Manifold Learning Homework 3

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习题 (48.1)**.** 证明. 首先，写出 $\mathbf{F}^T\mathbf{L}_W\mathbf{F}$ 第 (i,i) 个元素，有

$$\begin{aligned}\left(\mathbf{F}^T\mathbf{L}_W\mathbf{F}\right)_{i,i} &= \mathbf{f}_i^T\mathbf{L}_W\mathbf{f}_i \\ &= \mathbf{f}_i^T\mathbf{D}_W\mathbf{f}_i - \mathbf{f}_i^T\mathbf{W}\mathbf{f}_i\end{aligned}$$

其中

$$\begin{aligned}\mathbf{f}_i^T\mathbf{D}_W\mathbf{f}_i &= \left(\sum_j \mathbf{W}_{1,j}\mathbf{f}_i^1, \sum_j \mathbf{W}_{2,j}f_i^2, \ldots, \sum_j \mathbf{W}_{k,j}f_i^k\right)\mathbf{f}_i \\ &= \sum_k\sum_j \mathbf{W}_{k,j}\left(\mathbf{f}_i^k\right)^2\end{aligned}$$

$$\begin{aligned}\mathbf{f}_i^T\mathbf{W}\mathbf{f}_i &= \left(\sum_j \mathbf{W}j,1\mathbf{f}_i^j, \sum \mathbf{W}j,2\mathbf{f}_i^j, \ldots, \sum_j \mathbf{W}_{j,k}\mathbf{f}_i^j\right)\mathbf{f}_i \\ &= \sum_{j,k}\mathbf{W}_{j,k}\mathbf{f}_i^j\mathbf{f}_i^k\end{aligned}$$

所以有

$$\begin{aligned}\operatorname{tr}\left(\mathbf{F}^T\mathbf{L}_W\mathbf{F}\right) &= \sum_i\sum_j\sum_k \mathbf{W}_{k,j}\left(\left(\mathbf{f}_i^k\right)^2 - \mathbf{f}_i^k\mathbf{f}_i^j\right) \\ &= \frac{1}{2}\sum_k\sum_i\sum_j \mathbf{W}_{i,j}\left(\mathbf{f}_i - \mathbf{f}_j\right)_k^2 \\ &= \frac{1}{2}\sum_{i,j}\mathbf{W}_{i,j}\|\mathbf{f}_i - \mathbf{f}_j\|^2\end{aligned}$$

□

习题 (48.2)**.**