Contents

Chapter 1

Cyclotron

1.1 Introduction

Beyond simple knowledge and concepts, this Chapter introduces to the basic principles and formulas attached to the cyclotron, which will be manipulated during the user computer exercises. Deeper insight in the theory and technology of the cyclotron can be found in the brief series of refrences cited in this chapter. The candidate to the computer exercises which follow is encouraged to first read these references.

1.2 Cyclotron

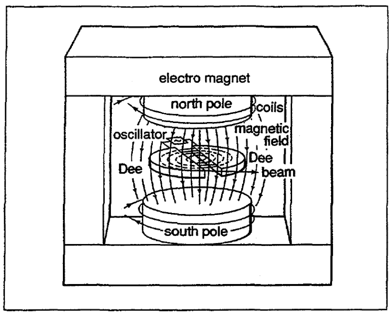

Fig. 1.1 A resonant acceleration device: the cyclotron [1].

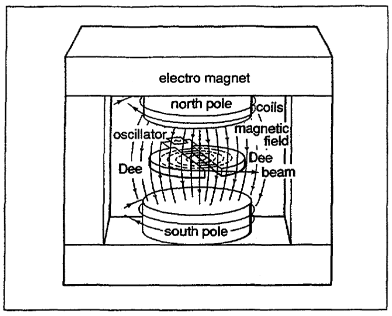

Fig. 1.2 The world greatest beam power cyclotron, at PSI.

In a uniform field B a particle with rest mass m_0, relativistic mass $m = \gamma m_0$, charge q, momentum $p = mv$, undergoes a circular motion characterized by its radius R, with a revolution frequency $f_{\rm rev}$ which depends on the field and on the particle charge and mass, but *not* on R neither on the particle velocity. Thus repeated acceleration can be provided by a system (the double-gap formed by the double dee in Fig. 1.1) with fixed frequency f_{RF} in resonance, at some harmonic integer h, with the orbital frequency of the particle. These three quantities satisfy, respectively

$$R = p/qB, \qquad 2\pi f_{\mathrm{rev}} = v/R = qB/m, \qquad f_{\mathrm{RF}} = h f_{\mathrm{rev}} \tag{1.1}$$

These expressions hold in both cases of *classical* cyclotron, which operates in sub-relativistic regime, relativistic $\gamma m_0 \sim m_0$, and *isochronous* cyclotron which operates in relativistic regime, $\gamma > 1$.

From design and beam simulation viewpoints, a cyclotron accelerator is a typical case where just putting together drifts and dipoles in a matrix transport code won't do. This may at best work for a particular orbit, and will not allow designing the accelerator.

The common solution os to use a field map, 2- or 3-dimensional. However, mathematical models for the field are also a good approach, especially in a preliminary design phase due to the flexibility it brings in tayloring the field so to achieve required constrains of orbit excursion limitations, focusing, isochronism, etc.

This is the techniques addressed in the exercises.

1.3 Classical cyclotron

The classical limit assumes less then $2-3\%$ increase in mass, $\gamma \approx 1.02$ 1.03. This means ∼25 MeV for protons, 50 MeV for D and α, That is enough energy to transmute all nuclei. As a matter of fact, the classical cyclotron allowed discovering oodles of nuclear reactions and isotopes.

1.3.1 *Closed orbit, time of flight*

• Exercise 1.3-1

Construct a 45-degree sector mid-plane 2-dimensional field map, with constant vertical field such as to achieve a R=0.5 m extraction radius for 10 MeV H- ions (an injector to a higher energy installation, typically, with stripping extraction). Use a uniform mesh in cylindrical coordinates. In a 8-sector configuration (8 times that map, so constituting a complete ring), track H- ions on their circular orbit, for a few different momenta ranging from 5 keV to 10 MeV. Plot, as a function of momentum, their equilibrium orbit radius R and the time of flight $T_{\rm rev}$ on that orbit, from both ray-tracing and theory on a same graphic. Study the effect of the mesh density on the accuracy of trajectory computation.

• Exercise 1.3-2 Zgoubi provides analytical models for the magnetic field $B_y(r,\theta)|_{y=0}$ in the median plane of a dipole magnet. The particular keyword `CYCLOTRON` is an instance. Use it to repoduce the results above. From the two series of results, comment on various pros and cons of the two methods, analytical field models and fieldmaps.

1.3.2 *Acceleration*

The RF gap provides a voltage

$$V_{\rm RF}(t) = \hat{V}\cos .... \tag{1.2}$$

Particles are accelerated as long as they belong in the $[-90, +90]$ degree phase interval, the closer to $\phi = 90$ deg, the smaller the number of turns (the time interval) necessary to reach the extraction radius of the cyclotron. A deviation of the field B from the isochronous value $2\pi m f_{\rm rev}/q$ will result in a shift in the arrival phase of the particle at the RF gap amounting to

$$\Delta(\sin\phi) = 2\pi h n \Delta B/B \tag{1.3}$$

******* prendre de valeurs R, B, etc. realistes, e.g. in ../biblio/22047216.pdf, 23001796.pdf*******

• Exercise 1.3.2-1 Assume an accelerating double-gap configuration as in Fig. 1.1. What is the minimum number of turns expected from 5 keV to 10 MeV ? Track a particle over that range, play with the RF phase, conclude on the expectations. In a V(t) diagram, plot the position of the particle along the V(t) curve at the accelerating gap, for a magnetic field defect $\Delta B/B = 10^{-4}$, homogeneous, in the previous sector map.

1.3.3 *Focusing*

Let B_r, B_y be the radial and axial components of the magnetic field, respectively, $x = r - R$ a small radial displacement with respect to the reference circular orbit, $\omega_{\rm rev} = 2\pi f_{\rm rev}$ the angular frequency of the circular motion. The radial and axial strengths experienced by a particle moving in the vicinity of that reference orbit write, to the first order in the radial, x, and axial, y, coordinates

$$F_x = m\ddot{x} = -qvB_y + m\frac{v^2}{r} \approx -qv(B_y|_{x=0} + \frac{\partial B_y}{\partial r}x) + m\frac{v^2}{R}(1-\frac{x}{R}),$$
$$\text{yielding } \ddot{x} + \omega_r^2 x = 0$$
$$F_y = m\ddot{y} = qvB_r \approx qv\frac{\partial B_r}{\partial y}y = qv\frac{\partial B_y}{\partial r}y, \text{ yielding } \ddot{y} - \omega_y^2 y = 0 \quad (1.4)$$

wherein $\omega_r^2 = \omega_{\rm rev}^2(1 + \frac{R}{B}\frac{\partial B_y}{\partial r})$, $\omega_y^2 = \omega_{\rm rev}^2\frac{R}{B}\frac{\partial B_y}{\partial r}$. Focusing by a restoring force appears owing to the use of a magnetic field with radial index $k = \frac{R}{B}\frac{\partial B_y}{\partial r}|_{x=0,y=0}$. The two quantities

$$\nu_r = \omega_r/\omega_{\rm rev} = \sqrt{1+k}, \qquad \nu_y = \omega_y/\omega_{\rm rev} = \sqrt{-k} \qquad (1.5)$$

are known respectively as the radial and the axial "wave number" of the oscilatory motion in the neiboring of the reference circular orbit. Note that $\nu_r^2 + \nu_y^2 = 1$. Vertical motion stability requires k to be negative : B_y (respectively, the magnet gap) is slowly decreasing (increasing) with radius, restoring force toward the median plane. Focussing in both radial and axial motions requires $0 < k < -1$, a conditon known as "weak focusing". Note that at low energy the electric field in the region of the accelerating gap also contributes to the focusing, an aspect omitted here.

• Exercise 1.3.3-1 Plot two particle trajectories that demonstrate the value of the radial wave number in the uniform field of Sec. 1.3. Conclude on orbit and horizontal motion stability. Derive the vertical transport matrix from ray-tracing, conclude on the stability of the vertical motion in a uniform field.

• Exercise 1.3.3-2 Back to the field map of exercise 1.3-1, or to the analytical model of exercise 1.3-2: introduce a field index $-1 < k < 0$. Plot the radial and vertical phase space of a 5 MeV ion on a 1μm normalized invariant. Compute its radial and axial motion wave numbers, ν_r and ν_y, using two different methods, namely, 1-turn mapping and Fourrier analysis of multi-turn motion. From multiturn tracking, generate the envelope of a 5 MeV beam around the ring cyclotron.

• Exercise 1.3.3-3 Using either the field map or the analytical model devised in the exercise 1.3.3-2, plot the energy dependence of the reference orbit radius, $R(E)$. Plot $\nu_r^2 + \nu_y^2$ as a function of radius, compare with the value of the field index. On a common graphic, plot the horizontal phase space of the 1, 10, 20, 50 MeV particle motion, assuming the latter on a 1μm normalized invariant in each case. Plot the vertical phase space motion for these very energies. Plot the components of the field vector experienced by a particle as a function of azimutahl angle, over a few turns.

1.3.4 *Isochronism*

The condition for vertical focusing, $-1 < k < 0$ spoils the isochronism: the guiding field B and thus $\omega_{\text{rev}} = qB/m$ decrease with R (with increasing momentum). As a consequence, the arrival time of a particle at the RF gap (by extension the "RF phase") is not constant (ABCDE path)

• Exercise 1.3.4-1

bunching : particle beam injected into the cyclotron necessarily gets bunched, at the frequency of the RF (the time interval between two bunches is an RF period)

Classical cyclotron $0 < n < 1$

Isochronous cyclotron, $0 < n < 1$ cannot be satisfied as $B \propto \gamma$. Introduce Thomas focusing, using sector magnets, no longer slow r-decrease, weak focusing uniform field.

1.3.5 *Resonant acceleration*

A tight tuning between particle time-of-flight, which measns in practice the field B, and the RF frequency is necessary in order to reach the resonance condiiton, $f_{\text{RF}} = hf_{\text{rev}}$.

The accelerating gap in the simulation provide $V_{\text{RF}}(t) = \hat{V}\sin(\omega_{\text{RF}}t)$.........

(i) Check the evolution of the particle acceleration when getting away from the resonant frequency:

- Plot the particle position along the $V_{\text{RF}}(t)$ curve in a $V_{\text{RF}}(t)$ diagram.
- Plot the final phase ϕ_{final} as a function of $\Delta B/B$.
- Find the tolerable deviation from the isochronous field, $\Delta B/B$, if particles are required to be dispersed in momentum by les than $dp/p = 10^{-3}$ at the end of the acceleration cycle.

1.4 Relativistic cyclotron

The bad news with relativistic energies, is, the cyclotron resonance $\omega_0 = qB/\gamma m_0$, with $R = \beta c/\omega_0$ yields

$$k = \frac{R}{B}\frac{\partial B}{\partial R} = \frac{\beta}{\gamma}\frac{\partial \gamma}{\partial \beta} = \beta^2\gamma^2$$

k is positive and increases with energy, the weak focussing condition $-1 < k < 0$ is not satisfied.

The time of flight on the equilibrium orbit, momentum $p = qB\rho$ and circumference $\mathcal{C}$, is $T = \mathcal{C}/\beta c = 2\pi\gamma m_0/qB$. Isochronism requires p-invariant time of flight, $dT/dp = 0$. Differentiating the previous expression, this requirement yields

$$B = \frac{B_0}{\gamma_0}\gamma$$

with B_0 and γ_0 some refence conditions and time of flight $T_0 = (2\pi m_0/q)(\gamma_0/B_0)$. In other words, isochronism requires $B(r) \propto \gamma$, which yields vertical defocusing.

That was the end of the story in the late 1930s. Hans Bethe (1937) : "... it seems useless to build cyclotrons of larger proportions than the existing ones... an accelerating chamber of 37 cm radius will suffice to produce deuterons of 11 MeV energy which is the highest possible...". Frank Cole : "If you went to graduate school in the 1940s, this inequality ($1 < k < 0$) was the end of the discussion of accelerator theory."

Until...

1.4.1 *Thomas focusing*

- 1938, L.H. Thomas, "The Paths of Ions in the Cyclotron", introduces the
  "Thomas focusing", based on separate sector bending, namely, "edge-focusing",
- 1954, Kerst, spiral edges increase vertical focusing further
  $\nu_z = \sqrt{-k + F^2(1 + 2\tan^2\xi)}, \qquad F = Flutter = \frac{<B^2> - <B>^2}{<B>^2}$
- That allowed having $B(r)$ increase in proportion to γ, so to ensure constant RF frequency ($\omega_0 = qB/\gamma m$),
  while *preserving vertical focusing.*
- Modern cyclotrons still rely on these principles

- Cyclotron is limited in energy by its field strength and magnet size.

Exercise

On RF harmonic 1, which RF frequency applied on dees is needed to accelerate a proton in a $\sim$uniform, 1 Tesla magnetic field ?

What is the energy gained by a proton when it reaches $r = 0.3\,\mathrm{m}$?

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Answers :

1/ $f_{rev.} = eB/2\pi m = c^2B/2\pi(mc^2/e) = 9\,10^{16} \times 1/2\pi 10^9 \approx 15\,\mathrm{MHz}$
Two accelerating gaps at 180 degrees $\Rightarrow f_{RF} = f_{rev.}$

2/ From $r = \frac{mv}{eB}$ one gets $\frac{1}{2}mv^2 \equiv E = \frac{1}{2}\frac{(eBr)^2}{m} = \frac{1}{2}\frac{eB^2r^2c^2}{(mc^2/e}$ hence $\frac{E}{e}(eV) = \frac{1}{2}\frac{B^2r^2c^2}{(mc^2/e)}$
$\frac{E}{e}(eV) = \frac{1}{2}\frac{1^2(0.3)^2(3\,10^8)^2}{10^9} \approx 4.3\,\mathrm{MeV}$.

1.4.2 *Orbits*

- find the orbits, energy dependence of radius, time of flight.
Plot these quantities against theoretical expectations

1.4.3 *Focusing*

Check $Q_x^2+Q_y^2$. Plot radial dependence of field index n(r), check `0<n(r) <1`.

1.4.4 *Acceleration*

Introduce a double-Dee simulation. Hint: use CAVITE

Accelerate a complete cycle, on h=1, h=3. In V(t) space, plot V(t) path of reference particle (use PRINT in CAVITE).

1.5 Exercises: hints, help

- Exercise 1.3.2-1

1.6 Bibliography

Stambach CAS
R Baartman Cyclotrons: Classic to FFAG