Warshall's Algorithm :

Let $G = (V, E)$ be a directed graph with n vertices.

Let $A[i, j]$ be a path adjacency matrix such that,

$A[i,j] = 1$, if path exists

$A[i,j] = 0$, if path doesnot exists.

The warshall's Algorithm to determine the matrix R, $R[i,j]$ such that, $R[i,j]$ is defined by connecting two vertices with $A[i,j] = 0$.

Algorithm :

Algorithm warshall (A, R, n)

// A[i, j] → path adjacency matrix

// A[i, j] = 1, if path exists.

// A[i, j] = 0, if path doesnot exists

// n → number of vertices

// R[i, j] → determination of path between i

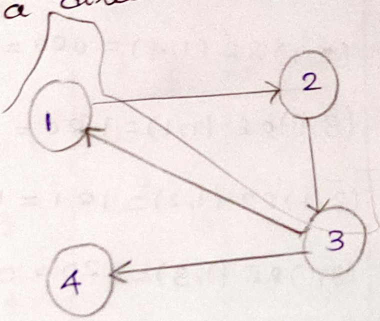
// and j with A[i, j] = 0

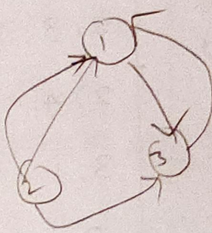
```
{
    for i:=1 to n do
        for j:=1 to n do
            R[i,j]:= A[i,j];
```

```
for K := 1 to n do
    for i := 1 to n do
        for j := 1 to n do
            R^(K)[i,j] := R^(K-1)[i,j] || R^(K-1)[i,K] && R^(K-1)[K,j];

    return R^(n);
}
```

Consider a directed graph





path adjacency matrix - A[i, j]

A[i, j]	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	1	0	0	1
4	0	0	0	0

A[i,j]	1	2	3
1	0		
2		0	
3			0

$R[i,j] = A[i,j]$

$R^{0}[i,j]$	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	1	0	0	1
4	0	0	0	0

Intermediate Vertex	$\langle i,j \rangle$	$R^{0}[i,j]$	$R^{0}[i,k]$ && $R^{0}[k,j]$	$R^{0}[i,j]$ || $R^{0}[i,k]$ && $R^{0}[k,j]$
	1, 1	0	(1,1) && (1,1) = 0&&0 = 0	0
	1, 2	1	(1,1) && (1,2) = 0&&1 = 0	1
	1, 3	0	(1,1) && (1,3) = 0&&0 = 0	0
	1, 4	0	(1,1) && (1,4) = 0&&0 = 0	0
	2, 1	0	(2,1) && (1,1) = 0&&0 = 0	0
	2, 2	0	(2,1) && (1,2) = 0&&1 = 0	0
	2, 3	1	(2,1) && (1,3) = 0&&0 = 0	1
k = 1	2, 4	0	(2,1) && (1,4) = 0&&0 = 0	0
	3, 1	1	(3,1) && (1,1) = 1&&0 = 0	1
	3, 2	0	(3,1) && (1,2) = 1&&1 = 1	$\boxed{1}$
	3, 3	0	(3,1) && (1,3) = 1&&0 = 0	0
	3, 4	1	(3,1) && (1,4) = 1&&0 = 0	1
	4, 1	0	(4,1) && (1,1) = 0&&0 = 0	0
	4, 2	0	(4,1) && (1,2) = 0&&1 = 0	0
	4, 3	0	(4,1) && (1,3) = 0&&0 = 0	0
	4, 4	0	(4,1) && (1,4) = 0&&0 = 0	0

$R^{1}[i,j]$	1	2	3	4
1	0	1	0	0
2	0	0	1	0
3	1	$\boxed{1}$	0	1
4	0	0	0	0

Intermediate vertex	<i, j>	R'[i,j]	R'[i,k] && R'[k,j]	R^2[i,j] = R'[i,j] || R'[i,k] && R'[k,j]
	1, 1	0	(1,2) && (2,1) = 1&0 = 0	0
	1, 2	1	(1,2) && (2,2) = 1&0 = 0	1
	1, 3	0	(1,2) && (2,3) = 1&1 = 1	1
	1, 4	0	(1,2) && (2,4) = 1&0 = 0	0
	2, 1	0	(2,2) && (2,1) = 0&0 = 0	0
	2, 2	0	(2,2) && (2,2) = 0&0 = 0	0
	2, 3	1	(2,2) && (2,3) = 0&1 = 0	1
	2, 4	0	(2,2) && (2,4) = 0&0 = 0	0
K=2	3, 1	1	(3,2) && (2,1) = 1&0 = 0	1
	3, 2	1	(3,2) && (2,2) = 1&0 = 0	1
	3, 3	0	(3,2) && (2,3) = 1&1 = 1	1
	3, 4	0	(3,2) && (2,4) = 1&0 = 0	0
	4, 1	0	(4,2) && (2,1) = 0&0 = 0	0
	4, 2	0	(4,2) && (2,2) = 0&0 = 0	0
	4, 3	0	(4,2) && (2,3) = 0&1 = 0	0
	4, 4	0	(4,2) && (2,4) = 0&0 = 0	0

R^2[i,j]	1	2	3	4
1	0	1	1	0
2	0	0	1	0
3	1	1	1	1
4	0	0	0	0

intermediate vertex	<i,j>	$R^3[i,j]$	$R^2(i,k)$ && $R^3(k,j)$	$R^2[i,j]$ || $R^2[i,k]$ && $R^2[k,j]$
	1,1	0	(1,3) && (3,1) = 1 & 1 = 1	$\boxed{1}$
	1,2	1	(1,3) && (3,2) = 1 & 1 = 1	1
	1,3	1	(1,3) && (3,3) = 1 & 1 = 1	1
	1,4	0	(1,3) && (3,4) = 1 & ~~0~~ 1 = ~~0~~ 1	~~0~~ $\boxed{1}$
	2,1	0	(2,3) && (3,1) = 1 & 1 = 1	$\boxed{1}$
	2,2	0	(2,3) && (3,2) = 1 & 1 = 1	$\boxed{1}$
	2,3	1	(2,3) && (3,3) = 1 & 1 = 1	1
	2,4	0	(2,3) && (3,4) = 1 & ~~0~~ 1 = ~~0~~ 1	~~0~~ $\boxed{1}$
K = 3	3,1	1	(3,3) && (3,1) = 1 & 1 = 1	1
	3,2	1	(3,3) && (3,2) = 1 & 1 = 1	1
	3,3	1	(3,3) && (3,3) = 1 & 1 = 1	1
	3,4	~~0~~ 1	(3,3) && (3,4) = 1 & ~~0~~ 1 = ~~0~~ 1	~~0~~ 1
	4,1	0	(4,3) && (3,1) = 0 & 1 = 0	0
	4,2	0	(4,3) && (3,2) = 0 & 1 = 0	0
	4,3	0	(4,3) && (3,3) = 0 & 1 = 0	0
	4,4	0	(4,3) && (3,4) = 0 & ~~0~~ 1 = 0	0

$R^3[i,j]$	1	2	3	4
1	1	1	1	~~0~~ 1
2	1	1	1	~~0~~ 1
3	1	1	1	~~0~~ 1
4	0	0	0	0

Intermediate vertex	<i, j>	$R^3(i,j)$	$R^3(i,k)$ && $R^3[k,j]$	$R^3[i,j]$ || $R^3(i,k)$ && $R^3(k,j)$
K=4	1,1	1	(1,4) && (4,1) = 0&0 = 0	1
	1,2	1	(1,4) && (4,2) = 0&0 = 0	1
	1,3	1	(1,4) && (4,3) = 0&0 = 0	1
	1,4	1	(1,4) && (4,4) = 0&0 = 0	1
	2,1	1	(2,4) && (4,1) = 0&0 = 0	1
	2,2	1	(2,4) && (4,2) = 0&0 = 0	1
	2,3	1	(2,4) && (4,3) = 0&0 = 0	1
	2,4	1	(2,4) && (4,4) = 0&0 = 0	1
	3,1	1	(3,4) && (4,1) = 0&0 = 0	1
	3,2	1	(3,4) && (4,2) = 0&0 = 0	1
	3,3	1	(3,4) && (4,3) = 0&0 = 0	1
	3,4	1	(3,4) && (4,4) = 0&0 = 0	0
	4,1	0	(4,4) && (4,1) = 0&0 = 0	0
	4,2	0	(4,4) && (4,2) = 0&0 = 0	0
	4,3	0	(4,4) && (4,3) = 0&0 = 0	0
	4,4	0	(4,4) && (4,4) = 0&0 = 0	0

$R^4[i,j]$	1	2	3	4
1	1	1	1	1
2	1	1	1	1
3	1	1	1	1
4	0	0	0	0

source vertex	destination vertex	~~Shortest~~ path travelled	~~Shortest~~ dist from i to j
{1}	{1}	1 - 2 - 3 - 1	1
{1}	{2}	1 - 2	1
{1}	{3}	1 - 2 - 3	1
{1}	{4}	1-2-3 - 4	1
{2}	{1}	2 - 3 - 1	1
{2}	{2}	2 - 3 - 1 - 2	1
{2}	{3}	2 - 3	1
{2}	{4}	2 - 3 - 4	1
{3}	{1}	3 - 1	1
{3}	{2}	3 - 1 - 2	1
{3}	{3}	3 - 1 - 2 - 3	1
{3}	{4}	3 - 4	1
{4}	{1}	—	0
{4}	{2}	—	0
{4}	{3}	—	0
{4}	{4}	—	0