Contents

Chapter 1

Scaling FFAG

1.1 Introduction

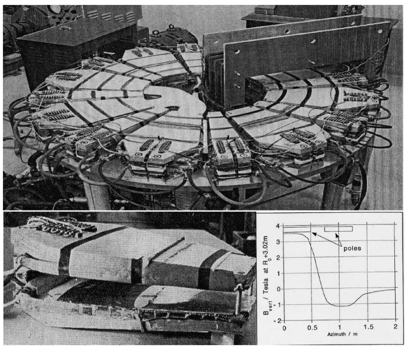

Fig. 1.1 Top: the DF lattice MURA MARK II and its induciton acceleration system, the first electron FFAG (first beam in 1956 ******) [**?**]. Bottom: variable gap, defocusing, sector dipole.

Fig. 1.2 The first proton FFAG (first beam 1999), particle source in the background, RF system on the right and bottom-right picture, DFD triplet sector dipole bottom-left picture [**?**].

Fixed Field Alternating Gradient (FFAG) accelerators are separated sector, fixed-field, ring accelerators. Fixed field technology allows high repetition rates. Gaps between sectors allow the insertion of dedicated equipmnet for injection, acceleration, extraction, diagnostics, vacuum (Figs. 1.1, 1.2). Trajectories in an FFAG lattice spiral under the effect of acceleration (outward as in cyclotrons, possibly inward based on particular lattice properties [**?**]).

Transverse focussing is based on alternating gradient and Thomas focussing. Typical lattice cells include FD (focussing-defocusing) radial sector doublet (Fig. 1.1), DFD radial sector triplet (Fig. 1.2), spiral sector dipole (Fig. **??**). FFAG optics is strong focussing (large betatron phase advance per cell, transverse horizontal and vertical wave numbers have large values) resulting in tight transverse bunch confinement.

FFAGs can operate in two ways: (i) like synchro-cyclotrons, the acceleration is cycled using frequency-modulated RF to follow the change in revolution period of the accelerated particles, the synchrotron motion ensures longitudinal pahse stability (longitudinal focussing), or (ii) in a *quasi-isochronous* mode using fixed frequency accelerating RF, for near-crest acceleration of small phase-extent bunches, no longitudinal phase stability in that case (Chapter **??**). Both methods result in beam being delivered in bunches, at a repetition rate which, in the former case is that of the cycling, in the $10^2 \sim 10^3$ Hz range, or in the latter case amounts to the accelerating RF frequency (MHz range) or a sub-harmonic.

1.2 Scaling FFAG

Scaling FFAG lattice is in general not isochronous (more in Sec. 1.2.5), by contrast with cyclotron. This results from a different use of the transverse field index k (Eq. refEqCycloRadialIndex), namely, to ensure constant focusing, whereas in cyclotrons it is used to ensure isochronism.

1.2.1 *Orbits*

• Exercise 1.2.1-1 As part of this exercise, plots will include theoretical expections from the formulas in the text, together with ray-tracing outcomes. Using the `FFAG` keyword, construct a DFD dipole-triplet cell of a 12-cell ring, with the D and F sectors respectively 10.24 and 3.43 degrees, half-drift 4.75 deg, identical D-F and F-D spacings, field index $k = 7.6$, injection energy 10 MeV on $R_{\rm inj} = 4.5 * * * *$ m radius and extraction energy 110 MeV. In the hard-edge magnet model, compute 51 orbits, evenly spaced in kinetic energy, in the range $R_{\rm inj} < r < R_{\rm ext}$ and plot $B(r)$ (Eq. EqFFAGB), orbit length $\mathcal{L}(r)$ (Eq. EqFFAGOrbitL), revolution period $T_{\rm rev}(r)$ (Eq. EqFFAGOrbitT). Repeat in a soft-dege magnet model using $C_0 - C_5$ = fringe-field coefficient values in `FFAG`.

Plot the field along the orbits at 10:110:20 MeV, in the hard-edge and soft-edge models.

Compute the momentum compaction (Eq. EqFFAGAlpha), the transition γ.

1.2.2 *Focusing*

• Exercise 1.2.1-2

Plot the betatron and dispersion functions at 12, 50 and 100 MeV for cpmparison (three separate graphs).

Plot the radial and axial wave numbers, and the chromaticity, as a function of energy or radius.

The hard-dege model has no B_s focussing. Please explain why, nevertheless, the model does provide first order vertical wedge focusing.

Why is the vertical tune not constant? What parameter can be used to restore constant vertical tune? Fing the corresponding gap shape, appply in the FFAG model and check the new evolution of the vertical tune so obtained.

1.2.3 *Acceleration*

The RF gap provides a voltage

$$V_{\mathrm{RF}}(t) = \hat{V} \cos .... \tag{1.1}$$

Particles are accelerated as long as they belong in the $[-90, +90]$ degree phase interval, the closer to $\phi = 90$ deg, the smaller the number of turns (the time interval) necessary to reach the extraction radius of the FFAG. A deviation of the field B from the isochronous value $2\pi m f_{\mathrm{rev}}/q$ will result in a shift in the arrival phase of the particle at the RF gap amounting to

$$\Delta(\sin\phi) = 2\pi h n \Delta B/B \tag{1.2}$$

******* prendre de valeurs R, B, etc. realistes, e.g. in ../biblio/22047216.pdf, 23001796.pdf*******

• Exercise 1.2.3-1 Assume an accelerating double-gap configuration as in Fig. **??**. What is the minimum number of turns expected from 5 keV to 10 MeV ? Track a particle over that range, play with the RF phase, conclude on the expectations. In a V(t) diagram, plot the position of the particle along the V(t) curve at the accelerating gap, for a magnetic field defect $\Delta B/B = 10^{-4}$, homogeneous, in the previous sector map.

1.2.4 *Focusing*

Let B_r, B_y be the radial and axial components of the magnetic field, respectively, $x = r - R$ a small radial displacement with respect to the reference circular orbit, $\omega_{\mathrm{rev}} = 2\pi f_{\mathrm{rev}}$ the angular frequency of the circular motion.

The radial and axial strengths experienced by a particle moving in the vicinity of that reference orbit write, to the first order in the radial, x, and axial, y, coordinates

$$F_x = m\ddot{x} = -qvB_y + m\frac{v^2}{r} \approx -qv(B_y|_{x=0} + \frac{\partial B_y}{\partial r}x) + m\frac{v^2}{R}(1-\frac{x}{R}),$$
$$\text{yielding } \ddot{x} + \omega_r^2 x = 0$$
$$F_y = m\ddot{y} = qvB_r \approx qv\frac{\partial B_r}{\partial y}y = qv\frac{\partial B_y}{\partial r}y, \text{ yielding } \ddot{y} - \omega_y^2 y = 0 \quad (1.3)$$

wherein $\omega_r^2 = \omega_{\rm rev}^2(1 + \frac{R}{B}\frac{\partial B_y}{\partial r})$, $\omega_y^2 = \omega_{\rm rev}^2\frac{R}{B}\frac{\partial B_y}{\partial r}$. Focusing by a restoring force appears owing to the use of a magnetic field with radial index $k = \frac{R}{B}\frac{\partial B_y}{\partial r}|_{x=0,y=0}$. The two quantities

$$\nu_r = \omega_r/\omega_{\rm rev} = \sqrt{1+k}, \qquad \nu_y = \omega_y/\omega_{\rm rev} = \sqrt{-k} \qquad (1.4)$$

are known respectively as the radial and the axial "wave number" of the oscilatory motion in the neiboring of the reference circular orbit. Note that $\nu_r^2 + \nu_y^2 = 1$. Vertical motion stability requires k to be negative : B_y (respectively, the magnet gap) is slowly decreasing (increasing) with radius, restoring force toward the median plane. Focussing in both radial and axial motions requires $0 < k < -1$, a conditon known as "weak focusing". Note that at low energy the electric field in the region of the accelerating gap also contributes to the focusing, an aspect omitted here.

• Exercise 1.2.4-1 Plot two particle trajectories that demonstrate the value of the radial wave number in the uniform field of Sec. **??**. Conclude on orbit and horizontal motion stability. Derive the vertical transport matrix from ray-tracing, conclude on the stability of the vertical motion in a uniform field.

• Exercise 1.2.4-2 Back to the field map of exercise **??**-1, or to the analytical model of exercise **??**-2: introduce a field index $-1 < k < 0$. Plot the radial and vertical phase space of a 5 MeV ion on a 1μm normalized invariant. Compute its radial and axial motion wave numbers, ν_r and ν_y, using two different methods, namely, 1-turn mapping and Fourrier analysis of multi-turn motion. From multiturn tracking, generate the envelope of a 5 MeV beam around the ring FFAG.

• Exercise 1.2.4-3 Using either the field map or the analytical model devised in the exercise 1.2.4-2, plot the energy dependence of the reference orbit radius, $R(E)$. Plot $\nu_r^2 + \nu_y^2$ as a function of radius, compare with the value of the field index. On a common graphic, plot the horizontal phase space of the 1, 10, 20, 50 MeV particle motion, assuming the latter on a 1μm normalized invariant in each case. Plot the vertical phase space motion for these very energies. Plot the components of the field vector experienced by a particle as a function of azimutahl angle, over a few turns.

1.2.5 *Quasi-isochronous scaling FFAG lattice*

With particular constraints on the field index, scaling FFAG lattices can be made *quasi-isochronous* so allowing the use, as in cyclotrons, of fixed-frequency RF for acceleration, with however poor isochronism overcome with brute force RF voltage.

1.3 Non-scaling FFAG

1.4 Bibliography

[BibFFAG-1] MURA, MARKII O Camelot ??

[BibFFAG-2] Theme Section: FFAG Accelerators, in Beam Dynamics Newsletter No. 43, Issue Editor C.R. Prior, August 2007.

1.5 Response to exercises

• Exercise **??**-2

The hard-dege model has no B_s focussing. Nevertheless, the model does provide first order vertical wedge focusing due to a kick transform, fortran procedure WEDGKI.

Why is the vertical tune not constant? Because the gap is assumed constant.

What parameter can be used to restore constant vertical tune? Gap shape gap $\sim 1/\mathrm{r}^{\kappa}$, with $\kappa \sim$ a few units to be determine. This will cause the firnge field extent to have the appropriate r-dependence.