

Manifold Learning Homework 1

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Ex 1

$$\text{tr}(A) = 15$$

Ex 2

$$A = \begin{pmatrix} 0.9487 & -0.1778 & 0.2615 \\ 0 & 0.8269 & 0.5623 \\ -0.3126 & -0.5335 & 0.7845 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0.6993 & 0 \\ 0 & 0 & 14.3007 \end{pmatrix} \begin{pmatrix} 0.9487 & 0 & -0.3126 \\ -0.1778 & 0.8269 & -0.5335 \\ 0.2615 & 0.5623 & 0.7845 \end{pmatrix}$$

Ex 3

$$A_{Full-SVD} = \begin{pmatrix} -0.1409 & 0.8247 & 0.5477 & -0.0037 \\ -0.3439 & 0.4263 & -0.1880 & 0.4131 \\ -0.5470 & 0.0278 & -0.1880 & -0.8153 \\ -0.7501 & -0.3706 & 0.3679 & 0.4058 \end{pmatrix} \begin{pmatrix} 25.4624 & 0 & 0 \\ 0 & 1.2907 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -0.5045 & -0.7608 & -0.4092 \\ -0.5745 & -0.0571 & 0.8165 \\ -0.6445 & 0.6465 & -0.4082 \end{pmatrix}$$

$$A_{Thin-SVD} = \begin{pmatrix} -0.1409 & 0.8247 & 0.5477 \\ -0.3439 & 0.4263 & -0.7276 \\ -0.5470 & 0.0278 & -0.1880 \\ -0.7501 & -0.3706 & 0.3679 \end{pmatrix} \begin{pmatrix} 25.4624 & 0 & 0 \\ 0 & 1.2907 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} -0.5045 & -0.7608 & -0.4092 \\ -0.5745 & -0.0571 & 0.8165 \\ -0.6445 & 0.6465 & -0.4082 \end{pmatrix}$$

Ex 4

$$P_x = \begin{pmatrix} 0.4794 & -0.1850 & -0.2398 & 0.3973 \\ -0.1850 & 0.9343 & -0.0852 & 0.1412 \\ -0.2398 & -0.0852 & 0.8896 & 0.1830 \\ 0.3973 & 0.1412 & 0.1830 & 0.6968 \end{pmatrix}$$

Ex 5 *Proof:*

$$(AXB)_k = AXB_k$$

$$\begin{aligned} &= A(x_1, x_2, x_3, \dots, x_p) \begin{pmatrix} b_{1k} \\ b_{2k} \\ b_{3k} \\ \vdots \\ b_{pk} \end{pmatrix} \\ &= (b_{1k}A, b_{2k}A, b_{3k}A, \dots, b_{pk}A) \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_p \end{pmatrix} \\ &= (b_{1k}, b_{2k}, b_{3k}, \dots, b_{pk}) A \text{Vec}(X) \\ &= (B_k^T \otimes A) \text{Vec}(X) \end{aligned}$$

$$\therefore \text{Vec}(AXB) = (B^T \otimes A) \text{Vec}(X)$$

Ex 6 (8) Proof:

两边同乘 $(A + UCV^T)^{-1}$, 有:

$$\begin{aligned} I &= I - U(C^{-1} + V^T A^{-1} U)^{-1} V^T A^{-1} \\ &\quad + UCV^T A^{-1} - UCV^T A^{-1} U(C^{-1} + V^T A^{-1} U) V^T A^{-1} \end{aligned}$$

故只需有

$$-U(C^{-1} + V^T A^{-1} U)^{-1} + UC - UCV^T A^{-1} U(C^{-1} + V^T A^{-1} U)^{-1} = 0$$

即

$$\begin{aligned} C - (C^{-1} + V^T A^{-1} U)^{-1} - CV^T A^{-1} U(C^{-1} + V^T A^{-1} U)^{-1} &= 0 \\ C - (I + CV^T A^{-1} U)(C^{-1} + V^T A^{-1} U)^{-1} &= 0 \end{aligned}$$

而

$$I + CV^T A^{-1} U = C(C^{-1} + V^T A^{-1} U)$$

故原式得证

$$\mathbf{Ex\ 7\ (21)\ 1-norm} = 30$$

$$\mathbf{2-norm} = 25.4624$$

$$\mathbf{\infty-norm} = 33$$

$$\mathbf{F1-norm} = 78$$

$$\mathbf{F2-norm} = 25.4951$$

$$\mathbf{F\infty-norm} = 12$$

$$\mathbf{Nuclear-norm} = 26.7531$$

$$\mathbf{(2,1)-norm} = 43.0445$$