Small Sample

When the size of sample is less than 30, it is called small sample otherwise it is large sample.

Uses of Student-'t' test

1) It is used to test the significance of the difference between the mean of a small sample and the mean of the population.

2) It is used to test the significance between the mean of two samples.

Test I

Test of significance between sample mean and population mean.

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}}$$

Where $\bar{x}$ - Sample mean
μ - Pop. mean
s - Sample S.D.

dof : $n-1$.

Table	
2 tail	0.05
1 tail	0.10

95% confidence limit are

$$\left(\bar{x} \pm 1.96 \frac{s}{\sqrt{n-1}}\right)$$

Problem

) A machine is designed to produce insulating for electrical devices of average thickness of 0.025 cm. A random sample of 10 washers was found to have a thickness of 0.024 cm with a standard deviation of 0.02 cm. Test the significance of deviation

(3)

Sol:

$n = 10 < 30$

Small sample

It is small sample.

Sample mean $\bar{x} = 0.024$ cm

Population mean $\mu = 0.025$ cm

s SD $= 0.002$ cm

dof $= n-1 = 10-1 = 9$.

H_0: ~~The difference~~ The difference between $\bar{x}$ and μ is not significant.

$\mu = 0.025$

H_1: $\mu \neq 0.025$

$t = \dfrac{\bar{x}-\mu}{S.D/\sqrt{n-1}} = \dfrac{\bar{x}-\mu}{s/\sqrt{n-1}}$

$$= \frac{0.024 - 0.025}{0.002/\sqrt{9}}$$

Table

2 tail	0.05
1 tail	0.10

$= -1.5$

$|t| = 1.5$

cal $t = 1.5$.

tab t (9 deg) $= 2.262$

cal t < tab t.

Accept H_0

∴ The difference between

$\bar{x}$ and μ is not

significant.

2) The (2) Mean weakly sales of soap bars departmental stores was 146.3 bars. After Advertisement, the sales in 22 stores was increased to 153.7 with S.D of 17.2. was the Advertisement successful.

Small Sample (2) test 1

Sol

$n = 22 < 30$

1) t test

Mean — 3
dof — 3
para — 1

$\bar{x} = 153.7$

$\mu = 146.3$

$S.D = 17.2$

$dof = n-1 = 22-1 = 21$

6

H_0: The advertising was not successful.

H_1: $\mu > 146.3$ (Right tail) $\bar{x} > \mu$ (right tail)

$$t = \frac{\bar{x} - \mu}{S.D/\sqrt{n-1}}$$

$$= \frac{153.7 - 146.3}{17.2/\sqrt{21}} = 1.97$$

Cal t = 1.97

tab t (21 deg) = 1.72

1 tail → 0.10

Cal t > tab t

Reject H_0.

The advertising was successful.

~~Reject H_0.~~ (5) ~~Test 1~~

1) The mean life time of a sample of 25 bulbs is found as 1550 hrs with a S.D of 120 hrs. The Company manufacturing the bulbs claims that the average life of their bulbs is 1600 hours. Is the claim acceptable at 5% LOS

Sol

$\bar{x} = 1550, \quad s = 120, \quad n = 25$

$\mu = 1600$

$H_0 : \bar{x} = \mu$

$H_1 = \bar{x} < \mu$. (one tailed) (left tail)

$dof = 25 - 1 = 24$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = -2.04$$

$t_{tab} = 1.71$.

$t > t_{tab}$.

Reject H_0

Problem [when mean & SD not given directly]

A random sample of 10 boys had the following I.Q's 70, 120, 110, 101, 88, 83, 95, 98, 107, 100. Do these data support the assumption of a population mean I.Q's of 100 and find the range

Sol

$$A = \frac{120+70}{2} = 95 \approx 100$$

x_i	$d_i = x_i - A$	d_i^2
70	-30	900
120	20	400
110	10	100
101	1	1
88	-12	144
83	-17	289
95	-5	25
98	-2	4
107	7	49
100	0	0
	-28	1912

$$\bar{d} = \frac{\Sigma d_i}{n} = \frac{-28}{10}$$

$$= -2.8$$

$$\bar{x} = \bar{d} + A$$

$$= -2.8 + 100$$

$$= 97.2$$

$$s^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2$$

$H_0 : \bar{x} = \mu$

$H_1 : \bar{x} \neq \mu.$

$$s^2 = \frac{1912}{10} - (-2.8)^2 = 183.36$$

$(\mu = 100)$

$s = 13.54$

$$t = \frac{\bar{x} - \mu}{s/\sqrt{n-1}} = \frac{97.2 - 100}{13.54/\sqrt{9}} = -0.62.$$

dof $\nu = n-1 = 9.$

$t_{tab} = 2.26$

$|t| < t_{tab}$ Accept H_0.

95% Confidence limits $\bar{x} \pm 2.26 \frac{s}{\sqrt{n-1}}$

$= 107.41$ & 86.99

Test-2

Test of significance of the difference between mean of two ~~sam~~ samples drawn from the same normal population

$$Z = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$$

Where $\sigma = \sqrt{\dfrac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$

dof $\nu = n_1 + n_2 - 2$.

Note

If $n_1 = n_2 = n$ and if the samples are independent.

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\dfrac{s_1^2 + s_2^2}{n-1}}} \qquad \text{dof } \nu = 2n-2.$$

1) Sample of two types of electric bulbs were tested for length of life and the following data were obtained

	Size	Mean	S.D
Sample I	8	1234	36
Sample II	7	1036	40

Is the difference in the means sufficient to warrant that type 1 bulbs are superior to type 2 bulbs

Sol

$s_1 = 36$

$\bar{x}_1 = 1234$

$n_1 = 8$

$s_2 = 40$

$\bar{x}_2 = 1036$

$n_2 = 7$

(11)

$H_0 : \bar{x}_1 = \bar{x}_2$ $\quad H_1 : \bar{x}_1 > \bar{x}_2$ (Right tail)

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\left(\frac{n_1 s_1^2 + n_2 s_2^2}{n_1+n_2-2}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \frac{198}{21.0807}$$

$\doteq 9.39$

dof $v = n_1 + n_2 - 2 = 13$.

$t_{abt} = 1.77$

$t > t_{abt}$.

Reject H_0

Accept H_1

(1)

2) Two indep. samples of sizes 8 and 7 contained the following values

Sample I : 19 17 15 21 16 18 16 14

Sample II : 15 14 15 19 15 18 16

Is the difference between the sample means significant.

Sol

$14 \leftrightarrow 21$

$\frac{14+21}{2} = 17.5$

$A = 18$

$14 \leftrightarrow 19$

$\frac{14+19}{2} = 16.5$

$A = 16$

Sample I

x_i	$d_i = x_i - 18$	d_i^2
19	1	1
17	-1	1
15	-3	9
21	3	9
16	-2	4
18	0	0
16	-2	4
14	-4	16
	-8	44

Sample II

x_i'	$d_i = x_i - 16$		d_i^2
15	-1	-2α	1
14	-2	-3	4
15	-1	-2	1
19	3	2α	9
15	-1	-4	1
18	2	+1α	4
16	0	-1α	0
	0		20

$$\bar{d_i} = \frac{\sum d_i}{n}$$

$$\bar{x}_1 = \bar{d}_1 + 18$$

$$= \frac{1}{8}(-8) + 18 = 17$$

$$x_2 = 16 + \bar{d}_2$$

$$= 16 + 0 = 16$$

$$S_1^2 = \frac{\sum d_1^2}{n_1} - \left(\frac{\sum d_1}{n_1}\right)^2$$

$$= \frac{1}{8}(44) - \left(\frac{1}{8}(-8)\right)^2$$

$$= 4.5$$

$$S_1 = 2.12$$

$$S_2^2 = \frac{\sum d_2^2}{n_2} - \left(\frac{\sum d_2}{n_2}\right)^2$$

$$= \frac{1}{7}(20) - \frac{1}{7}(0)$$

$$= 2.857$$

$$S_2 = 1.69$$

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 \neq \bar{x}_2$

$$t = \frac{\bar{x}_1 - \bar{x}_2}{\sigma\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}} = 0.96$$

13

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2 - 2}}$$

Table (0.05)

ν dof = $n_1 + n_2 - 2 = 13$ $t_{abt} = 2.16$

Accept H_0

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3) The mean height and SD height of eight randomly chosen soldiers are 166.9 cm and 8.29 cm resply. The corresponding values of six randomly chosen sailors are 170.3 cm and 8.50 cm resps. Based on this data can we conclude that soldiers are ingeneral shorter than sailors.

$\bar{x}_1 = 166.9$ $\bar{x}_2 = 170.3$

$s_1 = 8.29$ $s_2 = 8.50$

$n_1 = 8$ $n_2 = 6$

$H_0 : \bar{x}_1 = \bar{x}_2$

$H_1 : \bar{x}_1 < \bar{x}_2$ (Left tail)

$t_{10} = 1.78$ Accept H_0

4) ~~Test~~ The biological values of protein from cow's milk and buffalo's milk are given below. Examine if the average value of protein in the two sample significantly differ.

Cow's milk	1.82	2.02	1.88	1.61	1.81	1.54
Buffalo's milk	2.00	1.83	1.86	2.03	2.16	1.88

2

Sol

$n_1 = n_2 = 6.$

$s^2 = E(x^2) - (E(x))^2$

$\bar{x}_1 = 1.78. = \frac{\sum x_1}{6}$

$E(x^2) = \sum x^2 p(x)$

$s_1^2 = \frac{\sum x_1^2}{6} - \left(\frac{\sum x_1}{6}\right)^2 \quad \left(s_1^2 = \frac{\sum x_1^2}{n_1} - \left(\frac{\sum x_1}{n_1}\right)^2\right)$

$= \frac{1}{6}(19.167) - (1.78)^2 = 0.0261.$

$x_1 \quad x_1^2 \quad x_2 \quad x_2^2$

$s_2^2 = \frac{\sum x_2^2}{6} - \left(\frac{\sum x_2}{6}\right)^2$

3.3124

4.0804

3.5344

2.5921

3.2761

2.3716

19.167

$= \frac{1}{6}(23.2599) - \left(\frac{11.879}{6}\right)^2$

$= 0.0164$

$v = 2n - 2$

$= 10.$

$t = \frac{\bar{x}_1 - \bar{x}_2}{\sqrt{\frac{s_1^2 + s_2^2}{n-1}}}$

$t_{tab} = 2.23$

$H_0: \bar{x}_1 = \bar{x}_2$ $H_1: \bar{x}_1 \neq \bar{x}_2$

$t = -2.03$ $|t| < t_{abs}$

Accept H_0

Type 3 (Paired t test)

If $n_1 = n_2 = n$. and if the pair values of S_1 and S_2 are associated in some way

then $t = \dfrac{\bar{d}}{s/\sqrt{n-1}}$ where $\bar{d} = \bar{x} - \bar{y}$.

$d_i = x_i - y_i$

$s^2 = \frac{1}{n} \sum (d_i - \bar{d})^2$. $\quad v = n-1$

$s^2 = \frac{\sum d^2}{n} - \left(\frac{\sum d}{n}\right)^2$

1) The following data relate to the marks obtained by 11 students in two tests one held at the beginning of a year and the other at the end of the year after coaching. Do these data indicate the the students have benefitted by coaching

Test 1 : 19 23 16 24 17 18 20 18 21 19

Test 2 17 24 20 24 20 22 20 20 18 22

① (16)

Sol

Here we use Paired 't' test.

$$d_i = x_i - y_i$$

The values of d's are.

-2, -1, -4, 0, -3, -4, 0, -2, 3, -3, 1.

$\Sigma d = -11$ $\quad \Sigma d^2 = 69.$ $\quad \bar{d} = \frac{\Sigma d}{n} = \frac{-11}{11} =$

$$s^2 = \frac{\Sigma d^2}{n} - \left(\frac{\Sigma d}{n}\right)^2 = \frac{69}{11} - (-1)^2 = 5.27$$

$s = 2.26.$

$H_0: \bar{d} = 0. (\bar{x}_1 = \bar{x}_2)$ (the students have not benefited by coaching]

$H_1: \bar{d} < 0. (\bar{x}_1 < \bar{x}_2)$ (one tail test)

$$t = \frac{\bar{d}}{s/\sqrt{n-1}} = \frac{-1}{2.296/\sqrt{10}} = -1.38$$

$\nu = dof = n-1 = 1.81 \quad = 11-1 = 10$

$|t| <$ tabt

Accept Ho no difference

1) Memory capacity of 9 students were tested before and after a course of meditation for a month. State whether the course was effective or not from the data below.

Before :	10	15	9	3	7	12	16	17	4
After :	12	17	8	5	6	11	18	20	3