n- Queen Problem :

The n queen problem is to find all the ways to place n non-attacking queens in a n x n chess board.

The solution vector of n queen problem is represented by {x[1], x[2], ... x[n]} in which x[i] is the column of i^{th} row where i^{th} queen is placed.

x[i] will be distinct since no two queens can be placed in the same column or same diagonal. ~~The colum~~

The columnwise check of non-attacking queens can be done by placement of previous K-1 queens

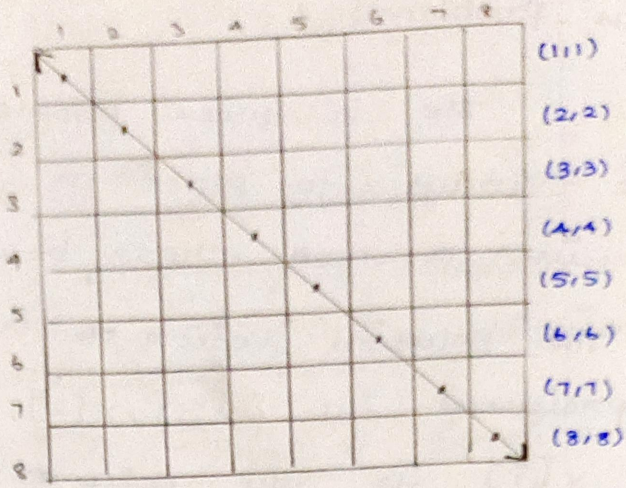
The diagonalwise check of non-attacking queens can be done in two forms

i) upper left to lower right

ii) lower left to upper right

upper left to lower right :

consider a 8 x 8 chess board



Let $[i,j]$ be the position of previous queen placed in chess board.

Let $[k,l]$ be the position of new queen ie) (current queen) yet to be placed, then $[i,j]$ and $[k,l]$ are on the same diagonal only if,

$$\boxed{j - i = l - k}$$

$$\Rightarrow \boxed{j - l = i - k} \quad \text{——— (1)}$$

example :

Let $[2,2]$ be the previous position of queen ie) $[i,j] = [2,2]$

Let $[3,3]$ be a position of new queen yet to be placed. ie) $[k,l] = [3,3]$

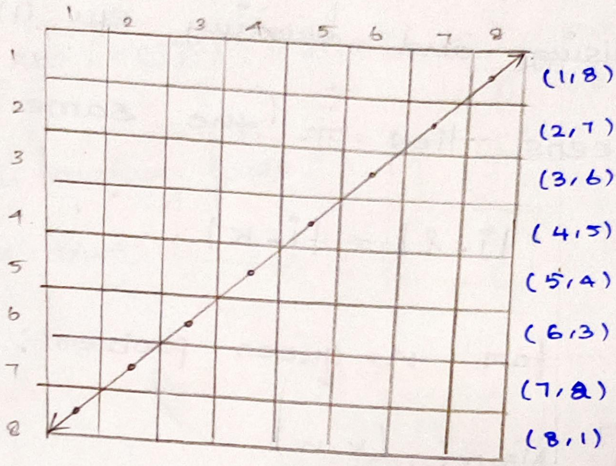
if $[\overset{i}{2}, \overset{j}{2}]$ and $[\overset{k}{3}, \overset{l}{3}]$ lie on the same diagonal, then

$$j - l = i - k$$

$$\Rightarrow 2 - 3 = 2 - 3$$

$$\Rightarrow \boxed{-1 = -1}$$

Lower left to upper right :



Let [i, j] be the position of previous Queen placed in chess board.

Let [k, l] be the position of new queen (current queen) yet to be placed, then [i, j] and [k, l] lie on the same diagonal only if,

$$\boxed{i + j = k + l}$$

$$\Rightarrow \boxed{j - l = k - i} \quad \text{——— (2)}$$

example :

Let [2,7] be the previous position of Queen ie) [i, j] = [2,7]

Let [3,6] be the position of new queen yet to be placed. ie) [k, l] = [3,6]

if $[\overset{i}{2}, \overset{j}{7}]$ & $[\overset{k}{3}, \overset{l}{6}]$ lie on the same diagonal, then

$$j - l = k - i$$

$\Rightarrow \quad 7 - 6 = 3 - 2$

$\Rightarrow \quad \boxed{1 = 1}$

generalising and equating equ (1) & (2)

two queens lies on the same diagonal only

if $\boxed{|j - l| = |i - k|}$

Algorithm for n-queen problem :

```
Algorithm Nqueen (k,n)
// possible placement n-non attacking queens in
// a nxn chess board using Backtrack
{
    for i:=1 to n do
    {
        if (Place(k,i)) then
        {
            x[k] := i;
            if (k=n) then write (x[1:n]);
            else
                Nqueen (k+1,n);
        }
    }
}
Algorithm place (k,i)
// return true if queen can be placed in
// kth row and ith column.
// x[] - a global array whose first k-1 values is (known)
```

// Abs(r) - returns the Absolute value of r (only magnitude)

```
{
    for j:=1 to k-1 do
    {
      if (x[j] = i)  // two queens lies in the same
                      // column.
      or (Abs(x[j]-i) = Abs(j-k)) // two queens lies
                      // in the same diagonal

      then return false;
    }
    return true

}
```

State space tree for 4 queen problem:

A 4 queen problem is to place 4 Non-attacking Queens in a 4×4 chess board such that no two queens should be placed in same column or same diagonal.

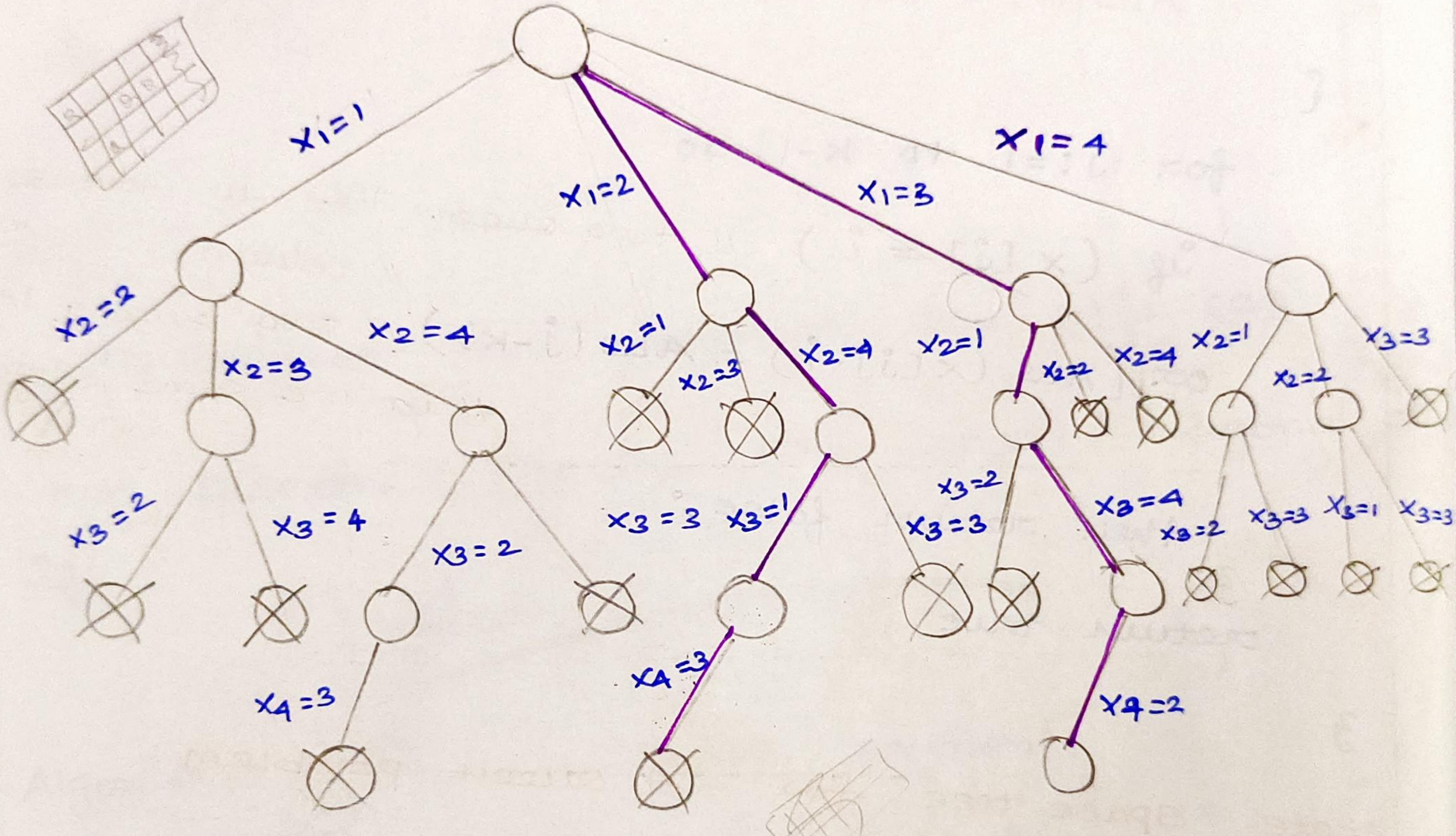
Two optimal solutions are provided for 4 queen problem

Soln 1:

	Q			$x_1 = 2$
			Q	$x_2 = 4$
Q				$x_3 = 1$
		Q		$x_4 = 3$

Soln 2:

		Q		$x_1 = 3$
Q				$x_2 = 1$
			Q	$x_3 = 4$
	Q			$x_4 = 2$



x1=1
x1=2
x1=3
x1=4
x2=2
x2=3
x2=4
x2=1
x2=3
x2=4
x2=1
x2=2
x2=4
x2=1
x2=2
x3=3
x3=2
x3=4
x3=2
x3=3
x3=1
x3=3
x3=2
x3=4
x3=2
x3=3
x3=1
x3=3
x4=3
x4=3
x4=2