ORBIT TRACKING IN FFAGS AND REVERSE-BEND CYCLOTRONS USING THE CYCLOPS CODE

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FFAG DESIGN TOOLS

FFAG designs have generally been developed using synchrotron lattice codes – or adaptations of them – perhaps because their designers have mostly come from a synchrotron background.

But synchrotron codes are poorly adapted for use in accelerators with fixed magnetic fields:

- The central orbit is a spiral – rather than a fixed-radius ring - with the equilibrium-orbit (E.O.) radius depending on energy;
- A wide radial region of magnetic field must be characterized.

As a result, special arrangements must be made to deal with momentum-dependent effects accurately.

ORBIT-TRACKING TOOLS

Méot et al[1]. have avoided these problems by using ZGOUBI:
- an orbit tracking code originally developed for the study and tuning of mass spectrometers and beam lines.

Here, we report studies made with the cyclotron orbit code CYCLOPS[2], which tracks particles through magnetic fields specified on a polar grid and determines the equilibrium orbits and their optical properties.

This has the advantages of:

- Being designed for multi-sector machines with wide aperture magnets
- Simultaneous computation of orbit properties at all energies
- Capability of tracking through measured magnetic fields
- Availability of the sister code GOBLIN for accelerated-orbit studies.

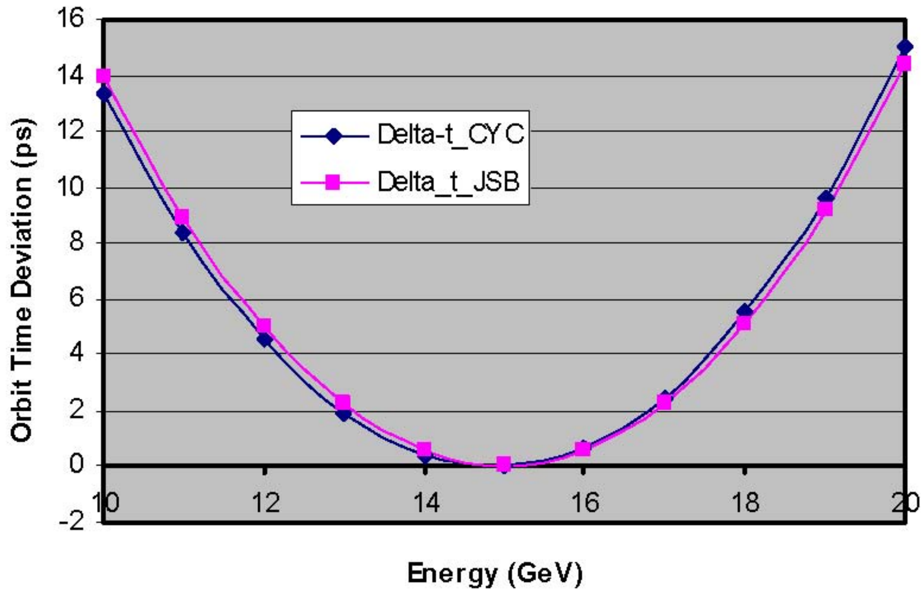
Some initial results were reported[3] at Cyclotrons'07.

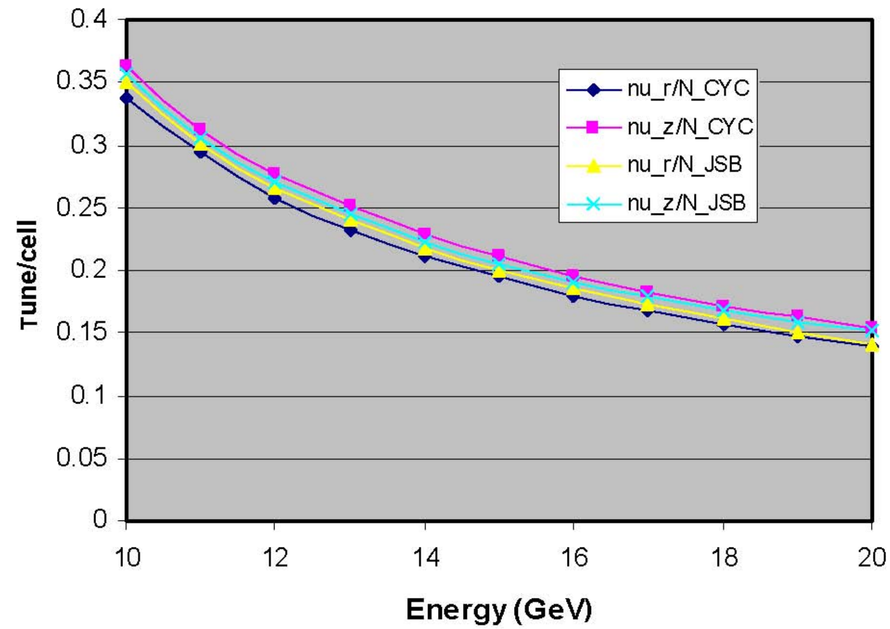
1. F. Lemuet, F. Méot, G. Rees, Proc. PAC'05, 2693 (2005).
2. M.M. Gordon, Part. Accel. **16**, 39 (1984).
3. M.K. Craddock, Y.-N. Rao, Cyclotrons'07, 370 (2007).

TEST RUNS WITH THE FODO-2 LATTICE

FODO-2 was one of several linear non-scaling FFAG lattices developed by Scott Berg in 2003-4 for 10-20 GeV muons:

- D is a positive-bending, and F a negative-bending, sector magnet.



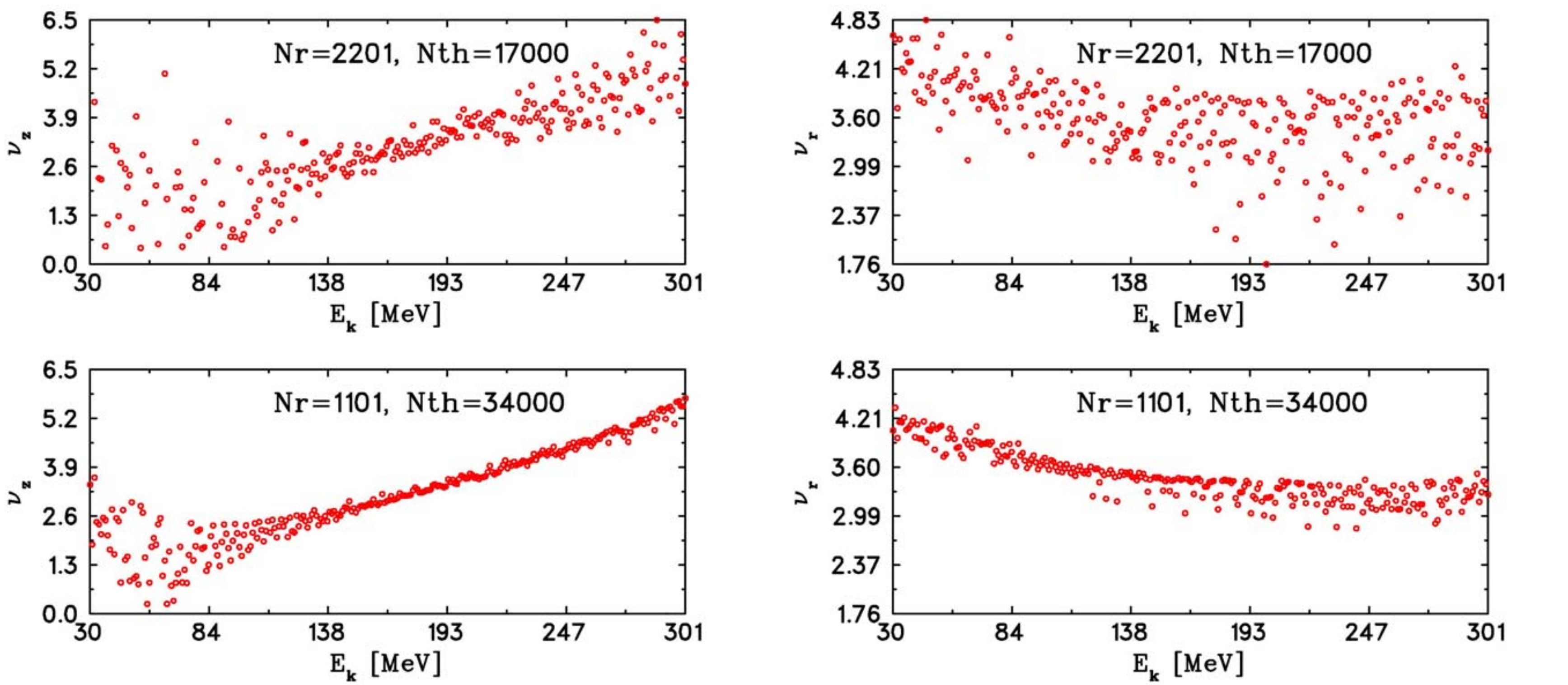


- In order to get good E.O. solutions with the hard magnet edges, a very fine field grid was required (800 θ values, 400 r values).
- The agreement between the CYCLOPS results (CYC) and Berg's (JSB) was very satisfactory.

JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (1)

Carol Johnstone and Shane Koscielniak have developed a LNS FFAG, based on a FODO lattice, for cancer therapy with 18-400 MeV/u carbon ions[4]. This uses edge- as well as gradient-focusing to minimize the tune variation.

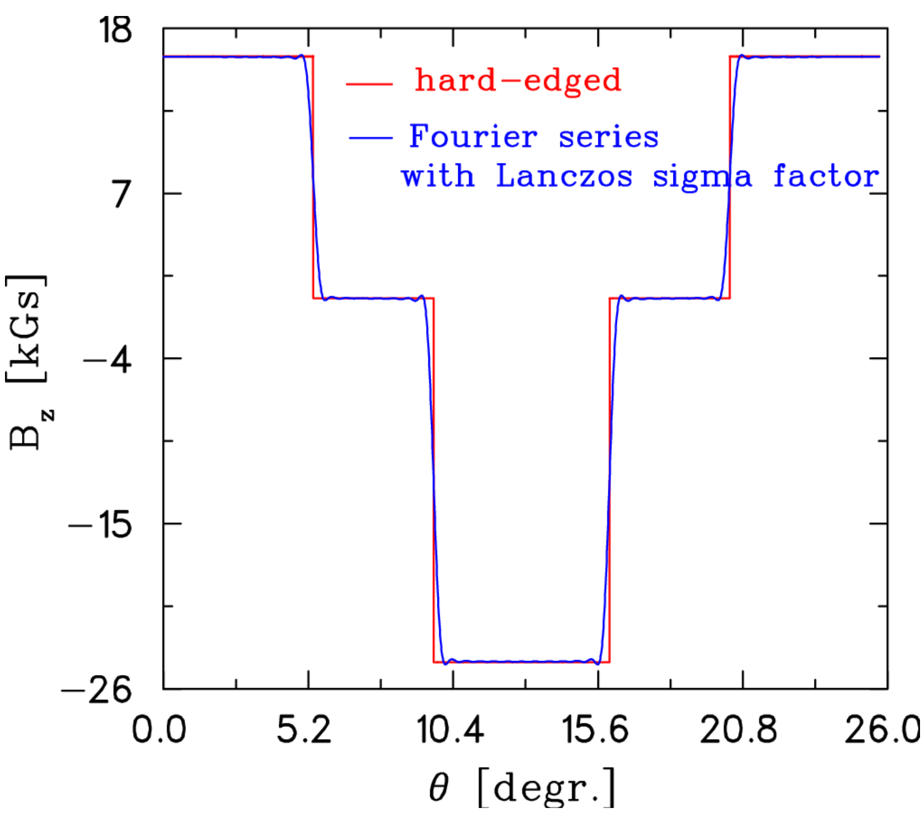
But non-radial hard magnet edges are tricky to model with a polar grid – and lead to noisy results from CYCLOPS – even with 37 million grid points!

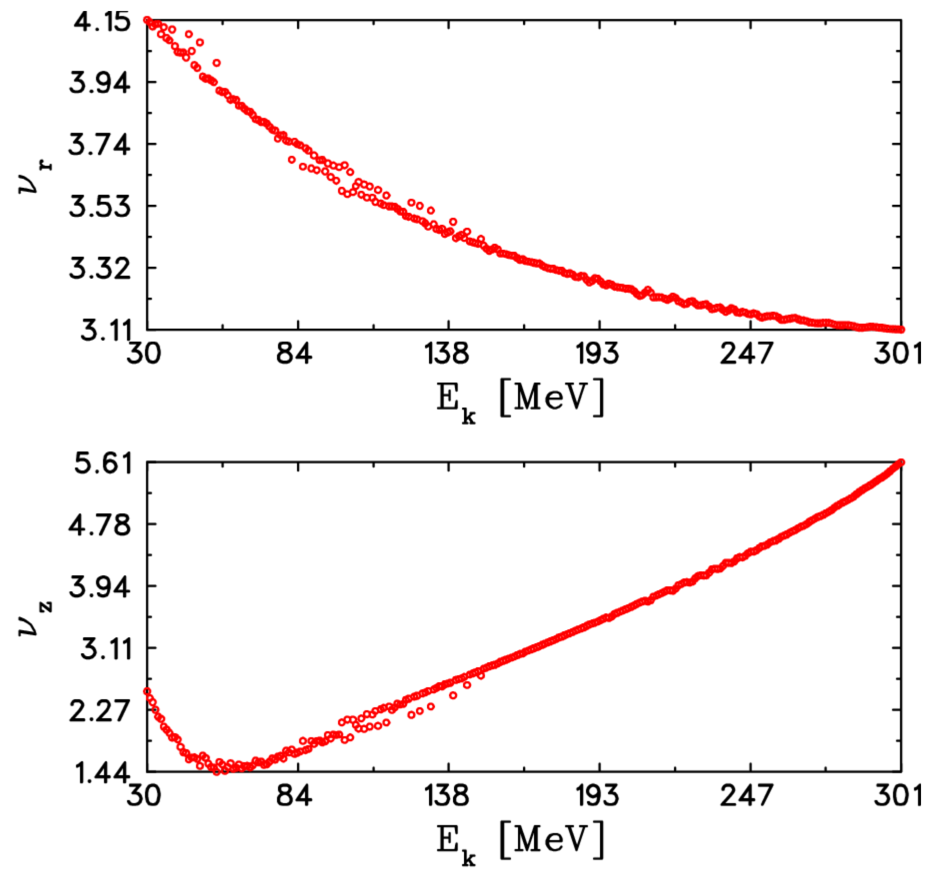


4. C. Johnstone, S.R. Koscielniak, *Proc. PAC'07*, 2951-3 (2007).

JOHNSTONE-KOSCIELNIAK MEDICAL FFAG (2)

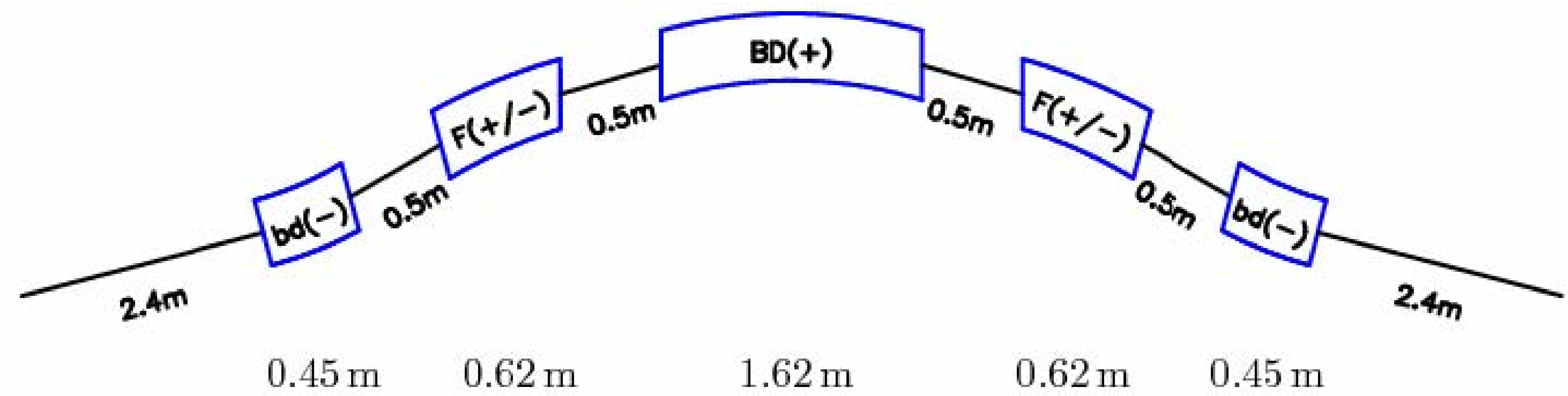
To smooth the field's hard edges our initial technique was to fit $B(\theta)$ at each radius by a Fourier series, combined with Lanczos σ-factors to eliminate ringing – simple and quite effective:





REES'S ISOCHRONOUS IFFAG

G.H. Rees[1,5] has designed several FFAGs using novel 5-magnet "pumplet" cells, in which variations in field gradient and sign enable each magnet's function to vary with radius – providing great flexibility.



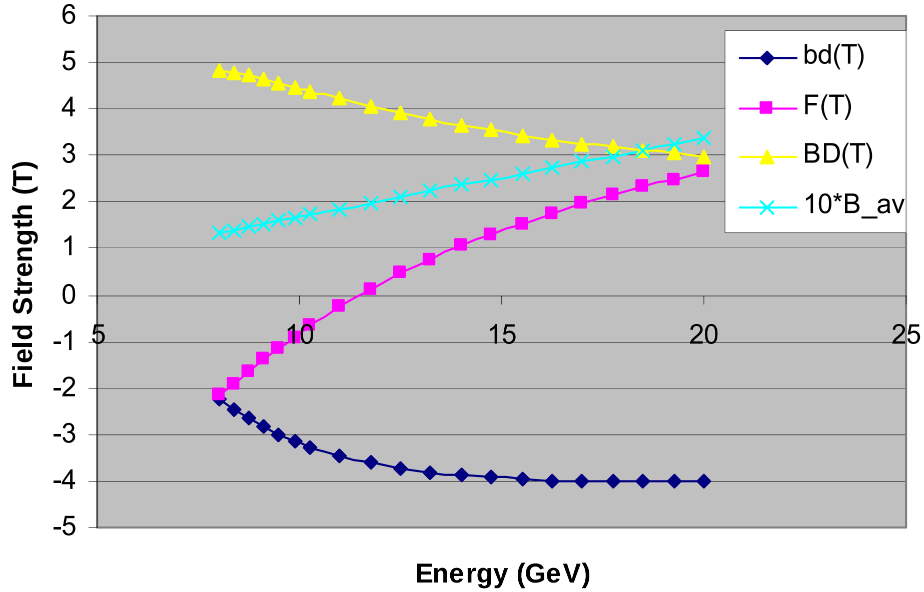
- The example shown is an isochronous design (IFFAG) for accelerating muons from 8-20 GeV in 16 turns.
- This is remarkable in achieving both isochronism and vertical focusing at highly relativistic energies ($77 \leq \gamma \leq 190$) without invoking spiral magnet edge focusing [recall isochronous $\Delta\nu_z^2 = -(r/B_{av})(dB_{av}/dr) = -\beta^2\gamma^2$].
- Highest energy spiral-sector isochronous cyclotron design had $\gamma \leq 15$.

5. G.H. Rees, *FFAG'04* (2004); *FFAG'05* (2005); *ICFA-Beam Dynamics Newsletter* **43**, 74 (2007)

IFFAG FIELDS & TUNES

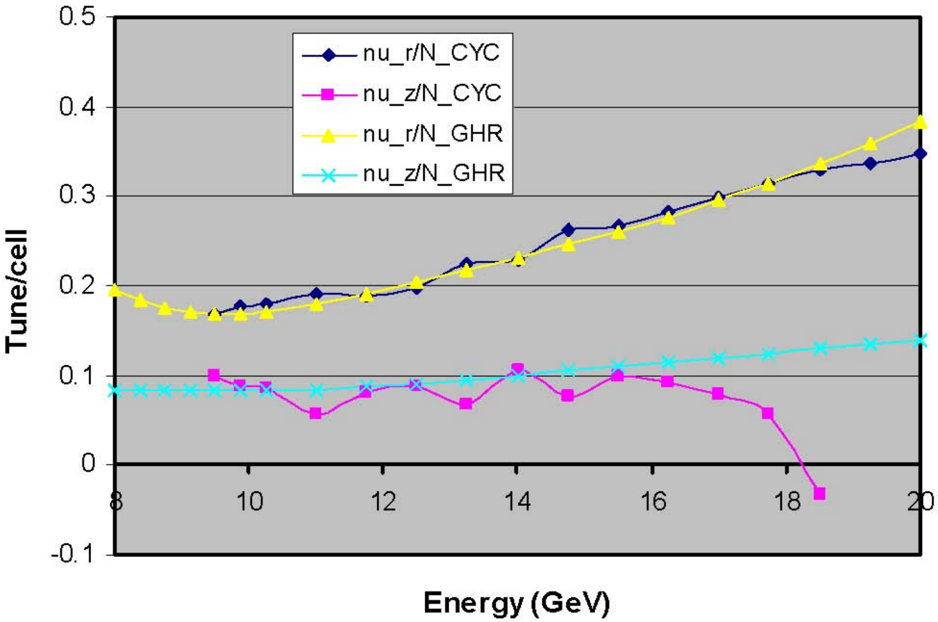
For the **fields**, note how:

- F reverses sign at ~11 GeV
- bd focusing vanishes at high E
- BD focusing vanishes at low E
- Bav rises linearly with E.



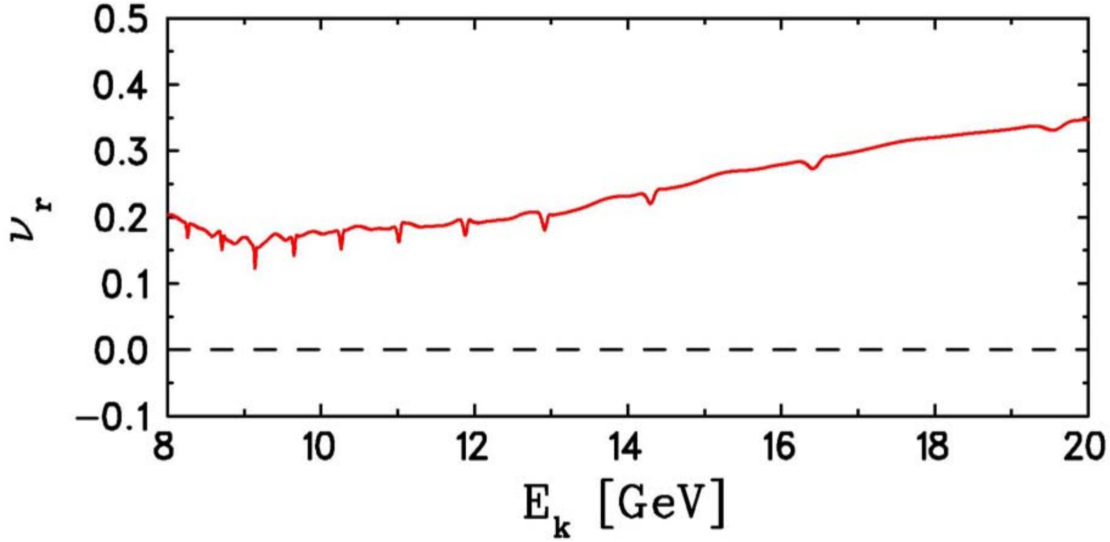
For the **tunes** our initial results[3] (♦■) were in general agreement with Rees's (▲ x) – except above 17 GeV - but rather noisy.

As before, a finer mesh is needed to map the non-radial hard edges adequately – requiring more grid points than CYCLOPS can handle.

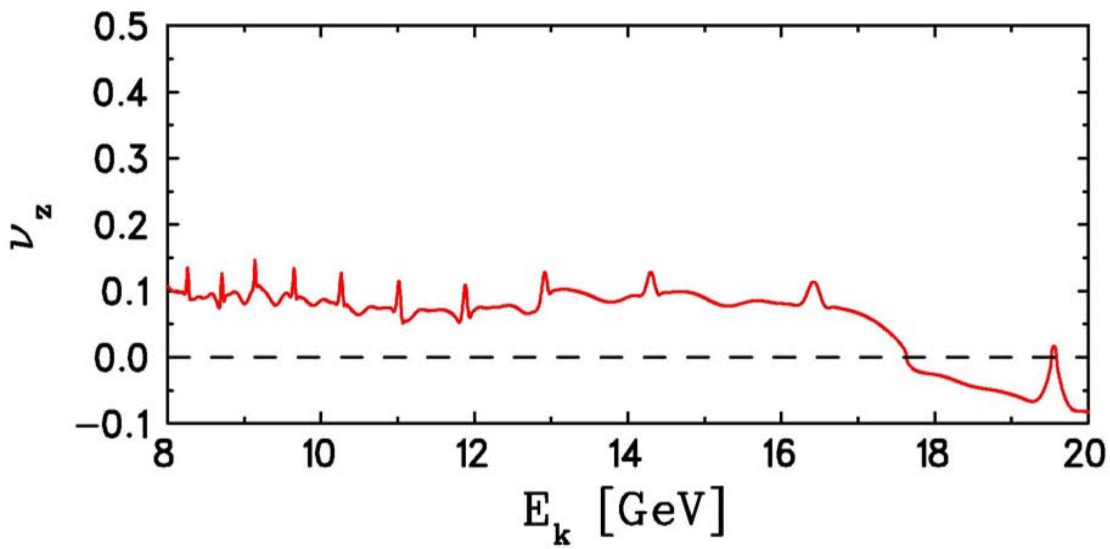


IFFAG WITH SOFTENED MAGNET EDGES

Smoothing the hard-edge data by Fourier-fitting them azimuthally gives less noisy results, but leaves regular spikes – probably due to the remaining radial steps in the data.



A scheme to smooth in both r and θ by replacing the hard edges in the raw data by sloping ones a few grid steps wide is under way. This should eliminate the spikes.



Note that the results for ν_z continue to disagree with Rees's above 17 GeV.

RADIAL-SECTOR CYCLOTRONS WITH REVERSE BENDS

The IFFAG is essentially an isochronous ring cyclotron with an unusually complicated magnet arrangement – 5 magnets/cell rather than 1.

An isochronous cyclotron's top energy is limited by vertical focusing:

$$\nu_z^2 \approx -\beta^2\gamma^2 + F^2(1 + 2\tan^2\varepsilon)$$

where ε is spiral angle and the magnetic "flutter" (mean square deviation)

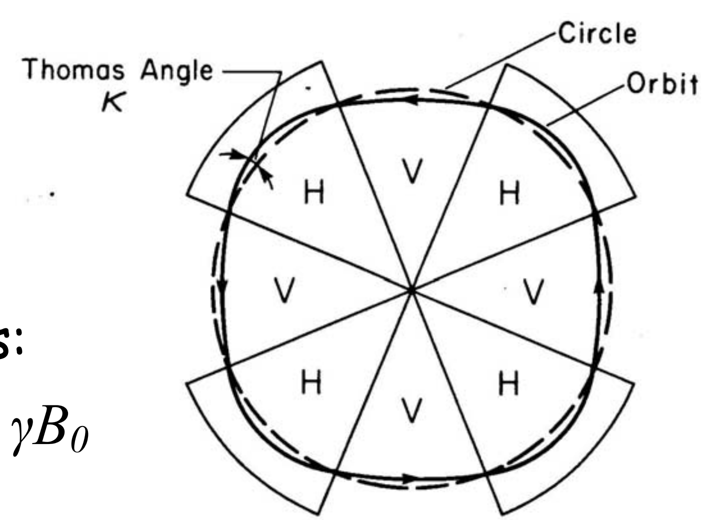
$$F^2 \equiv \left\langle \left(\frac{B(\theta)}{B_{av}} - 1 \right)^2 \right\rangle .$$

How high an energy could a radial-sector cyclotron reach by just converting the field-free "valley" to a reverse bend to maximize F^2?

To begin with, we assume:

- N radial sectors
- Hard-edge magnets
- No drift spaces
- Equal and opposite hill and valley fields:

$$B_h = -B_v = B(r) = \gamma B_0$$



Denoting the angular fraction of a sector taken up by a hill as h, and ignoring scalloping effects on the orbit length and average field around it:

$$B_{av} = 2(h - ½)B\ .$$

The flutter is determined entirely by h, and so is the same at all energies:

$$F^2 = ¼(h - ½)^{-2} - 1\ .$$

For the axial focusing to remain positive up to some maximum energy γ_m, but no further, the tune formula tells us that:

$$h - ½ = 1/2\gamma_m.$$

If the maximum magnetic field available, B_m, is applied at maximum energy γ_m , then the "central field" B_c and "cyclotron radius" R_c are given by:

$$B_c = B_m/\gamma_m{}^2$$

$$R_c = (m_0c/e)\,\gamma_m{}^2/B_m\ ;$$

i.e. the ring radius required increases as the square of the desired energy.

The recipe for hill and valley field strength is:

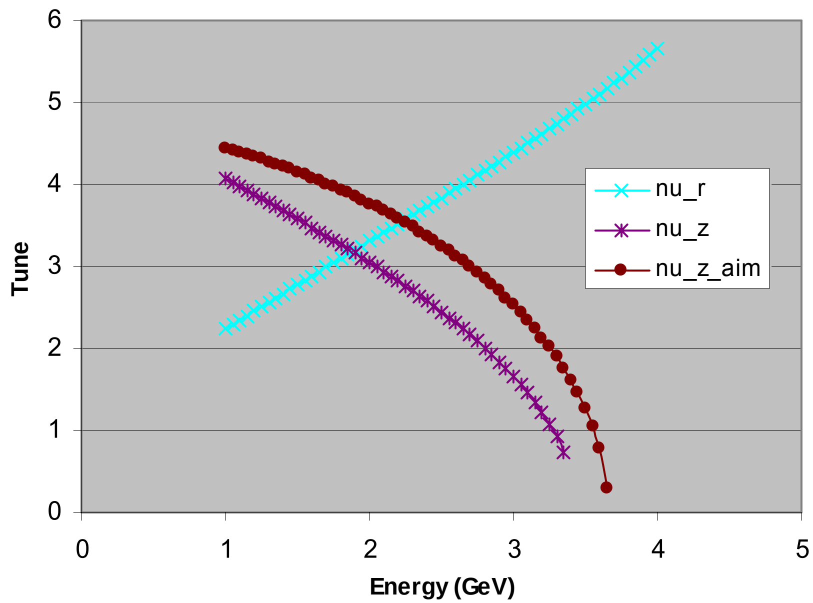
$$B(r) = (B_m/\gamma_m)/\sqrt{\{1 - (r/R_c)^2\}}.$$

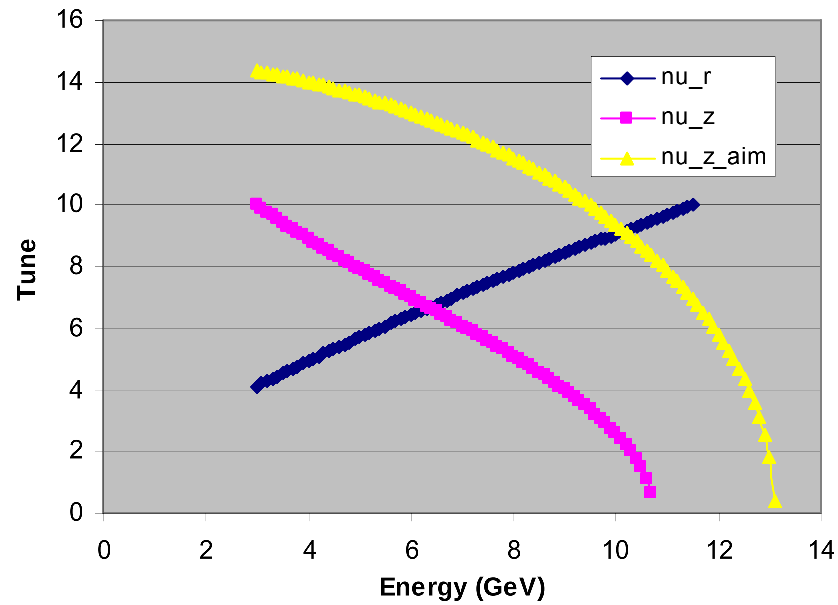
Symon's circumference factor, the ratio of the actual circumference to that obtainable with uniform B_m and no reverse bends: $C = \gamma_m$.

REVERSE-BEND CYCLOTRONS – SIMULATIONS (1)

We studied two cases with specifications similar to old spiral-cyclotron proposals[6,7], (with $N \approx 3\gamma_m$ so $\nu_r \approx \gamma$ stays well below the $N/2$ resonance):

E (GeV)	γ_m	N	h	F^2	$B_m(T)$	$B_c(T)$	$R_c(m)$
1 – 4	5	15	0.6	24	5	0.2	15.65
3 - 13	15	45	0.533·	224	5	0.022·	140.83



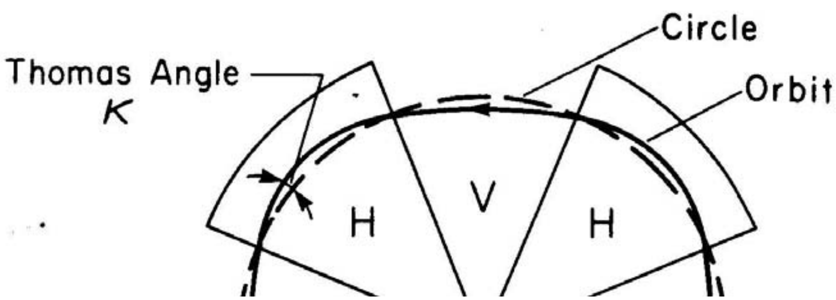


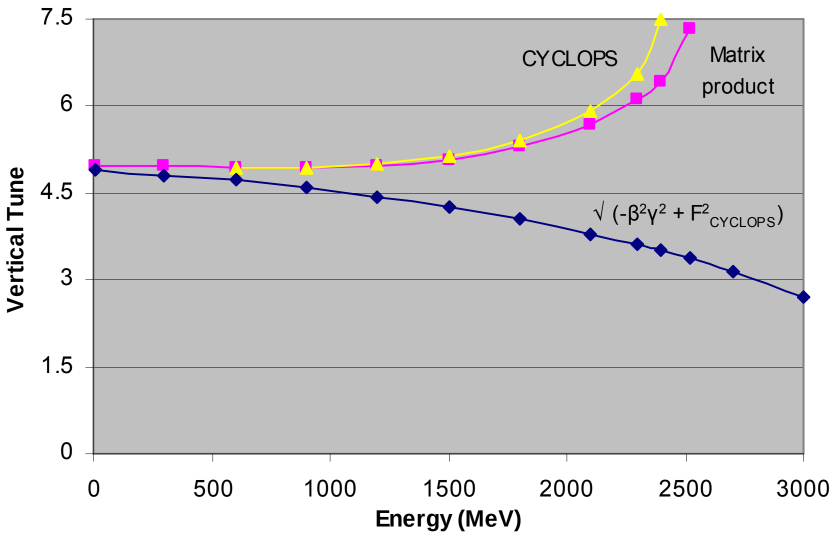
In both cases ν_z drops to 0 below the desired E_{max} – reflecting a drop in flutter due to the orbits not following arcs of constant B.

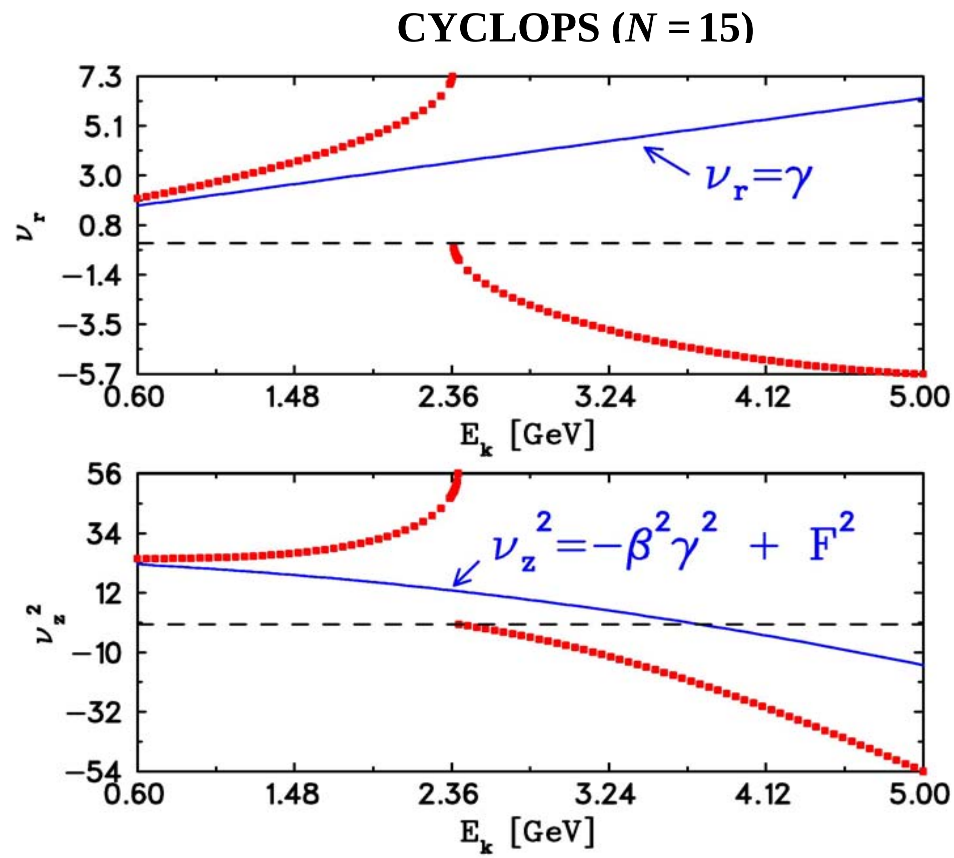
6. M.K. Craddock, C.J. Kost, J.R. Richardson, Proc.CYC'78, IEEE Trans. NS-26, 2065 (1979).
7. J.I.M. Botman, M.K.Craddock *et al.*, Proc. PAC'83, IEEE Trans. NS-30, 2007 (1983).

REVERSE-BEND CYCLOTRONS – SIMULATIONS (2)

The flutter and focusing can be restored by shaping the field contours to follow the scalloped orbits:







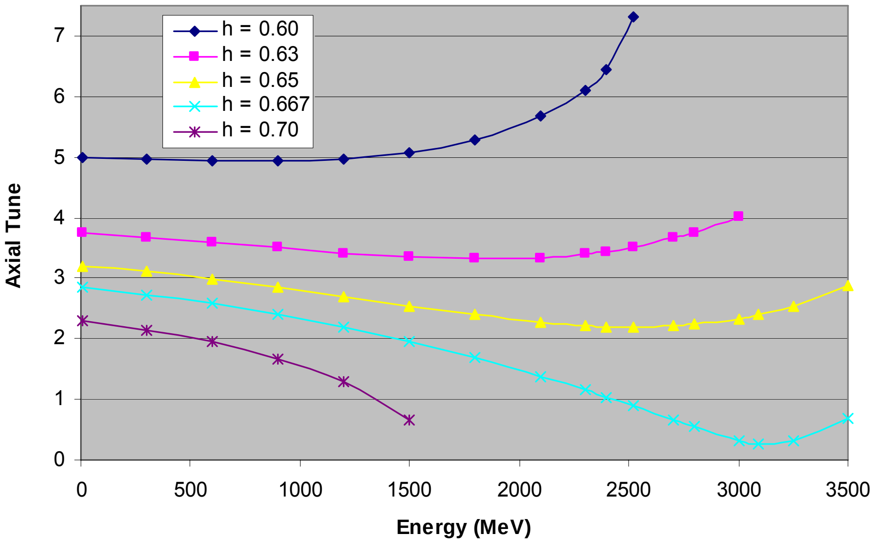
But the focusing is now too strong!

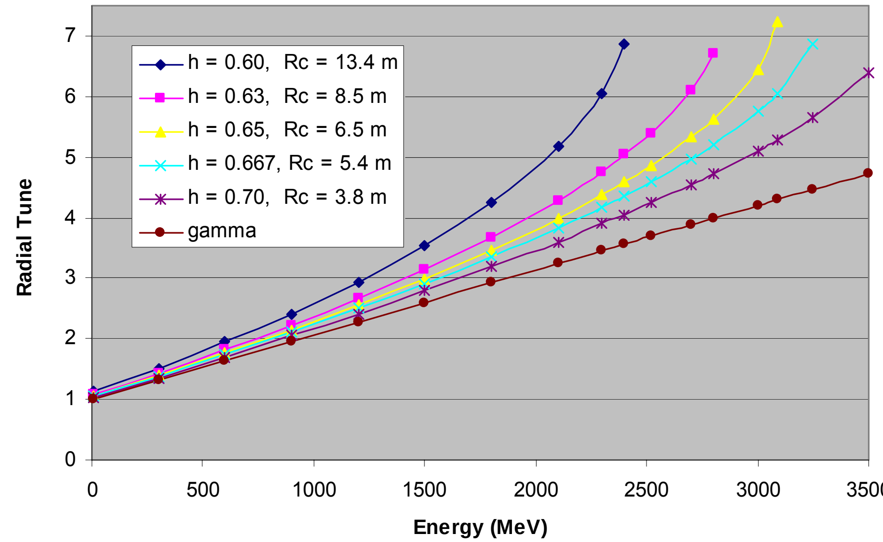
- the N/2 resonances ($\nu = 7.5$) are reached at only $\gamma \approx 3.5$;
- ν_z from CYCLOPS tracking and matrix multiplication agree fairly well;
- the tunes are driven well above the low-energy cyclotron formulae.

REVERSE-BEND CYCLOTRONS – SIMULATIONS (3)

The top energy can be raised either by widening the hills

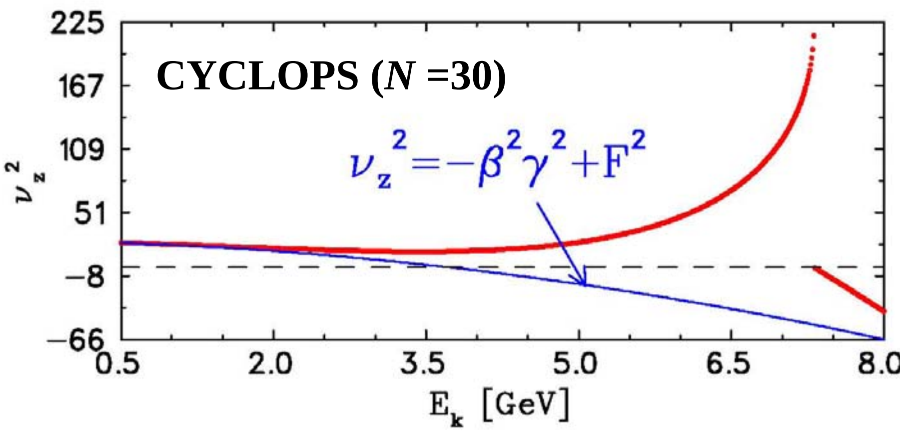
- reducing the radius and both tunes (say $h = 0.65$, $E = 3$ GeV, $R_c = 6.5$ m),

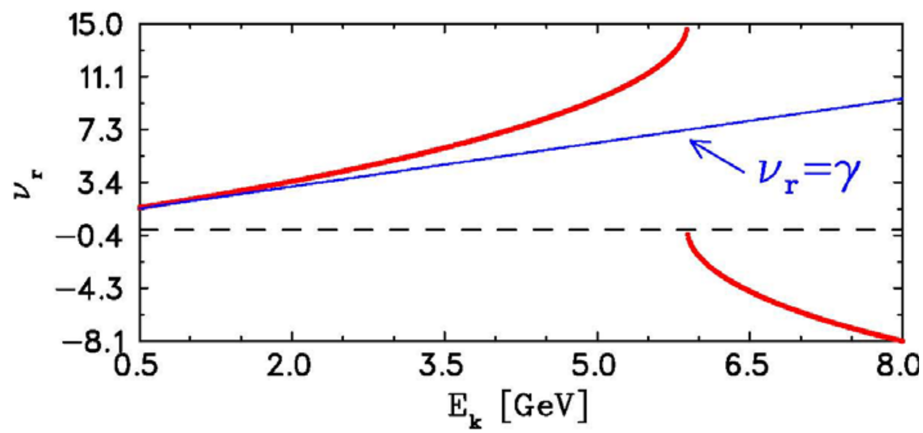




or by increasing the number of sectors (say to $N = 30$, with $h = 0.6$)

- a more effective way of repelling $\nu_r = N/2$ ($E = 6$ GeV, but $R_c = 14.9$ m).





SUMMARY

The cyclotron E.O. code CYCLOPS has been applied to several FFAGs:

- LNS FODO-2 for 10-20 GeV muons
- LNS edge-focusing FODO medical FFAG for 18-400 MeV/u C ions
- NLNS isochronous pumplet IFFAG for 8-20 GeV muons.

CYCLOPS is:

- designed for energy-dependent E.O.s in wide-aperture magnets
- ideal for finding E.O. properties in measured magnetic fields
- uncomfortable with hard-edge magnets – data softening needed!

Using reverse bends in radial-sector ring cyclotrons has also been studied:

- $N/2$ resonance makes simple tune formulae useless at higher energies;
- higher energies achievable if the field contours follow the orbits;
- with 5-T magnets & 0 drifts: 3 GeV for R_c = 6.5 m, 6 GeV for R_c = 15 m.