Hamiltonian cycle :

The Hamiltonian cycle is a round trip path along n edges of graph G that visits every vertex exactly once and return to its starting position. (initial vertex)

Let $G = (V, E)$ be a connected graph with n vertices.

A Hamiltonian cycle begins at $V_1 \in G$ and vertices of G are visited in order $V_1, V_2, V_3 \ldots V_n, V_{n+1}$.

Here, $\boxed{V_1 = V_{n+1}}$; the edges $(V_i, V_{i+1}) \in E(G)$.

ie) The vertices are visited in order

$$\boxed{V_1, V_2, V_3, \ldots V_n, V_1}$$

The Backtracking solution vector of a hamiltonian cycle is defined by n tuple $\{x_1, x_2, x_3 \ldots x_n\}$ such that x_i represents i th visited vertex of the proposed hamiltonian cycle.

Algorithm for Hamiltonian cycle :

```
Algorithm Hamiltanian( K )
// G[1:n, 1:n] - adjacency matrix of graph G
// G[i,j] = 1 ; if <i,j> ∈ E(G)
// G[i,j] = 0,  if <i,j> ∉ E(G)
// all cycles begin at node 1
{
  repeat
  {
     // generate value of x[K]
     Next value (K);
     // assign a legal value to x[K]
     if (x[K] = 0) then return;
     // no legal x[K] has been assigned.
     if (k = n) then write (x[1:n]);
     else
         Hamiltonian (K+1);
  } until (false);
}
```

```
Algorithm Nextvalue (k)
// x[1: k-1] - Path of k-1 distinct vertices
// if x[K] = 0 then no vertices has been assigned
// x[K] is assigned to next highest number vertex
// which doesnot already appeard in x[1:k-1]
```

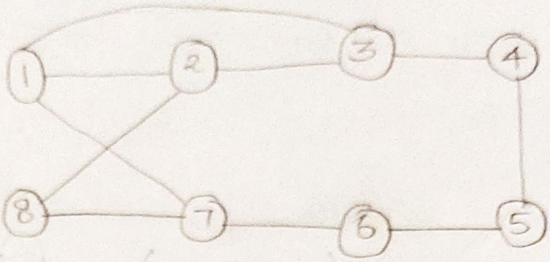
// an edge is connected from x[k] - x[k-1]

// if (k = n) and x[k] is connected to x[1] then

// there exist a hamiltonian cycle.

```
{
  repeat
  {
    x[k] := (x[k]+1) mod (n+1);
    // next highest number vertex.
    if (x[k]=0) then return;
    if (G[x[k-1], x[k]] ≠ 0) then
    // is there an edge
    {
      for j:=1 to k-1 do
      {
      if (x[j] := x[k]) then break;
      }
      // check for distinct vertices.
    if (j = k) then
    // if true then the vertex is distinct
  if ((k<n) or ((k = n) and (G [x[n], x[1]]
                                        ≠0)))
  then return;
      }
  } until (false);
}
```

Consider the graph G and its adjacency matrix.



$$
\begin{array}{c|c}
 & \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \end{matrix} &
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\end{array}
$$

Since all the Hamiltonian cycle starts with an initial Vertex $x[1] = 1$, the Backtracking prodecure starts with ~~Hamiltonian~~

- hamiltonian(2).

x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]	x[8]
1	0	0	0	0	0	0	0
1	2	3	4	5	6	7	8
1	2	8	0	0	0	0	0
1	2	8	7	6	5	4	3
1	3	0	0	0	0	0	0
1	3	2	8	7	6	5	4

	x[1]	x[2]	x[3]	x[4]	x[5]	x[6]	x[7]	x[8]	
	1	3	4	0	0	0	0	0	
✓	1	3	4	5	6	7	8	2	Solution B
	1	7	0	0	0	0	0	0	
✓	1	7	6	5	4	3	2	8	
	1	7	8	0	0	0	0	0	
✓	1	7	8	2	3	4	5	6	
	1	0	0	0	0	0	0	0	until (false)

State space tree for Hamiltonian cycle: