturing improvements do not reduce the incidence of burn-throughs, the coolant passage will have to be redesigned.

Two other factors which, in conjunction with the ANFA velocity problem, could produce the somewhat randomly occurring burn-through are:

(1) ANFA resinification on the outer wall of the thrust chamber producing a significant decrease in heat transfer capability of the ANFA coolant. If the wall temperatures become too high in the critical area, ANFA may resinificate on the outer wall.† The resinification would further decrease heat transfer capability of the ANFA and cause wall temperatures to rise close to the failure point.

*The most significant change in manufacturing procedures was establishing control on the helix depth in the critical area to 0.230 in. ± 0.030 in.

†Mason, D. M., Noel, M. B. and Whittick, J. S., "Factors Affecting Formation of Resins and Their Deposition on Heat Exchange Walls by Furfuryl Alcohol-Amline Mixtures," JPL Progress Report No. 20-210, Pasadena, Calif.: Jet Propulsion Laboratories, California Institute of Technology, December 15, 1953.

(2) Use of high temperature propellants (greater than 80°) in the rocket for flight. All burned-through thrust chambers have had relatively high propellant temperatures (Tables 2-4). Two Navy Aerobee flights in the Bahamas which also had short burning times were serviced with propellants with temperatures above 80°F. The high temperature propellants can decrease the efficiency of the fuel as a coolant and increase combustion, which would produce a higher combustion gas temperature.

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Appendix A

## Velocity Distribution in Cooling Passage of the Aerobee 150 Thrust Chamber, Flight 4.126 GG*

The following analysis of the velocity distribution in the cooling passage of Aerobee thrust chamber serial No. A-385 was undertaken as a result of a burn-through occurring in the convergent section of the nozzle during flight. Figure A1 shows a flow section of a geometry similar to that where burn-through occurred. The symbols used in this analysis are:

$H_1$ = Upstream Pressure (ft)

$H_2$ = Downstream Pressure (ft)

$f$ = Coefficient of Friction

$g$ = Gravitational Acceleration (32.174 ft/sec$^2$)

$L$ = Length of Fluid Passage (in.)

$V$ = Fluid Velocity (ft/sec)

$W$ = Rate of Flow (lb/sec.)

$R$ = Hydraulic Radius (in.)

$A$ = Flow Area (in.$^2$)

$P_W$ = Wetted Perimeter (in.)

$S$ = Specific Gravity of Fluid

$\nu$ = Kinematic Viscosity (centistokes)

$\gamma$ = Specific Weight of Fluid (lb/ft$^3$)

$N_R$ = Reynolds Number

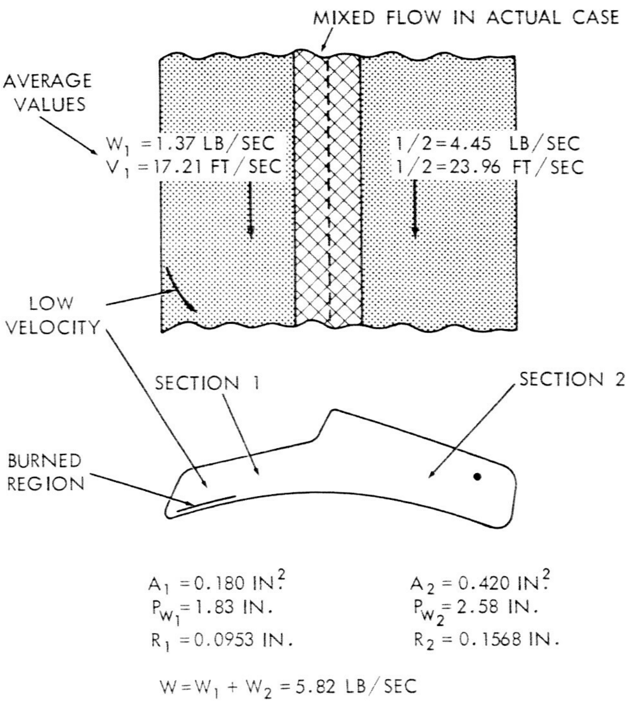


Figure A1—Cooling passage in region of burn-through.

Since no data were available to determine the exact distribution of pressure at the inlet to the cooling passage under consideration, uniform pressure on the upstream and downstream side of the fluid passage was assumed. Thus, based on the concept of the hydraulic radius, the

*Based on analysis made by Mr. N. M. Sverdrup at the Space General Corporation.

pressure loss through the portion of the cooling passage labelled Section 1 in Figure 1 is

$$H_1 - H_2 = f_1 \frac{L}{4R_1} \frac{V_1^2}{2g} \tag{A1}$$

and through Section 2,

$$H_1 - H_2 = f_2 \frac{L}{4R_2} \frac{V_2^2}{2g} \tag{A2}$$

Expressing the velocity in terms of the rate of flow and flow area, we have

$$V_1 = \frac{12^2 W_1}{A_1 \gamma} \tag{A3}$$

and

$$V_2 = \frac{12^2 W_2}{A_2 \gamma} \tag{A4}$$

Substituting these values in Equations 1 and 2, we obtain

$$H_1 - H_2 = f_1 \frac{L}{4R_1} \frac{1}{2g} \left(\frac{12^2 W_1}{A_1 \gamma}\right)^2 \tag{A5}$$

and

$$H_1 - H_2 = f_2 \frac{L}{4R_2} \frac{1}{2g} \left(\frac{12^2 W_1}{A_2 \gamma}\right)^2 \tag{A6}$$

Equating Equation 5 to 6 produces

$$f_1 \frac{L}{4R_1} \frac{1}{2g} \left(\frac{12^2 W_1}{A_1 \gamma}\right)^2 = f_2 \frac{L}{4R_2} \frac{1}{2g} \left(\frac{12^2 W_2}{A_2 \gamma}\right)^2$$

Simplifying and solving for the ratio $W_1/W_2$,

$$\frac{W_1}{W_2} = \frac{A_1}{A_2}\sqrt{\frac{f_2}{f_1}\frac{R_1}{R_2}} \tag{A7}$$

We also have

$$W = W_1 + W_2 , \tag{A8}$$

where $W$ is the total flow rate of cooling fluid.

From the above equations, average values of the rate of flow and the velocity distribution can be approximated.

The flow area of Section 1 $(A_1)$ in this illustration is 0.180 in.$^2$ and the wetted perimeter $(P_{W_1})$ is 1.83 in. Hence, the hydraulic radius $(R_1)$ is 0.0983 in. Similarly for Section 2 in this illustration, $A_2 = 0.420$ in.$^2$, $P_{W_2} = 2.68$ in. and $R_2 = 0.1568$ in. The total rate of flow $(W)$ of cooling fluid is 5.82 lb/sec. The specific gravity $(s)$ is assumed as 1.02 and the kinematic viscosity $(\nu) = 1.47$ centistokes.

Then substituting in Equations 7 and 8,

$$\frac{W_1}{W_2}\frac{0.180}{0.420}\sqrt{\frac{f_2}{f_1}\frac{0.0983}{0.1568}} = 0.3393\sqrt{\frac{f_2}{f_1}} \tag{A9}$$

$$W = W_1 + W_2 = 5.82 \text{ lb/sec} \tag{A10}$$

Since the coefficient of friction is a function of the Reynolds number, which is a function of $V_1$ and $V_2$ (unknown quantities), we must assume that $f_2 = f_1$.

Thus from Equations 9 and 10, we obtain

$$5.82 = W_2\, 0.3393\sqrt{\frac{f_2}{f_1}} + W_2 = 1.3393\, W_2 .$$

from which

$$W_2 = \frac{5.82}{1.3393} = 4.33 \text{ lb/sec for Section 2}$$

and

$$W_1 = 5.82 - 4.33 = 1.49 \text{ lb/sec for Section 1.}$$

By Equations 3 and 4, the average velocity,

$$V_1 = \frac{12^2 \times 1.49}{0.180 \times 62.427 \times 1.02} = 18.72 \text{ ft/sec for Section 1}$$

and

$$V_2 = \frac{12^2 \times 4.33}{0.420 \times 62.427 \times 1.02} = 23.31 \text{ ft/sec for Section 2.}$$

The Reynolds number is

$$N_R = 7741.92 \frac{V4R}{\nu} . \tag{A11}$$

Thus

$$N_{R_1} = 7741.92 \times \frac{18.72 \times 4 \times 0.0983}{1.47} = 38766, \ f_1 = 0.023 \text{ for Section 1}$$

and

$$N_{R_2} = 7741.92 \times \frac{23.31 \times 4 \times 0.1568}{1.47} = 76998, \ f_2 = 0.019 \text{ for Section 2.}$$

Substituting the values of $f_1$ and $f_2$ in Equation 9,

$$\frac{W_1}{W_2} = 0.3393\sqrt{\frac{0.019}{0.023}}$$

$$W = W_1 + W_2 = 5.82 \text{ lb/sec} ,$$

from which

$$5.82 = W_2\, 0.3393 \sqrt{\frac{0.019}{0.023}} + W_2$$

and

$$W_2 = \frac{5.82}{1 + 0.3393 \sqrt{\frac{0.019}{0.023}}} = \frac{5.82}{1 + 0.3084} = 4.45 \text{ lb/sec for Section 2.}$$

$$W_1 = 5.82 - 4.45 = 1.37 \text{ lb/sec for Section 1.}$$

Therefore, by Equations 3 and 4, the average velocities $V_1$ and $V_2$ are

$$V_1 = \frac{12^2 \times 1.37}{0.180 \times 62.427 \times 1.02} = 17.21 \text{ ft/sec for Section 1;}$$

$$V_2 = \frac{12^2 \times 4.45}{0.420 \times 62.427 \times 1.02} = 23.96 \text{ ft/sec for Section 2.}$$