Sum of Subsets :

Given n distinct positive numbers (usually called as weights) and desired to find all combination of those numbers (weights) whose sum equals to 'm' (maximum capacity). This is called Sum of Subsets.

In sum of subsets the solution vector x_i can be either '0' or '1'.

$\boxed{x_i = 1}$ ⇒ w_i is included.

&

$\boxed{x_i = 0}$ ⇒ w_i is not included.

where w_i represents weights or positive numbers of sum of subset problems.

for a node at level i, the left

child corresponds to $\boxed{x_i = 1}$ and the right child corresponds to $\boxed{x_i = 0}$.

The bounding function of sum of subset problem $B_K(x_1, x_2 \dots x_n)$ is true if and only if;

$$\boxed{\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i \geq m}$$

&

$$\boxed{\sum_{i=1}^{k} w_i x_i + w_{k+1} \leq m}$$

Algorithm for sum of subset problem:

Algorithm sum of sub (s, k, r)

// w[1:n] - given distinct positive numbers (or)
// weight
// find all subsets of w[1:n] that sums to m
// m - maximum capacity for sum of subsets.
// r - sum of all the weights or positive
// number in sum of subset problem,
// $r = \sum_{i=1}^{n} w[i]$
// x[j], 1 ≤ j < k - the solution vector for
// first (k-1) values which has be already
// determined $s = \sum_{j=1}^{k-1} w[j] * x[j]$ & $r = \sum_{j=k}^{n} w[j]$
// w[j] - The weights of sum of subset are
// arranged in non-decreasing order.

// First weight should less than or equal to m

// $w[1] \leq m$

$\sum_{i=1}^{n} w[i] \geq m$

{

// generate left child. ie) $s+w[k] \leq m$

x[k] := 1;

if (s + w[k] = m) then

write (x[1:k]);

else if (s + w[k] + w[k+1] ≤ m) then

sum of sub (s+w [k] , k+1, r - w[k]);

// generate right child.

if ((s+r - w[k] ≥ m) and (s+w[k+1] ≤ m)) then

{

x [k] = 0;

sum of sub (s, k+1 , r-w[k]);

}

}

consider the sum of subset problem

n = 6 whose weights are defined by

$w_i = \{5, 10, 12, 13, 15, 18\}$ capacity m = 30

intially s = 0 and k = 1 r

$r = \sum_{i=1}^{n} w_i$

$\Rightarrow r = w_1 + w_2 + w_3 + w_4 + w_5 + w_6$

$\boxed{r = 73}$

$s[j] = w[j] * x[j]$

$x[j] = 1 \Rightarrow$ generate left child and weight w_j is included.

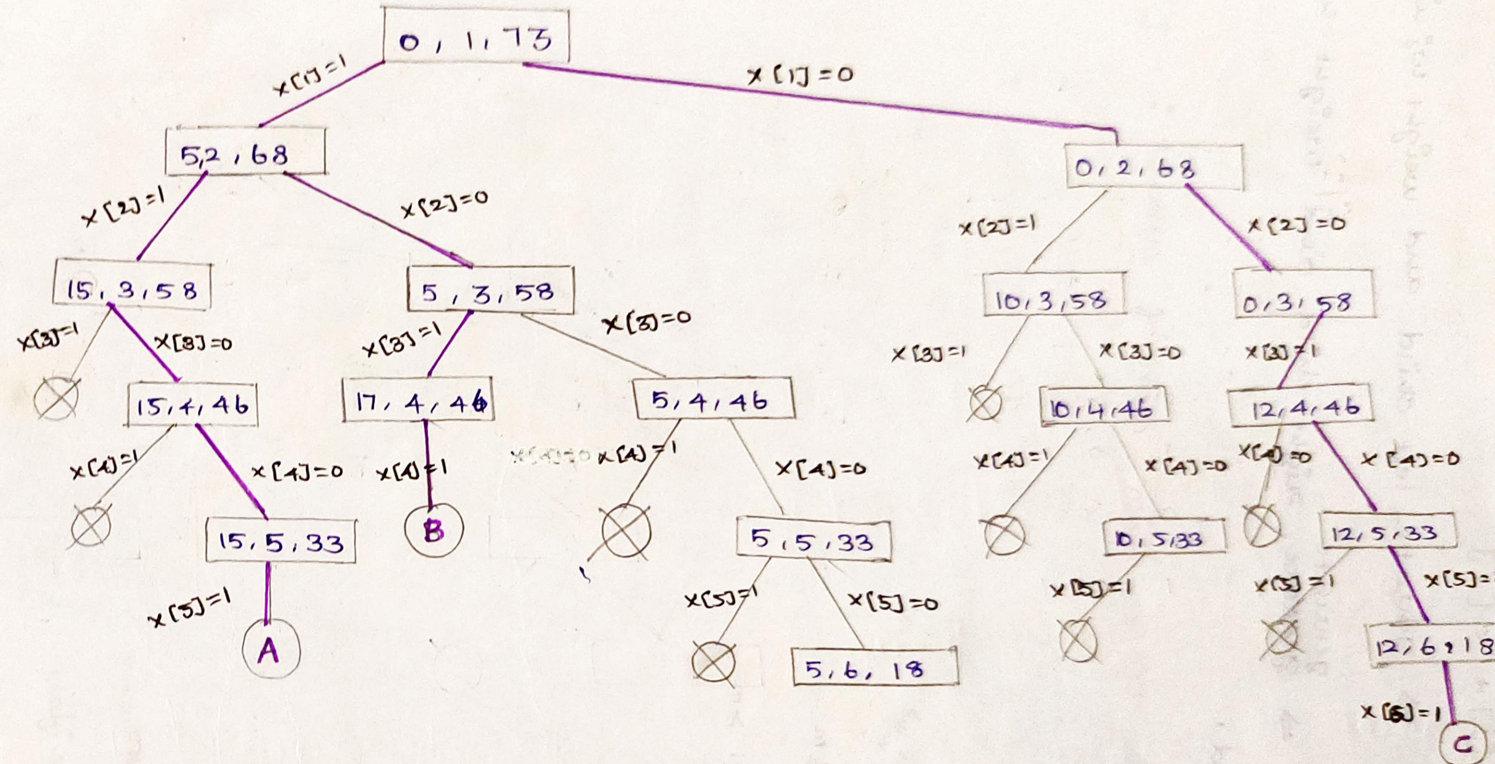
$x[j] = 0 \Rightarrow$ ~~generate~~ generate right child and weight w_j is not included.

state space tree for sum of subset :

condition where node is killed.

$$\sum_{i=1}^{k} w_i + w_{k+1} \leq m$$

$$\sum_{i=1}^{k} w_i x_i + \sum_{i=k+1}^{n} w_i > m$$



15+12

27

$\{5 \quad 10 \quad 12 \quad 13 \quad 15 \quad 18\}$

1 1 0 0 1 — A

1 0 1 1 — B

0 0 1 0 0 1 — C

The solution vector of sum of subset problem

is,

solution (A) $\Rightarrow x_i = \{1, 1, 0, 0, 1\}$

solution (B) $\Rightarrow x_i = \{1, 0, 1, 1\}$

solution (C) $\Rightarrow x_i = \{0, 0, 1, 0, 0, 1\}$