

Deep Generative Models

Lecture 12

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Recap of previous lecture

$$f(\mathbf{x}, \phi) = \mathbf{W}_{K+1} \sigma_K (\mathbf{W}_K \sigma_{K-1} (\dots \sigma_1 (\mathbf{W}_1 \mathbf{x}) \dots)).$$

- ▶ σ_k is a pointwise nonlinearities. We assume that $\|\sigma_k\|_L = 1$ (it holds for ReLU).
- ▶ $\mathbf{g}(\mathbf{x}) = \mathbf{W}\mathbf{x}$ is a linear transformation ($\nabla \mathbf{g}(\mathbf{x}) = \mathbf{W}$).

$$\|\mathbf{g}\|_L = \sup_{\mathbf{x}} \|\nabla \mathbf{g}(\mathbf{x})\|_2 = \|\mathbf{W}\|_2.$$

Critic spectral norm

$$\|f\|_L \leq \|\mathbf{W}_{K+1}\|_2 \cdot \prod_{k=1}^K \|\sigma_k\|_L \cdot \|\mathbf{W}_k\|_2 = \prod_{k=1}^{K+1} \|\mathbf{W}_k\|_2.$$

Spectral Normalization GAN

If we replace the weights in the critic $f(\mathbf{x}, \phi)$ by $\mathbf{W}_k^{SN} = \mathbf{W}_k / \|\mathbf{W}_k\|_2$, we will get $\|f\|_L \leq 1$.

Power iteration approximates the value of $\|\mathbf{W}\|_2$.

Recap of previous lecture

f-divergence minimization

$$D_f(\pi || p) = \mathbb{E}_{p(\mathbf{x})} f\left(\frac{\pi(\mathbf{x})}{p(\mathbf{x})}\right) \rightarrow \min_p .$$

Here $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ is a convex, lower semicontinuous function satisfying $f(1) = 0$.

Variational divergence estimation

$$D_f(\pi || p) \geq \sup_{T \in \mathcal{T}} [\mathbb{E}_\pi T(\mathbf{x}) - \mathbb{E}_p f^*(T(\mathbf{x}))],$$

Fenchel conjugate

$$f^*(t) = \sup_{u \in \text{dom}_f} (ut - f(u)), \quad f(u) = \sup_{t \in \text{dom}_{f^*}} (ut - f^*(t))$$

Note: To evaluate lower bound we only need samples from $\pi(\mathbf{x})$ and $p(\mathbf{x})$. Hence, we could fit implicit generative model.

Recap of previous lecture

Let take some pretrained image classification model to get the conditional label distribution $p(y|\mathbf{x})$ (e.g. ImageNet classifier).

Evaluation of likelihood-free models

- ▶ Sharpness \Rightarrow low $H(y|\mathbf{x}) = - \sum_y \int_{\mathbf{x}} p(y, \mathbf{x}) \log p(y|\mathbf{x}) d\mathbf{x}$.
- ▶ Diversity \Rightarrow high $H(y) = - \sum_y p(y) \log p(y)$.

Inception Score

$$IS = \exp(H(y) - H(y|\mathbf{x})) = \exp(\mathbb{E}_{\mathbf{x}} KL(p(y|\mathbf{x}) || p(y)))$$

Frechet Inception Distance

$$D^2(\pi, p) = \|\mathbf{m}_\pi - \mathbf{m}_p\|_2^2 + \text{Tr} \left(\boldsymbol{\Sigma}_\pi + \boldsymbol{\Sigma}_p - 2\sqrt{\boldsymbol{\Sigma}_\pi \boldsymbol{\Sigma}_p} \right).$$

FID is related to moment matching.

Salimans T. et al. *Improved Techniques for Training GANs*, 2016

Heusel M. et al. *GANs Trained by a Two Time-Scale Update Rule Converge to a Local Nash Equilibrium*, 2017

Recap of previous lecture

- ▶ $\mathcal{S}_\pi = \{\mathbf{x}_i\}_{i=1}^n \sim \pi(\mathbf{x})$ – real samples;
- ▶ $\mathcal{S}_p = \{\mathbf{x}_i\}_{i=1}^n \sim p(\mathbf{x}|\theta)$ – generated samples.

Embed samples using pretrained classifier network (as previously):

$$\mathcal{G}_\pi = \{\mathbf{g}_i\}_{i=1}^n, \quad \mathcal{G}_p = \{\mathbf{g}_i\}_{i=1}^n.$$

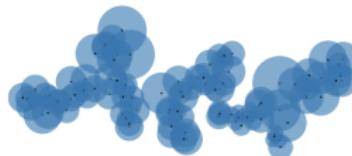
Define binary function:

$$f(\mathbf{g}, \mathcal{G}) = \begin{cases} 1, & \text{if exists } \mathbf{g}' \in \mathcal{G} : \|\mathbf{g} - \mathbf{g}'\|_2 \leq \|\mathbf{g} - \text{NN}_k(\mathbf{g}, \mathcal{G})\|_2; \\ 0, & \text{otherwise.} \end{cases}$$

$$\text{Precision}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_p} f(\mathbf{g}, \mathcal{G}_\pi); \quad \text{Recall}(\mathcal{G}_\pi, \mathcal{G}_p) = \frac{1}{n} \sum_{\mathbf{g} \in \mathcal{G}_\pi} f(\mathbf{g}, \mathcal{G}_p).$$



(a) True manifold

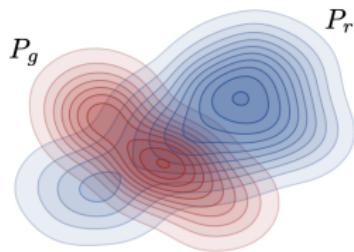


(b) Approx. manifold

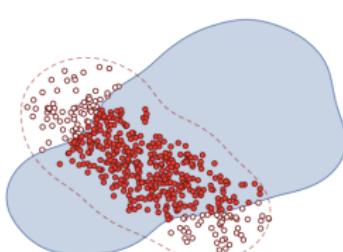
Precision-Recall for Generative Models

What do we want from samples

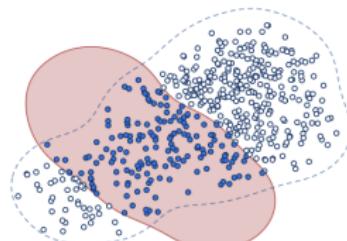
- ▶ **Sharpness:** generated samples should be of high quality.
- ▶ **Diversity:** their variation should match that observed in the training set.



(a) Example distributions



(b) Precision



(c) Recall

- ▶ **Precision** denotes the fraction of generated images that are realistic.
- ▶ **Recall** measures the fraction of the training data manifold covered by the generator.

Precision-Recall for Generative Models

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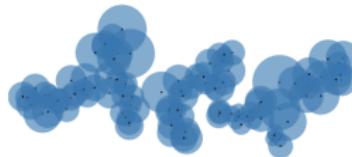
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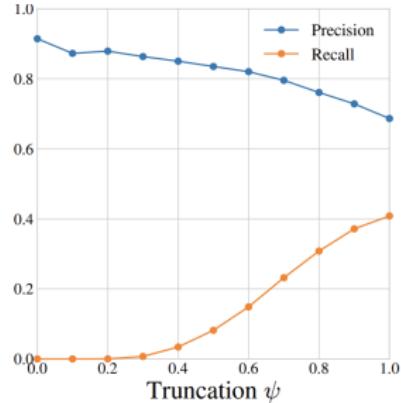
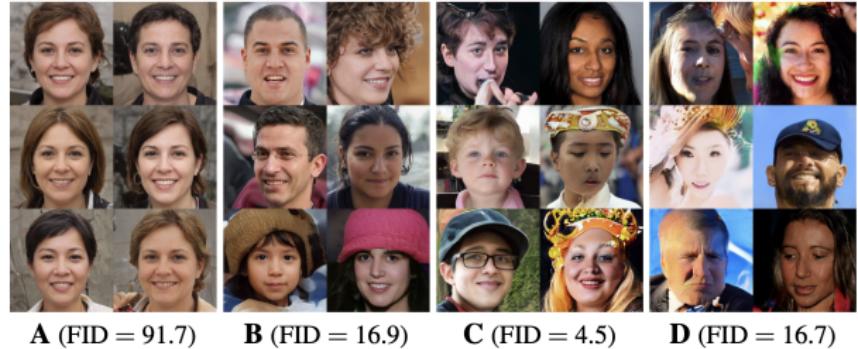
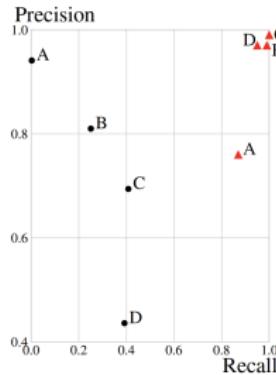


(a) True manifold



(b) Approx. manifold

Precision-Recall for Generative Models



Kynkäanniemi T. et al. Improved precision and recall metric for assessing generative models, 2019

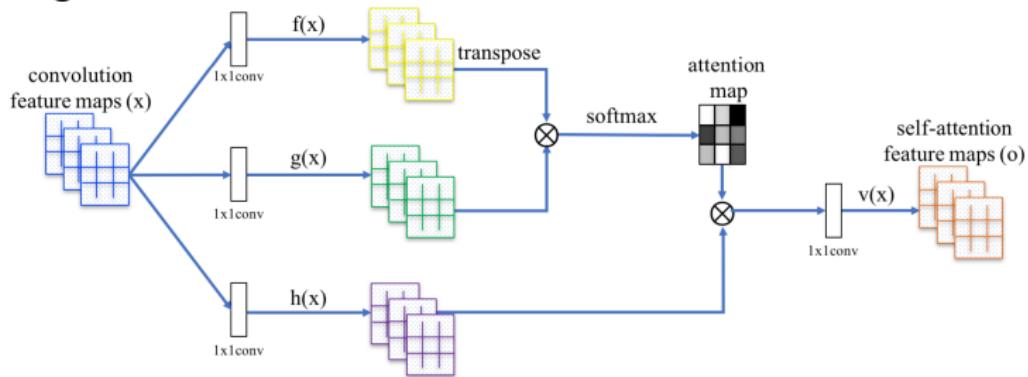
Evolution of GANs



- ▶ **Standard GAN** <https://arxiv.org/abs/1406.2661>
- ▶ **DCGAN** <https://arxiv.org/abs/1511.06434>
- ▶ **CoGAN** <https://arxiv.org/abs/1606.07536>
- ▶ **ProGAN** <https://arxiv.org/abs/1710.10196>
- ▶ **StyleGAN** <https://arxiv.org/abs/1812.04948>

Self-Attention GAN

Convolutional layers process the information in a local neighborhood \Rightarrow inefficient for modeling long-range dependencies in images.

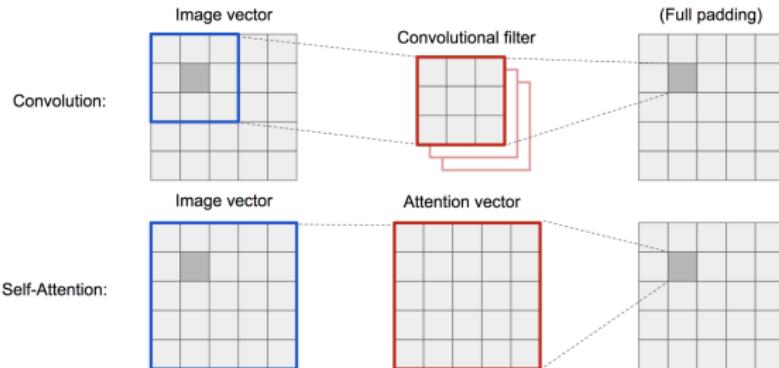


$$\mathbf{f}(\mathbf{x}) = \mathbf{W}_f \mathbf{x}, \quad \mathbf{g}(\mathbf{x}) = \mathbf{W}_g \mathbf{x}, \quad \mathbf{h}(\mathbf{x}) = \mathbf{W}_h \mathbf{x}, \quad \mathbf{v}(\mathbf{x}) = \mathbf{W}_v \mathbf{x}$$

$$s_{ij} = \mathbf{f}(\mathbf{x}_i)^T \mathbf{g}(\mathbf{x}_j), \quad a_{ij} = \frac{\exp s_{ij}}{\sum_{i=1}^N \exp s_{ij}}, \quad \mathbf{o}_j = \mathbf{v} \left(\sum_{i=1}^N a_{ij} \mathbf{h}(\mathbf{x}_i) \right)$$

Self-Attention GAN

Convolution vs Attention



Visualization of attention maps



image credit: <https://lilianweng.github.io/lil-log/2018/06/24/attention-attention.html>
Zhang H. et al. Self-Attention Generative Adversarial Networks, 2018

BigGAN

Batch-size is matter

Batch	Ch.	Param (M)	Shared	Skip- z	Ortho.	Itr $\times 10^3$	FID	IS
256	64	81.5	SA-GAN Baseline			1000	18.65	52.52
512	64	81.5	X	X	X	1000	15.30	58.77(± 1.18)
1024	64	81.5	X	X	X	1000	14.88	63.03(± 1.42)
2048	64	81.5	X	X	X	732	12.39	76.85(± 3.83)
2048	96	173.5	X	X	X	295(± 18)	9.54(± 0.62)	92.98(± 4.27)

Samples (512x512)



Progressive Growing GAN

Problems with HR image generation

- ▶ Disjoint manifolds \Rightarrow gradient problem.
- ▶ Small minibatch \Rightarrow training instability.

Samples (1024x1024)

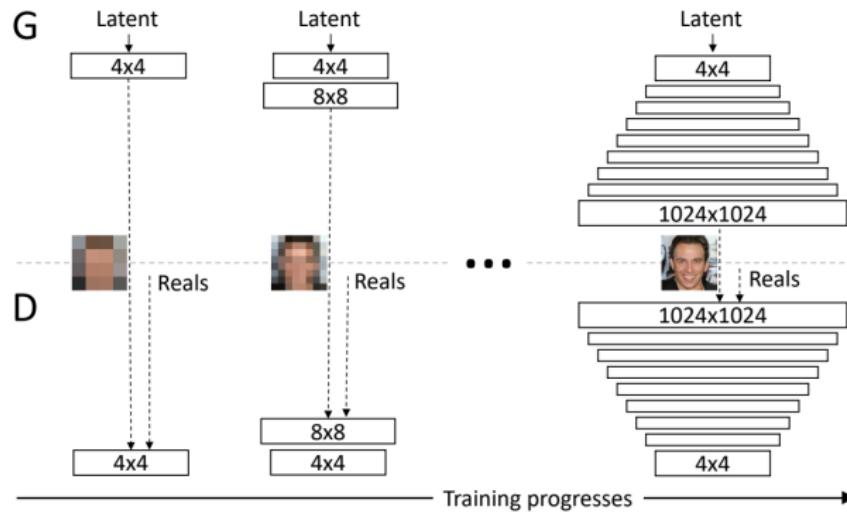


Karras T. et al. *Progressive Growing of GANs for Improved Quality, Stability, and Variation*, 2017

Progressive Growing GAN

Grow both the generator and discriminator progressively, new layers will introduce higher-resolution details as the training progresses.

- ▶ Train GAN which generate 4x4 images (2 convs for G and D).
- ▶ Add upsampling layers to G, downsampling layers to D.
- ▶ Train GAN which generate 8x8 images.
- ▶ etc.



Karras T. et al. *Progressive Growing of GANs for Improved Quality, Stability, and Variation*, 2017

StyleGAN

- ▶ Generating of HR images is hard.
- ▶ Progressive growing greatly simplifies the task.
- ▶ The ability to control specific features of the generated image is very limited.

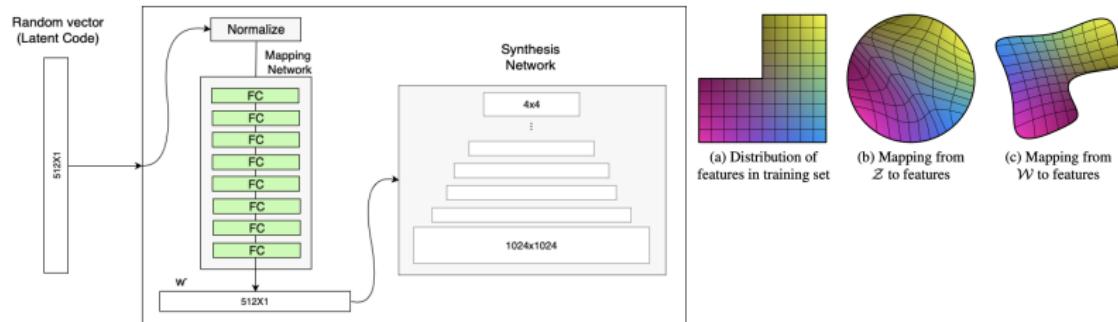
Face image features

- ▶ Coarse (pose, general hair style, face shape). Resolution $4^2 - 8^2$.
- ▶ Middle (finer facial features, hair style, eyes open/closed). Resolution $16^2 - 32^2$.
- ▶ Fine (color scheme (eye, hair and skin) and micro features). Resolution $64^2 - 1024^2$.

StyleGAN

Mapping Network

- ▶ Generator input is likely to be **disentangled**. Each component of input vector \mathbf{z} should be responsible for one generative factor.
- ▶ Mapping network $f : \mathcal{Z} \rightarrow \mathcal{W}$ is used to reduce correlations between components of \mathbf{z} .



Truncation trick

BigGAN: truncated normal sampling

$$p(\mathbf{z}|b) = \mathcal{N}(\mathbf{z}|0, 1) / \int_{-\infty}^b \mathcal{N}(\mathbf{z}|0, 1) d\mathbf{z}$$

Components of $\mathbf{z} \sim \mathcal{N}(0, \mathbf{I})$ which fall outside a predefined range are resampled.

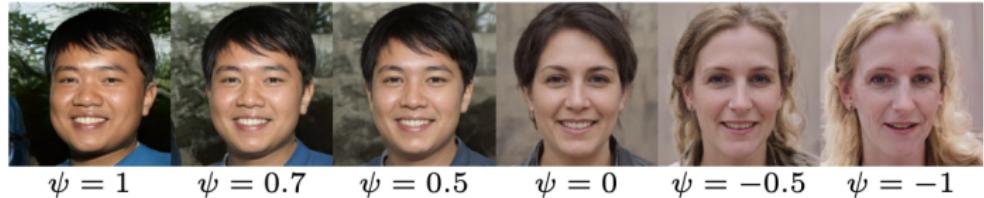
StyleGAN

$$\mathbf{w}' = \hat{\mathbf{w}} + \psi \cdot (\mathbf{w} - \hat{\mathbf{w}}), \quad \hat{\mathbf{w}} = \mathbb{E}_{\mathbf{z}} p(f(\mathbf{z}))$$

- ▶ Constant ψ is a tradeoff between diversity and fidelity.
- ▶ $\psi = 0.7$ is used for most of the results.
- ▶ Truncation is done only at the low-resolution layers.

StyleGAN

Truncation trick

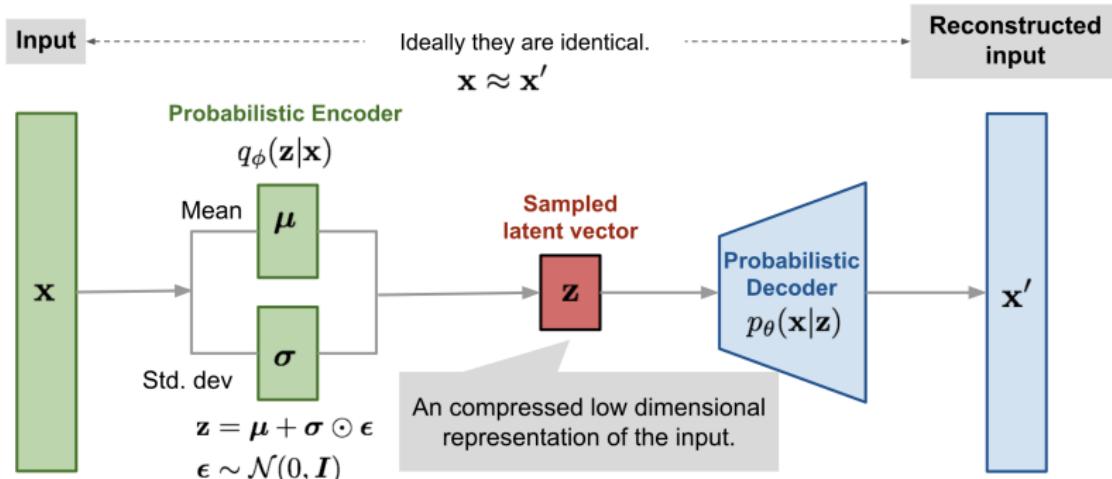


Samples (1024x1024)



Karras T., Laine S., Aila T. A Style-Based Generator Architecture for Generative Adversarial Networks, 2018

VAE recap



- ▶ Encoder $q(\mathbf{z}|\mathbf{x}, \phi) = \mathcal{N}(\mathbf{z}|\mu_\phi(\mathbf{x}), \sigma_\phi(\mathbf{x}))$.
- ▶ Variational posterior $q(\mathbf{z}|\mathbf{x}, \phi)$ originally approximates the true posterior $p(\mathbf{z}|\mathbf{x}, \theta)$.
- ▶ Which methods are you already familiar with to make the posterior more flexible?

image credit:

<https://lilianweng.github.io/lil-log/2018/08/12/from-autoencoder-to-beta-vae.html>

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} [\log p(x|z, \theta) + \log p(z) - \log q(z|x, \phi)] \rightarrow \max_{\phi, \theta} .$$

What is the problem to make the variational posterior model an implicit model?

- ▶ The first term is reconstruction loss that needs only samples from $q(z|x, \phi)$ to evaluate.
- ▶ Reparametrization trick allows to get gradients of reconstruction loss

$$\begin{aligned}\nabla_{\phi} \int q(z|x, \phi) f(z) dz &= \nabla_{\phi} \int r(\epsilon) f(z) d\epsilon \\ &= \int r(\epsilon) \nabla_{\phi} f(g(x, \epsilon, \phi)) d\epsilon \approx \nabla_{\phi} f(g(x, \epsilon^*, \phi)),\end{aligned}$$

where $\epsilon^* \sim r(\epsilon)$, $z = g(x, \epsilon, \phi)$, $z \sim q(z|x, \phi)$.

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} [\log p(x|z, \theta) + \log p(z) - \log q(z|x, \phi)] \rightarrow \max_{\phi, \theta}.$$

What is the problem to make the variational posterior model an implicit model?

- ▶ The third term requires the explicit value of $q(z|x, \phi)$.
- ▶ We could join second and third terms:

$$\mathbb{E}_{q(z|x, \phi)} \log \frac{p(z)}{q(z|x, \phi)} = \mathbb{E}_{q(z|x, \phi)} \log \frac{p(z)\pi(x)}{q(z|x, \phi)\pi(x)}.$$

- ▶ We have to estimate density ratio

$$r(x, z) = \frac{q_1(x, z)}{q_2(x, z)} = \frac{p(z)\pi(x)}{q(z|x, \phi)\pi(x)}.$$

Density ratio trick

Consider two distributions $q_1(\mathbf{x})$, $q_2(\mathbf{x})$ and probabilistic model

$$p(\mathbf{x}|y) = \begin{cases} q_1(\mathbf{x}), & \text{if } y = 1, \\ q_2(\mathbf{x}), & \text{if } y = 0, \end{cases} \quad y \sim \text{Bern}(0.5).$$

Density ratio

$$\begin{aligned} \frac{q_1(\mathbf{x})}{q_2(\mathbf{x})} &= \frac{p(\mathbf{x}|y=1)}{p(\mathbf{x}|y=0)} = \frac{p(y=1|\mathbf{x})p(\mathbf{x})}{p(y=1)} \Big/ \frac{p(y=0|\mathbf{x})p(\mathbf{x})}{p(y=0)} = \\ &= \frac{p(y=1|\mathbf{x})}{p(y=0|\mathbf{x})} = \frac{p(y=1|\mathbf{x})}{1 - p(y=1|\mathbf{x})} = \frac{D(\mathbf{x})}{1 - D(\mathbf{x})} \end{aligned}$$

Here $D(\mathbf{x})$ is a discriminator model the output of which is a probability that \mathbf{x} is a sample from $q_1(\mathbf{x})$ rather than from $q_2(\mathbf{x})$.

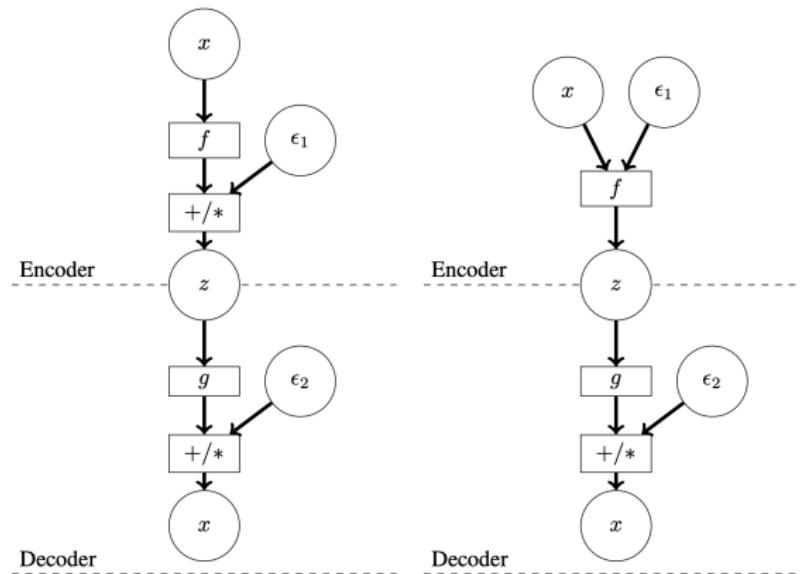
Adversarial Variational Bayes

$$\max_D \left[\mathbb{E}_{\pi(\mathbf{x})} \mathbb{E}_{q(\mathbf{z}|\mathbf{x}, \phi)} \log D(\mathbf{x}, \mathbf{z}) + \mathbb{E}_{\pi(\mathbf{x})} \mathbb{E}_{p(\mathbf{z})} \log(1 - D(\mathbf{x}, \mathbf{z})) \right]$$

Adversarial Variational Bayes

ELBO objective

$$\mathcal{L}(\phi, \theta) = \mathbb{E}_{q(z|x, \phi)} \left[\log p(x|z, \theta) + \log \frac{p(z)}{q(z|x, \phi)} \right] \rightarrow \max_{\phi, \theta} .$$



Summary

- ▶ Precision-recall allows to select model with compromise with sample quality and sample diversity.
- ▶ Self-Attention GAN allows to make huge receptive field and reduce convolution inductive bias.
- ▶ BigGAN shows that large batch size increase model quality gradually.
- ▶ Progressive growing for GAN learning allows to make training more stable.
- ▶ StyleGAN introduces mapping network to get more disentangled latent representation.
- ▶ Adversarial Variational Bayes uses density ratio trick to get more powerful variational posterior.