An Introduction to Particle Accelerators // - Playing Accelerators, on Computer -

Contents

1 Cyclotron - resonant acceleration

- Non-relativistic cyclotron

  - orbit : $\boxed{r = v/\omega_0 = mv/qB}$
  - focusing (1) :
    $F_z = qvB_r \approx qv\frac{\partial B_r}{\partial z}z \equiv qv\frac{\partial B_z}{\partial r}z$
    $\ddot{z} - \frac{qv}{m}\frac{\partial B_z}{\partial r}z = 0 \rightarrow \omega_z^2/\omega_0^2 = \nu_z^2 = -\frac{r}{B_z}\frac{\partial B_z}{\partial r} = -k, \quad \boxed{\nu_z = \sqrt{-k}}$.
    hence the field index k needs be negative : B_z **is slowly decreased with radius**.
    Similarly, $\boxed{\nu_r = 1 + k}$. This sets the requirement $\boxed{-1 < k < 0}$
  - focusing (2) : is also ensured at lower energy by the electric field.
  - isochronism :
    The condition for vertical focusing, $-1 < k < 0$ (B is not constant),
    spoils the isochronism.
    As a consequence, the phase is not constant (ABCDE path)
  - bunching : particle beam injected into the cyclotron necessarily gets bunched, at the frequency of the RF (the
    an RF period)

- The classical limit ($\gamma \approx 1$) is $\sim$25 MeV for protons, 50 MeV for D and α, (about $2-3$% increase in mass), GANII
  100 MeV/u...

- That was enough energy to transmute all nuclei... The classical cyclotron allowed discovering oodles of nuclear reac

  Yet, let's keep in mind : transmutation was not the all story

Cyclotron (3/5) - classical

- Relativistic energies, the bad news :

  - The cyclotron resonance $\omega_0 = qB/\gamma m$, with $r = \beta c/\omega_0$ yields $\boxed{k = \frac{\beta}{\gamma}\frac{\partial\gamma}{\partial\beta} = \beta^2\gamma^2}$

  - so k cannot satisfy $\boxed{-1 < k < 0}$,

    isochronism requires that $B(r) \propto \gamma$, which yields vertical defocusing...

- That was the end of the story, ~ 25 MeV protons, etc... :
  Hans Bethe (1937) :
  "... it seems useless to build cyclotrons of larger proportions than the existing ones... an accelerating chamber of deuterons of 11 MeV energy which is the highest possible..."

  Frank Cole : "If you went to graduate school in the 1940s, this inequality ($1 < k < 0$) was the end of the discussion

- Until...

Cyclotron (4/5) - Thomas focusing

- 1938, L.H. Thomas, “The Paths of Ions in the Cyclotron”, introduces the “Thomas focusing”, based on separate sector bending, namely, “edge-focusing”,

- 1954, Kerst, spiral edges increase vertical focusing further
  $\boxed{\nu_z = \sqrt{-k + F^2(1 + 2\tan^2\xi)}}$, $\quad F = Flutter = \frac{<B^2> - <B>^2}{<B>^2}$

- That allowed having $B(r)$ increase in proportion to γ, so to ensure constant RF frequency ($\omega_0 = qB/\gamma m$), while *preserving vertical focusing*.

- Modern cyclotrons still rely on these principles

- Cyclotron is limited in energy by its field strength and magnet size.

Cyclotron (5/5), exercise

On RF harmonic 1, which RF frequency applied on dees is needed to accelerate a proton in a ∼uniform, 1 Tesla magnetic

What is the energy gained by a proton when it reaches $r = 0.3\,\mathrm{m}$?

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Answers :

1/ $f_{rev.} = eB/2\pi m = c^2 B/2\pi(mc^2/e) = 9\,10^{16} \times 1/2\pi 10^9 \approx 15\,\mathrm{MHz}$
Two accelerating gaps at 180 degrees $\Rightarrow f_{RF} = f_{rev.}$

2/ From $r = \frac{mv}{eB}$ one gets $\frac{1}{2}mv^2 \equiv E = \frac{1}{2}\frac{(eBr)^2}{m} = \frac{1}{2}\frac{eB^2r^2c^2}{(mc^2/e}$ hence $\frac{E}{e}(eV) = \frac{1}{2}\frac{B^2r^2c^2}{(mc^2/e)}$
$\frac{E}{e}(eV) = \frac{1}{2}\frac{1^2(0.3)^2(3\,10^8)^2}{10^9} \approx 4.3\,\mathrm{MeV}.$