ii. The first four moments of a distribution about X = 4 are 1, 4, 10, 45. Find mean, variance, μ_3 and μ_4.

29. a. Find the MGF of binomial distribution and hence find the mean and variance.

(OR)

b. If X is a normal variate with mean 30 and standard deviation 5. Find the probabilities that $(i)\ 26 \le X \le 40$ $(ii)\ X \ge 45$ $(iii)\ |X-30| \le 5$ $(iv)\ |X-30| \ge 5$.

30.a.i The SD of a random sample of 1000 is found to be 2.6 and the SD of another random sample of 500 is 2.7. Assuming the samples to be independent, find whether the two samples could have come from the populations with the same standard deviation?

ii. Theory predicts that the proportion of beans in four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

(OR)

b. Two independent samples of sizes 8 and 7 gave the following data:

Sample I	19	17	15	21	16	18	16	14
Sample II	15	14	15	19	15	18	15	

Test whether the samples are drawn from the same normal population.

31. a. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and the service time is an exponential random variable with mean 8 minutes

(i) Find the average number of customers in the shop.
(ii) Find the average number of customers in the queue.
(iii) Find the average waiting time of a customer spend in the shop.
(iv) Find the average waiting time of a customer spend in queue?
(v) What is the probability that the server is idle?

(OR)

b. Patients arrive at a clinic according to a Poisson distribution at a rate of 30 patients per hour. The waiting room cannot accommodate more than 14 patients. Examination time per patient is exponential at a rate of 20 per hour.

(i) Find the effective arrival rate at the clinic.
(ii) What is the probability that an arriving patient will not wait?
(iii) What is the expected waiting time until a patient is discharged from the clinic?

32. a. The tpm of a Markov chain $\{X_n\}, n=1,2,3......$ having three states 1, 2, 3 is $P=\begin{bmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{bmatrix}$ and the initial distribution is $p^{(0)}=[0.7 \quad 0.2 \quad 0.1]$. Find $(i)\ P[X_2=3]$ $(ii)\ P[X_3=2, X_2=3, X_1=3, X_0=2]$.

(OR)

b. The three state Markov chain is given by the tpm

$$P=\begin{bmatrix}0 & \frac{2}{3} & \frac{1}{3}\\ \frac{1}{2} & 0 & \frac{1}{2}\\ \frac{1}{2} & \frac{1}{2} & 0\end{bmatrix}$$

(i) Classify the states of the Markov chain.
(ii) Find the steady state distribution of the chain.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, DECEMBER 2017
Third/ Fourth/ Fifth Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A random variable X has the probability density function $f(x)=\begin{cases} e-x, & x\ge 0\\ 0, & x<0\end{cases}$. The MGF of f (x) is
(A) $\frac{2}{2-t}$ (B) $2(1-t)$
(C) $\frac{1}{1-t}$ (D) $\frac{2}{2+t}$

2. Let X be a discrete RV with possible values 1, 2, 3 associated with probabilities $\frac{1}{4},\frac{1}{2},\frac{1}{4}$ respectively. Then E(X) is
(A) 2 (B) 1
(C) 4 (D) 3

3. Let X be a continuous RV with pdf $f(x)=\begin{cases}\frac{k}{6}, & 1<x<5\\ 0, & otherwise\end{cases}$. The value of k is
(A) $\frac{1}{4}$ (B) $\frac{1}{2}$
(C) 2 (D) 4

4. If $Var(X)=4$, then find $Var(4X+5)$ where X is a random variable
(A) 16 (B) 64
(C) 69 (D) 21

5. If X is uniformly distributed in $(-3,3)$ then its probability density function $f(x)$ is given by
(A) $\frac{1}{6}$ (B) $\frac{3}{4}$
(C) $\frac{1}{3}$ (D) $\frac{1}{9}$

6. In a normal distribution about 99% of the observation lie between
(A) $\mu\pm\sigma$ (B) $\mu\pm3\sigma$
(C) $\mu\pm35$ (D) $\mu\pm\sigma^2$

7. The mean of the binomial distribution is
(A) ηp (B) ηpq
(C) $\sqrt{\eta p}$ (D) $\sqrt{\eta pq}$

8. The MGF of the Poisson distribution with parameter λ is.
(A) e^{λ} (B) $e^{\lambda}\left(e^{t}-1\right)$
(C) $e^{1-\lambda}$ (D) e^{t-1}

9. The chi-square goodness of fit test can be used to test for
(A) Significance of sample statistics (B) Difference between population means
(C) Normality (D) Probability

10. A type II error occurs when
(A) The null hypothesis is incorrectly accepted when it is false (B) The null hypothesis is incorrectly rejected when it is true
(C) The sample mean differs from the population mean (D) The test is biased

11. The 't' distribution tends to __________ distribution for sufficiently large value of γ.
(A) Uniform (B) Exponential
(C) Standard normal (D) Poisson

12. The degrees of freedom for testing a sample mean of a sample of size 'n' is
(A) n (B) n-1
(C) –n (D) n+1

13. Single server Poisson queue with finite capacity of the Markove model is
(A) $(M/M/1):(\infty/FIFO)$ (B) $(M/M/S):(k/FIFO)$
(C) $(M/M/1):(k/FIFO)$ (D) $(M/M/S):(\infty/FIFO)$

14. The arrival pattern in queuing theory follows
(A) Binomial distribution (B) Normal distribution
(C) Geometric distribution (D) Poisson distribution

15. The symbol 'c' in the queueing model $(a/b/c):(d/e)$ stands for
(A) Arrival pattern (B) Service pattern
(C) System capacity (D) Number of servers

16. In a queuing system the arrival and inter service rate respectively are denoted as
(A) λ, μ (B) $\frac{1}{\mu}, \frac{1}{\lambda}$
(C) $\mu, \frac{1}{\lambda}$ (D) $\lambda, \frac{1}{\mu}$

17. The period d_i of a Markov chain is given by
(A) $GCD\left\{m, pii^{(m)}<0\right\}$ (B) $GCD\left\{m, pij^{(m)}<0\right\}$
(C) $GCD\left\{m, pii^{(m)}>0\right\}$ (D) $GCD\left\{m, pij^{(m)}>0\right\}$

18. A statistic matrix p is said to be a regular matrix if all the entries of p^m are.
(A) Negative (B) Regular
(C) Positive (D) Symmetric

19. The steady state probability vector π of a discrete Markov chain with tpm satisfies the matrix equation.
(A) $\pi p=0$ (B) $\pi p=p$
(C) $\pi p=1$ (D) $\pi(1+p)=0$

20. A markov chain is said to be absorbing if it has __________ one absorbing rate
(A) Atmost (B) Atleast.
(C) Exactly (D) Nearly

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If X has the probability distribution

x:	−1	0	1	2
p(x)	0.3	0.1	0.4	0.2

Find $E[X], E\left[X^2\right], VAR[X]$.

22. Given the random variable X with density function
$f(x)=\begin{cases}4x, & 0<x<2\\ 0, & elsewhere\end{cases}$. Find the pdf of $y=2X^3$.

23. Buses arrive at a specified stop at 15 mins interval starting at 6 am (i.e) arrive at 6 am, 6.15 am,6.30 am and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 6 and 6.30 am, find the probability that he waits less than 5 mins for a bus.

24. The mileage which car owner get with a certain kind of radial tyre is a random variable having an exponential distribution with mean 40,000km. find the probabilities that one of these tyres will last (i) atleast 20,000 kms (ii) atmost 30,000 kms.

25. A sample of 900 members is found to have a mean of 3.4 cms, can it be reasonably regarded as a simple sample from a large population with mean 3.2 cms and SD 2.3 cms?

26. A student's study habits are as follows. If he studies one night, he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next night as well. Write TPM of the Markov chain.

27. Explain the symbolic representation of a queuing model.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution

x	−2	−1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	3k

(i) Find k (ii) Evaluate $P[X>2]$ (iii) $P[-2<X<2]$ (iv) Find the cumulative distribution function of X.

(OR)

b.i A fair dice is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

b. A and B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are 3/5 and 5/7 respectively. Find the probability that B will require more shots than A.

30. a. Before an increase in excise duty on tea, 800 people out of a sample of 1000 were consumer's of tea. After the increase in duty, 800 people were consumers of tea in a sample of 1200 persons. Find whether there is a significant decrease in the consumption of tea after the increase in duty.

(OR)

b. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In a experiment among 1600 beans, the numbers in the four groups were 882, 313,287 and 118. Does the experiment support the theory?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 min (i) Find the average number of persons waiting in the system (ii) what is the probability that a person arriving at the booth will have to wait in the queue? (iii) Estimate the fraction of the day when the phone will be in use (iv) what is the average length of the queue that forms from time to time.

(OR)

b. The local on-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customer arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time) (i) what percentage of time is the barber idle? (ii) what fraction of the potential customers are turned away? (iii) what is the expected number of customers waiting for a haircut (iv) how much time can a customer expected to spend in the barber shop?

32. a. The transition probability matrix of a Markov chain $\{X_n\}$, n = 1, 2, ,3 .....having 3 states 1, 2, and 3 is

$$P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$.

Find (i) $P\{X_2 = 3\}$ *and (ii)* $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. show that the process is Markovian. Find the transition matrix and classify the states.

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B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
3rd to 7th Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If c is a constant (non random variable) then E(c) is
(A) 0 (B) 1
(C) cf(c) (D) c

2. $Var(4X+8)$ is
(A) $12Var(X)$ (B) $4Var(X)+8$
(C) $16Var(X)$ (D) $16Var(X)+8$

3. The expectation of the number on a die when thrown
(A) 1 (B) 7/2
(C) 3 (D) 2

4. the $E[X^2]=8$ and E(X)=2, then Var(X) is
(A) 3 (B) 2
(C) 1 (D) 4

5. The MGF of binomial distribution is
(A) $(p+q^{et})^n$ (B) $(pe^t+q)^n$
(C) $(pe^t+qe^{-t})^{-n}$ (D) $(pe^{-t}+q)^n$

6. Mean of the Poisson distribution is
(A) λ (B) $\lambda+1$
(C) λ^2 (D) $\lambda-1$

7. If the probability of success on each trial is 1/3. What is the expected no of trials required for the first success
(A) 2 (B) 3
(C) 4 (D) 5

8. If X is uniform distributed in (0, 10) then P(X>8) is
(A) 1/5 (B) 1/10
(C) 3/5 (D) 1/3

9. The form of the alternative hypothesis can be
(A) One-tailed (B) Two-tailed
(C) Neither one nor two-tailed (D) One or two tailed

10. What is the standard deviation of a sampling distribution called?
(A) Sampling error (B) Sample error
(C) Standard error (D) Simple error

11. A failing student is passed by an examiner, it is an example of
(A) Type I error (B) Type II error
(C) Unbiased decision (D) Difficult to tel

12. The degree of freedom for t-test based on n observations is
(A) 2n−1 (B) n−2
(C) 2(n−1) (D) n−1

13. What stands for 'd' in the queue model $(a/b/c:d/e)$
(A) Queue discipline (B) System capacity
(C) Service time (D) Number of servers

14. The probability of no customer in the system in $(M/M/I):(\infty/FIFO)$ model is
(A) λ/μ (B) $\frac{\lambda}{\mu}-1$
(C) $1-\frac{\lambda}{\mu}$ (D) $\frac{\lambda}{\mu}+1$

15. The probability that the number of customers in the system exceeds K, in $(M/M/I):(\infty/FIFO)$ model
(A) $\left(\frac{\lambda}{\mu}\right)^{K+1}$ (B) $\left(\frac{1}{\mu}\right)^{K-2}$
(C) $\left(\frac{\lambda}{\mu}\right)^{K+2}$ (D) $\left(\frac{\lambda}{\mu}\right)^{K}$

16. The average waiting time of a customer in the system in $(M/M/1:\infty/FIFO)$ model ______.
(A) $\frac{1}{\mu-\lambda}$ (B) $\frac{1}{\lambda-\mu}$
(C) $\frac{1}{\lambda+\mu}$ (D) $\frac{1}{\lambda^2}$

17. If P is a tpm of the regular chain, then
(A) $p\pi=\pi+1$ (B) $\pi p=\pi$
(C) $\pi p^2=\pi$ (D) $\pi p=\pi-1$

18. Ergodic means
(A) Irreducible and periodic (B) Irreducible and aperiodic
(C) Not irreducible (D) Regular

19. In a transition probability matrix, the sum of all elements of any row is
(A) 0 (B) 1
(C) 2 (D) −1

20. If the one step transition probability does not depend on the step, then the Markov chain is
(A) Reducible (B) Regular
(C) Homogeneous (D) Non homogeneous

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A RV X has mean $\mu = 12$ and variance $\sigma^2 = 9$ and an unknown probability distribution. Find $P(6<X<18)$.

22. If a boy is throwing stones at a target, what is the probability that his 10th throw is his 5th hit, if the probability of hitting the target at any trial is 1/2?

23. A salesman in a departmental store claims that atmost 60% of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of salesman? Use 5% LOS.

24. In the usual notation of a $(M/M/I):(\infty/FIFO)$ queue system is $\lambda = 12$ per hour and $\mu = 24$ per hour, find the average number of customers in the system and in the queue.

25. A gambler has ₹2. He bets ₹1 at a time and wins ₹1 with probability 1/2. He stops playing if he loses ₹2 or wins ₹4. What is the tpm of the related Markov chain?

26. Given the random variable X with density function
$f(x)=\begin{cases}2x, & 0<x<1\\ 0, & elsewhere\end{cases}$ find the pdf of $Y = 8X^3$.

27. X is normally distributed and the mean of X is 12 and standard deviation is 4. Find (i) $P(X\geq 20)$ (ii) $P(X\leq 20)$

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution.

x	−2	−1	0	1	2	3
p(x)	0.1	K	0.2	2K	0.3	3K

(i) Find K (ii) Evaluate P(X < 2) and P(−2 <X < 2) (iii) Find cdf of X (iv) evaluate the mean of X.

(OR)

b. The cdf of a continuous random variable X is given by
$$F(x) = 0,\ x<0$$
$$= x^2,\ 0\leq x<1/2$$
$$= 1-\tfrac{3}{25}(3-x)^2,\ \tfrac{1}{2}\leq x<3$$
$$= 1, x\geq 3$$
Find the pdf of X and evaluate $P(|X|\leq 1)$ and $P\left(\frac{1}{3}\leq X<4\right)$ using both the pdf and cdf.

29. a. Fit a Poisson distribution for the following distribution.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

(OR)

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B.Tech. DEGREE EXAMINATION, NOVEMBER 2016
Third Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 onwards)
(Statistical tables to be supplied)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A RV X has the probability density function $f(x)=\begin{cases} ke^{-x/2}, & x\ge 0 \\ 0, & x<0 \end{cases}$. The value of k is
   (A) 2 (B) $\frac{1}{2}$
   (C) 1 (D) $\frac{-1}{2}$

2. Let X be a continuous RV then
   (A) $F(-\infty)=0$ (B) $F(\infty)=0$
   (C) $F(\infty)=-1$ (D) $F(\infty)=2$

3. Let X be a discrete RV with possible values 1,2,3 associated with probabilities $\frac{1}{4},\frac{1}{2},\frac{1}{4}$ respectively. Then E(X) is
   (A) 1 (B) 2
   (C) 3 (D) 4

4. Given $E(X)=1$, $E(X^2)=2$ for a discrete RV X, then Var(3X+1) is
   (A) 9 (B) 18
   (C) 27 (D) 3

5. If X is uniformly distributed in (–3, 3) then its probability density function f(x) is given by
   (A) $\frac{1}{6}$ (B) $\frac{3}{4}$
   (C) $\frac{1}{3}$ (D) $\frac{1}{9}$

6. The normal probability curve is symmetrical about
   (A) $Z=1$ (B) $Z=6$
   (C) $X=\mu$ (D) $X=\sigma$

7. If the RV X follows a Poisson distribution with mean $\frac{1}{2}$ then P(X=0) is given by

(A) e^{-1}

(B) $e^{-\frac{1}{2}}$

(C) $e^{\frac{1}{2}}$

(D) e^{2}

8. If the MGF of a distribution is $e^{3(e^t-1)}$ then the variance is

(A) $\frac{1}{3}$

(B) $\frac{1}{9}$

(C) 9

(D) 3

9. The Chi-square test is used to test

(A) The difference between population means

(B) The difference between population variances

(C) The goodness of fit

(D) The difference between proportions

10. If a researcher rejects a null hypothesis which is true then it is a

(A) Type 1 error

(B) Type 2 error

(C) Types A error

(D) Standard error

11. The 't' distribution tends to _________ distribution for sufficiently large value of γ

(A) Uniform

(B) Exponential

(C) Standard normal

(D) Poisson

12. _______ is used to test the difference between the population variances

(A) 't'-test

(B) Chi-Square test

(C) Z-test

(D) F-test

13. The symbol c in the queueing model (a|b|c): (d|e) stands for

(A) System capacity

(B) Number of servers

(C) Queue size

(D) Queue discipline

14. In a queueing system the arrival and inter service rate respectively are denoted as

(A) λ, μ

(B) $\frac{1}{\mu}, \frac{1}{\lambda}$

(C) $\mu, \frac{1}{\lambda}$

(D) $\lambda, \frac{1}{\mu}$

15. If the behavior of the queueing system does not depend on time, then the system is said to be in

(A) Transient state

(B) Idle state

(C) Steady state

(D) Busy state

16. If $\lambda = 8 / hr \text{ and } E(N) = 4$ customers for (M|M|1):(∞|FIFO) queue system then μ is given by

(A) 9

(B) 10

(C) 11

(D) 12

17. A Markov chain is said to be a-periodic if

(A) $d_i = 1$

(B) $d_i = 2$

(C) $d_i = 0$

(D) 3

18. If the tpm of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$ *and* $p^{(0)} = \left(\frac{5}{6}, \frac{1}{6}\right)$ then $p^{(1)}$ is

(A) $\left[\frac{1}{12}, \frac{11}{12}\right]$ (B) $\left[\frac{11}{24}, \frac{13}{24}\right]$

(C) $\left[\frac{11}{12}, \frac{1}{12}\right]$ (D) $\left[\frac{13}{24}, \frac{11}{24}\right]$

19. The reaction to state i is uncertain if $F_{ij} = \sum_{i=1}^{n} f_{ii}^{(n)}$ is

(A) Greater than 1 (B) 0

(C) 1 (D) Less than 1

20. A Markov chain is said to be absorbing if it has __________ one absorbing rate

(A) Atmost (B) Atleast

(C) Exactly (D) Nearly

PART – B (5 × 4 = 20 Marks)

Answer **ANY FIVE** Questions

21. The probability distribution of X is

x:	0	2	4	6
P(x):	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{3}{8}$

Find the mean and variance of X.

22. Buses arrive at a specified stop at 15 min intervals starting at 7 A.M, that is they arrive at 7, 7:15, 7:30, 7:45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7:30 A.M. Find the probability that he waits atleast 12 min for a bus.

23. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samples be from populations with the same variance?

24. A student's study habit's are as follows: If he studies one night he is 70% sure not to study the next night. On the other hand, if he does not study one night, he is 60% sure not to study the next as well. Write the tpm of the Markov Chain.

25. Explain the symbolic representation of a Queuing model.

26. Given the RV X with density function $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$ find the pdf of $Y = 8X^3$.

27. State and prove memoryless property of exponential distribution.

PART – C (5 x 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. A continuous RV X has pdf f(x)=k (1–x) for $0 < x < 1$. Find the rth moment about the origin. Hence find mean, variance.

ii. If X denote the number in a throw of a fair die find E(X), E(9x+2), Var (X).

(OR)

b. A fair die is tossed 600 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 80 to 120 sixes.

29. a. Find the MGF of Binomial distribution and hence find the mean and variance.

(OR)

b. In a normal distribution, 7% of the times are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

30. a. A group of 5 patients treated with medicine A weigh 42,39,48,60 and 41 kg, a second group of 7 patients from the same hospital treated with medicine B weigh 38,42,56,64,68,69 and 62. Do you agree with the claim that medicine B increases the weight significantly?

(OR)

b. In a locality 100 persons were randomly selected and asked about their educational achievements. The results are given as

		Education			
		Middle	High school	College	Total
Sex	Male	10	15	25	50
	Female	25	10	15	50
	Total	35	25	40	100

Can you say that education depends on sex?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 10 min between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 3 min
   (i) Find the average number of persons waiting in the system
   (ii) What is the probability that a person arriving at the booth will have to wait in the queue?
   (iii) What is the probability that it will take him more than 10 min altogether to wait for phone and complete his call?
   (iv) The telephone department will install a second booth when convinced that an arrival has to wait on the average for atleast 3 min for phone. By how much the flow of arrivals should increase in order to justify a second booth?

(OR)

b. In a single server queuing system with Poisson input and exponential service times. If the mean arrival rate is 3 calling units per hr, the mean service rate is 4 per hr and the maximum number of calling units in the system is 2, find
   (i) $P_n(n \geq 0)$
   (ii) Average number of calling units in the system
   (iii) Average waiting time in the system
   (iv) Average waiting time in the queue.

32. a. The tpm of a Markov chain $\{X_n, n \geq 0\}$ having three states 0, 1 and 2 is $P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$.

The initial distribution is given by $P^{(0)} = (0.5\ 0.3\ 0.2)$ find

(i) $P(X_2 = 2)$

(ii) $P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 2)$.

(OR)

b. A salesman's territory consists of 3 cities A, B and C. He never sells in the same city on successive days. If he sells in city A, then the next day he sells in B. However, if he sells either in B of C, then the next day he is twice as likely to sell in city A as in the other city. How often does he sell in each of the cities in the steady state?

* * * * *

ii. In a normal distribution, 7% of the items are under 35 and 89% are under 63. What are the mean and standard deviation of the distribution?

30. a.i. In a large city A, 20% of a random sample of 900 school boys had a slight physical defect. In another large city B, 18.5% of a random sample of 1600 school boys had the same defect. Is the difference between the proportions significant?

ii. The mean height and the SD height of 8 randomly chosen soldiers are 166.9 and 8.29 cms respectively. The corresponding values of 6 randomly chosen sailors are 170.3 and 8.50 cm respectively. Based on these data, can we conclude that soldiers are, in general shorter than sailors?

(OR)

b. A total number of 3759 individuals were interviewed in a public opinion survey on a political proposal. Of them, 1872 were men and the rest women. A total of 2257 individuals were in favour of the proposal and 917 were opposed to it. A total of 243 men were undecided and 424 women were opposed to the proposal. Do you justify or contradict the hypothesis that there is no association between sex and attitude?

31. a. Customers arrive at a one-man barber shop according to a Poisson process with a mean inter arrival time of 12 min. Customers spend an average of 10 min in the barber's chair.
(1) What is the expected number of customers in the barber shop and in the queue?
(2) Calculate the percentage of time an arrival can walk straight into the barber's chair without having to wait.
(3) How much time can a customer expect to spend in the barber's shop?
(4) What is the average time customers spend in the queue?
(5) What is the probability that the waiting time in the system is greater than 30 min?
(6) Calculate the percentage of customers who have to wait prior to getting into the barber's chair.

(OR)

b. The local one-person barber shop can accommodate and maximum of 5 people at a time (4 waiting and 1 getting hair-cut). Customers arrive according to a Poisson distribution with mean 15 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time)
(1) What percentage of time is the barber idle?
(2) What fraction of the potential customers are turned away?
(3) What is the expected number of customers waiting for a hair-cut?
(4) How much time can a customer expect to spend in the barber shop?

32. a. A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find (i) the probability that he takes a train on the third day (ii) the probability that he drives to work in the long run.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, MAY 2017
Third / Fourth Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 onwards)
(Statistical table to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A random variable X has the following probability function

x:	0	1	2	3	4
P(x):	k	2k	5k	7k	9k

The value of K is.
(A) 2/24 (B) 21/24
(C) 7/12 (D) 1/24

2. $\text{var}[4X+8]$ is
(A) $12\,\text{var}[X]$ (B) $4\,\text{var}[X]+8$
(C) $16\,\text{var}[X]$ (D) $16\,\text{var}[X]+8$

3. When a die is thrown, X denotes the number that turns up, find the mean
(A) 7/2 (B) 3/4
(C) 4/5 (D) –4/5

4. Given that the PDF of a random variable X is $f(x)=2x;\ 0<x<1$, find $P(X>0.5)$
(A) 1/2 (B) 2/3
(C) 3/4 (D) 4/5

5. If on an average, 9 ships out of 10 arrive safely to a port, then mean and SD of the number of ships returning safely out of 150 ships are
(A) 135, 2.674 (B) 125, 3.674
(C) 135, 3.674 (D) 125, 2.674

6. Mean of the Poisson distribution is
(A) λ (B) $\lambda+1$
(C) $1/\lambda$ (D) λ^2

7. The MGF of Geometric distribution is
(A) $\dfrac{1}{1-qe^t}$ (B) $\dfrac{1}{1-pe^t}$
(C) $\dfrac{q}{1-pe^t}$ (D) $\dfrac{pe^t}{1-qe^t}$

8. The variance of Uniform distribution $U(a,b)$ is
(A) $\frac{1}{12}(b-a)^2$ (B) $\frac{1}{8}(b-a)^2$
(C) $\frac{1}{2}(b+a)$ (D) $\frac{1}{2}(b-a)$

9. The value set for α is known as
(A) Rejection level (B) Acceptance level
(C) Significance level (D) Hypothetical level

10. The standard deviation of a sampling distribution is called as
(A) Sampling error (B) Standard error
(C) Simple error (D) Sample error

11. A __________ is a subset of a ____________
(A) Sample, population (B) Population, sample
(C) Statistic, parameter (D) Parameter, statistic

12. The hypothesis that an analyst is trying to prove is called
(A) Elective hypothesis (B) Alternate hypothesis
(C) Null hypothesis (D) Optional hypothesis

13. The symbolic notation of queuing model is represented by
(A) Kendall (B) Euler
(C) Fisher (D) Neuman

14. The interval between two consecutive arrivals of a Poisson process follows ________ distribution.
(A) Binomial (B) Uniform
(C) Normal (D) Exponential

15. The probability of 'n' customers in the system P_n =__________
(A) $\left(\frac{\lambda}{\mu}\right)P_0$ (B) $\left(\frac{\lambda}{\mu}\right)^n P_0$
(C) $\left(\frac{\mu}{\lambda}\right)^n P_0$ (D) $\left(\frac{\mu}{\lambda}\right)P_0$

16. Which term refers to "a customer who leaves the queue because the queue is too long"
(A) Balking (B) Reneging
(C) Jockeying (D) Leaving

17. The sum of all the elements of any row of the transition probability matrix is
(A) 0 (B) 0.5
(C) 0.75 (D) 1

18. The steady state probability vector π of a discrete markov chain with transition probability matrix satisfies the matrix equation.
(A) $\pi P = 0$ (B) $\pi P = \pi$
(C) $\pi(1-P) = 0$ (D) $\pi P = 1$

19. A non-null persistent and aperiodic state is called
(A) Empty (B) Finite
(C) Ergodic (D) Full

20. A state 'i' is said to be aperiodic with period 'di' if
(A) di < 1 (B) di = 1
(C) di > 1 (D) di = 0

PART – B (5 x 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A continuous random variable x has a pdf $f(x) = kx^2e^{-x}; x \geq 0$. Find the value of k.

22. Given the random variable x with density function $f(x) = \begin{cases} 2x & 0 < x < 1 \\ 0 & elsewhere \end{cases}$. Find the pdf of Y = $8X^3$.

23. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the fourth trial (ii) in fewer than 4 trials.

24. The mileage which can owner get with a certain kind of radial tire is a random variable having an exponential distribution with mean 40, 000 km. Find the probabilities that one of these tires will last (i) atleast 20,000kms (ii) atmost 30, 000kms.

25. Define (i) null hypothesis (ii) alternate hypothesis (iii) type I error (iv) type II error.

26. What do the letters in the symbolic representation $(a/b/c):(d/e)$ of a queuing model represent?

27. If the tpm of a Markov chain is $\begin{bmatrix} 0 & 1 \\ 1/2 & 1/2 \end{bmatrix}$. Find the steady state distribution of the chain.

PART – C (5 x 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution

x:	–2	–1	0	1	2	3
P(x):	0.1	k	0.2	2k	0.3	3k

(i) Find k (ii) evaluate P(X < 2) (iii) P(–2 < X < 2) (iv) CDF of X (v) mean of X (vi) variance of X.

(OR)

b.i If X is a random variable with E(X)=3 and $E(X^2)$=13, find the lower bound for P(–2 < X< 8), using Tchebycheff's inequality.

ii. If X represents the outcome, when a fair die is tossed, find the MGF of X and hence find E(X) and var (X).

29. a. Out of 800 families with 4 children each, how many families would be expected to have (i) 2 boys and 2 girls (ii) atleast 1 boy (iii) atmost 2 girls (iv) children of both sexes. Assume equal probabilities for boys and girls.

(OR)

b.i. Buses arrive at a specified stop at 15 mins interval starting at 6 am (ie) arrive at 6am, 6.15 am, 6.30 am and so on. If a passenger arrives at the stop at a time that is uniformly distributed between 6 and 6.30 am, find the probability that he waits (1) less than 5 mins for a bus (2) more than 10 mins for a bus.

Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
1^{st} to 6^{th} Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 to 2017-2018)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. $\lim_{x\to\infty} F(x) =$
   (A) 0 (B) 2
   (C) 1 (D) –1

2. If C is a constant non random variable, then E(c) is
   (A) 0 (B) 1
   (C) Cf(c) (D) C

3. The expectation of the number on a die when thrown
   (A) 1 (B) 7/2
   (C) 3 (D) 2

4. If the exponential distribution is given as $f(x) = e^{-x}; 0 \le x \le \infty$, then the mean of the distribution is
   (A) 1 (B) 0
   (C) 2 (D) –1

5. The MGF of binomial distribution is
   (A) $(p+q^{et})^n$ (B) $(pe^t+q)^n$
   (C) $(pe^t+q)^{-n}$ (D) $(pe^{-t}+q)^n$

6. If the probability of a target to the destroyed on any one shot is 0.5. What is the probability that it would be destroyed on 6^{th} attempt?
   (A) $(0.5)^4$ (B) $(0.5)^5$
   (C) $(0.5)^6$ (D) $(0.5)^7$

7. For a standard normal variable the mean and variance are respectively
   (A) 1 and 0 (B) μ and σ^2
   (C) 0 and 1 (D) μ and σ

8. If X is uniformly distributed in (0, 10) then P(X>8) is
   (A) 1/5 (B) 1/10
   (C) 3/5 (D) 1/3

9. A type II error occurs when
   (A) The null hypothesis is incorrectly accepted when it is false
   (B) The null hypothesis is incorrectly rejected when it is true
   (C) The sample mean differ from the population mean
   (D) The test is biased

10. The value set for α is known as
   (A) The rejection level
   (B) The acceptance level
   (C) The significance level
   (D) The error in the hypothesis test

11. A _____ is a subset of a _______.
   (A) Sample, population
   (B) Population, sample
   (C) Statistics, parameter
   (D) Parameter, statistic

12. The degree of freedom for t-test based on n-observation is
   (A) 2n–1
   (B) n–2
   (C) 2(n–1)
   (D) n–1

13. In which basis the service is provided in queuing theory
   (A) LCFO
   (B) LIFO
   (C) FCFS
   (D) FCLS

14. The interval between two consecutive arrivals of Poisson process follows _____ distribution.
   (A) Binomial
   (B) Uniform
   (C) Normal
   (D) Exponential

15. In $(M/M/1):(K/FIFO)$ model, if $\lambda = 3/hour$, $\mu = 4/hour$ and effective mean arrival rate of a customer is 2.88/hours then what is P_0?
   (A) 0.18
   (B) 0.28
   (C) 0.38
   (D) 0.48

16. The average waiting time of a customer in the system in $(M/M/1:\infty/FIFO)$ model ______.
   (A) $\dfrac{1}{\mu-\lambda}$
   (B) $\dfrac{1}{\lambda-\mu}$
   (C) $\dfrac{1}{\lambda+\mu}$
   (D) $\dfrac{\lambda}{\lambda+\mu}$

17. A Markov chain is said to be a periodic if
   (A) $d_i = 10$
   (B) $d_i = 1$
   (C) $d_i = 2$
   (D) $d_i = 0$

18. Chapman-Kolmogrov theorem states that
   (A) $\left[p_{ij}^{(n)}\right] = \left[p_{ij}\right]^n$
   (B) $\left[p(n)\right] = \left[p_{ij}^{(n)}\right]$
   (C) $\left[n\,p_{ij}\right]^{n+1} = n\left[p_{ij}\right]^n$
   (D) $\left[p_{ij}\right]^{n+1} = n\left[p_{ij}\right]^n$

19. The steady state probability vector π of a discrete Markov chain with transition probability matrix satisfies the matrix equation

(A) $\pi P = 1$ (B) $\pi P = \pi$

(C) $\pi = \pi - P$ (D) $\pi(1-P) = \pi$

20. The tpm of a Markov chain is stochastic matrix, because

(A) $p_{ij} \geq 0$ and $\sum\limits_{j} p_{ij} = 1$ (B) $p_{ij} \leq 0$ and $\sum\limits_{j} p_{ij} = 1$

(C) $p_{ij} \geq 0$ and $\sum\limits_{j} p_{ij} = 2$ (D) $p_{ij} \geq 0$ and $\sum\limits_{i} p_{ij} = 1$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The probability function of an infinity discrete distribution is given by $P(X=i) = \frac{1}{2i} (i = 1, 2, \ldots. \infty)$. (i) Find mean of X (ii) Find P(X is even).

22. Buses arrive at a specified stop at 15 min intervals starting at 7am that is, they arrive at 7, 7.15, 7.30, 7.45 and so on. If a passenger arrives at the stop at a random time that is uniformly distributed between 7 and 7.30am. Probability that he waits (i) less than 5 min for a bus, and (ii) atleast 12 min for a bus.

23. A mechanist is expected to make engine parts with axle diameter of 1.75cm. A random sample of 10 parts shows a mean diameter 1.85 cm with a S.D of 0.1 cm on the sample. Would you say that the work of the machinist is inferior?

24. If λ = 3 per hour, μ = 4 per hour and maximum capacity k = 7 in a $(M/M/1):(K/FIFO)$ system, find the average number of customers in the system.

25. If the tpm of a Markov chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$ find the steady state distribution of the chain.

26. A salesman in a departmental store claims that atmost 60% of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the claim of the salesman? Use the level of significance of 0.05.

27. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test in fewer than 4 trials?

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. In a continuous distribution, the probability density is given by $f(x) = Kx(2-x), 0 < x < 2$. Find K, mean, variance and the distribution function. (8 Marks)

ii. If X is uniformly distributed in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$, find the pdf of Y = tanX. (4 Marks)

(OR)

b.i. If the MGF of a RV X is $\frac{2}{2-t}$ find the SD of X. (4 Marks)

ii. If X denotes the sum of the numbers obtained when 2 dice are thrown, obtain an upper bound for $P\{|X-7| \geq 4\}$. Compare with the exact probability. (8 Marks)

29. a. Fit a Poisson distribution for the following data.

x	0	1	2	3	4	5
f	142	156	69	27	5	1

(OR)

b. In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he score less than 45%, between 45% and 60%, between 60% and 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentage of students who have got first class and second class (assume normal distribution of marks).

30. a. The following table gives a sample of a married women, the level of education and the marriage adjustment score:

		Marriage adjustment			
		Very low	Low	High	Very high
Level of education	College	24	97	62	58
	High school	22	28	30	41
	Middle school	32	10	11	20

Can you conclude from the above data that the higher level of education, the greater is the degree of adjustment in marriage?

(OR)

b.i. A simple sample of heights of 6400 Englishmen has a mean of 170cm and a SD of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D of 6.3 cm. Do the data indicate that on the average, taller than Englishmen? (8 Marks)

ii. A sample of size 13 gave an estimated population variance of 3.0, while another sample of size 15 gave an estimate of 2.5. Could both samplc bc from population with the same variance? (4 Marks)

31. a. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic.

(ii) What is the probability that an arriving patient will not wait?

(iii) What is the expected waiting time until a patient is discharged from the clinic?

(OR)

b. If people arrive to purchase cinema tickets at the average rate of 6 per minute it takes an average of 7.5 seconds to purchase a ticket. If a person arrives 2 min before the picture starts and if it takes exactly 1.5 min to reach the correct seat after purchasing the ticket (i) can he expect to be seated for the start of the picture (ii) what is the probability that he will be seated for the start of the picture (iii) how early must he arrive in order to be 99% sure of being seated for the start of the picture?

32. a. Three boys A, B and C are throwing a ball to each other. A always throw the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. show that the process in Markovian. Find the transition matrix and classify the states.

(OR)

b. A fair dice is tossed repeatedly, if X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Find also P^2 and $P(X_2=6)$.

* * * * *

ii. A continuous random variable X has a pdf $f(x)=kx^2e^{-x}$, find k, mean and variance.

29. a. Fit a binomial distribution for the following data:

x	0	1	2	3	4	5	6	Total
f	5	18	28	12	7	6	4	80

(OR)

b. In an engineering examination, a student is considered to have failed, secured second class, first class and distinction according as he scores less than 45%, between 45% and 60%, between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class.

30.a. 15.5 percent of a random sample of 1600 undergraduates were smokers, whereas 20% of a random sample of 900 post-graduates were smokers in a state. Can we conclude that less number of undergraduates are smokers than the postgraduates?

(OR)

b. Theory predicts that the proportion of beans in four groups A, B, C and D should be 9:3:3:1. In an experiment among 1600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.
   (i) Find the average number of persons waiting in the system.
   (ii) What is the probability that a person arriving at the booth will have to wait in the queue?
   (iii) Estimate the fraction of the day when the phone will be in use.
   (iv) What is the probability that it will take him more than 10 minutes altogether to wait for the phone and complete his call?

(OR)

b. The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1getting hair-cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour.
   (i) What percentage of time is the barber idle?
   (ii) What fraction of the potential customers are turned away?
   (iii) What is the expected number of customers waiting for a hair-cut?
   (iv) How much time can a customer expect to spend in the barber shop?

32. a. A gambler has ₹2 he bets ₹1 at a time and wins ₹1 with probability 1/2. He stops playing if he loses ₹2 or wins ₹4.
   (i) What is the tpm of the related Markov chain?
   (ii) What is the probability that he has lost his money at the end of 5 plays?
   (iii) What is the probability that the game lasts more than 7 plays?

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process in Markovian. Find the transition matrix and classify the states.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2018
First to Sixth Semester

15MA207 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2015 – 2016 onwards)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If C is a constant then E(c) is
   (A) 0 (B) 1
   (C) cf(c) (D) c

2. $Var(4X+8)$ is
   (A) $12\,Var(X)$ (B) $4\,Var(X)+8$
   (C) $16Var(X)$ (D) $16Var(X)+8$

3. A random variable X has mean $\mu = 12$ and variance $\sigma^2 = 9$ and an unknown probability distribution, then $P(6<X<18)$ is
   (A) $\frac{1}{2}$ (B) $\frac{3}{4}$
   (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

4. If $E(X^2)=8$ *and* $E(X)=2$, then Var(X) is
   (A) 3 (B) 2
   (C) 1 (D) 4

5. The mean and variance of a binomial distribution is
   (A) $\mu=np,\ \sigma^2=npq$ (B) $\mu=npq,\ \sigma^2=np$
   (C) $\mu=nq,\ \sigma^2=npq$ (D) $\mu=np,\ \sigma^2=pq$

6. Mean of the Poisson distribution is
   (A) λ (B) $\lambda+1$
   (C) λ^2 (D) $\lambda-1$

7. The MGF of geometrical distribution is
   (A) $\frac{1}{1-qe^t}$ (B) $\frac{1}{1-pe^t}$
   (C) $\frac{q}{1-pe^t}$ (D) $\frac{pe^t}{1-qe^t}$

8. If X is uniformly distributed in (0, 10), then P(X > 8) is
(A) 1/5
(B) 1/10
(C) 3/5
(D) 1/3

9. The form of the alternative hypothesis can be
(A) One-tailed
(B) Two-tailed
(C) Neither one nor two tailed
(D) One or two tailed

10. What is the standard deviation of a sampling distribution called?
(A) Sampling error
(B) Sample error
(C) Standard error
(D) Simple error

11. When the researcher rejects a true null hypothesis, a _______ error occurs.
(A) Type I
(B) Type A
(C) Type B
(D) Type II

12. Student's t-distribution has (n−1) degrees of freedom. When all the n observations in the sample are
(A) Dependent
(B) Independent
(C) Maximum
(D) Minimum

13. What stand for 'd' in the queue model $(a/b/c:d/e)$
(A) Queue discipline
(B) System capacity
(C) Service time
(D) Number of servers

14. The average number of customers in the system in $(M/M/1:\infty/FIFO)$ model is
(A) $\frac{\lambda}{\mu-\lambda}$
(B) $\frac{\mu}{\lambda-\mu}$
(C) $\frac{\lambda}{\mu+\lambda}$
(D) $\frac{\mu}{\lambda+\mu}$

15. Which term refers to 'a customer who leaves the queue because the queue is too long'?
(A) Balking
(B) Reneging
(C) Jockeying
(D) Leaving

16. In $(M/M/1:K/FIFO)$ model, if $\lambda = 3$ per hour, $\mu = 4$ per hour and effective mean arrival rate of a customer is 2.88 per hour then what P_n?
(A) 0.18
(B) 0.28
(C) 0.38
(D) 0.48

17. Markov proves is one in which the future value is independent of____values.
(A) Present
(B) Past
(C) Future
(D) Outcome

18. Ergodic means
(A) Irreducible and periodic
(B) Irreducible and aperiodic
(C) Not irreducible
(D) Regular

19. If one step transition probability does not depend on the step, then the Markov chain is called
(A) Reducible
(B) Regular
(C) Homogeneous
(D) Non homogeneous

20. The steady state probability vector π of A discrete Markov chain with TPM satisfies the matrix equation
(A) $\pi P = 0$
(B) $\pi P = P$
(C) $\pi P = \pi$
(D) $\pi(1+P) = 0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If X is uniformly distributed in $\left(\frac{-\pi}{2}, \frac{-\pi}{2}\right)$ find the pdf of $Y = \tan X$

22. If the probability that an applicant for a driver's license will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test (i) on the fourth trial and (ii) in fewer than 4 trials?

23. The fatality rate of typhoid patients is believed to be 17.26 percent. In a certain year 640 patients suffering from typhoid were treated in metropolitan hospital and only 63 patients died. Can you consider the hospital efficient?

24. In the usual notation of a $(M/M/1):(\infty/FIFO)$ queue system if $\lambda = 12$ per hour and $\mu = 24$ per hour, find the average number of customers in the system and in the queue.

25. The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3....$ having 3 states 1, 2, and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p(0) = (0.7, 0.2, 0.1)$. Find $P\{X_2 = 3\}$.

26. If X is a random variable whose density function is $f(x) = e^{-x}, 0 < x < \infty$, $= 0$ *otherwise*. Find rth moment about origin and hence find μ_1' *and* μ_2'.

27. State and prove memoryless property of the exponential distribution.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. If the random variable X takes the values 1, 2, 3 and 4 such that $2P(X=1) = 3P(X=2) = P(X=3) = 5P(X=4)$, find the probability distribution and cumulative distribution function of X.

(OR)

b.i A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

ii. If X is uniformly distributed over (0, 10) find (i) P (X<4) (ii) P(X>6) (iii) P(2 < X< 5).

(OR)

b.i. Derive Moment Generating Function and hence find mean and variance of a Poisson Distribution.

ii. Assume the mean height of soldiers to be 68.22 inches with variance 10.8 (inch). How many soldiers in a regiment of 1000, would you expected to be over 6 feet tall.

30. a. Fit a Poisson distribution for the following data and also test the goodness of fit.

X	0	1	2	3	4	5	Total
F(x)	142	156	69	27	5	1	400

(OR)

b. Two random samples gave the following data.

Sample	Size	Mean	Standard Deviation
1	100	61	4
2	200	63	6

Can we conclude that the two samples have been drawn from the same normal population?

31. a. Patients arrive at a clinic according to Poisson Distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination per patient is exponential with mean rate of 20 per hour
   (i) Find the effective arrival rate at the clinic.
   (ii) What is the probability that an arriving patient will not wait?
   (iii) What is the expected waiting time until a patient is discharged from the clinic?

(OR)

b. A departmental store has a single cashier. During the rush hours, customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour (i) What is the probability that the cashier is idle (ii) What is the average number of customer in the queuing system? (iii) What is the average time a customer spends in the system? (iv) What is the average time a customer spends in the queue, waiting for service?

32. a. The transition probability matrix of a Markov chain $\{X_n\}: n=1,2,3...$ having 3 states 1, 2 and 3 is $P=\begin{pmatrix}0.1 & 0.5 & 0.4\\ 0.6 & 0.2 & 0.2\\ 0.3 & 0.4 & 0.3\end{pmatrix}$. Initial distribution is $P^{(0)}=(0.7 \quad 0.2 \quad 0.1)$. Find (i) $P\{X_2=3\}$ (ii) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$.

(OR)

b. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing a ball from one to the other. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand, each girly throws the ball to each boy with probability 1/2 and never to the other girl. In the long run, how often does each receive the ball?

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Reg. No.														

B.Tech. DEGREE EXAMINATION, DECEMBER 2017
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. $\underset{x\to\infty}{Lt}\ F(x)=e^{-x}$ is
   (A) 0 (B) 2
   (C) 1 (D) −1

2. $Var(4x+8)$ is
   (A) $12Var(X)$ (B) $4Var(X)+8$
   (C) $16Var(X)$ (D) $16Var(X)+8$

3. If the exponential distribution is given as $f(x)=e^{-x};\ 0\le x\le\infty$, then the mean of the distribution is
   (A) 1 (B) 0
   (C) 2 (D) −1

4. If $E[x^2]=8$ and $E[x]=2$, then Var(X) is
   (A) 3 (B) 2
   (C) 1 (D) 4

5. If the random variable X has the pdf
   $f(x)=\begin{cases}ax^3 & ; \quad 0<x<1\\ 0 & ; \quad otherwise\end{cases}$
   then the value of a is
   (A) 3 (B) 4
   (C) 1/2 (D) 3/4

6. The MGF of binomial distribution is
   (A) $(p+qe^t)^n$ (B) $(pe^t+q)^n$
   (C) $(p+qe^{-t})^n$ (D) $(p+qe^t)^n$

7. Poisson distribution is limiting case of
   (A) Geometric distribution (B) Normal distribution
   (C) Binomial distribution (D) Exponential distribution

8. If the probability of a target to be destroyed on any one shot is 0.5. What is the probability that it would by destroyed on 6th attempt?
   (A) $(0.5)^4$ (B) $(0.5)^5$
   (C) $(0.5)^6$ (D) $(0.5)^7$

9. The mean of the exponential distribution with pdf $\lambda e^{-\lambda x}; x > 0$ is
(A) λ
(B) $1/\lambda$
(C) $1/\lambda^2$
(D) 1

10. In a normal distribution about 99% of the observation lies between
(A) $\mu \pm 2\sigma$
(B) $\mu \pm \sigma$
(C) $\mu \pm 3\sigma$
(D) $\mu \pm \sigma^2$

11. The form of the alternative hypothesis can be
(A) One-tailed
(B) Two-tailed
(C) Neither one nor two-tailed
(D) One or two tailed

12. The Chi-square test is not very effective if the sample is
(A) Small
(B) Large
(C) Irregular
(D) Heterogeneous

13. What is the standard deviation of a sampling distribution called?
(A) Sampling error
(B) Sample error
(C) Standard error
(D) Simple error

14. The degrees of freedom for t-test based on n-observation is
(A) 2n–1
(B) N – 1
(C) 2(n–1)
(D) n–1

15. In queuing theory, generally the service rate is denoted by
(A) λ
(B) μ
(C) $\lambda\mu$
(D) λ/μ

16. The overall effective arrival rate is $\lambda' =$
(A) $\mu(1-P_0)$
(B) $\lambda(1-P_0)$
(C) μP_0
(D) λP_0

17. The traffic intensity of a queuing system is
(A) λ
(B) μ
(C) λ/μ
(D) μ/λ

18. A state i is said to be periodic with period d_i if
(A) $d_i < 1$
(B) $d_i = 1$
(C) $d_i > 1$
(D) $d_i = 0$

19. If the transition probability matrix of a Markov chain is $\begin{pmatrix} 0 & 1 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$. Find the steady state distribution
(A) $\left(\frac{1}{3}, \frac{2}{3}\right)$
(B) $\left(\frac{1}{2}, \frac{1}{3}\right)$
(C) $\left(\frac{1}{2}, \frac{1}{2}\right)$
(D) $\left(\frac{1}{2}, \frac{1}{4}\right)$

20. Markov analysis is useful for predicting the state of the system at some
(A) Future time
(B) Present time
(C) Past time
(D) Present and past time

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The first three moments about the origin are 5, 26, 78. Find the first three moments about the value 3.

22. Given the random variable X with density function
$$f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & elsewhere \end{cases}$$
Find the probability density of $Y = 8X^3$.

23. If X is a Poisson Random Variable such that $P(X=2) = 9P(x=4) + 90P(x=6)$, find λ.

24. In a particular manufacturing process it is found that, on the average 1% of the items is defective. What is the probability that the fifth item inspected is the first defective item?

25. Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles only 50 are of top quality. Show that either the production of the day chosen was not a representative sample on the hypothesis of 20% was wrong.

26. What is the average waiting time of a customer in a single server, infinite capacity, Poisson queue, if he happens to wait, given that $\lambda = 6$ per hour and $\mu = 4$ per hour.

27. What is the average waiting time of a customer in a single server, infinite capacity, Poisson queue, if he happens to wait, given that $\lambda = 6$ per hour and $\mu = 4$ per hour.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution.

X	–2	–1	0	1	2	3
P(X= x)	0.1	K	0.2	2K	0.3	3K

(i) Find K
(ii) Evaluate P(X < 2)
(iii) Evaluate P(–2 < X < 2)
(iv) Find F(X)
(v) Find Mean and Variance of X

(OR)

b.i. A Random variable X have pdf $f(x) = \frac{1}{2}e^{-x/2}, x > 0$. Find the moment generating function, mean and variance of X.

ii. In a continuous distribution, pdf is given by
$$f(x) = \begin{cases} k\, x(2-x), & 0 < x < 2 \\ 0 & otherwise \end{cases}$$
Find (i) K (ii) Mean (iii) Variance

29. a.i. Fit a binomial distribution for the following data.

x	0	1	2	3	4	5	6	Total
P(x)	5	18	28	12	7	6	4	80

ii. Derive the moment generating function of uniform distribution and hence find mean and variance of the distribution.

(OR)

b.i. If X has a geometric distribution, then for any two positive integers m and n, prove that $P[X > m+n / X > m] = P[X > n]$.

ii. Let X be is a normal variate with mean 30 and standard deviation is 5. Find the following $P[26 < X < 40]$, $P[X \geq 45]$ *and* $P[|X-30|] > 5$.

30. a. Two random samples gave the following data.

Sample	Size	Mean	Variance
1	8	9.6	1.2
2	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population?

(OR)

b.i. Test the significance of the difference between the means of the samples, drawn from two normal populations with the same standard deviation using following data.

Sample	Size	Mean	SD
1	100	61	4
2	200	63	6

ii. The following data give the number of air-craft accidents that occurred during the various days of a week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed.

31. a. A departmental store has a single cashier. During the rush hours, customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. (i) what is the probability that the cashier is idle? (ii) what is the average number of customer in the queuing system? (iii) what is the average time a customer spends in the system? (iv) what is the average number of customers in the queue? (v) what is the average time a customer spends in the queue, waiting for service?

(OR)

b. The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time). (i) what percentage of time is the barber idle? (ii) what fraction of the potential customers are turned away? (iii) what is the expected number of customers waiting for a hair cut? (iv) how much time can a customer expected to spend in the barber shop?

32. a. The transition probability matrix of a Markov chain $\{X_n\}$: n = 1, 2, 3... having 3 states 1, 2, and 3 is $\rightarrow$ $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Initial distribution is $P^{(0)}$ = (0.7 0.2 0.1). Find (i) $P\{X_2=3\}$ (ii) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

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Reg. No.														

B.Tech. DEGREE EXAMINATION, NOVEMBER 2015
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014)
(Statistical table is to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours — Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The probability density function of a continuous random variable is $f(x) = ce^{-|x|}; -\infty < x < \infty$ then the value of C is
(A) 1/2 (B) 1/4
(C) 3/4 (D) 1/6

2. If $f(x) = \begin{cases} x/6 & x = 1,2,3 \\ 0 & otherwise \end{cases}$, then E(X) is
(A) 0.33 (B) 1.33
(C) 2.33 (D) 3.33

3. *Var*(4X+8) is
(A) 12*Var*(X) (B) 4*Var*(X)+8
(C) 16*Var*(X) (D) 16*Var*(X)+8

4. If X is a random variable and r is an integer, then $E(X-\overline{X})^r$ represents
(A) r^th central moment (B) r^th factorial moment
(C) r^th raw moment (D) (r−1)^th central moment

5. Which distribution has equal mean and variance?
(A) Binomial (B) Poisson
(C) Exponential (D) Normal

6. If the probability that a target is destroyed on any one shot is 0.5, then the probability that it would be destroyed on 6th attempt is
(A) 0.1065 (B) 0.0165
(C) 0.1056 (D) 0.0156

7. The cumulative distribution function of f(x) of an exponential distribution is
(A) $e^{-\lambda x} - 1;\ x \geq 0$ (B) $e^{\lambda x} - 1;\ x \geq 0$
(C) $1 - e^{-\lambda x};\ x \geq 0$ (D) $1 - e^{\lambda x};\ x \geq 0$

8. The probability density function of a standard normal distribution is
(A) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}; -\infty < z < \infty$ (B) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}; 0 < z < \infty$
(C) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2/2}; 0 < z < \infty$ (D) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2/2}; -\infty < z < \infty$

9. The test used to test the equality of variance of the populations from two small samples is
(A) t-test (B) F-test
(C) χ^2 -test (D) Independent of attributes

10. In large samples, to test the significance of the difference between sample mean $\overline{X}$ and population mean μ, the test statistic (sample standard deviation s is known) is
(A) $z=\frac{\overline{X}-\mu}{s/\sqrt{n}}$ (B) $z=\frac{\mu-\overline{X}}{s/\sqrt{n}}$
(C) $z=\frac{\overline{X}-\mu}{s/\sqrt{n-1}}$ (D) $z=\frac{\mu-\overline{X}}{s/\sqrt{n-1}}$

11. 95% confidence limits for the population proportions is
(A) $p+2.58\sqrt{\frac{pq}{n}}$ (B) $p-2.58\sqrt{\frac{pq}{n}}$
(C) $1.96\sqrt{\frac{pq}{n}}$ (D) $p\pm1.96\sqrt{\frac{pq}{n}}$

12. The application of one-tailed or two-tailed test depends upon the nature of the
(A) Null hypothesis (B) Alternative hypothesis
(C) Level of Significance (LOS) (D) Degrees of freedom

13. In queuing system, the number of customers serviced per unit has a Poisson distribution with mean
(A) μ (B) 1/μ
(C) 2/μ (D) 3/μ

14. In the symbolic representation of a queue model $(a/b/c):(d/e)$, where c denotes.
(A) The type of distribution of the no of arrivals per unit time (B) The type of distribution of the no of services per unit time
(C) No of servers (D) The capacity of the system

15. Arrivals at a telephone booth are considered to the Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. What is the average number of persons waiting in the system?
(A) 0.5 (B) 1.0
(C) 1.5 (D) 2.0

16. Probability that the number of customers in the system exceeds k (i.e) P(N > k) is
(A) λ/μ (B) $(\lambda/\mu)^k p_0$
(C) $(\lambda/\mu)^{k+1} p_0$ (D) $(\lambda/\mu)^{k-1} p_0$

17. The conditional probability that the process is in state a_j at step n, given that it was in state a_i at step 0 is called
(A) One-step transition probability (B) Two-step transition probability
(C) (n−1) step transition probability (D) n-step transition probability

18. The *tpm* of a Markov chain is stochastic matrix, because
(A) $p_{ij}\geq 0\ \&\sum_j p_{ij}=1$ (B) $p_{ij}\leq 0\ \&\sum_j p_{ij}=1$
(C) $p_{ij}\geq 0\ \&\sum_i p_{ij}=1$ (D) $p_{ij}\leq 0\ \&\sum_i p_{ij}=1$

19. If $p_{ij}^{(n)}>0$ for some n and for all *i* and *j* then every state can be reached from every other state. When this condition is satisfied, the Markov Chain is said to
(A) Reducible (B) Irreducible
(C) Periodic (D) Aperiodic

20. State *i* is said to be aperiodic with period *di*, if
(A) $d_i > 1$ (B) $d_i < 1$
(C) $d_i = 1$ (D) $d_i = 0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A random variable X has a mean μ = 12 and variance σ^2 = 9 and an unknown probability distribution. Find P(6<X<18).

22. Given the random variable X with density function $f(x)=\begin{cases}2x & 0<x<1\\ 0 & elsewhere\end{cases}$.
Find the probability density of $Y = 8X^3$.

23. A travel company has two cars for hiring. The demand for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which some demand is refused.

24. The time (in hours) required to repair a machine is exponentially distributed with parameter λ = 1/2. What is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours?

25. Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20% was wrong.

26. In a railway marshalling yard goods trains arrive at rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time is also exponential with an average of 36 minutes. Calculate expected queue size (line length).

27. If the transition probability matrix of a Markov Chain is $\begin{pmatrix}0 & 1\\ 1/2 & 1/2\end{pmatrix}$, find the steady state distribution of the chain.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. A random variable X has the following probability distribution.

X	−2	−1	0	1	2	3
P(X=x)	0.1	K	0.2	2K	0.3	3K

(A) Find K (B) Evaluate P(X< 2) (C) Evaluate P(−2<X<2) (D) Find the cumulative distribution function of X (E) Evaluate the mean and variance of X.

(OR)

b.i. The probability function of an infinite discrete distribution is given by $P(X=j)=\frac{1}{2^j}(j=1,2,....\infty)$. Verify that the total probability is 1 and find the mean and variance of the distribution.

ii. If $f(x)=\begin{cases}xe^{-x^2/2}; & x\geq 0\\ 0 \quad ; & x<0\end{cases}$
(1) Show that f(x) is a probability density function of a continuous random variable X
(2) Find its distribution F(x).

29. a.i. Fit a binomial distribution for the following data.

x	0	1	2	3	4	5	6	Total
f(x)	5	18	28	12	7	6	4	80

b.i. Fit a Poisson distribution to the following data and calculate the theoretical frequencies.

x	0	1	2	3	4
f	122	60	15	2	1

ii. The time (in hours) required to repair a machine is exponentially distributed with parameter $\lambda = 1/2$. What is the probability that a repair takes atleast 10 hours given that the duration exceeds 9 hours.

30. a. The following data related to the marks obtained by 11 students in 2 tests one held at the beginning of the year and the other at the end of the year, after intensive coaching. Do the data indicate that the students have benefited by coaching.

Test 1	19	23	16	24	17	18	20	18	21	19	20
Test 2	17	24	20	24	20	22	20	20	18	22	19

(OR)

b.i. In a certain sample of 2000 families, 1400 people are consumers of tea. Out of 1800 Hindu families, 1236 Hindu families consume tea. Use Chi square test and state whether there is any significant difference in the consumption of tea in Hindu and non-hindu families.

ii. The sales manager for a large company conducted a sample survey in states A and B taking 400 samples in each case. The results were

Average Sales	₹2500	₹2200
Standard Deviation	₹400	₹550

Test whether the average sales is the same in the 2 states at 1% LOS.

31. a. In a city airport, flights arrive at a rate of 24 flights per day. It is known that the inter-arrival time follows an exponential distribution, and the service time distribution is also an exponential with an average of 30 minutes, find the following
(i) The probability that the system will be idle.
(ii) The mean queue size.
(iii) The average number of flights in queue.
(iv) The probability that the queue size exceeds 7.

(OR)

b. Patients arrive at a clinic according to Poisson distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Investigation time per patient is an exponential with mean rate of 20 per hour.
(i) Determine the effective arrival rate.
(ii) What is the probability that an arriving patient will not wait?
(iii) What is the expected waiting time until patients discharged from the clinic?

32. a. The transition probability matrix of a Markov chain $\{X_n\} n = 1, 2, 3 \ldots$ having 2 states 1, 2, and 3 is

$$P = \begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$$ and the initial distribution is $P^{(0)} = (0.7, 0.2, 0.1)$. Find (i) $P(X_2 = 3)$

(ii) $P\{X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2\}$ (iii) $P(X_3 = 3/X_2 = 1, X_1 = 2)$.

(OR)

b. A gambler has ₹2. He bets ₹1 at a time and wins ₹1 with probability ½. He stops playing if he loses ₹2 or wins ₹4 what is the transition probability matrix of the related Markov chain? What is the probability that he lost his money at the end of 5 plays. What is the probability game more than 7 plays?

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Reg. No.														

B.Tech. DEGREE EXAMINATION, JUNE 2017
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. If the random variable X has the pdf $f(x) = \begin{cases} ax^3; & a < x < 1 \\ 0 & ; otherwise \end{cases}$ then the value of a is
(A) 3 (B) 4
(C) ½ (D) 3/4

2. Variance (4X+8) is
(A) 14Var(X) (B) 4Var(X)+8
(C) 16Var(X) (D) 16Var(X)+8

3. If X is a random variable which can take only non-negative values, then
(A) $E(X^2) = [E(X)]^2$ (B) $E(X^2) \geq [E(X)]^2$
(C) $E(X^2) \leq [E(X)]^2$ (D) $E(X^2) = E(X)$

4. If $E[X^2] = 8$ and $E(X) = 2$, then Var(X) is
(A) 3 (B) 2
(C) 1 (D) 4

5. The MGF of normal distribution is
(A) $e^{\mu t + \frac{\sigma^2 t^2}{2}}$ (B) $e^{\frac{\mu t + \sigma^2 t^2}{2}}$
(C) $e^{\mu t + \frac{\sigma t}{2}}$ (D) $e^{\mu t + \frac{\sigma t^2}{2}}$

6. If X is Poisson variate such that P(X=0) = 0.5 then Var(X) is
(A) e^2 (B) log 2
(C) 0.5 (D) log 4

7. If X has uniform distribution (–1, 3), then P(X > 0) is
(A) 1/2 (B) 3/4
(C) 1/3 (D) 1/4

8. If X is exponentially distributed with mean 10 then the pdf is
(A) $10e^{-10x}, x \ge 0$
(B) $\frac{1}{10}e^{-10x}, x \ge 0$
(C) $\frac{1}{10}e^{\frac{x}{10}}, x \ge 0$
(D) $\frac{1}{10}e^{\frac{-x}{10}}, x \ge 0$

9. Which hypothesis is always in an equality form
(A) Null hypothesis
(B) Alternative hypothesis
(C) Simple hypothesis
(D) Composite hypothesis

10. _____result if you fail to reject the null hypothesis actually false.
(A) Type I error
(B) Type II error
(C) Type III error
(D) Types IV error

11. Which of the following values is not typically used for α?
(A) 0.01
(B) 0.05
(C) 0.10
(D) 0.25

12. A_______is a numerical characteristic of a sample and a ______is a numerical characteristics of a population.
(A) Sample, population
(B) Population, sample
(C) Statistic, parameter
(D) Parameter, statistic

13 If the behaviour of the system is independent of time, then the system is said to be
(A) Steady state
(B) Transient state
(C) Unsteady state
(D) Steady and transient state

14. What stands for 'e' in the queue model $(a/b/c:d/e)$
(A) Queue discipline
(B) System capacity
(C) Maximum queue size
(D) Service time

15. The overall effective arrival rate is $\lambda' =$
(A) $\mu(1-p_n)$
(B) $\lambda(1-p_n)$
(C) μp_0
(D) λp_0

16. Which term refers to A customer who leaves the queue because the queue is too long
(A) Balking
(B) Reneging
(C) Jockeying
(D) Leaving

17. Markov process one in which the future value is independent of_______values.
(A) Most recent previous
(B) Past
(C) Future
(D) Past and future

18. Chapman-Kolomogrov theorem states that
(A) $\left[Pij^{(n)}\right]=\left[P_{ij}\right]^n$
(B) $\left[P(n)\right]=\left[P_{ij}\right]^n$
(C) $\left[nPij\right]=\left[P_{ij}\right]^n$
(D) $Pij[n]=\left[P_{ij}\right]^n$

19. Transition matrix is a_____ with sum of rows as 1.
(A) Zero matrix
(B) Square matrix
(C) Rectangular matrix
(D) Null matrix

20. Ergodic means
(A) Irreducible and periodic 1
(B) Reducible and periodic
(C) Not irreducible
(D) Regular

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The pdf of a continuous random variable X is given by $f(x)=\begin{cases} ax & 0\le x\le 1 \\ a & 1\le x\le 2 \\ 3a-ax & 2\le x\le 3 \\ 0 & otherwise \end{cases}$.
Find the value of 'a', find the cdf of X.

22. Find the MGF, mean and variance of Geometric distribution.

23. The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient? Assume 1% level of significance.

24. A telephone exchange receives one call for every 4 minutes and connects one call for every 3 minutes. If the rate of arrivals follows Poisson distribution and service time follows exponential distribution find out (i) Expected waiting time for a call (ii) Expected time in the system (iii) Expected number of customers in the system.

25. A housewife buys 3 kinds of cereals A, B and C. she never buys the same cereals in successive weeks if she buys cereal A the next week she buys cereal B. however, if she buys B or C the next week she is 3 times as likely to buy A as the other cereal. How often she buys each of the cereals?

26. If the pdf of X is $f(X) = e^{-x}, x > 0$ find the value of $y = 2X+1$.

27. A random variable X has a uniform distribution over (–3, 3), compute $P(|X|<2), P(|X-2|<2)$.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for probability of getting 100 to 140 sixes.

(OR)

b. Find the MGF of a random variable X whose pdf is defined by $f(x)=\begin{cases} x & ; 0\le x\le 1 \\ 2-x & , 1\le x\le 2 \\ 0 & , otherwise \end{cases}$
and also find mean and variance of X.

29. a. In a test on 2000 electric bulbs, it was found that the life of a particular make was normally distributed with an average of 2040 hours and SD of 60 hours. Estimate the number of bulbs likely to burn for (i) more than 2150 hours (ii) less than 1950 hours (iii) more than 1920 hours but less than 2160 hours.

(OR)

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B.Tech. DEGREE EXAMINATION, MAY 2018
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)
(Statistical tables to be supplied)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The probability density function of a continuous random variable is $f(x)=Ce^{-|x|}, -\infty<x<\infty$ then the value of C = ________.
   (A) $\frac{1}{2}$ (B) $\frac{1}{4}$
   (C) $\frac{3}{4}$ (D) $\frac{1}{6}$

2. If the continuous random variable X has pdf $f(x)=\begin{cases}3x, 0<x<1\\ 0, \quad otherwise\end{cases}$ then the pdf of $y=4x+3$ is
   (A) $\frac{3}{4}(y-3)$ (B) $\frac{3}{16}(y-3)$
   (C) $\frac{1}{4}(y-3)$ (D) $\frac{3}{2}(y-3)$

3. A random variable X has the pdf given by $f(x)=\begin{cases}2e^{-2x}, x\geq 0\\ 0, \quad x<0\end{cases}$, then MGF is
   (A) $\frac{2}{2-t}$ (B) $\frac{3}{3-t}$
   (C) $2(2-t)^{-3}$ (D) $3(3-t)^{-2}$

4. A random variable X has mean $\mu=12$ and variance $\sigma^2=9$ and an unknown probability distribution, then $P(6<x<18)$ is
   (A) $\frac{1}{2}$ (B) $\frac{3}{4}$
   (C) $\frac{1}{4}$ (D) $\frac{1}{8}$

5. If X is a Poisson variate such that $E(X^2)=6$ then $E(X)$ is
   (A) 3 (B) 2
   (C) 1 (D) 0

6. The mean of the exponential distribution with pdf $\lambda e^{-\lambda x}, x > 0$

(A) λ (B) $\frac{1}{\lambda}$

(C) $\frac{1}{\lambda^2}$ (D) 1

7. A random variable X has a uniform distribution over $(-3, 3)$. The value of k for which $P(x > k) = \frac{1}{3}$ is

(A) 3 (B) 2

(C) 1 (D) −2

8. Normal distribution is the limiting form of ________ distribution under suitable statistical conditions.

(A) Binomial (B) Poisson

(C) Geometric (D) Uniform

9. The Chi-square goodness of fit test can be used to test for

(A) Significance of sample statistics (B) Difference between population means

(C) Normality (D) Probability

10. Which of the following values is not typically used for α?

(A) 0.01 (B) 0.05

(C) 1.10 (D) 0.25

11. The degrees of freedom for t test based on n observations is

(A) $2n-1$ (B) $n-2$

(C) $2(n-1)$ (D) $n-1$

12. When the researcher rejects a true null hypothesis a ________ error occurs?

(A) Type I (B) Type A

(C) Type II (D) Type B

13. The probability of no customers in the system in $(M/M/1:\infty/FIFO)$ model ________.

(A) $\frac{\lambda}{u}$ (B) $\frac{\lambda}{u}-1$

(C) $1-\frac{\lambda}{u}$ (D) $\frac{\lambda}{u}+1$

14. The probability that the number of customers in the system exceeds k, in $(M/M/1:\infty|FIFO)$ model

(A) $\left(\frac{\lambda}{\mu}\right)^{k+1}$ (B) $\left(\frac{\lambda}{\mu}\right)^{k+2}$

(C) $\left(\frac{\lambda}{\mu}\right)^{k+3}$ (D) $\left(\frac{\lambda}{\mu}\right)^{k}$

15. What is the mean of the Poisson process?
(A) λ (B) λt
(C) λ / t (D) t / λ

16. The traffic intensity of a queuing system is
(A) λ (B) μ
(C) λ / μ (D) μ / λ

17. A state i is said to be aperiodic with periodic di if
(A) $di < 1$ (B) $di = 1$
(C) $di > 1$ (D) $di = 0$

18. A non null persistent and aperiodic state is called
(A) Empty (B) Finite
(C) Ergodic (D) 1

19. The state i is said to be non null persistent if its mean recurrence time μ_{ii} is
(A) Finite (B) Infinite
(C) Empty (D) 1

20. The steady state probability vector π of a discrete Markov chain with transition probability matrix p satisfies the matrix equation
(A) $\pi p = 0$ (B) $\pi p = \pi$
(C) $\pi(1-p) = 0$ (D) $\pi p' = 0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If a random variable X has moment generating function $Mx(t) = \frac{3}{3-t}$. Obtain the standard deviation of X.

22. Trains arrive at a station at 15 minutes intervals starting at 4am. If a passenger arrive to the statin at a time that is uniformly distributed between 9.00 and 9.30 find the probability that he has to wait for the train for (i) less than 6 minutes (ii) more than 10 minutes.

23. Theory predicts that the proportion of beans in four groups A, B, C, D should be 9:3:3:1. In an experiment among 1600 beans, the numbers in the four groups were 882, 313, 287 and 118. Does the experiment support the theory?

24. Explain Kendall's notation for queuing models.

25. Let $p = \begin{pmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{pmatrix}$ be the TPM of a two state Markov chain. Find the stationary probabilities of the chain.

26. State and prove memoryless property of geometric distribution.

27. Find MGF of the random variable whose moments are $\mu_r' = (r+1)3^r$ and find its mean.

PART – C (5 × 12 = 60 Marks)

Answer **ALL** Questions

28. a.i If X is random variable having the density function

$$f(x)=\begin{cases} x/6, x=1,2,3 \\ 0, \quad otherwise \end{cases}$$

Find $E\left(x^3+2x+7\right)$ and $\operatorname{var}(4x+5)$.

(8 Marks)

ii. If the continuous random variable X has pdf $f(x)=2/9(x+1), -1<x<2$ find the pdf of $y=x^2$. (4 Marks)

(OR)

b.i The distribution function of random variable X is given by $F(x)=1-(1+x)e^{-x}, x\geq 0$. Find the density function, mean, variance of X. (8 Marks)

ii. A fair die is tossed 720 times. Use Tchebycheff inequality to find a lower bound for the probability of getting 100 to 140 sixes. (4 Marks)

29. a.i Fit a Poisson distribution for the following data:

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

Find the probability mass function and then find the theoretical frequencies. (8 Marks)

ii. The time (in hours) required to repair a machine is exponential distributed with parameter $\lambda=1/2$. What is the conditional probability that a repair takes atleast 10h given that its duration exceeds 9h? (4 Marks)

(OR)

b.i In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less that 45%, between 45% and 60% between 60% and 75% and above 75% respectively. In a particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class. (8 Marks)

ii. If the probability that an applicant for a driving license will pass the road test on any given trial is 0.8. What is the probability that he will finally pass the test on the fourth trial?

(4 Marks)

30. a.i Random samples of 400 men and 600 women were asked whether they would like to have a school near their residence 200 men and 325 women were in favour of the proposal. Test the hypothesis that the proportions of men and women in favour of the proposal are same, at 5% level of significance. (8 Marks)

ii. A group of five patients treated with the medicine 'A' weights 42, 39, 48, 60 and 41 kg; a second group of 7 patients from the same hospital treated with the medicine 'B' weights 38, 42, 56, 64, 68, 69 and 62 kg. Do you agree with the claim that medicine B increases the weight significantly? (4 Marks)

(OR)

b.i The mean of 2 large samples 1000 and 2000 members are 67.5 inches and 68 inches respectively. Can the samples be regarded as drawn from the same population of standard deviation = 2.5 inches? (8 Marks)

ii. Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables

Sample 1	18	13	12	15	12	14	16	14	15
Sample 2	16	19	13	16	18	13	15	-	-

Do the estimates of the population variance differ significantly at 5% level? (4 Marks)

31. a. Customers arrive at a watch repair shop according to a Poisson process at a rate of one per every 10 minutes and service time is an exponential random variable with mean 8 minutes.
    (i) Find the average number of customers Ls in the shop.
    (ii) Find the average time a customer spend in the shop Ws.
    (iii) Find the average time that a customers spends in the queue Wq.
    (iv) Find the average number of customers in the queue Lq.
    (v) What is the probability that the server is idle?

(OR)

b. In a car wash service facility, cars arrive for service according to Poisson distribution with mean 5 per hour. The time for washing and cleaning each car has exponential distribution with mean 10 minutes per car. The facility cannot handle more than one car at a time and has a total of 5 parking spaces.
    (i) Find the effective arrival rate.
    (ii) What is the probability that an arriving car will get service immediately upon arrival?
    (iii) Find the expected number of parking spaces occupied.

32. a. The transition probability matrix of a Markov chain $\{X_n\}, n = 1,2,3......$ having 3 states 1, 2 and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 2, 0.1)$. Find
(i) $P(x_2 = 3)$ (ii) $P(X_3 = 2, X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$.

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throws the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the status.

* * * * *

b. A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

29. a. Assume that the chance of an individual coal miner being killed in an accident during a year is 1/1400. Using Poisson distribution calculate the probability that in a minor employing 350 miners, there will be (i) atleast one fatal accident in a year (ii) exactly 3 fatal accidents in a year. (iii) atmost one fatal accident in a year.

(OR)

b.i. State and prove memoryless property of exponential distribution.

ii. A normal distribution has mean $\mu = 20$ and S.D $\sigma = 10$. Find $P(15 \le X \le 40)$.

30. a. Two independent samples of sizes 5 and 6 contained the following values.

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Is the difference between the means significant?

(OR)

b. The following data is collected on two characters. Based on this can you say that there is no relation between smoking and literacy?

	Smokers	Non smokers
Literates	83	57
Illiterates	45	68

31. a. A super market has a single cashier, during the peak hours, customers arrive at a rate of 20 customer per hour. The average number of customers that can be processed by the cashier is 24/hr. Find (i) the probability that the cahier is idle (ii) the average number of customers in the queue (iii) the average number of customers in the system (iv) the average time a customer spends in the system and (v) the average time a customer spends in the queue.

(OR)

b. A one man barber shop can accommodate a maximum of 5 people at a time, 4 waiting and 1 getting hair cut. Customers arrive following Poisson distribution with an average of 5/ hr and service is rendered according to exponential distribution at an average rate of 15 min (i) what is the percentage of idle time? (ii) what fraction of the potential customers are turned away (iii) what is the expected number of customers waiting in the system?

32. a. A fair dice is tossed repeatedly. If X_n denotes the maximum of the numbers occurring in the first n tosses, find the transition probability matrix P of the Markov chain $\{X_n\}$. Find the P^2 and $P(X_2 = 6)$.

(OR)

b. Three bags A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

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B.Tech. DEGREE EXAMINATION, NOVEMBER 2018
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. If $E(X^2)=8$ and $E(X)=2$ then Var(2X) is
(A) 140 (B) 120
(C) 100 (D) 130

2. A random variable X has the pdf $f(x)=\begin{cases} 2e^{-2x}, & x\ge 0 \\ 0 & , x<0 \end{cases}$ then the MGF is
(A) $\frac{3}{3-t}$ (B) $\frac{2}{2-t}$
(C) $\frac{2}{(2-t)^2}$ (D) $\frac{3}{(3-t)^2}$

3. If $p(x)=\frac{x}{6}, x=1,2,3$ then E(X) is
(A) 2.33 (B) 0.33
(C) 1.33 (D) −1.33

4. If CDF of a continuous random variable X is given by $F(x)=\begin{cases} 0 & , x<0 \\ 1-e^{-x/5}, & x\ge 0 \end{cases}$ then its pdf is
(A) $e^{-x/5}$ (B) $\frac{1}{10}e^{x/5}$
(C) $\frac{1}{10}e^{-x/10}$ (D) $\frac{1}{5}e^{-x/5}$

5. If the random variable x follows Poisson distribution with mean 3 than P(X=2) is
(A) $\frac{9}{2}e^{-3}$ (B) $\frac{3}{2}e^{-3}$
(C) $\frac{1}{4}e^{-3}$ (D) $9e^{-3}$

6. The MGF of a binomial distribution is
(A) $(q+pe^t)^n$ (B) $(p+qe^t)^n$
(C) $(q+e^t)^n$ (D) $(p+e^t)^n$

7. If X is uniformly distributed over (0, 5) then P(X > 2) is
(A) 1/5 (B) 2/5
(C) 5 (D) 3/5

8. Passion distribution is the limiting form of _____ distribution under suitable statistical conditions.
(A) Uniform (B) Geometric
(C) Binomial (D) Exponential

9. If a teacher is trying to prove that new method of teaching mathematics is more effective than traditional one, then the teacher will conduct a
(A) Two tailed test (B) One tailed test
(C) Neither one tailed nor 2 tailed test (D) Both one tailed and 2 tailed test

10. A type II error occurs when
(A) H0 is incorrectly rejected when it is true (B) H0 is incorrectly accepted when it is false
(C) The sample mean differs from population mean (D) The test is biased

11. Which of the following is true about the null and alternative hypothesis?
(A) Exactly one hypothesis must be true (B) Both hypothesis must be true
(C) It is possible for both hypothesis to be true (D) It is possible for neither hypothesis to be true

12. The form of the alternative hypothesis can be
(A) One tailed (B) Two tailed
(C) Neither one nor two tailed (D) One or two tailed

13. In a watch repair shop customers arrive according to Poisson process at a rate of 8/hr and μ = 10/hr. The value of ρ is
(A) 2/10 (B) 4/10
(C) 8/10 (D) 6/10

14. One of the Little's formula is
(A) $E(Nq) = \lambda E(Ws)$ (B) $E(Ns) = \lambda E(Ws)$
(C) $\lambda = \mu(1 - P_0)$ (D) $\mu = \lambda(1 - P_0)$

15. In a $(M/M/1):(K/FIFO)$ queue what does K stand for
(A) Number of customers (B) Number of servers
(C) Service rate (D) Arrival rate

16. _____ should specify the number of servers and the arrangement of servers
(A) Queue discipline (B) Arrival pattern
(C) Service pattern (D) Customers behavior

17. Markov analysis is useful for predicting the state of the system related to
(A) Future time (B) Present time
(C) Past time (D) Present and past time

18. A state i is said to be periodic with period d_i if
(A) $d_i < 1$ (B) $d_i = 1$
(C) $d_i > 1$ (D) $d_i = 0$

19. The sum of all the elements of any row of the tpm is
(A) 0 (B) 0.5
(C) 0.25 (D) 1

20. A _____ and aperiodic state is called ergodic.
(A) Null persistent (B) Non null persistent
(C) Transient (D) persistent

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. The probability densityfunction of a continuous random variable X is given by
$$f(x) = \begin{cases} K, & 0 < x < 1 \\ 2K, & 1 < x < 2 \\ 0, & elsewhere \end{cases}$$
Find (i) K (ii) $P(1/2 < X < 3/2)$.

22. A die is cast until 6 appears. What is the probability that it must be cast more than 3 times.

23. A sample of 400 individuals is found to have a mean height of 67.17 inches. Can it be reasonably regarded as a sample from a large population with mean height of 67.39 inches and S.D. of 1.30 inches.

24. In a $(M/M/1):(\infty/FIFO)$ queuing system if $\lambda = 3/hr$ and $\mu = 4/hr$ find (i) $P(N \geq 5)$ (ii) $P(W > t)$.

25. Consider the Markov chain with two states and transition probability matrix $P = \begin{bmatrix} 3/4 & 1/4 \\ 1/2 & 1/2 \end{bmatrix}$. Find the stationary distribution of the chain.

26. If X is a Poisson variate such that $P(X = 1) = \frac{3}{10}$ and $P(X = 2) = \frac{1}{5}$. Find P(X=0) and P(X=3).

27. Find the MGF of binomial distribution.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A discrete random variable X has the following probability distribution.

x	0	1	2	3	4	5	6	7	8
p(x)	a	3a	5a	7a	9a	11a	13a	15a	17a

Find (i) the value of a (ii) P(0 < X < 3) (iii) P(X ≥ 3) (iv) the distribution function of X (v) P(3 < X < 6 / X > 4).

(OR)

ii. Derive the moment generating function of uniform distribution and hence find mean and variance of the distribution.

(OR)

b.i. If X has a geometric distribution, then for any two positive integers m and n, prove that $P[X > m+n / X > m] = P[X > n]$.

ii. Let X be is a normal variate with mean 30 and standard deviation is 5. Find the following $P[26 < X < 40]$, $P[X \geq 45]$ *and* $P[|X-30|] > 5$.

30. a. Two random samples gave the following data.

Sample	Size	Mean	Variance
1	8	9.6	1.2
2	11	16.5	2.5

Can we conclude that the two samples have been drawn from the same normal population?

(OR)

b.i. Test the significance of the difference between the means of the samples, drawn from two normal populations with the same standard deviation using following data.

Sample	Size	Mean	SD
1	100	61	4
2	200	63	6

ii. The following data give the number of air-craft accidents that occurred during the various days of a week.

Days	Mon	Tues	Wed	Thurs	Fri	Sat
No. of accidents	15	19	13	12	16	15

Test whether the accidents are uniformly distributed.

31. a. A departmental store has a single cashier. During the rush hours, customers arrive at the rate of 20 customers per hour. The average number of customers that can be processed by the cashier is 24 per hour. (i) what is the probability that the cashier is idle? (ii) what is the average number of customer in the queuing system? (iii) what is the average time a customer spends in the system? (iv) what is the average number of customers in the queue? (v) what is the average time a customer spends in the queue, waiting for service?

(OR)

b. The local one-person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour (exponential service time). (i) what percentage of time is the barber idle? (ii) what fraction of the potential customers are turned away? (iii) what is the expected number of customers waiting for a hair cut? (iv) how much time can a customer expected to spend in the barber shop?

32. a. The transition probability matrix of a Markov chain $\{X_n\}$: n = 1, 2, 3... having 3 states 1, 2, and 3 is $\rightarrow$ $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$. Initial distribution is $P^{(0)}$ = (0.7 0.2 0.1). Find (i) $P\{X_2=3\}$ (ii) $P\{X_3=2, X_2=3, X_1=3, X_0=2\}$

(OR)

b. Three boys A, B and C are throwing a ball to each other. A always throws the ball to B and B always throws the ball to C, but C is just as likely to throw the ball to B as to A. Show that the process is Markovian. Find the transition matrix and classify the states.

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Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2015
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014)
(Statistical table is to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. The probability density function of a continuous random variable is $f(x) = ce^{-|x|}; -\infty < x < \infty$ then the value of C is
(A) 1/2 (B) 1/4
(C) 3/4 (D) 1/6

2. If $f(x) = \begin{cases} x/6 & x = 1,2,3 \\ 0 & otherwise \end{cases}$, then E(X) is
(A) 0.33 (B) 1.33
(C) 2.33 (D) 3.33

3. *Var*(4X+8) is
(A) 12*Var*(X) (B) 4*Var*(X)+8
(C) 16*Var*(X) (D) 16*Var*(X)+8

4. If X is a random variable and r is an integer, then $E(X-\overline{X})^r$ represents
(A) r^{th} central moment (B) r^{th} factorial moment
(C) r^{th} raw moment (D) $(r-1)^{th}$ central moment

5. Which distribution has equal mean and variance?
(A) Binomial (B) Poisson
(C) Exponential (D) Normal

6. If the probability that a target is destroyed on any one shot is 0.5, then the probability that it would be destroyed on 6^{th} attempt is
(A) 0.1065 (B) 0.0165
(C) 0.1056 (D) 0.0156

7. The cumulative distribution function of f(x) of an exponential distribution is
(A) $e^{-\lambda x} - 1; x \geq 0$ (B) $e^{\lambda x} - 1; x \geq 0$
(C) $1 - e^{-\lambda x}; x \geq 0$ (D) $1 - e^{\lambda x}; x \geq 0$

8. The probability density function of a standard normal distribution is
(A) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}; -\infty < z < \infty$ (B) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-z^2/2}; 0 < z < \infty$
(C) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2/2}; 0 < z < \infty$ (D) $\phi(z) = \frac{1}{\sqrt{2\pi}} e^{z^2/2}; -\infty < z < \infty$

9. The test used to test the equality of variance of the populations from two small samples is
(A) t-test (B) F-test
(C) χ^2-test (D) Independent of attributes

10. In large samples, to test the significance of the difference between sample mean $\overline{X}$ and population mean μ, the test statistic (sample standard deviation s is known) is
(A) $z=\dfrac{\overline{X}-\mu}{s/\sqrt{n}}$ (B) $z=\dfrac{\mu-\overline{X}}{s/\sqrt{n}}$
(C) $z=\dfrac{\overline{X}-\mu}{s/\sqrt{n-1}}$ (D) $z=\dfrac{\mu-\overline{X}}{s/\sqrt{n-1}}$

11. 95% confidence limits for the population proportions is
(A) $p+2.58\sqrt{\dfrac{pq}{n}}$ (B) $p-2.58\sqrt{\dfrac{pq}{n}}$
(C) $1.96\sqrt{\dfrac{pq}{n}}$ (D) $p\pm1.96\sqrt{\dfrac{pq}{n}}$

12. The application of one-tailed or two-tailed test depends upon the nature of the
(A) Null hypothesis (B) Alternative hypothesis
(C) Level of Significance (LOS) (D) Degrees of freedom

13. In queuing system, the number of customers serviced per unit has a Poisson distribution with mean
(A) μ (B) 1/μ
(C) 2/μ (D) 3/μ

14. In the symbolic representation of a queue model $(a/b/c):(d/e)$, where c denotes.
(A) The type of distribution of the no of arrivals per unit time (B) The type of distribution of the no of services per unit time
(C) No of servers (D) The capacity of the system

15. Arrivals at a telephone booth are considered to the Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes. What is the average number of persons waiting in the system?
(A) 0.5 (B) 1.0
(C) 1.5 (D) 2.0

16. Probability that the number of customers in the system exceeds k (i.e) P(N > k) is
(A) λ/μ (B) $(\lambda/\mu)^k p_0$
(C) $(\lambda/\mu)^{k+1} p_0$ (D) $(\lambda/\mu)^{k-1} p_0$

17. The conditional probability that the process is in state a_j at step n, given that it was in state a_i at step 0 is called
(A) One-step transition probability (B) Two-step transition probability
(C) (n−1) step transition probability (D) n-step transition probability

18. The *tpm* of a Markov chain is stochastic matrix, because
(A) $p_{ij}\ge0\ \&\sum_j p_{ij}=1$ (B) $p_{ij}\le0\ \&\sum_j p_{ij}=1$
(C) $p_{ij}\ge0\ \&\sum_i p_{ij}=1$ (D) $p_{ij}\le0\ \&\sum_i p_{ij}=1$

19. If $p_{ij}^{(n)}>0$ for some n and for all *i* and *j* then every state can be reached from every other state. When this condition is satisfied, the Markov Chain is said to
(A) Reducible (B) Irreducible
(C) Periodic (D) Aperiodic

20. State *i* is said to be aperiodic with period *di*, if
(A) $d_i>1$ (B) $d_i<1$
(C) $d_i=1$ (D) $d_i=0$

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. A random variable X has a mean μ = 12 and variance σ^2 = 9 and an unknown probability distribution. Find P(6<X<18).

22. Given the random variable X with density function $f(x)=\begin{cases}2x & 0<x<1\\ 0 & elsewhere\end{cases}$.
Find the probability density of $Y=8X^3$.

23. A travel company has two cars for hiring. The demand for a car on each day is distributed as Poisson variate with mean 1.5. Calculate the proportion of days on which some demand is refused.

24. The time (in hours) required to repair a machine is exponentially distributed with parameter λ = 1/2. What is the conditional probability that a repair takes atleast 10 hours given that its duration exceeds 9 hours?

25. Experience has shown that 20% of a manufactured product is of top quality. In one day's production of 400 articles only 50 are of top quality. Show that either the production of the day chosen was not a representative sample or the hypothesis of 20% was wrong.

26. In a railway marshalling yard goods trains arrive at rate of 30 trains per day. Assuming that the inter-arrival time follows an exponential distribution and the service time is also exponential with an average of 36 minutes. Calculate expected queue size (line length).

27. If the transition probability matrix of a Markov Chain is $\begin{pmatrix}0 & 1\\ 1/2 & 1/2\end{pmatrix}$, find the steady state distribution of the chain.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. A random variable X has the following probability distribution.

X	−2	−1	0	1	2	3
P(X=x)	0.1	K	0.2	2K	0.3	3K

(A) Find K (B) Evaluate P(X< 2) (C) Evaluate P(−2<X<2) (D) Find the cumulative distribution function of X (E) Evaluate the mean and variance of X.

(OR)

b.i. The probability function of an infinite discrete distribution is given by $P(X=j)=\dfrac{1}{2^j}(j=1,2,....\infty)$. Verify that the total probability is 1 and find the mean and variance of the distribution.

ii. If $f(x)=\begin{cases}xe^{-x^2/2}; & x\ge0\\ 0 \quad ; & x<0\end{cases}$
(1) Show that f(x) is a probability density function of a continuous random variable X
(2) Find its distribution F(x).

29. a.i. Fit a binomial distribution for the following data.

x	0	1	2	3	4	5	6	Total
f(x)	5	18	28	12	7	6	4	80

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B.Tech. DEGREE EXAMINATION, MAY 2016
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)
(Statistical tables to be supplied)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45^{th} minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours | Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer **ALL** Questions

1. A random variable X has the probability density function $f(x)=\begin{cases} e^{-x}, x\ge 0 \\ 0,\ x<0 \end{cases}$ the MGF of f(x) is
   (A) $\frac{1}{1-t}$ (B) $\frac{2}{2-t}$
   (C) $2(1-t)$ (D) $(1-t)$

2. If X is a discrete RV taking values $x_1, x_2 ...$ where $x_1 < x_2 < x_3 < .....x_{i-1} < x_i < ...$ then $P(X = x_i) =$
   (A) $F(x_i)$ (B) $F(x_i) - F(x_{i-1})$
   (C) $F(x_{i-1})$ (D) $F(x_{i-1}) - F(x_i)$

3. Let X be a continuous RV with pdf $f(x)=\begin{cases} \frac{k}{2}; 1<x<5 \\ 0;\ \text{otherwise} \end{cases}$ the value of K is
   (A) $\frac{1}{4}$ (B) $\frac{1}{2}$
   (C) 4 (D) 2

4. Given $E(X^2) = 4$, $E(X) = 0$ for a discrete RV X then var (2x +3) is
   (A) 4 (B) 8
   (C) 16 (D) 19

5. If X is uniformly distributed in $(-1, 3)$ then its probability density function f(x) is given by
   (A) $\frac{1}{2}$ (B) $\frac{3}{4}$
   (C) $\frac{1}{3}$ (D) $\frac{1}{4}$

6. In a normal distribution about 99% of the observation lie between

(A) $\mu \pm 25$ (B) $\mu \pm \sigma$

(C) $\mu \pm 3\sigma$ (D) $\mu \pm \sigma^2$

7. If the RV X follows a Poisson distribution with mean 2, then P(X =1) is

(A) $\frac{2}{e^2}$ (B) $\frac{1}{e^2}$

(C) $\frac{1}{2e^2}$ (D) $\frac{e^2}{2}$

8. The mean of the exponential distribution with pdf $f(x) = \begin{cases} \frac{1}{\theta} e^{\frac{-x}{\theta}}, x > 0 \\ 0, \text{ otherwise} \end{cases}$ is

(A) $\frac{1}{\theta}$ (B) θ

(C) $\frac{1}{\theta^2}$ (D) θ^2

9. The Chi-square goodness of fit test can be used to test

(A) Significance of sample statistic (B) Difference between population means

(C) Normality (D) Probability

10. A ________ error occurs when the researcher rejects a null hypothesis when it is true

(A) Type 1 error (B) Type 2 error

(C) Type A error (D) Type B error

11. For sufficiently large value of n, the t-distribution tends to

(A) Uniform distribution (B) Standard normal distribution

(C) Geometric distribution (D) Binomial distribution

12. ___________ is used to test whether the differences between observed and expected frequencies are significant or not

(A) Chi-square (B) t -test

(C) F-test (D) Z -test

13. The symbol d in the queuing model (a/b/c):(d/e) stands for

(A) Queue discipline (B) System capacity

(C) Maximum queue size (D) Service time

14. If the behaviour of the queuing system depends on time, then the system is said to be in

(A) Steady state (B) Transient state

(C) Idle state (D) Busy state

15. In a queuing system the arrival and inter service rate respectively are denoted as

(A) λ, μ (B) $\frac{1}{\mu}, \frac{1}{\lambda}$

(C) $\mu, \frac{1}{\lambda}$ (D) $\lambda, \frac{1}{\mu}$

16. The probability that a customer has to wait more than 15 min to get his service completed in a (M/M/1):(∞/FIFO) queue system if λ =6/hr and μ =10/hr is ________

(A) 0.3769
(B) 0.3976
(C) 0.3697
(D) 0.3679

17. The period d_i of a Markov chain is given by

(A) $GCD\{m : P_{ii}^{(m)} \geq 0\}$
(B) $GCD\{m : P_{ij}^{(m)} > 0\}$
(C) $GCD\{m : P_{ii}^{(m)} > 0\}$
(D) $GCD\{m : P_{ij}^{(m)} \geq 0\}$

18. The return to state i is certain if $F_{ii} = \sum_{i=1}^{n} f_{ii}^{(n)}$ is

(A) 0
(B) 1
(C) Less than 1
(D) Greater than 1

19. A non null persistent and a periodic state is said to be

(A) Ergodic
(B) Recurrent
(C) Transient
(D) Irreducible

20. If the one step transition probability does not depend on the step, then the Markov chain is called

(A) Reducible
(B) Regular
(C) Homogeneous
(D) Non homogeneous

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Given the random variable X with density function $f(x) = \begin{cases} 4x, & 0 < x < 2 \\ 0, & \text{elsewhere} \end{cases}$.
Find the pdf of $Y = 2X^3$.

22. In 100 sets of 10 tosses of an unbiased coin, how many cases do you expect to get atleast 7 heads.

23. A sample of 900 members is found to have a mean of 3.4 cms, can it be reasonably regarded as a simple sample from a large population with mean 3.2 cms and SD 2.3 cms.

24. In the usual notation of a (M/M/1):(∞/FIFO) queue system if λ =12/hr and μ =24/hr find
    (i) The average waiting time of a customer in the queue if he has to wait
    (ii) The probability that the number of customers in the system exceeds 3.

25. If the initial state probability distribution of a Markov chain $P^{(0)} = \left(\frac{3}{4}, \frac{1}{4}\right)$ and the tpm of the chain is $\begin{pmatrix} 1 & 0 \\ \frac{1}{2} & \frac{1}{2} \end{pmatrix}$ find the probability distribution of the chain after 2 steps.

26. Find the MGF of exponential distribution and hence find its mean and variance.

27. A random sample of 400 mangoes was taken from a big consignment and 40 were found to be bad. Prove that the percentage of bad mangoes in the consignment will, lie between 5.5 and 14.5.

PART – C ($5 \times 12 = 60$ Marks)
Answer **ALL** Questions

28. a. The probability distribution of a discrete RV X is given below

x:	0	1	2	3	4	5	6	7	8
p(x):	k	3k	5k	7k	9k	11k	13k	15k	17k

Find

(i) k

(ii) $P(0 < X \leq 6)$

(iii) $P(X < 3)$

(iv) $P(1.5 < X < 4.5 / X > 2)$

(v) $E(X^2)$.

(OR)

b. For the number of points X on a die, find by using Chebychev's inequality $P\{|X - E(X)| > 2.5\}$.

ii. The probability distribution function of a continuous RV X is $f(x) = \begin{cases} x \ \ for\ 0 \leq x < 1 \\ 2x \ \ for\ 1 \leq x < 2 \\ 0\ for\ x \geq 2 \end{cases}$

Compute the cumulative distribution function of X.

29. a. A and B shoot independently until each has hit his own target. The probabilities of their hitting the target at each shot are $\frac{3}{5}$ and $\frac{5}{7}$ respectively. Find the probability that B will require more shots than A.

(OR)

b. In an examination it is laid down that a student passes if he secures 30 percent or more marks. He is placed in the first, second or third division according as he secures 60% or more marks, between 45% and 60% marks and marks between 30% and 45% respectively. He gets distinction in case he secures 80% or more marks. It is noticed from failed result that 10% of the students failed in the examination, whereas 5% of them obtained distinction. Calculate the percentage of students placed in the second division (Assume normal distribution of marks).

30. a. Two independent samples of rats chosen among a group of rats had the following increase in weights when fed on 2 different diets. Can you say that the mean increase in weight differs significantly

Diet A: 96 88 97 89 92 95 90
Diet B: 112 80 98 100 84 82 89 95 100 96

(OR)

b. For the data in the following table, test for independence between a person's ability in mathematics and interest in Economics.

		Ability in mathematics		
		Low	Average	High
Interest in Economics	Low	63	42	15
	Average	58	61	31
	High	14	47	29

31. a. The customers arrive at the ATM machine of a bank according to a Poisson process at an average rate of 15/hr. It is known that the average time taken by each customer is an exponential random variable with mean 2 minutes. Find the following

   (i) The probability that an arriving customer will find the ATM machine occupied
   (ii) Average number of customers in the queue
   (iii) Average waiting time in the system
   (iv) The bank has the policy of installing additional ATM machines if customers wait at an average of 3 or more minutes in the queue. Find the average arrival rate of customers required to justify an additional ATM machine.

(OR)

b. A stenographer is attached to 5 offices for whom she performs stenographic work. She gets call from the offices at the rate of 4 per hour and takes on the average 10 min to attend to each call. If arrival rate is Poisson and service time exponential find

   (i) The average number of waiting calls
   (ii) The average waiting time for an arriving call and
   (iii) The average time an arriving call spends in the system.

32. a. The tpm of a Markov chain $\{X_n, n \geq 0\}$ having three states 0, 1 and 2 is

$P = \begin{bmatrix} 0.2 & 0.3 & 0.5 \\ 0.1 & 0.6 & 0.3 \\ 0.4 & 0.3 & 0.3 \end{bmatrix}$. The initial distribution is given by $P^{(0)} = (0.5\ 0.3\ 0.2)$ find

   (i) $P(X_2 = 2)$
   (ii) $P(X_3 = 3, X_2 = 2, X_1 = 1, X_0 = 2)$.

(OR)

b. A man either drives a car or catches a bus to go to office each day. He never goes to 2 days in a row by bus but if he drives one day, then the next day he is just as likely to drive again as he is to travel by bus. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find

   (i) The probability that he takes a bus on the fourth day
   (ii) The probability that he travels by bus in the long run.

* * * * *

b. i. If X is the number obtained in a throw of a fair die, show that the Tchebycheff's inequality gives $P\{|X-\mu|>2.5\}<0.47$, while the actual probability is zero.

ii. In a continuous distribution, the probability density function is given by $f(x)=\begin{cases} kx(2-x), & 0<x<2 \\ 0 & otherwise \end{cases}$

Find: (A) K (B) Mean (C) Variance (D) Distribution function F(x)

29. a.i. Derive moment generating function (MGF) and hence find mean and variance for a Binomial Distribution.

ii. Assume the mean heights of soldiers to be 68.22 inches with a variance of 10.8(inch)^2. How many soldiers in a regiment of 1000, would you expect to be over 6 feet tall?

(OR)

b. i. Derive memoryless property for exponential distribution.

ii. A candidate applying for a driving license has the probability of 0.58 in passing the road test in a given trial? What is the probability that he will pass the test (A) on the fourth trail (B) in less than four trials?

30. a. Fit a Poisson Distribution for the following data and also test the goodness of fit.

x	0	1	2	3	4	5	Total
f	142	156	69	27	5	1	400

(OR)

b. The nicotine contents in two random samples of tobacco are given below.

Sample 1	21	24	25	26	27	
Sample 2	22	27	28	30	31	36

Can you say that the two samples come from the same population?

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be exponentially distributed with mean 3 minutes.

(i) Find the average number of persons waiting in the system.
(ii) What is the probability that a person arriving at the booth will have to wait in the queue?
(iii) What is the probability that it will take him more than 10 minutes altogether to wait for phone and complete his call?
(iv) The telephone department will install a second booth when convinced that an arrival has to wait on the average 3 minutes for phone. By how much the flow of arrivals should increase in order to justify a second booth?

(OR)

b. Patients arrive at a clinic according to Poisson Distribution at a rate of 30 patients per hour. The waiting room does not accommodate more than 14 patients. Examination time per patient is exponential with mean rate of 20 per hour.

(i) Find the effective arrival rate at the clinic?
(ii) What is the probability that an arriving patient will not wait?
(iii) What is the expected waiting time until a patient is discharged from the clinic?

32. a. A fair die is tossed repeatedly. If $\{X_n\}$ denotes the maximum of the number occurring in the first n trials, find the transition probability matrix P of the Markov chain. Also find P^2 and $P(X_2=6)$

(OR)

b. Two boys B_1 and B_2 and two girls G_1 and G_2 are throwing a ball from one to the other. Each boy throws the ball to the other boy with probability 1/2 and to each girl with probability 1/4. On the other hand, each girl throws the ball to each boy with probability 1/2 and never to the other girl. In the long run, how often does each receive the ball?

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Reg. No.														

B.Tech. DEGREE EXAMINATION, MAY 2015
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted from the academic year 2013 – 2014 onwards)
(Statistical table is to be provided)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. If $E[X^2]=8$ and $E[X]=2$ then Var(X) is
(A) 3 (B) 2
(C) 1 (D) 4

2. A random variable X has the *pdf* $f(x)=\begin{cases} 2e^{-2x}, & x\geq 0 \\ 0, & x<0 \end{cases}$ then the *mgf* is
(A) $\frac{2}{2-t}$ (B) $\frac{3}{3-t}$
(C) $2(2-t)^{-2}$ (D) $3(3-t)^{-2}$

3. If $f(x)=\begin{cases} \frac{x}{6}, & x=1,2,3 \\ 0, & otherwise \end{cases}$ then E(X) is
(A) 0.33 (B) 2.33
(C) 1.33 (D) −1.33

4. If cdf of a continuous random variable is given by $F(x)=\begin{cases} 0 & x<0 \\ 1-e^{-x/5} & 0\leq x\leq\infty \end{cases}$ then its *pdf* is
(A) $\frac{1}{5}e^{\frac{-1}{5}x}$ (B) $\frac{1}{10}e^{\frac{1}{5}x}$
(C) $e^{\frac{-1}{5}x}$ (D) $\frac{1}{2}e^{\frac{-1}{2}x}$

5. If the random variable X follows a Poisson distribution with mean 3, then P(X = 0) is
(A) e^{-3} (B) e^3
(C) e^2 (D) E

6. The *mgf* of a geometrical distribution is
(A) $\frac{1}{1-qe^t}$ (B) $\frac{1}{1-pe^t}$
(C) $\frac{q}{1-pe^t}$ (D) $\frac{pe^t}{1-qe^t}$

7. If X is uniformly distributed over (0, 10) then P(X > 8) is
(A) 1/3 (B) 3/5
(C) 1/5 (D) 1/10

8. Normal distribution is the limiting form of _______ distribution under suitable statistical conditions
(A) Uniform (B) Exponential
(C) Geometric (D) Binomial

9. If a teacher is trying to prove that new method of teaching mathematics is more effective than traditional one, he/she will conduct a
(A) One-tailed test (B) Two-tailed test
(C) Neither one tailed nor two-tailed test (D) Both one tailed and two tailed test

10. A type-II error occurs when
(A) The null hypothesis is incorrectly rejected when it is true (B) The null hypothesis is incorrectly accepted when it is false
(C) The sample mean differs from population mean (D) The test is biased

11. The Chi-square test can be used for
(A) Significance of sample mean (B) Difference between population means
(C) Goodness of fit (D) Difference between population variances

12. Which of the following is true of the null and alternative hypotheses
(A) Exactly one hypothesis must be true (B) Both hypothesis must be true
(C) It's possible for both hypothesis to be true (D) It's possible for neither hypothesis to be true

13. Consider a watch repair shop where customers arrive according to Poisson process at a rate of one for every 10 min with $\mu = 60/8$ *per hour* then the average number of customers in the queue is
(A) 3.2 (B) 2.3
(C) 3.0 (D) 2.0

14. Consider an $(M/M/1):(\infty/FIFO)$ queue. The probability that the waiting time of a customer in the system exceeds t, $(P(W_s > t)$ is
(A) $e^{-(\lambda-\mu)t}$ (B) $e^{-\lambda t}$
(C) $e^{-(\mu-\lambda)t}$ (D) $e^{-\mu t}$

15. In an $(M/M/1):(k/FIFO)$ queue, with k = 5, λ = 5/hr, service time = 10 min/person, the percentage of time, the system is idle.
(A) 20.05% (B) 20.55%
(C) 25.50% (D) 25.05%

16. A petrol pump station with only one pump can accommodate 5 cars. The arrival of the cars is Poisson with μ = 30/hours, λ = 10/hours. How many cars are there in the petrol pump station on an average?
(A) 0.942 (B) 0.492
(C) 0.294 (D) 0.429

17. The steady state probability vector π of a Discrete Markov Chain with transition probability matrix P satisfies the matrix equation.
(A) $P\pi = 0$ (B) $P\pi = \pi$
(C) $P^t\pi = 0$ (D) $\pi(1-P) = 0$

18. A factory has a large number of machines which can be in one of two states, operating or broken. The probability that an operating machine is still operating is 0.99 and the probability that a broken machine is repaired is 0.88. then the *tpm* is
(A) $\begin{pmatrix} 0.99 & 0.88 \\ 0.01 & 0.12 \end{pmatrix}$ (B) $\begin{pmatrix} 0.01 & 0.88 \\ 0.99 & 0.12 \end{pmatrix}$
(C) $\begin{pmatrix} 0.99 & 0.01 \\ 0.88 & 0.12 \end{pmatrix}$ (D) $\begin{pmatrix} 0.99 & 0.88 \\ 0.12 & 0.11 \end{pmatrix}$

19. If the tpm of a Markov Chain is $\begin{pmatrix} 0 & 1 \\ 1/2 & 1/2 \end{pmatrix}$, the steady state distribution of the chain is
(A) $\left(\frac{1}{3}, \frac{2}{3}\right)$ (B) $\left(\frac{1}{3}, 0\right)$
(C) $\left(0, \frac{2}{3}\right)$ (D) $\left(\frac{4}{3}, \frac{2}{3}\right)$

20. The one step tpm of a Markov chain $\{X_n; n=0, 1, 2, \ldots\}$ having state space S= {1, 2, 3} is $\begin{bmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{bmatrix}$ with $p_0 = (0.7, 0.2, 0.1), P(X_2 = 3) =$
(A) 0.279 (B) 0.219
(C) 0.729 (D) 0.799

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. If the *cdf* of a random variable is given by $F(x) = \begin{cases} 0 & x < 0 \\ x^2/16 & 0 \le x \le 4 \\ 1 & x \ge 4 \end{cases}$. Find $P(X > 1 / X < 3)$.

22. Given a random variable X with density function $f(x) = \begin{cases} 2x, & 0 < x < 1 \\ 0, & otherwise \end{cases}$. Find the *pdf* of $Y = 8X^3$.

23. In a book of 520 pages 390 typographical errors occur. Assuming X as a Poisson variate for number of errors per page. Find the probability that a random sample of 5 pages will contain no error.

24. A bus arrives every 20 minutes, at a specified stop, beginning at 6.40 a.m and continuing until 8.40 a.m. A passenger arrives randomly between 7.00 am and 7.30a.m. What is the probability that the passenger has to wait for more than 5 minutes for a bus?

25. The mean weekly sales of soap bars in departmental stores is 145 bars per store. After an advertising campaign, the mean weekly sales in 17 stores for typical week increased to 155 and showed a standard deviation of 16. Was the advertising campaign successful?

26. The *tpm* of a Markov chain with states 0, 1, 2 is $P = \begin{pmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{pmatrix}$ and the initial state distribution of the chain is $P(X_0 = i) = 1/3, i = 0,1,2$. Find $P(X_2 = 2)$.

27. What is the average waiting time of a customer in a single server, infinite capacity, Poisson queue, if he happens to wait, given that λ = 6 per hour and μ = 4 per hour.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a.i. A fair die is tossed 720 times. Use Tchebycheff's inequality to find a lower bound for the probability of getting 100 to 140 sixes.

ii. A random variable X has the following probability distribution:

x	−2	−1	0	1	2	3
p(x)	0.1	1K	0.2	2K	0.3	3K

(A) Find K (B) Evaluate P(−2 < X < 2) (C) Find the *cdf* of X

(OR)

(OR)

b. Buses arrive at a specified stop at 15 minute intervals starting at 7 a.m, that is, they arrive at 7, 7.15, 7,30, 7.45 and so on. If a passenger arrives at a stop at a random time that is uniformly distributed between 7 and 7.30, find the probability that he waits for
   (i) Less than 5 minutes for a bus
   (ii) Moe than 10 minutes for a bus.

30. a. A simple sample of heights of 6400 English men has a mean of 170 cm and a S.D of 6.4 cm, while a simple sample of heights of 1600 Americans has a mean of 172 cm and a S.D of 6.3 cm. Do the data indicate that Americans are on the average, taller than the English men.

(OR)

b. The following data is collected on two characters. Based on this, can you say that there is no relation between smoking and literacy?

	Smokers	Non Smokers
Literates	83	57
Illiterates	45	68

31. a. Arrivals at a telephone booth are considered to be Poisson with an average time of 12 minutes between one arrival and the next. The length of a phone call is assumed to be distributed exponentially with mean 4 minutes.
   (i) Find the average number of persons waiting in the system.
   (ii) Estimate the fraction of the day when the phone will be in use.
   (iii) What is the probability that a person arriving at the booth will have to wait in the queue?
   (iv) What is the average length of the queue that forms from time to time?

(OR)

b. The local one –person barber shop can accommodate a maximum of 5 people at a time (4 waiting and 1 getting hair –cut). Customers arrive according to a Poisson distribution with mean 5 per hour. The barber cuts hair at an average rate of 4 per hour
   (i) What percentage of time is the barber idle?
   (ii) What fraction of the potential customers are turned away?
   (iii) What is the expected number of customers waiting for a hair –cut?
   (iv) How much time can a customer expect to spend in the barber shop?

32. a. A gambler has Rs.2/-. He bets Re 1 at a time and wins Re 1 with probability $\frac{1}{2}$. He stops playing if he loses Rs 2 or wins Rs 4.
   (i) What is the tpm of the related Markov chain?
   (ii) What is the probability that he has lost his money at the end of 4 plays?

(OR)

b. Find the nature of the states of the Markov chain with the tpm.

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{pmatrix} 0 & 1 & 2 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

Also find he steady –state distribution of the Markov chain.

* * * * *

Reg. No.															

B.Tech. DEGREE EXAMINATION, NOVEMBER 2016
Fourth Semester

MA1014 – PROBABILITY AND QUEUING THEORY
(For the candidates admitted during the academic year 2013 – 2014 and 2014 -2015)
(Statistical tables to be permitted)

Note:
(i) **Part - A** should be answered in OMR sheet within first 45 minutes and OMR sheet should be handed over to hall invigilator at the end of 45th minute.
(ii) **Part - B** and **Part - C** should be answered in answer booklet.

Time: Three Hours Max. Marks: 100

PART – A (20 × 1 = 20 Marks)
Answer ALL Questions

1. A random variable X has the following probability function

x	0	1	2	3	4
p(x)	k/2	k/4	k	k/4	k/2

The value of K is equal to
(A) 0.1 (B) 0.2
(C) 0.3 (D) 0.4

2. Which of the following random variable is not discrete?
(A) The price of a litre of milk (B) The number of boys in the family
(C) The volume of petrol used by a family each week (D) The number of children in the family who watch TV

3. A random variable X is such that its mean is 4 and its standard deviation is 3. $E(X^2)$ is equal to
(A) 1 (B) 5
(C) 7 (D) 25

4. The r^{th} moment about origin is denoted by
(A) μ_r' (B) μ_r
(C) μ_{r+1} (D) μ_{r-1}

5. The mean of the binomial distribution is
(A) np (B) $\sqrt{np}$
(C) npq (D) $\sqrt{npq}$

6. The moment generating function of the geometric distribution with probability function pq^{x-1}, x =1, 2, 3,…… is
(A) pe^t (B) qe^t
(C) $\frac{pe^t}{1-qe^t}$ (D) $1-qe^t$

7. Variance of the Poisson distribution is
(A) λ^2 (B) $\lambda+1$
(C) λ (D) $\frac{1}{\lambda^2}$

8. The probability density function of the uniform distribution defined in the interval (a, b) is
(A) $\frac{1}{a-b}$ (B) $\frac{1}{a+b}$
(C) $\frac{1}{b-a}$ (D) $\frac{1}{a^2+b^2}$

9. The rejection of the null hypothesis when it is correct is called
(A) Type I error (B) Type II error
(C) Type III error (D) Type IV error

10. One –tailed or two tailed test depends upon the nature of the
(A) Null hypothesis (B) Alternative hypothesis
(C) Level of significance (D) Critical region

11. F test is used to test the equality of the
(A) Mean (B) Mode
(C) Median (D) Variance

12. χ^2 test of goodness of fit is based on the condition
(A) $\Sigma E_i = \Sigma 0_i$ (B) $\Sigma E_i \neq \Sigma 0_i$
(C) $\Sigma E_i > \Sigma 0_i$ (D) $\Sigma E_i < \Sigma 0_i$

13. The utilization factor for a system represents
(A) The steady state average waiting time (B) The probability that one is in the system
(C) The probability that the service facility is being used (D) The average number of customers in the queue

14. Single server Poisson queue with infinite capacity of the Markov model is
(A) $[(M/M/1):(\infty/FIFO)]$ (B) $[(M/M/1):(k/FIFO)]$
(C) $[(M/M/S):(\infty/FIFO)]$ (D) $[(M/M/1):(k/FIFO)]$

15. The arrival pattern in queuing theory follows
(A) Binomial distribution (B) Normal distribution
(C) Geometric distribution (D) Poisson distribution

16. The probability of n customers at time t +Δt is given by
(A) $P_n(t+\Delta t)$ (B) $P_n(t-\Delta T)$
(C) $P_n(t)$ (D) $P_n(1+\Delta t)$

17. A stochastic matrix P is said to be a regular matrix if all the entries of P^m are
(A) Negative (B) Regular
(C) Positive (D) Symmetric

18. The state i is said to be periodic if
(A) $d_i = 1$ (B) $d_i < 0$
(C) $d_i = 0$ (D) $d_i > 1$

19. In Markov chain tpm stands for
(A) Transportation matrix (B) Transition probability matrix
(C) Transpose matrix (D) Transition power matrix

20. A non null persistent and a periodic state is called
(A) Periodic (B) Ergodic
(C) Reducible (D) Irreducible

PART – B (5 × 4 = 20 Marks)
Answer **ANY FIVE** Questions

21. Given the random variable X with density function
$f(x) = \begin{cases} 2x, 0 < x < 1 \\ 0, \ \text{elsewhere} \end{cases}$ Find the probability density function of Y = $8X^3$.

22. X is a normal variate with mean 30 and S.D 5. Find
(i) $P(26 \le X \le 40)$
(ii) $P(X \ge 45)$

23. The fatality rate of typhoid patients is believed to be 17.26%. In a certain year 640 patients suffering from typhoid were treated in a metropolitan hospital and only 63 patients died. Can you consider the hospital efficient?

24. In the usual notation of a $[(M/M/1):(\infty/FIFO)]$ queue system if λ=12 per hour and μ =24 per hour,find the average number of customers in the system and in the queue.

25. The tpm of a Markov chain with three states 1, 2, 3 is
$$P = \begin{pmatrix} \frac{3}{4} & \frac{1}{4} & 0 \\ \frac{1}{4} & \frac{1}{2} & \frac{1}{4} \\ 0 & \frac{3}{4} & \frac{1}{4} \end{pmatrix}$$
and the initial state distribution of the chain is $P\{X_0 = i\} = \frac{1}{3}, i = 1,2,3$ find $P\{X_2 = 2\}$.

26. A fair die is tossed 720 times. Use Chebycheff's in equality to find a lower bound for the probability of getting 100 to 140 sixes.

27. If the probability that an applicant for a driver's licecse will pass the road test on any given trial is 0.8, what is the probability that he will finally pass the test
(i) On the fourth trial
(ii) In fewer than 4 trials.

PART – C (5 × 12 = 60 Marks)
Answer **ALL** Questions

28. a. A random variable X has the following probability distribution

x:	−2	−1	0	1	2	3
p(x)	0.1	k	0.2	2k	0.3	3k

(i) Find k
(ii) Evaluate P(X<2)
(iii) P(−2< X<2)
(iv) Find the cumulative distribution function of X.

(OR)

b. A continuous random variable X has pdf $f(x) = kx(2-x)$, $0 \le x \le 2$.
(i) Find k
(ii) r^{th} moment about origin
(iii) Find the first for central moments.

29. a. Fit a binomial distribution for the following data.

x:	0	1	2	3	4	5	6	Total
f:	5	18	28	12	7	6	4	80