

# Deep Generative Models

## Lecture supp

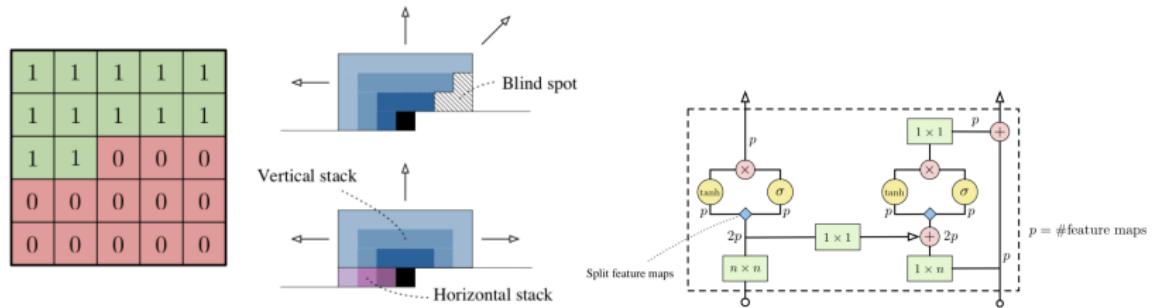
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Autumn, 2021

# GatedPixelCNN

# GatedPixelCNN (2016)

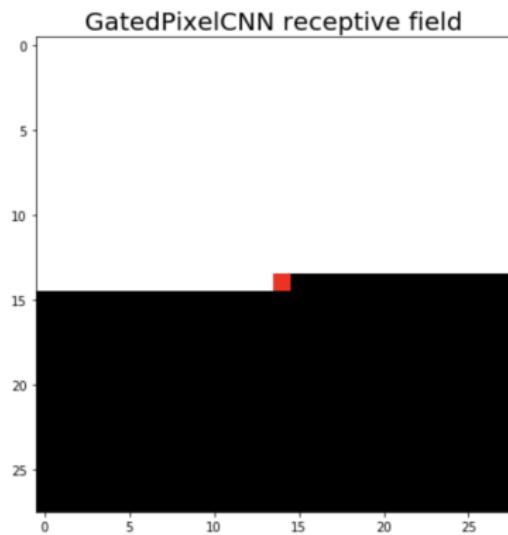
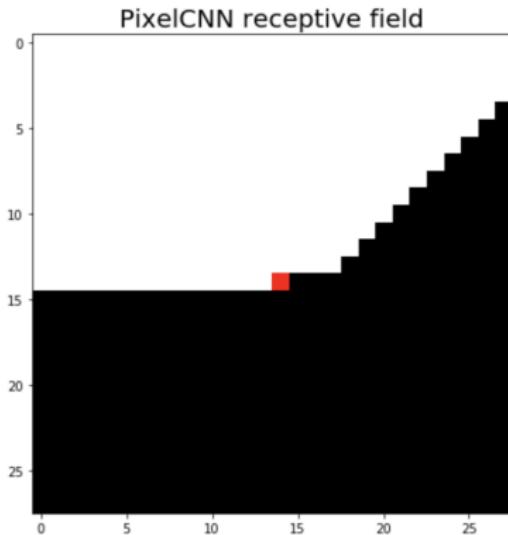


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Van den Oord A. et al. Conditional image generation with pixelcnn decoders

<https://arxiv.org/pdf/1606.05328.pdf>

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## Extensions

- ▶ **PixelCNN++**: *Improving the PixelCNN with Discretized Logistic Mixture Likelihood and Other Modifications*  
<https://arxiv.org/pdf/1701.05517.pdf>  
(mixture of logistics instead of softmax);
- ▶ **PixelSNAIL**: *An Improved Autoregressive Generative Model*  
<https://arxiv.org/pdf/1712.09763.pdf>  
(self-attention to learn optimal autoregression ordering).

## ELBO gradient, Log derivative trick

## ELBO gradient (E-step, $\nabla_{\phi}\mathcal{L}(\phi, \theta)$ )

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_q \log p(\mathbf{X}|\mathbf{Z}, \theta) - KL(q(\mathbf{Z}|\mathbf{X}, \phi) || p(\mathbf{Z})) \rightarrow \max_{\phi, \theta} .$$

Difference from M-step: density function  $q(\mathbf{z}|\mathbf{x}, \phi)$  depends on the parameters  $\phi$ , it is impossible to use Monte-Carlo estimation:

$$\nabla_{\phi}\mathcal{L}(\phi, \theta) = \int \nabla_{\phi}q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta) d\mathbf{Z} - \nabla_{\phi}KL$$

### Log-derivative trick

$$\nabla_{\xi}q(\eta|\xi) = q(\eta|\xi) \left( \frac{\nabla_{\xi}q(\eta|\xi)}{q(\eta|\xi)} \right) = q(\eta|\xi) \nabla_{\xi} \log q(\eta|\xi).$$

$$\nabla_{\phi}q(\mathbf{Z}|\mathbf{X}, \phi) = q(\mathbf{Z}|\mathbf{X}, \phi) \nabla_{\phi} \log q(\mathbf{Z}|\mathbf{X}, \phi).$$

## ELBO gradient (E-step, $\nabla_{\phi}\mathcal{L}(\phi, \theta)$ )

$$\begin{aligned}\nabla_{\phi}\mathcal{L}(\phi, \theta) &= \int \nabla_{\phi}q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta) d\mathbf{Z} - \nabla_{\phi}KL = \\ &= \int q(\mathbf{Z}|\mathbf{X}, \phi) [\nabla_{\phi} \log q(\mathbf{Z}|\mathbf{X}, \phi) \log p(\mathbf{X}|\mathbf{Z}, \theta)] d\mathbf{Z} - \nabla_{\phi}KL\end{aligned}$$

After applying log-reparametrization trick, we are able to use Monte-Carlo estimation:

$$\begin{aligned}\nabla_{\phi}\mathcal{L}(\phi, \theta) &\approx n \nabla_{\phi} \log q(\mathbf{z}_i^* | \mathbf{x}_i, \phi) \log p(\mathbf{x}_i | \mathbf{z}_i^*, \theta) - \nabla_{\phi}KL, \\ \mathbf{z}_i^* &\sim q(\mathbf{z}_i | \mathbf{x}_i, \phi).\end{aligned}$$

### Problem

Unstable solution with huge variance.

### Solution

Reparametrization trick

## Mean field approximation

# Bayesian framework

## Bayes theorem

$$p(\mathbf{t}|\mathbf{x}) = \frac{p(\mathbf{x}|\mathbf{t})p(\mathbf{t})}{p(\mathbf{x})} = \frac{p(\mathbf{x}|\mathbf{t})p(\mathbf{t})}{\int p(\mathbf{x}|\mathbf{t})p(\mathbf{t})d\mathbf{t}}$$

- ▶  $\mathbf{x}$  – observed variables,  $\mathbf{t}$  – unobserved variables (latent variables/parameters);
- ▶  $p(\mathbf{x}|\mathbf{t})$  – likelihood;
- ▶  $p(\mathbf{x}) = \int p(\mathbf{x}|\mathbf{t})p(\mathbf{t})d\mathbf{t}$  – evidence;
- ▶  $p(\mathbf{t})$  – prior distribution,  $p(\mathbf{t}|\mathbf{x})$  – posterior distribution.

## Meaning

We have unobserved variables  $\mathbf{t}$  and some prior knowledge about them  $p(\mathbf{t})$ . Then, the data  $\mathbf{x}$  has been observed. Posterior distribution  $p(\mathbf{t}|\mathbf{x})$  summarizes the knowledge after the observations.

## Variational Lower Bound

We have set of objects  $\mathbf{X} = \{\mathbf{x}_i\}_{i=1}^n$ . The goal is to perform Bayesian inference on the unobserved variables  $\mathbf{T} = \{\mathbf{t}_i\}_{i=1}^n$ .

### Evidence Lower Bound (ELBO)

$$\begin{aligned}\log p(\mathbf{X}) &= \log \frac{p(\mathbf{X}, \mathbf{T})}{p(\mathbf{T}|\mathbf{X})} = \\&= \int q(\mathbf{T}) \log \frac{p(\mathbf{X}, \mathbf{T})}{p(\mathbf{T}|\mathbf{X})} d\mathbf{T} = \int q(\mathbf{T}) \log \frac{p(\mathbf{X}, \mathbf{T})q(\mathbf{T})}{p(\mathbf{T}|\mathbf{X})q(\mathbf{T})} d\mathbf{T} = \\&= \int q(\mathbf{T}) \log \frac{p(\mathbf{X}, \mathbf{T})}{q(\mathbf{T})} d\mathbf{T} + \int q(\mathbf{T}) \log \frac{q(\mathbf{T})}{p(\mathbf{T}|\mathbf{X})} d\mathbf{T} = \\&= \mathcal{L}(q) + KL(q(\mathbf{T})||p(\mathbf{T}|\mathbf{X})) \geq \mathcal{L}(q).\end{aligned}$$

We would like to maximize lower bound  $\mathcal{L}(q)$ .

## Mean field approximation

### Independence assumption

$$q(\mathbf{T}) = \prod_{i=1}^k q_i(\mathbf{T}_i), \quad \mathbf{T} = [\mathbf{T}_1, \dots, \mathbf{T}_k], \quad \mathbf{T}_j = \{\mathbf{t}_{ij}\}_{i=1}^n, \quad \mathbf{t}_i = \{\mathbf{T}_{ij}\}_{j=1}^k.$$

Block coordinate optimization of ELBO for  $q_j(\mathbf{T}_j)$

$$\begin{aligned} \mathcal{L}(q) &= \int q(\mathbf{T}) \log \frac{p(\mathbf{X}, \mathbf{T})}{q(\mathbf{T})} d\mathbf{T} = \int \left[ \prod_{i=1}^k q_i(\mathbf{T}_i) \right] \log \frac{p(\mathbf{X}, \mathbf{T})}{\left[ \prod_{i=1}^k q_i(\mathbf{T}_i) \right]} \prod_{i=1}^k d\mathbf{T}_i = \\ &= \int \left[ \prod_{i=1}^k q_i \right] \log p(\mathbf{X}, \mathbf{T}) \prod_{i=1}^k d\mathbf{T}_i - \sum_{i=1}^k \int \left[ \prod_{j=1}^k q_j \right] \log q_i \prod_{j=1}^k d\mathbf{T}_j = \\ &= \int q_j \left[ \int \log p(\mathbf{X}, \mathbf{T}) \prod_{i \neq j} q_i d\mathbf{T}_i \right] d\mathbf{T}_j - \\ &\quad - \int q_j \log q_j d\mathbf{T}_j + \text{const}(q_j) \rightarrow \max_{q_j} \end{aligned}$$

## Mean field approximation

Block coordinate optimization of ELBO for  $q_j(\mathbf{T}_j)$

$$\begin{aligned}\mathcal{L}(q) &= \int q_j \left[ \int \log p(\mathbf{X}, \mathbf{T}) \prod_{i \neq j} q_i d\mathbf{T}_i \right] d\mathbf{T}_j - \int q_j \log q_j d\mathbf{T}_j + \text{const}(q_j) = \\ &= \int q_j \log \hat{p}(\mathbf{X}, \mathbf{T}_j) d\mathbf{T}_j - \int q_j \log q_j d\mathbf{T}_j + \text{const}(q_j) \rightarrow \max_{q_j}.\end{aligned}$$

Here we introduce

$$\log \hat{p}(\mathbf{X}, \mathbf{T}_j) = \int \log p(\mathbf{X}, \mathbf{T}) \prod_{i \neq j} q_i d\mathbf{T}_i = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}(q_j)$$

Final ELBO derivation for  $q_j(\mathbf{T}_j)$

$$\begin{aligned}\mathcal{L}(q) &= \int q_j(\mathbf{T}_j) \log \hat{p}(\mathbf{X}, \mathbf{T}_j) d\mathbf{T}_j - \int q_j(\mathbf{T}_j) \log q_j(\mathbf{T}_j) d\mathbf{T}_j + \text{const}(q_j) = \\ &\quad \int q_j(\mathbf{T}_j) \log \frac{\hat{p}(\mathbf{X}, \mathbf{T}_j)}{q_j(\mathbf{T}_j)} d\mathbf{T}_j + \text{const}(q_j) = \\ &= -KL(q_j(\mathbf{T}_j) || \hat{p}(\mathbf{X}, \mathbf{T}_j)) + \text{const}(q_j) \rightarrow \max_{q_j}.\end{aligned}$$

## Mean field approximation

Independence assumption

$$q(\mathbf{T}) = \prod_{i=1}^k q_i(\mathbf{T}_i), \quad \mathbf{T} = [\mathbf{T}_1, \dots, \mathbf{T}_k], \quad \mathbf{T}_j = \{\mathbf{t}_{ij}\}_{i=1}^n.$$

ELBO

$$\mathcal{L}(q) = -KL(q_j(\mathbf{T}_j) || \hat{p}(\mathbf{X}, \mathbf{T}_j)) + \text{const}(q_j) \rightarrow \max_{q_j}.$$

Solution

$$q_j(\mathbf{T}_j) = \text{const} \cdot \hat{p}(\mathbf{X}, \mathbf{T}_j)$$

$$\log \hat{p}(\mathbf{X}, \mathbf{T}_j) = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}$$

$$\log q_j(\mathbf{T}_j) = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}$$

# Mean field approximation

## ELBO

$$\mathcal{L}(q) = -KL(q_j(\mathbf{T}_j) || \hat{p}(\mathbf{X}, \mathbf{T}_j)) + \text{const}(q_j) \rightarrow \max_{q_j}.$$

## Solution

$$\log q_j(\mathbf{T}_j) = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}$$

Assumptions:

- ▶  $\mathbf{T} = [\mathbf{T}_1, \mathbf{T}_2] = [\mathbf{Z}, \boldsymbol{\theta}]$ ,  $q(\mathbf{T}) = q(\mathbf{T}_1) \cdot q(\mathbf{T}_2) = q(\mathbf{Z}) \cdot q(\boldsymbol{\theta})$ .
- ▶ restrict a class of probability distributions for  $\boldsymbol{\theta}$  to Dirac delta functions:

$$q_2 = q(\mathbf{T}_2) = q(\boldsymbol{\theta}) = \delta(\boldsymbol{\theta} - \boldsymbol{\theta}^*).$$

Under the restrictions the exact solution for  $q_2$  is not reached ( $KL$  can be greater than 0).

## Mean field approximation

General solution

$$\log q_j(\mathbf{T}_j) = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}$$

Solution for  $q_1 = q(\mathbf{Z})$

$$\begin{aligned}\log q(\mathbf{Z}) &= \int q(\boldsymbol{\theta}) \log p(\mathbf{X}, \mathbf{Z}, \boldsymbol{\theta}) d\boldsymbol{\theta} + \text{const} = \\ &= \int \delta(\boldsymbol{\theta} - \boldsymbol{\theta}^*) \log p(\mathbf{X}, \mathbf{Z}, \boldsymbol{\theta}) d\boldsymbol{\theta} + \text{const} = \\ &= \log p(\mathbf{Z}|\mathbf{X}, \boldsymbol{\theta}^*) + \text{const.}\end{aligned}$$

EM-algorithm (E-step)

$$q(\mathbf{Z}) = \arg \max_q \mathcal{L}(q, \boldsymbol{\theta}^*) = \arg \min_q KL(q||p) = p(\mathbf{Z}|\mathbf{X}, \boldsymbol{\theta}^*).$$

## Mean field approximation

### ELBO

$$\mathcal{L}(q) = -KL(q_j(\mathbf{T}_j) || \hat{p}(\mathbf{X}, \mathbf{T}_j)) + \text{const}(q_j) \rightarrow \max_{q_j}.$$

ELBO maximization w.r.t.  $q_2 = q(\boldsymbol{\theta}) = \delta(\boldsymbol{\theta} - \boldsymbol{\theta}^*)$

$$\begin{aligned}\mathcal{L}(q_1, q_2) &= -KL(q(\boldsymbol{\theta}) || \hat{p}(\mathbf{X}, \boldsymbol{\theta})) + \text{const}(\boldsymbol{\theta}^*) \\ &= \int q(\boldsymbol{\theta}) \log \frac{\hat{p}(\mathbf{X}, \boldsymbol{\theta})}{q(\boldsymbol{\theta})} d\boldsymbol{\theta} + \text{const}(\boldsymbol{\theta}^*) \\ &= \int q(\boldsymbol{\theta}) \log \hat{p}(\mathbf{X}, \boldsymbol{\theta}) d\boldsymbol{\theta} - \int q(\boldsymbol{\theta}) \log q(\boldsymbol{\theta}) d\boldsymbol{\theta} + \text{const}(\boldsymbol{\theta}^*) \\ &= \int \delta(\boldsymbol{\theta} - \boldsymbol{\theta}^*) \log \hat{p}(\mathbf{X}, \boldsymbol{\theta}) d\boldsymbol{\theta} + \text{const}(\boldsymbol{\theta}^*) \rightarrow \max_{\boldsymbol{\theta}^*}\end{aligned}$$

## Mean field approximation

ELBO maximization w.r.t.  $q_2 = q(\theta) = \delta(\theta - \theta^*)$

$$\begin{aligned}\mathcal{L}(q_1, q_2) &= \int \delta(\theta - \theta^*) \log \hat{p}(\mathbf{X}, \theta) d\theta + \text{const} = \log \hat{p}(\mathbf{X}, \theta^*) + \text{const} \\ &= \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const} = \mathbb{E}_{q_1} \log p(\mathbf{X}, \mathbf{Z}, \theta^*) + \text{const} \\ &= \int q(\mathbf{Z}) \log p(\mathbf{X}, \mathbf{Z} | \theta^*) d\mathbf{Z} + \log p(\theta^*) + \text{const} \rightarrow \max_{\theta^*}\end{aligned}$$

EM-algorithm (M-step)

$$\begin{aligned}\mathcal{L}(q, \theta) &= \int q(\mathbf{Z}) \log \frac{p(\mathbf{X}, \mathbf{Z} | \theta)}{q(\mathbf{Z})} d\mathbf{Z} \\ &= \int q(\mathbf{Z}) \log p(\mathbf{X}, \mathbf{Z} | \theta) d\mathbf{Z} + \text{const} \rightarrow \max_{\theta}\end{aligned}$$

# Mean field approximation

## Solution

$$\log q_j(\mathbf{T}_j) = \mathbb{E}_{i \neq j} \log p(\mathbf{X}, \mathbf{T}) + \text{const}$$

## EM algorithm (special case)

- ▶ Initialize  $\theta^*$ ;
- ▶ E-step

$$q(\mathbf{Z}) = \arg \max_q \mathcal{L}(q, \theta^*) = \arg \min_q KL(q||p) = p(\mathbf{Z}|\mathbf{X}, \theta^*);$$

- ▶ M-step
$$\theta^* = \arg \max_{\theta} \mathcal{L}(q, \theta);$$
- ▶ Repeat E-step and M-step until convergence.

## Flows intuition

## Flows intuition

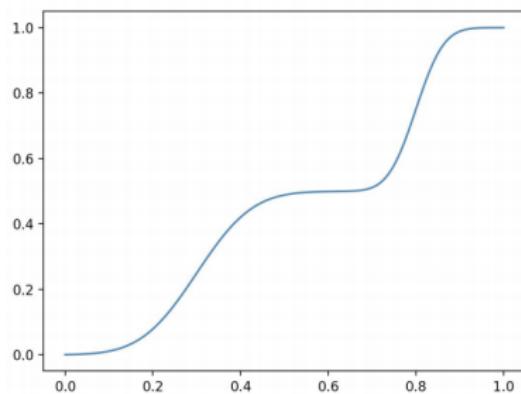
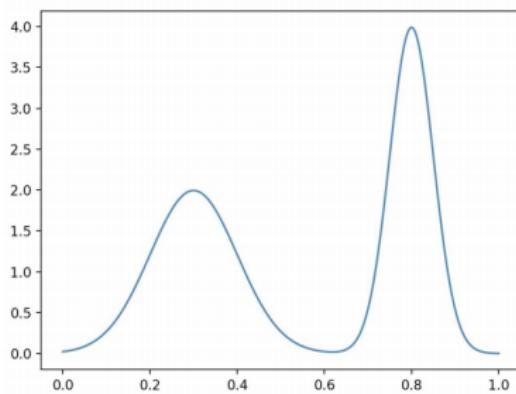
Let  $\xi$  be a random variable with density  $p(\xi)$ . Then

$$\eta = F(\xi) = \int_{-\infty}^{\xi} p(t)dt \sim U[0, 1].$$

$$P(\eta < y) = P(F(\xi) < y) = P(\xi < F^{-1}(y)) = F(F^{-1}(y)) = y$$

Hence

$$\eta \sim U[0, 1]; \quad \xi = F^{-1}(\eta) \quad \Rightarrow \quad \xi \sim p(\xi).$$



## Flows intuition

- ▶ Let  $z \sim p(z)$  is a random variable with base distribution  $p(z) = U[0, 1]$ .
- ▶ Let  $x \sim p(x)$  is a random variable with complex distribution  $p(x)$  and cdf  $F(x)$ .
- ▶ Then noise variable  $z$  can be transformed to  $x$  using inverse cdf  $F^{-1}$  ( $x = F^{-1}(z)$ ).

How to transform random variable  $z$  which has a distribution different from uniform to  $x$ ?

- ▶ Let  $z \sim p(z)$  is a random variable with base distribution  $p(z)$  and cdf  $G(z)$ .
- ▶ Then  $z_0 = G(z)$  has base distribution  $p(z_0) = U[0, 1]$ .
- ▶ Let  $x \sim p(x)$  is a random variable with complex distribution  $p(x)$  and cdf  $F(x)$ .
- ▶ Then noise variable  $z$  can be transformed to  $x$  using cdf  $G$  and inverse cdf  $F^{-1}$  ( $x = F^{-1}(z_0) = F^{-1}(G(z))$ ).

## ELBO interpretations

$$\log p(\mathbf{x}|\theta) = \mathcal{L}(q, \theta) + KL(q(\mathbf{z}|\mathbf{x}, \phi)||p(\mathbf{z}|\mathbf{x}, \theta)).$$

$$\mathcal{L}(q, \theta) = \int q(\mathbf{z}|\mathbf{x}, \phi) \log \frac{p(\mathbf{x}, \mathbf{z}|\theta)}{q(\mathbf{z}|\mathbf{x}, \phi)} d\mathbf{z}.$$

- ▶ Evidence minus posterior KL

$$\mathcal{L}(q, \theta) = \log p(\mathbf{x}|\theta) - KL(q(\mathbf{z}|\mathbf{x}, \phi)||p(\mathbf{z}|\mathbf{x}, \theta)).$$

- ▶ Average negative energy plus entropy

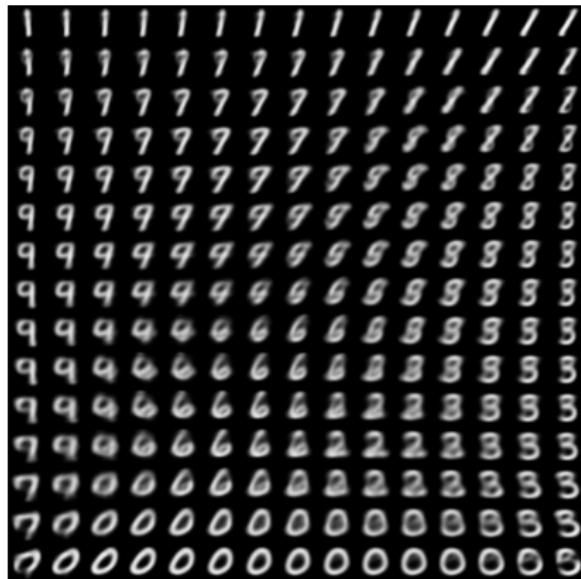
$$\begin{aligned}\mathcal{L}(q, \theta) &= \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} [\log p(\mathbf{x}, \mathbf{z}|\theta) - \log q(\mathbf{z}|\mathbf{x}, \phi)] \\ &= \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} \log p(\mathbf{x}, \mathbf{z}|\theta) + \mathbb{H}[q(\mathbf{z}|\mathbf{x}, \phi)].\end{aligned}$$

- ▶ Average reconstruction minus KL to prior

$$\begin{aligned}\mathcal{L}(q, \theta) &= \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} [\log p(\mathbf{x}|\mathbf{z}, \theta) + \log p(\mathbf{z}) - \log q(\mathbf{z}|\mathbf{x}, \phi)] \\ &= \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} \log p(\mathbf{x}|\mathbf{z}, \theta) - KL(q(\mathbf{z}|\mathbf{x}, \phi)||p(\mathbf{z})).\end{aligned}$$

# Variational Autoencoder

Generated images for latent objects  $\mathbf{z}$  sampled from prior  $\mathcal{N}(0, \mathbf{I})$



## IWAE, active units, posterior collapse

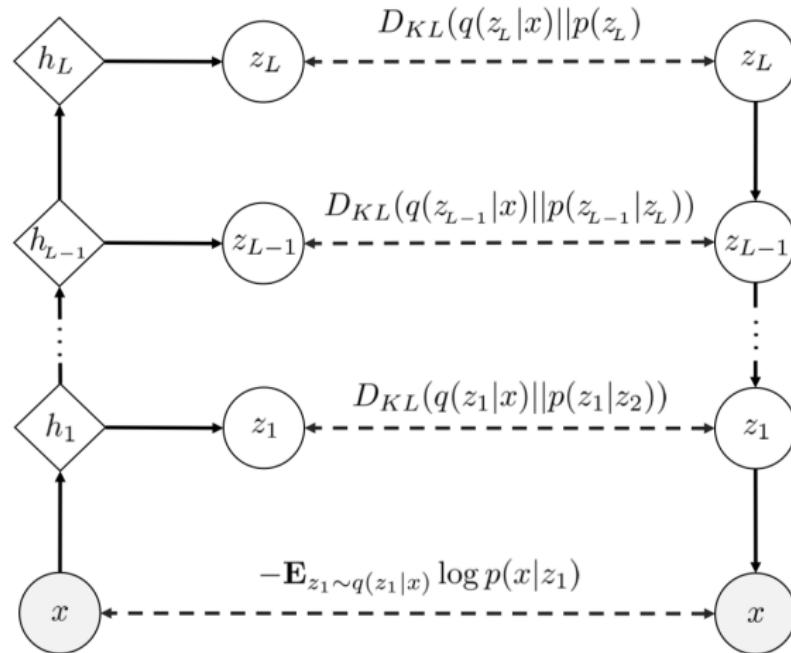
How to determine whether all VAE latent variables are informative?

$$A_i = \text{cov}_{\mathbf{x}} (\mathbb{E}_{q(z_i|\mathbf{x})}[z_i]) > 0.01 \Leftrightarrow z_i \text{ is active}$$

| # stoch.<br>layers | k  | MNIST |                 |       |                 | OMNIGLOT |                 |        |                 |
|--------------------|----|-------|-----------------|-------|-----------------|----------|-----------------|--------|-----------------|
|                    |    | VAE   |                 | IWAE  |                 | VAE      |                 | IWAE   |                 |
|                    |    | NLL   | active<br>units | NLL   | active<br>units | NLL      | active<br>units | NLL    | active<br>units |
| 1                  | 1  | 86.76 | 19              | 86.76 | 19              | 108.11   | 28              | 108.11 | 28              |
|                    | 5  | 86.47 | 20              | 85.54 | 22              | 107.62   | 28              | 106.12 | 34              |
|                    | 50 | 86.35 | 20              | 84.78 | 25              | 107.80   | 28              | 104.67 | 41              |
| 2                  | 1  | 85.33 | 16+5            | 85.33 | 16+5            | 107.58   | 28+4            | 107.56 | 30+5            |
|                    | 5  | 85.01 | 17+5            | 83.89 | 21+5            | 106.31   | 30+5            | 104.79 | 38+6            |
|                    | 50 | 84.78 | 17+5            | 82.90 | 26+7            | 106.30   | 30+5            | 103.38 | 44+7            |

## PixelVAE, Hierarchical VAE

## Hierarchical VAE



## Hierarchical decomposition

$$p(\mathbf{z}_1, \dots, \mathbf{z}_L) = p(\mathbf{z}_L)p(\mathbf{z}_{L-1}|\mathbf{z}_L)\dots p(\mathbf{z}_1, \mathbf{z}_2);$$

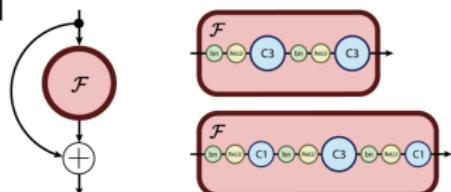
$$q(\mathbf{z}_1, \dots, \mathbf{z}_L|\mathbf{x}) = q(\mathbf{z}_1|\mathbf{x})\dots q(\mathbf{z}_L|\mathbf{x}).$$

## ELBO

$$\begin{aligned}\mathcal{L}(q, \theta) &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \theta) - KL(q(\mathbf{z}_1, \dots, \mathbf{z}_L|\mathbf{x})||p(\mathbf{z}_1, \dots, \mathbf{z}_L)) \\ &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \theta) - \int \prod_{j=1}^L q(\mathbf{z}_j|\mathbf{x}) \sum_{i=1}^L \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_1 \dots d\mathbf{z}_L \\ &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \theta) - \sum_{i=1}^L \int \prod_{j=1}^L q(\mathbf{z}_j|\mathbf{x}) \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_1 \dots d\mathbf{z}_L \\ &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \theta) - \sum_{i=1}^L \int q(\mathbf{z}_{i+1}|\mathbf{x}) q(\mathbf{z}_i|\mathbf{x}) \log \frac{q(\mathbf{z}_i|\mathbf{x})}{p(\mathbf{z}_i|\mathbf{z}_{i+1})} d\mathbf{z}_i d\mathbf{z}_{i+1} \\ &= \mathbb{E}_{q(\mathbf{z}_1|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}_1, \theta) - \sum_{i=1}^L \mathbb{E}_{q(\mathbf{z}_{i+1}|\mathbf{x})} [KL(q(\mathbf{z}_i|\mathbf{x})||p(\mathbf{z}_i|\mathbf{z}_{i+1}))]\end{aligned}$$

## RevNet, i-RevNet

- ▶ Modern neural networks are trained via backpropagation.
- ▶ Residual networks are state of the art in image classification.
- ▶ Backpropagation requires storing the network activations.



## Problem

Storing the activations imposes an increasing memory burden.

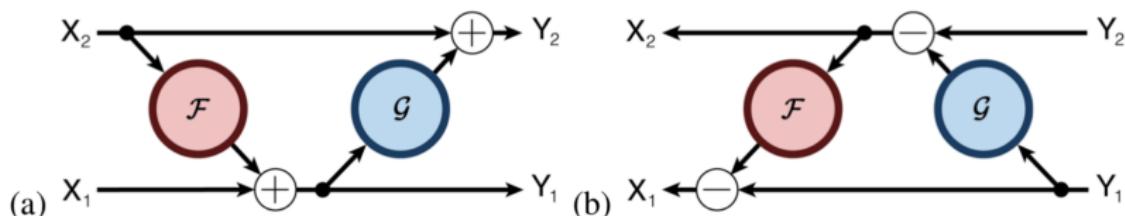
GPUs have limited memory capacity, leading to constraints often exceeded by state-of-the-art architectures (with thousand layers).

## NICE

$$\begin{cases} \mathbf{z}_1 = \mathbf{x}_1; \\ \mathbf{z}_2 = \mathbf{x}_2 + \mathcal{F}(\mathbf{x}_1, \theta); \end{cases} \Leftrightarrow \begin{cases} \mathbf{x}_1 = \mathbf{z}_1; \\ \mathbf{x}_2 = \mathbf{z}_2 - \mathcal{F}(\mathbf{z}_1, \theta). \end{cases}$$

## RevNet

$$\begin{cases} \mathbf{y}_1 = \mathbf{x}_1 + \mathcal{F}(\mathbf{x}_2, \theta); \\ \mathbf{y}_2 = \mathbf{x}_2 + \mathcal{G}(\mathbf{y}_1, \theta); \end{cases} \Leftrightarrow \begin{cases} \mathbf{x}_2 = \mathbf{y}_2 - \mathcal{F}(\mathbf{y}_1, \theta); \\ \mathbf{x}_1 = \mathbf{y}_1 - \mathcal{G}(\mathbf{x}_2, \theta). \end{cases}$$



| Architecture | CIFAR-10 [15] |              | CIFAR-100 [15] |               |
|--------------|---------------|--------------|----------------|---------------|
|              | ResNet        | RevNet       | ResNet         | RevNet        |
| 32 (38)      | <b>7.14%</b>  | 7.24%        | 29.95%         | <b>28.96%</b> |
| 110          | <b>5.74%</b>  | 5.76%        | 26.44%         | <b>25.40%</b> |
| 164          | 5.24%         | <b>5.17%</b> | <b>23.37%</b>  | 23.69%        |

- ▶ If the network contains non-reversible blocks (poolings, strides), activations for these blocks should be stored.
- ▶ To avoid storing activations in the modern frameworks, the backward pass should be manually redefined.

## Hypothesis

The success of deep convolutional networks is based on progressively discarding uninformative variability about the input with respect to the problem at hand.

- ▶ It is difficult to recover images from their hidden representations.
- ▶ Information bottleneck principle: an optimal representation must reduce the MI between an input and its representation to reduce uninformative variability + maximize the MI between the output and its representation to preserve each class from collapsing onto other classes.

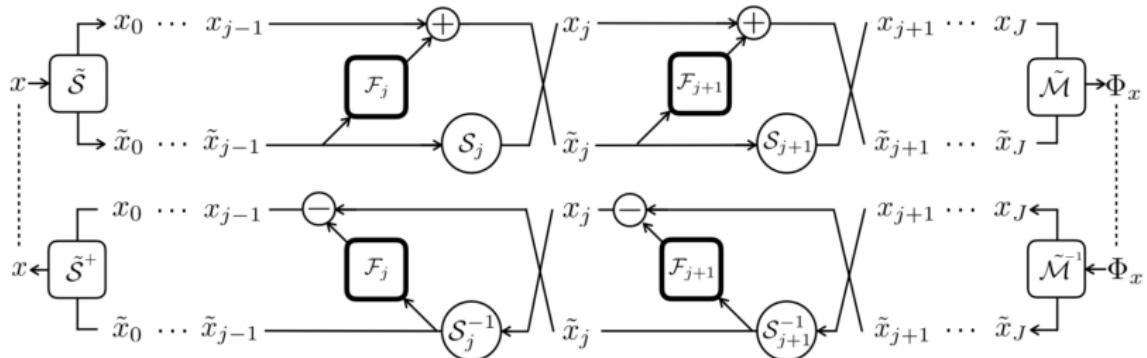
## Hypothesis

The success of deep convolutional networks is based on progressively discarding uninformative variability about the input with respect to the problem at hand.

## Idea

Build a cascade of homeomorphic layers (i-RevNet), a network that can be fully inverted up to the final projection onto the classes, i.e. no information is discarded.

# i-RevNet, 2018



| Architecture         | Injective | Bijective | Top-1 error | Parameters |
|----------------------|-----------|-----------|-------------|------------|
| ResNet               | -         | -         | 24.7        | 26M        |
| RevNet               | -         | -         | 25.2        | 28M        |
| <i>i</i> -RevNet (a) | yes       | -         | 24.7        | 181M       |
| <i>i</i> -RevNet (b) | yes       | yes       | 26.7        | 29M        |

## Posterior collapse, toy example

## Posterior collapse: toy example

Let define latent variable model in the following way:

$$p(\mathbf{x}|\boldsymbol{\theta}) = \int p(\mathbf{x}, \mathbf{z}|\boldsymbol{\theta}) d\mathbf{z} = \int p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) p(\mathbf{z}) d\mathbf{z}$$

- ▶ prior distribution  $p(\mathbf{z}) = \mathcal{N}(\mathbf{z}|0, \mathbf{I})$ ;
- ▶ probabilistic model  $p(\mathbf{x}|\mathbf{z}, \boldsymbol{\theta}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}_{\boldsymbol{\theta}}(\mathbf{z}), \boldsymbol{\sigma}_{\boldsymbol{\theta}}(\mathbf{z}))$  (diagonal covariance);
- ▶ variational posterior  $q(\mathbf{z}|\mathbf{x}, \boldsymbol{\phi}) = \mathcal{N}(\mathbf{z}|\boldsymbol{\mu}_{\boldsymbol{\phi}}(\mathbf{x}), \boldsymbol{\sigma}_{\boldsymbol{\phi}}(\mathbf{x}))$  (diagonal covariance).

Let data distribution is  $\pi(\mathbf{x}) = \mathcal{N}(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$ . Possible cases:

- ▶ covariance matrix  $\boldsymbol{\Sigma}$  is diagonal (univariate case);
- ▶ covariance matrix  $\boldsymbol{\Sigma}$  is **not** diagonal (multivariate case).

What is the difference?

# ELBO surgery, 2016

$$\mathcal{L}(q, \theta) = \int q(\mathbf{Z}|\mathbf{X}) \log \frac{p(\mathbf{X}, \mathbf{Z}|\theta)}{q(\mathbf{Z}|\mathbf{X})} d\mathbf{Z}.$$

## ELBO interpretations

- ▶ Evidence minus posterior KL

$$\mathcal{L}(q, \theta) = \log p(\mathbf{X}|\theta) - KL(q(\mathbf{Z}|\mathbf{X})||p(\mathbf{Z}|\mathbf{X}, \theta)).$$

- ▶ Average negative energy plus entropy

$$\mathcal{L}(q, \theta) = \mathbb{E}_{q(\mathbf{Z}|\mathbf{X})} p(\mathbf{X}, \mathbf{Z}|\theta) + \mathbb{H}[q(\mathbf{Z}|\mathbf{X})].$$

- ▶ Average term-by-term reconstruction minus KL to prior

$$\mathcal{L}(q, \theta) = \frac{1}{n} \sum_{i=1}^n [\mathbb{E}_{q(\mathbf{z}_i|\mathbf{x}_i)} \log p(\mathbf{x}_i|\mathbf{z}_i, \theta) - KL(q(\mathbf{z}_i|\mathbf{x}_i)||p(\mathbf{z}_i))].$$

# ELBO surgery, 2016

$$\mathcal{L}(q, \theta) = \frac{1}{n} \sum_{i=1}^n [\mathbb{E}_{q(\mathbf{z}_i|\mathbf{x}_i)} \log p(\mathbf{x}_i|\mathbf{z}_i, \theta) - KL(q(\mathbf{z}_i|\mathbf{x}_i)||p(\mathbf{z}_i))].$$

## Theorem

$$\frac{1}{n} \sum_{i=1}^n KL(q(\mathbf{z}_i|\mathbf{x}_i)||p(\mathbf{z}_i)) = KL(q(\mathbf{z})||p(\mathbf{z})) + \mathbb{I}_{q(i,\mathbf{z})}[i, \mathbf{z}],$$

where  $i$  is treated as random variable:

$$q(i, \mathbf{z}) = q(i)q(\mathbf{z}|i); \quad p(i, \mathbf{z}) = p(i)p(\mathbf{z}); \quad q(i) = p(i) = \frac{1}{n}; \quad q(\mathbf{z}|i) = q(\mathbf{z}|\mathbf{x}_i).$$

$$q(\mathbf{z}) = \sum_{i=1}^n q(i, \mathbf{z}) = \frac{1}{n} \sum_{i=1}^n q(\mathbf{z}|\mathbf{x}_i); \quad \mathbb{I}_{q(i,\mathbf{z})}[i, \mathbf{z}] = \mathbb{E}_{q(i,\mathbf{z})} \log \frac{q(i, \mathbf{z})}{q(i)q(\mathbf{z})}.$$

# ELBO surgery, 2016

## Theorem

$$\frac{1}{n} \sum_{i=1}^n KL(q(\mathbf{z}_i | \mathbf{x}_i) || p(\mathbf{z}_i)) = KL(q(\mathbf{z}) || p(\mathbf{z})) + \mathbb{I}_{q(i, \mathbf{z})}[i, \mathbf{z}].$$

## Proof

$$\begin{aligned}\frac{1}{n} \sum_{i=1}^n KL(q(\mathbf{z}_i | \mathbf{x}_i) || p(\mathbf{z}_i)) &= \sum_{i=1}^n \int q(i) q(\mathbf{z}|i) \log \frac{q(\mathbf{z}|i)}{p(\mathbf{z})} d\mathbf{z} = \\&= \sum_{i=1}^n \int q(i, \mathbf{z}) \log \frac{q(i, \mathbf{z})}{p(\mathbf{z})p(i)} d\mathbf{z} = \int \sum_{i=1}^n q(i, \mathbf{z}) \log \frac{q(\mathbf{z})q(i|\mathbf{z})}{p(\mathbf{z})p(i)} d\mathbf{z} = \\&= \int q(\mathbf{z}) \log \frac{q(\mathbf{z})}{p(\mathbf{z})} d\mathbf{z} + \int \sum_{i=1}^n q(i|\mathbf{z})q(\mathbf{z}) \log \frac{q(i|\mathbf{z})}{p(i)} d\mathbf{z} = \\&= KL(q(\mathbf{z}) || p(\mathbf{z})) - \mathbb{E}_{q(\mathbf{z})} \mathbb{H}[q(i|\mathbf{z})] + \log n.\end{aligned}$$

# ELBO surgery, 2016

## Theorem

$$\frac{1}{n} \sum_{i=1}^n KL(q(\mathbf{z}_i | \mathbf{x}_i) || p(\mathbf{z}_i)) = KL(q(\mathbf{z}) || p(\mathbf{z})) + \mathbb{I}_{q(i, \mathbf{z})}[i, \mathbf{z}].$$

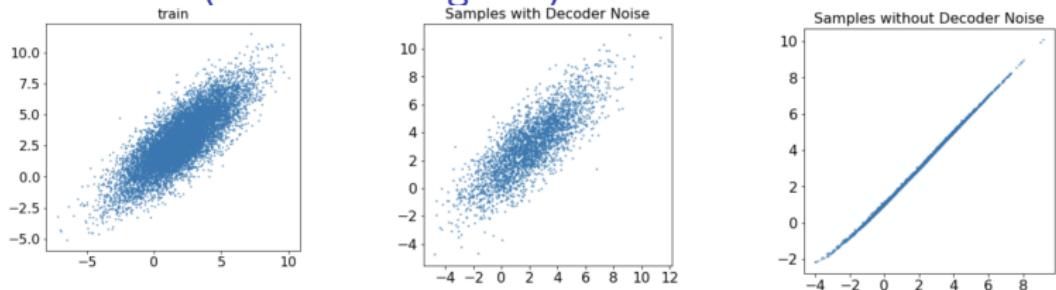
## Proof (continued)

$$\frac{1}{n} \sum_{i=1}^n KL(q(\mathbf{z}_i | \mathbf{x}_i) || p(\mathbf{z}_i)) = KL(q(\mathbf{z}) || p(\mathbf{z})) - \mathbb{E}_{q(\mathbf{z})} \mathbb{H}[q(i|\mathbf{z})] + \log n$$

$$\begin{aligned}\mathbb{I}_{q(i, \mathbf{z})}[i, \mathbf{z}] &= \mathbb{E}_{q(i, \mathbf{z})} \log \frac{q(i, \mathbf{z})}{q(i)q(\mathbf{z})} = \mathbb{E}_{q(\mathbf{z})} \mathbb{E}_{q(i|\mathbf{z})} \log \frac{q(i|\mathbf{z})q(\mathbf{z})}{q(i)q(\mathbf{z})} = \\ &= \mathbb{E}_{q(\mathbf{z})} \mathbb{E}_{q(i|\mathbf{z})} \log \frac{q(i|\mathbf{z})}{q(i)} = -\mathbb{E}_{q(\mathbf{z})} \mathbb{H}[q(i|\mathbf{z})] + \log n.\end{aligned}$$

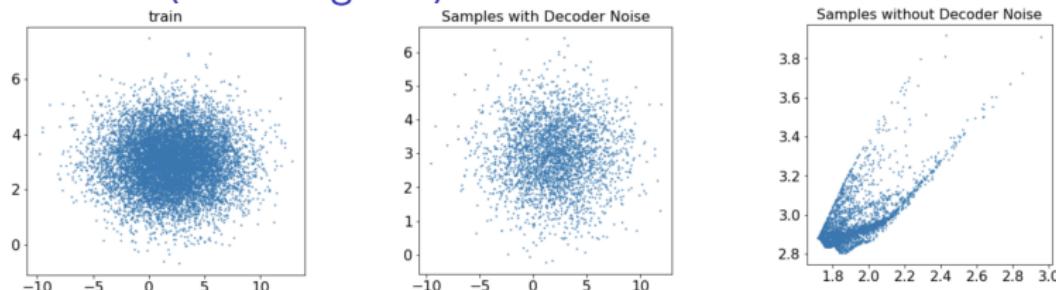
# Posterior collapse: toy example

## Multivariate ( $\Sigma$ is non-diagonal)



The encoder uses latent variables to model data.

## Univariate ( $\Sigma$ is diagonal)



Latent variables are not used, since the decoder could model the data without the encoder.

## Posterior collapse, VLAE

# Variational Lossy AutoEncoder

Lossy code via explicit information placement

$$p(\mathbf{x}|\mathbf{z}, \theta) = \prod_{i=1}^m p(x_i|\mathbf{z}, \mathbf{x}_{\text{WindowAround}(i)}, \theta).$$

- ▶  $\text{WindowAround}(i)$  restricts the receptive field (it forbids to represent arbitrarily complex distribution over  $\mathbf{x}$  without dependence on  $\mathbf{z}$ ).
- ▶ Local statistics of 2D images (texture) will be modeled by a small local window.
- ▶ Global structural information (shapes) is long-range dependency that can only be communicated through latent code  $\mathbf{z}$ .

# Variational Lossy AutoEncoder

- ▶ Can VLAE learn lossy codes that encode global statistics?
- ▶ Does using AF priors improves upon using IAF posteriors as predicted by theory?
- ▶ Does using autoregressive decoding distributions improve density estimation performance?

## CIFAR10

### MNIST

| Model                                       | NLL Test     |
|---|--------------|
| Normalizing flows (Rezende & Mohamed, 2015) | 85.10        |
| DRAW (Gregor et al., 2015)                  | < 80.97      |
| Discrete VAE (Rollef, 2016)                 | 81.01        |
| PixelRNN (van den Oord et al., 2016a)       | 79.20        |
| IAF VAE (Kingma et al., 2016)               | 79.88        |
| AF VAE                                      | 79.30        |
| VLAE  | <b>79.03</b> |

| Method  | bits/dim $\leq$ |
|---|-----------------|
| <i>Results with tractable likelihood models:</i>                  |                 |
| Uniform distribution [1]  | 8.00            |
| Multivariate Gaussian [1]   | 4.70            |
| NICE [2]  | 4.48            |
| Deep GMMS [3]   | 4.00            |
| Real NVP [4]  | 3.49            |
| PixelCNN [1]  | 3.14            |
| Gated PixelCNN [5]  | 3.03            |
| PixelRNN [1]  | 3.00            |
| PixelCNN++ [6]  | <b>2.92</b>     |
| <i>Results with variationally trained latent-variable models:</i> |                 |
| Deep Diffusion [7]  | 5.40            |
| Convolutional DRAW [8]  | 3.58            |
| ResNet VAE with IAF [9]   | 3.11            |
| ResNet VLAE   | 3.04            |
| DenseNet VLAE   | <b>2.95</b>     |

# Disentanglement, InfoGAN

# InfoGAN

## GAN objective

$$\min_G \max_D V(G, D)$$

$$V(G, D) = \mathbb{E}_{\pi(x)} \log D(x) + \mathbb{E}_{p(z)} \log(1 - D(G(z)))$$

Latent vector  $\mathbf{z}$  is not imposed to be disentangled.

InfoGAN decomposes input vector:

- ▶  $\mathbf{z}$  – incompressible noise;
- ▶  $\mathbf{c}$  – structured latent code  $p(\mathbf{c}) = \prod_{j=1}^d p(c_j)$ .

## Information-theoretic regularization

$$\max I(\mathbf{c}, G(\mathbf{z}, \mathbf{c}))$$

Information in the latent code  $\mathbf{c}$  should not be lost in the  
generation process.

Chen X. et al. InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets, 2016

# InfoGAN

## Objective

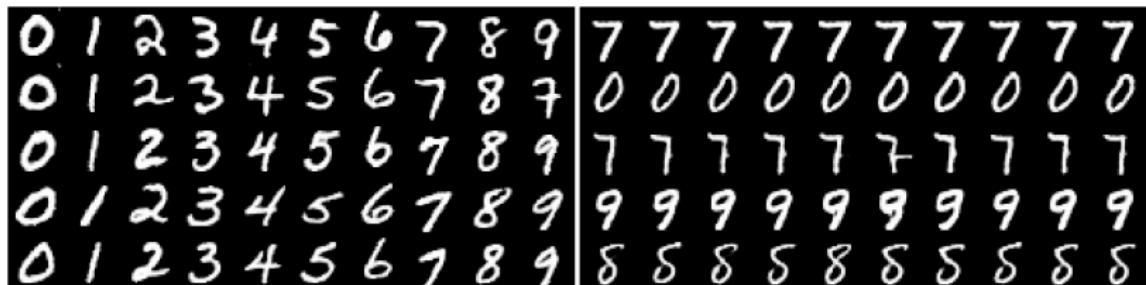
$$\min_G \max_D V(G, D) - \lambda I(\mathbf{c}, G(\mathbf{z}, \mathbf{c}))$$

## Variational Information Maximization

$$\begin{aligned} I(\mathbf{c}, G(\mathbf{z}, \mathbf{c})) &= H(\mathbf{c}) - H(\mathbf{c}|G(\mathbf{z}, \mathbf{c})) = \\ &= H(\mathbf{c}) + \mathbb{E}_{\mathbf{x} \sim G(\mathbf{z}, \mathbf{c})} [\mathbb{E}_{\mathbf{c}' \sim p(\mathbf{c}|\mathbf{x})} \log p(\mathbf{c}'|\mathbf{x})] = \\ &= H(\mathbf{c}) + \mathbb{E}_{\mathbf{x} \sim G(\mathbf{z}, \mathbf{c})} KL(p(\mathbf{c}'|\mathbf{x}) || q(\mathbf{z}'|\mathbf{x})) + \\ &\quad + \mathbb{E}_{\mathbf{x} \sim G(\mathbf{z}, \mathbf{c})} \mathbb{E}_{\mathbf{c}' \sim p(\mathbf{c}|\mathbf{x})} \log q(\mathbf{c}'|\mathbf{x}) \geq \\ &\geq H(\mathbf{c}) + \mathbb{E}_{\mathbf{x} \sim G(\mathbf{z}, \mathbf{c})} \mathbb{E}_{\mathbf{c}' \sim p(\mathbf{c}|\mathbf{x})} \log q(\mathbf{c}'|\mathbf{x}) = \\ &\quad H(\mathbf{c}) + \mathbb{E}_{\mathbf{c} \sim p(\mathbf{c})} \mathbb{E}_{\mathbf{x} \sim G(\mathbf{z}, \mathbf{c})} \log q(\mathbf{c}|\mathbf{x}) \end{aligned}$$

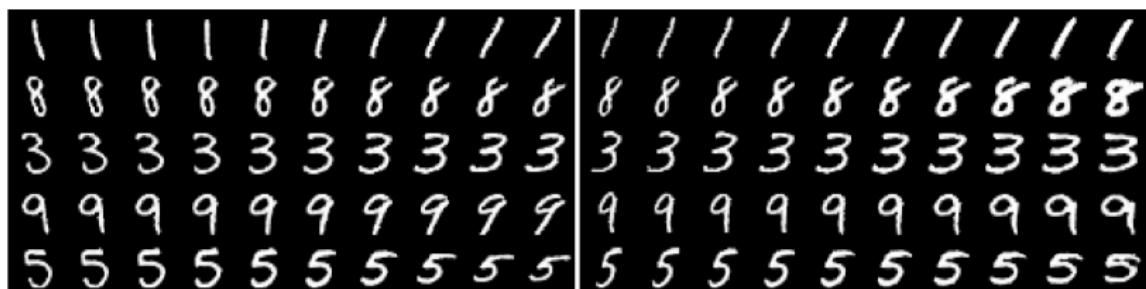
# InfoGAN

## Latent codes on MNIST



(a) Varying  $c_1$  on InfoGAN (Digit type)

(b) Varying  $c_1$  on regular GAN (No clear meaning)



(c) Varying  $c_2$  from -2 to 2 on InfoGAN (Rotation)

(d) Varying  $c_3$  from -2 to 2 on InfoGAN (Width)

# InfoGAN

## Latent codes on 3D Faces



(a) Azimuth (pose)

(b) Elevation



(c) Lighting

(d) Wide or Narrow

## $\beta$ -VAE, Disentanglement metric

# $\beta$ -VAE

## Disentangling metric

1. Generate two sets of objects

$$\mathbf{x}_{li} \sim \text{Sim}(\mathbf{v}_{li}, \mathbf{w}_{li}); \quad \mathbf{x}_{lj} \sim \text{Sim}(\mathbf{v}_{lj}, \mathbf{w}_{lj}); \quad y_{ij} \sim U[1, d].$$

$$\mathbf{v}_{li} \sim p(\mathbf{v}); \quad \mathbf{v}_{lj} \sim p(\mathbf{v}) ([v_{li}]_y = [v_{lj}]_y); \quad \mathbf{w}_{li}, \mathbf{w}_{lj} \sim p(\mathbf{w}).$$

2. Find representations

$$q(\mathbf{z}|\mathbf{x}) = \mathcal{N}(\mu(\mathbf{x})|\sigma^2(\mathbf{x})) ; \quad \mathbf{z}_{li} = \mu(\mathbf{x}_{li}); \quad \mathbf{z}_{lj} = \mu(\mathbf{x}_{lj}).$$

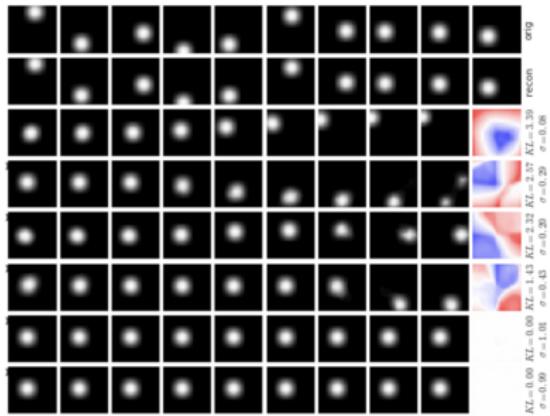
3. Use accuracy of classifier  $p(y|\mathbf{z}_{\text{diff}})$  with a low VC-dimension as metric of disentanglement

$$\mathbf{z}_{\text{diff}} = \frac{1}{L} \sum_{l=1}^L |\mathbf{z}_{li} - \mathbf{z}_{lj}|.$$

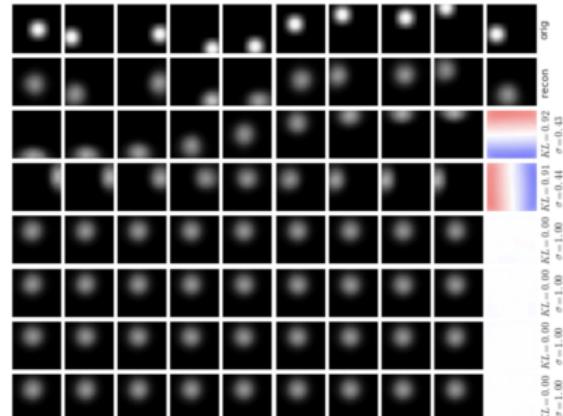
# $\beta$ -VAE

- ▶ **Top row:** original images.
- ▶ **Second row:** the corresponding reconstructions.
- ▶ **Remaining rows:** latent traversals ordered by KL divergence with the prior.
- ▶ **Heatmaps:** latent activations for each 2D position.

$\beta = 1$



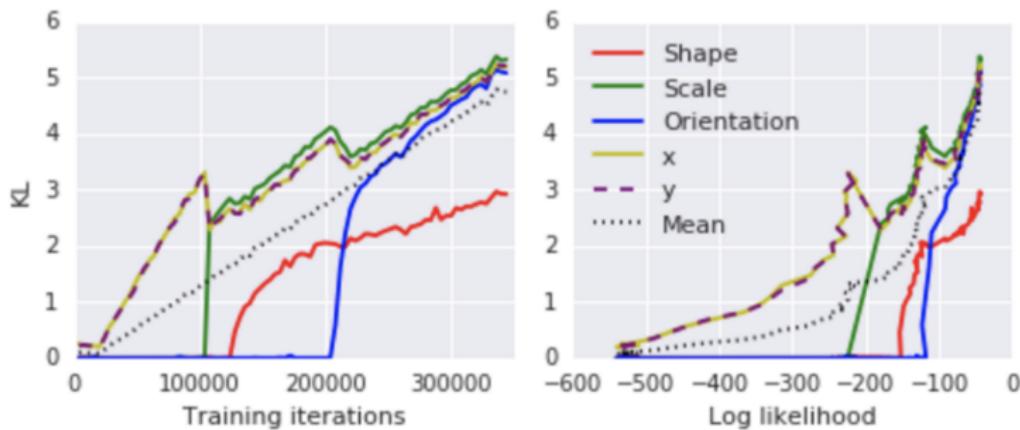
$\beta = 150$



# $\beta$ -VAE

## Controlled encoding capacity

$$\mathcal{L}(q, \theta, \beta) = \mathbb{E}_{q(\mathbf{z}|\mathbf{x})} \log p(\mathbf{x}|\mathbf{z}, \theta) - [KL(q(\mathbf{z}|\mathbf{x})||p(\mathbf{z})) - C].$$

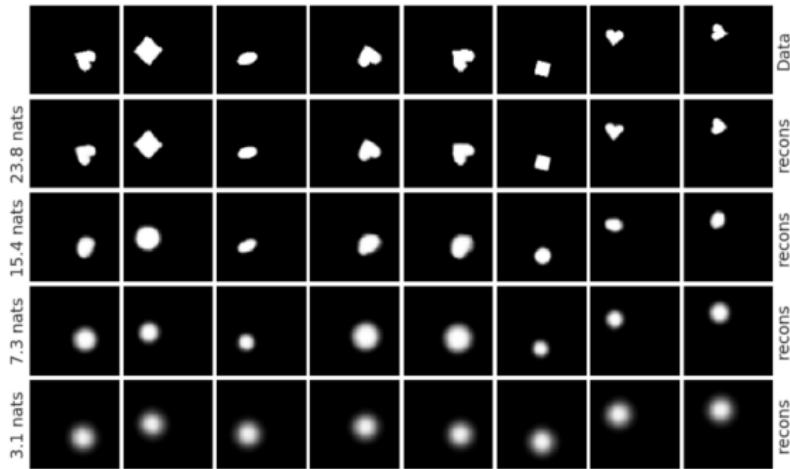


The early capacity is allocated to positional latents only, followed by a scale latent, then shape and orientation latents.

# $\beta$ -VAE

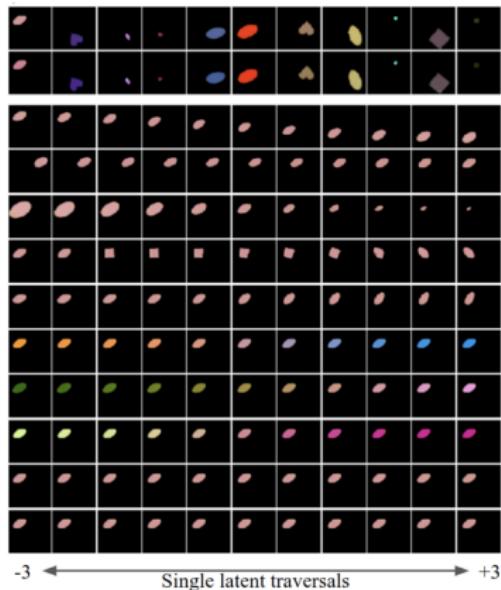
## Controlled encoding capacity

$$\mathcal{L}(q, \theta, \beta) = \mathbb{E}_{q(z|x)} \log p(x|z, \theta) - |KL(q(z|x)||p(z)) - C|.$$

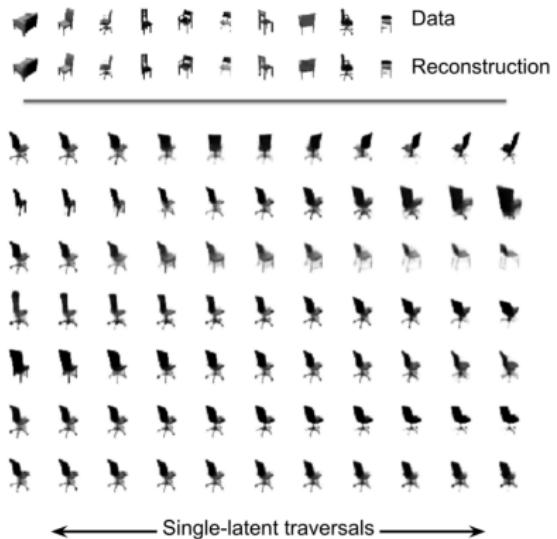


# $\beta$ -VAE

(a) Coloured dSprites



(b) 3D Chairs



# FactorVAE

## FactorVAE

Disentangled aggregated variational posterior

$$q(\mathbf{z}) = \frac{1}{n} \sum_{i=1}^n q(\mathbf{z}|\mathbf{x}) = \prod_{j=1}^d q(z_j)$$

Total correlation regularizer

$$\min KL(q(\mathbf{z}) || \prod_{j=1}^d q(z_j))$$

FactorVAE objective

$$\min_{\phi, \theta} \mathcal{L}(\phi, \theta) - \gamma \cdot KL(q(\mathbf{z}) || \prod_{j=1}^d q(z_j))$$

- ▶ The last term is intractable.
- ▶ FactorVAE uses density ratio trick for estimation.

## FactorVAE

Consider two distributions  $q_1(\mathbf{x})$ ,  $q_2(\mathbf{x})$  and probabilistic model

$$p(\mathbf{x}|y) = \begin{cases} q_1(\mathbf{x}), & \text{if } y = 1, \\ q_2(\mathbf{x}), & \text{if } y = 0, \end{cases} \quad y \sim \text{Bern}(0.5).$$

### Density ratio trick

$$\begin{aligned} \frac{q_1(\mathbf{x})}{q_2(\mathbf{x})} &= \frac{p(\mathbf{x}|y=1)}{p(\mathbf{x}|y=0)} = \frac{p(y=1|\mathbf{x})p(\mathbf{x})}{p(y=1)} \Big/ \frac{p(y=0|\mathbf{x})p(\mathbf{x})}{p(y=0)} = \\ &= \frac{p(y=1|\mathbf{x})}{p(y=0|\mathbf{x})} = \frac{p(y=1|\mathbf{x})}{1 - p(y=1|\mathbf{x})} = \frac{D(\mathbf{x})}{1 - D(\mathbf{x})} \end{aligned}$$

Here  $D(\mathbf{x})$  could be treated as a discriminator a model the output of which is a probability that  $\mathbf{x}$  is a sample from  $q_1(\mathbf{x})$  rather than from  $q_2(\mathbf{x})$ .

# FactorVAE

## FactorVAE objective

$$\min_{\theta, \phi} \text{ELBO}(\theta, \phi) - \gamma \cdot KL(q(\mathbf{z}) || \prod_{j=1}^d q(z_j))$$

## Total correlation regularizer

$$\begin{aligned} KL(q(\mathbf{z}) || \prod_{j=1}^d q(z_j)) &= KL(q(\mathbf{z}) || \bar{q}(\mathbf{z})) = \\ &= \mathbb{E}_{q(\mathbf{z})} \log \frac{q(\mathbf{z})}{\bar{q}(\mathbf{z})} \approx \mathbb{E}_{q(\mathbf{z})} \log \frac{D(\mathbf{z})}{1 - D(\mathbf{z})} \end{aligned}$$

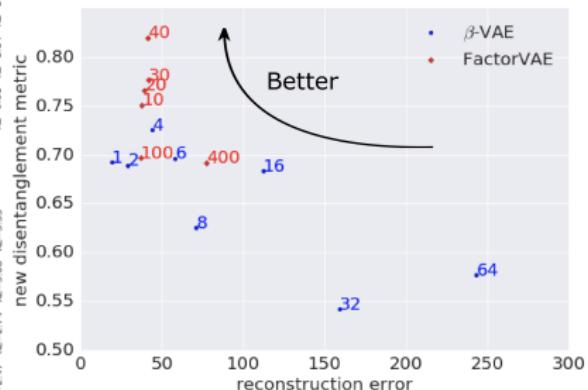
VAE and GAN are trained simultaneously.

# FactorVAE

$\beta$ -VAE ( $\beta = 8$ )



FactorVAE ( $\gamma = 10$ )



## Improved techniques for training GANs

# Improved techniques for training GANs

- ▶ Feature matching

$$\mathcal{L}_G = \|\mathbb{E}_{\pi(\mathbf{x})} \mathbf{d}(\mathbf{x}) - \mathbb{E}_{p(\mathbf{z})} \mathbf{d}(G(\mathbf{z}))\|_2^2$$

Here  $\mathbf{d}(\mathbf{x})$  – intermediate layer of discriminator. Matching the learned discriminator statistics instead of the output of the discriminator. Helps to avoid the vanishing gradients for sufficiently good discriminator.

- ▶ Historical averaging adds extra loss term for generator and discriminator losses

$$\|\boldsymbol{\theta} - \frac{1}{T} \sum_{t=1}^T \boldsymbol{\theta}_t\|_2^2.$$

Here  $\boldsymbol{\theta}_t$  – value of parameters at the previous step  $t$ . It allows to stabilize training procedure.

# Improved techniques for training GANs

- ▶ One-sided label smoothing. Instead of using one-hot labels in classification, use  $(1 - \alpha)$  for real data (the generated samples are not smoothed).

$$D^*(\mathbf{x}) = \frac{(1 - \alpha)\pi(\mathbf{x})}{\pi(\mathbf{x}) + p(\mathbf{x}|\theta)}$$

- ▶ Virtual batch normalization. BatchNorm makes samples within minibatch are highly correlated.



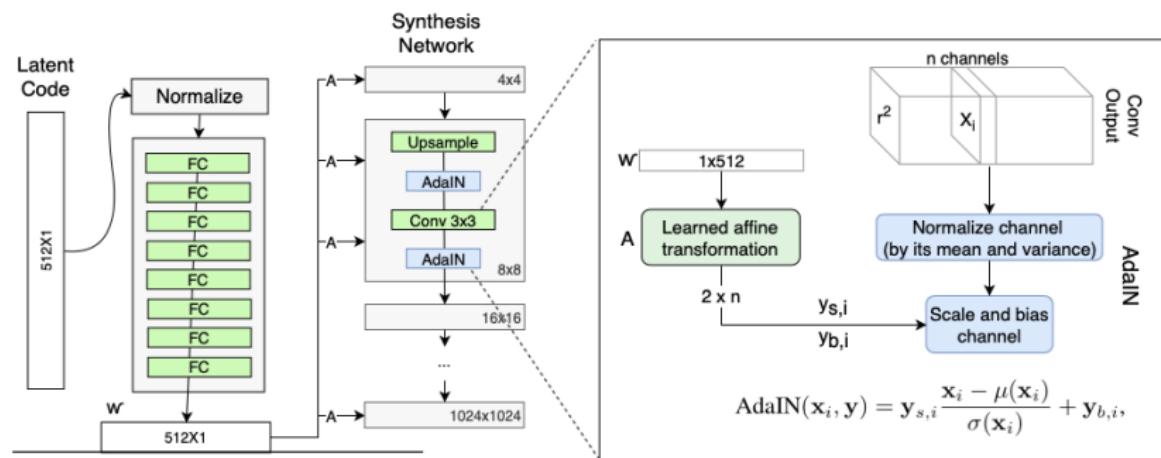
Use reference fixed batch to compute the normalization statistics. To avoid overfitting construct batch with the reference batch and the current sample.

# StyleGAN

# StyleGAN

## Step 2: Style modulation

- ▶ Adaptive Instance Normalization transfers the  $\mathbf{w}$  vector to the synthesis Network.
- ▶ The module is added to each resolution to define the visual expression of the features.

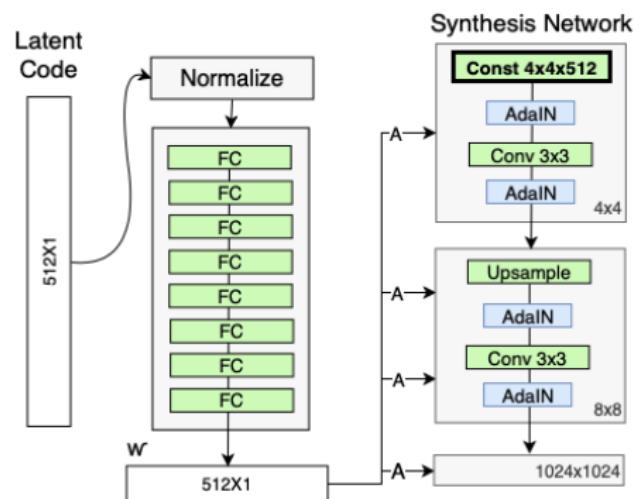


Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

# StyleGAN

## Step 3: Remove traditional input

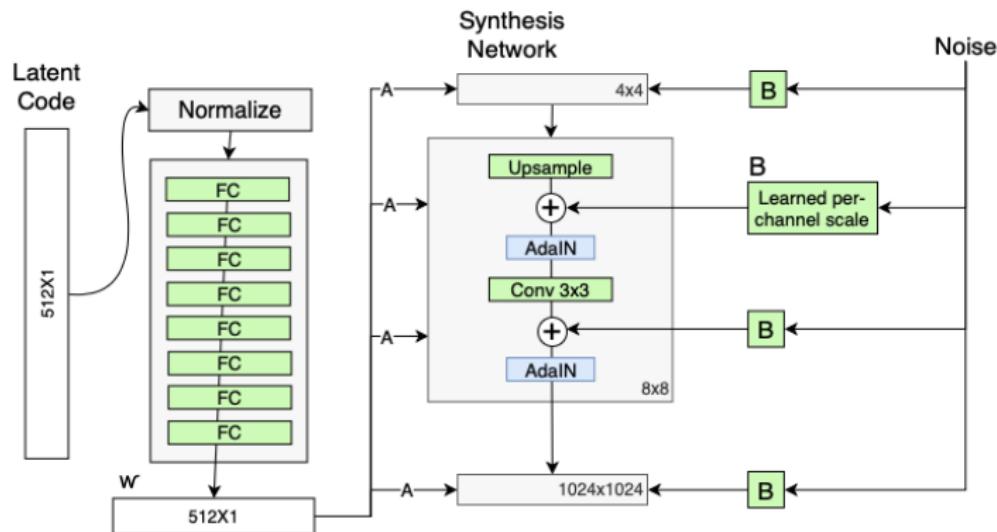
Mapping network provides stochasticity to different stages of the synthesis network. Input of the synthesis network is a trainable vector.



# StyleGAN

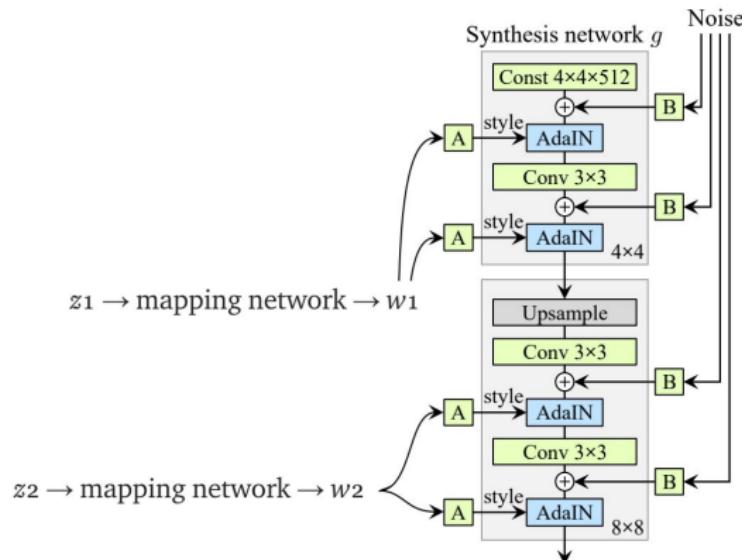
## Step 4: Stochastic variation

Inject random noise to add small aspects, such as freckles, exact placement of hairs, wrinkles, features which make the image more realistic and increase the variety of outputs.



# StyleGAN

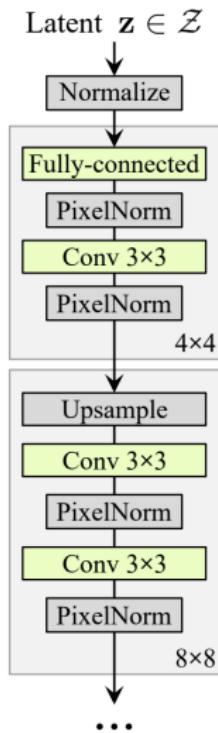
## Step 4: Style Mixing



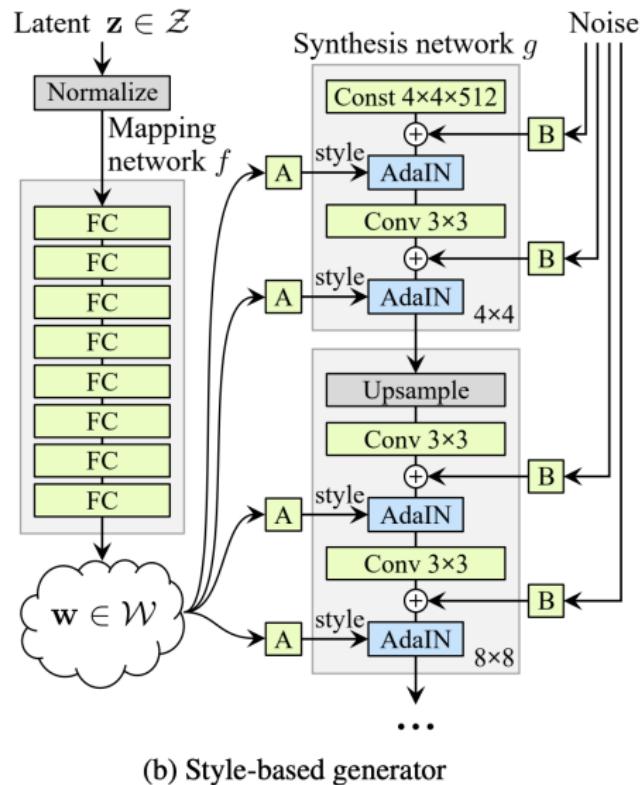
- ▶ Makes different levels of synthesis network to be independent.
- ▶ Allows to couple different styles.

Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

# StyleGAN

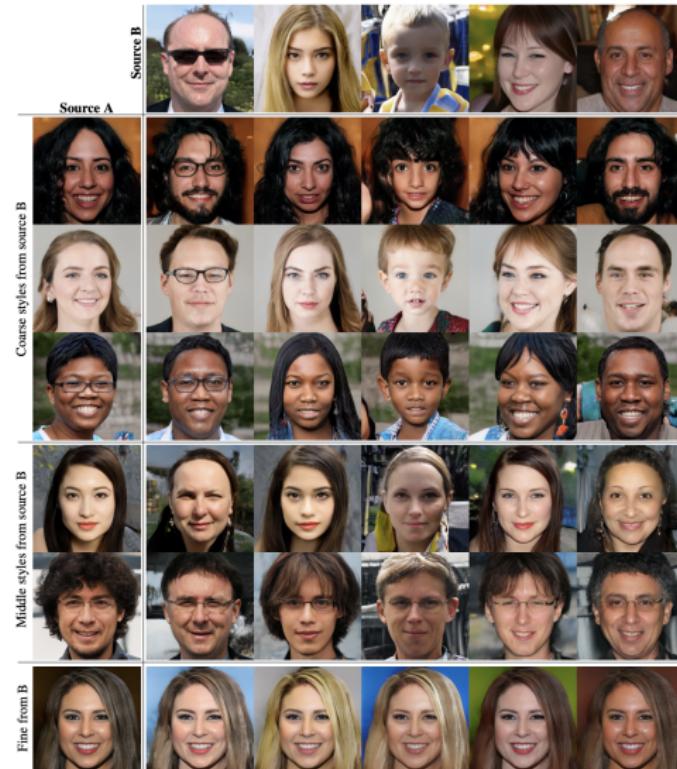


(a) Traditional



(b) Style-based generator

# StyleGAN



Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

# Feature Quantized GAN

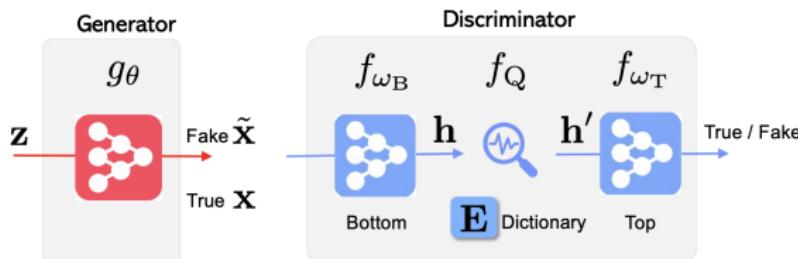
# Feature Quantized GAN

- ▶ GAN tries to find Nash equilibrium, minibatch training is unstable. GAN relies heavily on the minibatch statistics.
- ▶ Lots of feature matching strategies were proposed to stabilize the training.

## Feature quantized GAN discriminator

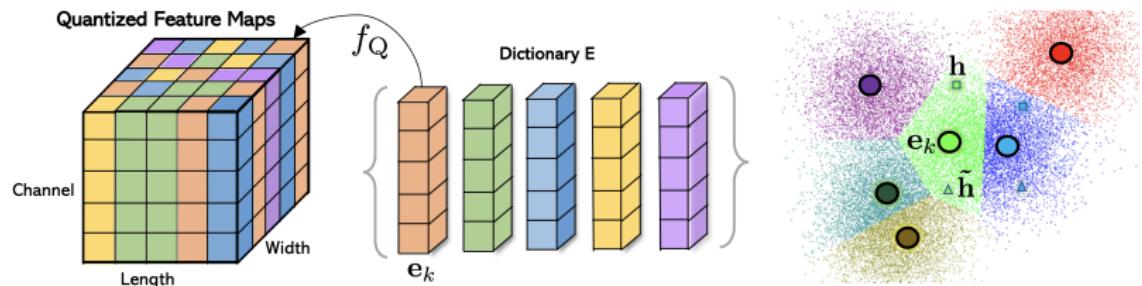
$$D(\mathbf{x}) = f_{\mathbf{w}_T} \circ f_{\mathbf{w}_B}(\mathbf{x}) \quad \Rightarrow \quad D(\mathbf{x}) = f_{\mathbf{w}_T} \circ f_Q \circ f_{\mathbf{w}_B}(\mathbf{x}).$$

Here  $f_Q$  is a vector quantization operation.



# Feature Quantized GAN

## Quantization procedure



## Quantized features

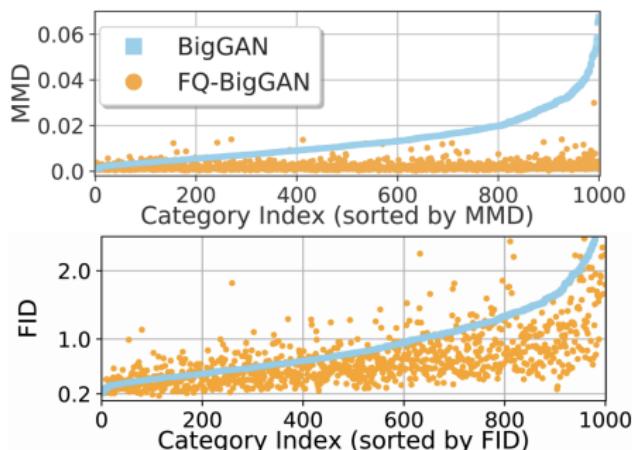


# Feature Quantized GAN

## ImageNet-1000

| Models | 64 × 64        |                           | 128 × 128                       |  |
|--------|----------------|---------------------------|---------------------------------|--|
|        | FID* ↓ / IS* ↑ |                           | FID* ↓ / IS* ↑                  |  |
| Half   | TAC-GAN        | -                         | 23.75 / 28.86±0.29 <sup>‡</sup> |  |
|        | BigGAN         | 12.75 / 21.84±0.34        | 22.77 / 38.05±0.79 <sup>‡</sup> |  |
| 256K   | FQ-BigGAN      | <b>12.62 / 21.99±0.32</b> | <b>19.11 / 41.92±1.15</b>       |  |
|        | BigGAN         | 10.55 / 25.43±0.15        | 14.88 / 63.03±1.42 <sup>†</sup> |  |
|        | FQ-BigGAN      | <b>9.67 / 25.96±0.24</b>  | <b>13.77 / 54.36±1.07</b>       |  |

## Per class metrics for ImageNet



## FFHQ

| Resolution  | 32 <sup>2</sup> | 64 <sup>2</sup> | 128 <sup>2</sup> | 1024 <sup>2</sup> |
|-------------|-----------------|-----------------|------------------|-------------------|
| StyleGAN    | 3.28            | 4.82            | 6.33             | 5.24              |
| FQ-StyleGAN | <b>3.01</b>     | <b>4.36</b>     | <b>5.98</b>      | <b>4.89</b>       |