A Bayesian-inspired approach to passive radar detection

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***Abstract*—This paper considers the passive detection of a signal common to two multi-sensor arrays. We consider Gaussian received signals and noises with positive-definite, but otherwise unstructured covariance matrices. Under the null hypothesis, the composite covariance matrix for the two arrays is block-diagonal with arbitrary positive definite (PD) blocks, whereas under the alternative, it is modeled as an unstructured covariance matrix. Assuming complex inverse-Wishart priors for the unknown covariance matrices, the proposed test relies on the marginalized likelihood ratio, where the unknown parameters (i.e., the covariance matrices) are integrated out. A proper choice of hyper-parameters of the prior distribution shows that the Bayesian-inspired test reduces to a regularized canonical correlation analysis (CCA) detector. Simulation results show the superior performance of the proposed method compared to the generalized likelihood ratio test (GLRT), which is given by a function of the canonical correlations.**

***Index Terms*—Coherence, complex inverse-Wishart distribution, marginal likelihood ratio, multi-sensor array, passive radar.**

I. Introduction

Over the past few decades, researchers have paid a lot of attention to passive radar systems [1]–[5]. These bistatic radars [6] operate without control over transmitted signals, i.e., they use non-cooperative transmitters, known as illuminators of opportunity, such as terrestrial TV [7] and FM broadcast transmitters [8], mobile phone base transceiver stations (e.g., 4G/5G base stations), and communication or navigation satellites [6]. The key advantages of passive radar systems include covert operation, simplicity, cost-effectiveness, and energy efficiency.

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In passive radar (PR) and passive source localization (PSL),[1] the transmitted signal, received at the surveillance and reference sensor arrays, is unknown. It is measured in additive noise. For PR/PSL, researchers have proposed numerous ad-hoc detection algorithms [10]–[13]. Additionally, detectors have also been proposed based on first principles. If the signal and channels are considered unknown parameters, then the detection problem is addressed in a first-order model. The work in [5] considers a first-order generalized likelihood ratio test (GLRT) and [14] derives a Rao test.

It is possible to derive alternative detectors by constraining the unknown signal. Detectors considering a priori signal information, such as frequency domain sparsity [15], [16] or known signal format [17], have been proposed. Another way to include prior information is to say that the unknown signal is drawn from a prior distribution. Then, the joint distribution of the measurement and signal may be marginalized with respect to the prior distribution for the signal. When measurement and signal are multivariate normal (MVN), this marginalization produces a MVN measurement model in which the measurement covariance matrix is a computable function of the noise covariance matrix and the signal covariance matrix. But still, the covariance matrix for the signal is unknown. If this unknown covariance matrix is estimated to maximize the likelihood, then the resulting likelihood ratio detector may be called a second-order GLRT [18], [19]. This gambit amounts to replacing a large number of unknown signal values by a smaller number of unknown values, or parameters, in the unknown covariance matrix.

This would seem to be the end of the story, but if the unknown signal can be constrained by a prior distribution, why can't the unknown covariance matrix be constrained by a prior distribution on covariance matrices. These ideas originate in the case of one-channel detection problems and the literature is much scarcer. One of the few exceptions is the work in [20], which derives a marginalized likelihood ratio for the detection of a signal in the presence of uncorrelated Gaussian noise, or [21], which considers Bayesian detectors for MIMO radar.

[1]In our parlance, passive radar differs from passive source localization, where signals are only received in both channels when a reflecting target is present [9].

This paper aims to extend the results of [18], where the second-order GLRT for PR is shown to be a (reduced-rank) canonical correlation detector, to the case where the unknown signal covariance matrix is given an inverse complex Wishart prior distribution [22]. The resulting marginal likelihood ratio detector is shown to be a *regularized*, canonical correlation detector, where the canonical correlations of [18] are replaced by regularized canonical correlations computed from regularized sample covariance matrices at the reference array and at the surveillance array, with a regularization parameter that appears naturally in the derivation and automatically adapts to the number of samples, thus controlling the trade-off between information provided by the observations and the information provided by the prior. That is, for large number of samples, this parameter takes smaller values than for a low number of samples. The performance of the proposed detector is evaluated by means of Monte Carlo simulations, and compared to that of the generalized likelihood ratio test [18] and the cross-correlation detector [11].

II. Problem formulation

In this paper, we address the detection of a Gaussian signal in independent noises at the surveillance and reference arrays, each with arbitrary spatial covariance matrix. In particular, after propagation delay and Doppler shift compensation, we consider the detection problem

$$\begin{aligned}\mathcal{H}_1 &: \begin{cases}\mathbf{y}_{s,n} = \mathbf{H}_s\mathbf{x}_n + \mathbf{n}_{s,n},\\ \mathbf{y}_{r,n} = \mathbf{H}_r\mathbf{x}_n + \mathbf{n}_{r,n},\end{cases}\\ \mathcal{H}_0 &: \begin{cases}\mathbf{y}_{s,n} = \mathbf{n}_{s,n},\\ \mathbf{y}_{r,n} = \mathbf{H}_r\mathbf{x}_n + \mathbf{n}_{r,n},\end{cases}\end{aligned} \tag{1}$$

where $n = 1, \ldots, N$, $\mathbf{y}_{s,n} \in \mathbb{C}^{L_s}$, and $\mathbf{n}_{s,n} \in \mathbb{C}^{L_s}$ are the received signal and noise in the surveillance channel, $\mathbf{H}_s \in \mathbb{C}^{L_s \times p}$, and $\mathbf{x}_n \in \mathbb{C}^p$ is the unknown transmitted waveform. Analogously, $\mathbf{y}_{r,n} \in \mathbb{C}^{L_r}$, $\mathbf{n}_{r,n} \in \mathbb{C}^{L_r}$, and $\mathbf{H}_r \in \mathbb{C}^{L_r \times p}$ are defined for the reference channel. Furthermore, similar to [18], [19], we model the transmitted signal $\mathbf{x}_n$ as a spatially and temporally white Gaussian process, $\mathbf{x}_n \sim \mathcal{CN}_p(\mathbf{0}, \mathbf{I})$, while $\mathbf{n}_{s,n}$ and $\mathbf{n}_{r,n}$ are independent Gaussian noises that are temporally white but each of them may present unknown spatial correlation. That is, the covariance matrices of $\mathbf{n}_{s,n}$ and $\mathbf{n}_{r,n}$, denoted by $\mathbf{\Sigma}_s$ and $\mathbf{\Sigma}_r$, respectively, are positive definite (PD). Finally, we assume that the transmitted signal does not induce any low-rank spatial structure, that is, $p \geq \min(L_s, L_r)$.

Since the transmitted signal, $\mathbf{x}_n$, and noises are Gaussian, the test in (1) becomes

$$\begin{aligned}\mathcal{H}_1 &: \mathbf{y}_n \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}),\\ \mathcal{H}_0 &: \mathbf{y}_n \sim \mathcal{CN}(\mathbf{0}, \mathbf{D}),\end{aligned} \tag{2}$$

where $\mathbf{y}_n = [\mathbf{y}_{s,n}^T \ \mathbf{y}_{r,n}^T]^T$. The covariance matrix under $\mathcal{H}_1$ is

$$\mathbf{R} = \begin{bmatrix}\mathbf{H}_s\mathbf{H}_s^H + \mathbf{\Sigma}_s & \mathbf{H}_s\mathbf{H}_r^H\\ \mathbf{H}_r\mathbf{H}_s^H & \mathbf{H}_r\mathbf{H}_r^H + \mathbf{\Sigma}_r\end{bmatrix},$$

and the covariance matrix under $\mathcal{H}_0$ is

$$\mathbf{D} = \begin{bmatrix}\mathbf{\Sigma}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{H}_r\mathbf{H}_r^H + \mathbf{\Sigma}_r\end{bmatrix}.$$

The matrices $\mathbf{R}$ and $\mathbf{D}$ belong, respectively, to the following sets of structured matrices: $\mathcal{R}_1 = \{\mathbf{R} \mid \mathbf{R} \succeq \mathbf{0}\}$, and

$$\mathcal{R}_0 = \left\{\mathbf{D} = \begin{bmatrix}\mathbf{D}_s & \mathbf{0}\\ \mathbf{0} & \mathbf{D}_r\end{bmatrix} \mid \mathbf{D}_s \succeq \mathbf{0}, \mathbf{D}_r \succeq \mathbf{0}\right\}.$$

Thus, the detection problem boils down to a test for block-diagonality, for which the generalized likelihood ratio (GLR) test is [23]

$$\lambda = \det(\mathbf{C}), \tag{3}$$

where the coherence matrix is [19]

$$\mathbf{C} = \begin{bmatrix}\mathbf{S}_{ss} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_{rr}\end{bmatrix}^{-1/2} \begin{bmatrix}\mathbf{S}_{ss} & \mathbf{S}_{sr}\\ \mathbf{S}_{sr}^H & \mathbf{S}_{rr}\end{bmatrix} \begin{bmatrix}\mathbf{S}_{ss} & \mathbf{0}\\ \mathbf{0} & \mathbf{S}_{rr}\end{bmatrix}^{-1/2},$$

with the composite sample covariance matrix

$$\mathbf{S} = \frac{1}{N}\mathbf{Y}\mathbf{Y}^H = \frac{1}{N}\sum_{n=1}^{N}\mathbf{y}_n\mathbf{y}_n^H = \begin{bmatrix}\mathbf{S}_{ss} & \mathbf{S}_{sr}\\ \mathbf{S}_{sr}^H & \mathbf{S}_{rr}\end{bmatrix}.$$

Here, the data matrix is $\mathbf{Y} = [\mathbf{Y}_s^T \ \mathbf{Y}_r^T]^T$, with $\mathbf{Y}_i = [\mathbf{y}_{i,1} \cdots \mathbf{y}_{i,N}]$. The statistic in (3) is a canonical correlation analysis (CCA) detector as it can be rewritten in terms of the canonical correlations [19]

$$\lambda = \prod_{l=1}^{\min(L_s,L_r)} (1 - k_l^2),$$

where $k_l = sv_l(\mathbf{S}_{ss}^{-1/2}\mathbf{S}_{sr}\mathbf{S}_{rr}^{-1/2})$, with $sv_l(\cdot)$ denoting the lth singular value, are the canonical correlations. For $p < \min(L_s, L_r)$, the GLR only depends of the p largest canonical correlation [18].

III. Derivation of the Bayesian-inspired detector

Generalized likelihood ratio (GLR) detectors rely on the maximum likelihood (ML) estimates of the unknown parameters, $\mathbf{R}$ and $\mathbf{D}$. Instead of computing the ML estimates, in this paper we follow a Bayesian-inspired approach that places a prior on the unknown parameters and marginalizes them out from the likelihood ratio to obtain what is often referred to in the literature as a marginalized likelihood ratio. As we shall show in Section IV, this approach yields better performance than the GLR and other previously proposed techniques.

The marginal likelihood ratio is given by

$$\lambda_m = \frac{p(\mathbf{Y} \mid \mathcal{H}_0)}{p(\mathbf{Y} \mid \mathcal{H}_1)}, \tag{4}$$

where the marginal likelihoods are

$$p(\mathbf{Y} \mid \mathcal{H}_1) = \int_{\mathcal{R}_1} p(\mathbf{Y} \mid \mathcal{H}_1, \mathbf{R})p(\mathbf{R})d\mathbf{R}, \tag{5}$$

and

$$p(\mathbf{Y} \mid \mathcal{H}_0) = \int_{\mathcal{R}_0} p(\mathbf{Y} \mid \mathcal{H}_0, \mathbf{D})p(\mathbf{D})d\mathbf{D}.$$

Here, $p(\mathbf{Y} \mid \mathcal{H}_1, \mathbf{R})$ is the likelihood under $\mathcal{H}_1$

$$p(\mathbf{Y} \mid \mathcal{H}_1, \mathbf{R}) = \frac{1}{\pi^{LN}[\det(\mathbf{R})]^N}\exp\left\{-\frac{1}{N}\operatorname{tr}\left(\mathbf{R}^{-1}\mathbf{S}\right)\right\},$$

and $p(\mathbf{Y} \mid \mathcal{H}_0, \mathbf{D})$ is the likelihood under $\mathcal{H}_0$, which can be factored as $p(\mathbf{Y} \mid \mathcal{H}_0, \mathbf{D}) = p(\mathbf{Y}_s \mid \mathcal{H}_0, \mathbf{D}_s) p(\mathbf{Y}_r \mid \mathcal{H}_0, \mathbf{D}_r)$ and yields $p(\mathbf{Y} \mid \mathcal{H}_0) = p(\mathbf{Y}_s \mid \mathcal{H}_0) p(\mathbf{Y}_r \mid \mathcal{H}_0)$, where

$$p(\mathbf{Y}_i \mid \mathcal{H}_0) = \int_{\mathbf{D}_i \succ \mathbf{0}} p(\mathbf{Y}_i \mid \mathcal{H}_0, \mathbf{D}_i) p(\mathbf{D}_i) d\mathbf{D}_i. \qquad (6)$$

To avoid the numerical integration required for the computation of the aforementioned marginal likelihoods, we choose conjugate priors for the likelihoods in (2). Concretely, for a complex Gaussian likelihood with zero mean and unknown covariance matrix, the conjugate prior is the complex inverse-Wishart distribution, which under $\mathcal{H}_1$ is given by [22]

$$\begin{aligned} p(\mathbf{R}) &= \mathcal{CW}_L^{-1}(\mathbf{R}; \breve{\nu}, \breve{\mathbf{R}}) \\ &= \frac{[\det(\breve{\mathbf{R}})]^{\breve{\nu}} [\det(\mathbf{R})]^{-(\breve{\nu}+L)}}{\mathcal{C}\Gamma_L(\breve{\nu})} \exp\left\{ -\operatorname{tr}\left(\mathbf{R}^{-1} \breve{\mathbf{R}} \right) \right\}, \end{aligned}$$

where $L = L_s + L_r$, $\breve{\nu}$ is the number of degrees of freedom, $\breve{\mathbf{R}}$ is the PD scale matrix, and $\mathcal{C}\Gamma_L(\cdot)$ is the complex multivariate gamma function [24]

$$\begin{aligned} \mathcal{C}\Gamma_L(a) &= \int_{\mathbf{A} \succ \mathbf{0}} \exp\{-\operatorname{tr}(\mathbf{A})\} [\det(\mathbf{A})]^{a-L} d\mathbf{A} \\ &= \pi^{L(L-1)/2} \prod_{i=1}^{L} \Gamma(a - i + 1), \end{aligned}$$

with $\Gamma(\cdot)$ the gamma function. Similarly, under $\mathcal{H}_0$, we can place independent priors on $\mathbf{D}_i, i = \{s, r\}$, which are given by $p(\mathbf{D}_i) = \mathcal{CW}_{L_i}^{-1}(\mathbf{D}_i; \breve{\nu}_i, \breve{\mathbf{D}}_i)$.

In the following, we shall sketch how to solve the integral in (5). An equivalent procedure can be followed to compute (6). The first step is noting that the integrand of (5) is the joint distribution $p(\mathbf{Y}, \mathbf{R} \mid \mathcal{H}_1)$, which can be rewritten as

$$p(\mathbf{Y}, \mathbf{R} \mid \mathcal{H}_1) = p(\mathbf{R} \mid \mathcal{H}_1, \mathbf{Y}) p(\mathbf{Y} \mid \mathcal{H}_1).$$

After some straightforward manipulations of $p(\mathbf{Y}, \mathbf{R} \mid \mathcal{H}_1)$, it can be shown that

$$p(\mathbf{R} \mid \mathcal{H}_1, \mathbf{Y}) = \mathcal{CW}_L^{-1}(\mathbf{R}; \hat{\nu}, \hat{\mathbf{R}}),$$

where $\hat{\nu} = \breve{\nu} + N$ and $\hat{\mathbf{R}} = \breve{\mathbf{R}} + N\mathbf{S}$. That is, as should happen with conjugate priors, the posterior of $\mathbf{R}$ belongs to the prior distribution family. Then, the marginal likelihood becomes

$$p(\mathbf{Y} \mid \mathcal{H}_1) = \frac{[\det(\breve{\mathbf{R}})]^{\breve{\nu}} \mathcal{C}\Gamma_L(\hat{\nu})}{\pi^{LN} [\det(\hat{\mathbf{R}})]^{\hat{\nu}} \mathcal{C}\Gamma_L(\breve{\nu})}, \qquad (7)$$

and, similarly,

$$p(\mathbf{Y}_i \mid \mathcal{H}_0) = \frac{[\det(\breve{\mathbf{D}}_i)]^{\breve{\nu}_i} \mathcal{C}\Gamma_{L_i}(\hat{\nu}_i)}{\pi^{L_i N} [\det(\hat{\mathbf{D}}_i)]^{\hat{\nu}_i} \mathcal{C}\Gamma_{L_i}(\breve{\nu}_i)}, \qquad (8)$$

where $\hat{\nu}_i = \breve{\nu}_i + N$ and $\hat{\mathbf{D}}_i = \breve{\mathbf{D}}_i + N\mathbf{S}_{ii}$.

Plugging (7) and (8) into (4), the marginal likelihood ratio is

$$\lambda_m = \frac{[\det(\hat{\mathbf{R}})]^{\hat{\nu}}}{[\det(\hat{\mathbf{D}}_s)]^{\hat{\nu}_s} [\det(\hat{\mathbf{D}}_r)]^{\hat{\nu}_r}}. \qquad (9)$$

Moreover, it is easy to show that for isotropic scaling matrices, $\breve{\mathbf{R}} = \eta \mathbf{I}, \breve{\mathbf{D}}_i = \eta \mathbf{I}$, and $\breve{\nu} = \breve{\nu}_i$, (9) becomes

$$\lambda_m = \det(\tilde{\mathbf{C}}),$$

where $\tilde{\mathbf{C}}$ is a regularized coherence matrix, given by

$$\tilde{\mathbf{C}} = \begin{bmatrix} \tilde{\mathbf{S}}_{ss} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_{rr} \end{bmatrix}^{-1/2} \begin{bmatrix} \tilde{\mathbf{S}}_{ss} & \mathbf{S}_{sr} \\ \mathbf{S}_{sr}^H & \tilde{\mathbf{S}}_{rr} \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{S}}_{ss} & \mathbf{0} \\ \mathbf{0} & \tilde{\mathbf{S}}_{rr} \end{bmatrix}^{-1/2},$$

with the regularized sample covariance matrices $\tilde{\mathbf{S}}_{ii} = \mathbf{S}_{ii} + \frac{\eta}{N}\mathbf{I}, i = \{s, r\}$. Similar to the GLR, λ_m can also be expressed in terms of the regularized canonical correlations

$$\lambda_m = \prod_{l=1}^{\min(L_s, L_r)} (1 - \tilde{k}_l^2),$$

where $\tilde{k}_l = sv_l(\tilde{\mathbf{S}}_{ss}^{-1/2} \mathbf{S}_{sr} \tilde{\mathbf{S}}_{rr}^{-1/2})$. This makes the marginal likelihood ratio λ_m a regularized CCA detector, where the regularization parameter appears naturally and it adapts automatically to the number of samples. That is, for large values of N, the regularization plays a small role and this is automatically taken into account as N appears in the denominator of the regularization parameter. Moreover, if there is no additional a priori information, assuming isotropic scaling matrices and identical $\breve{\nu}$'s is appropriate since it does not favor any direction for the covariance matrices.

IV. Numerical results

This section studies, by means of Monte Carlo simulations, the performance of the proposed marginal likelihood ratio test (9) and compares it with the performance of the GLRT (3) and the performance of the cross-correlation detector [11], $T_{cc} = \|\mathbf{S}_{sr}\|_F^2$. In the experiments, we consider that channels and noise covariance matrices are randomly generated in each Monte Carlo simulation. Concretely, the covariance matrices are randomly generated with uniformly distributed eigenvalues between 0.2 and 1, and the eigenvector matrices are uniformly distributed unitary matrices, while the channels are generated as $[\mathbf{H}_i]_{lk} \sim \mathcal{CN}(0, 1), i = \{s, r\}, l = 1, \ldots, L_i, k = 1, \ldots, p$, which are scaled to achieve the desired signal-to-noise ratio (SNR), defined as

$$\text{SNR}_i = 10 \log_{10} \left(\frac{\operatorname{tr}(\mathbf{H}_i \mathbf{H}_i^H)}{\operatorname{tr}(\mathbf{\Sigma}_{ii})} \right).$$

Additionally, in all examples, we have selected $\eta = 5$. However, we have also evaluated other values in $[0.5, 10]$, with no significant differences.

The first experiment compares the receiver operating characteristic (ROC) curves in a scenario with $N = 50, L_s = L_r = p = 10, \text{SNR}_s = -7$ and $\text{SNR}_r = 8$ dBs. The curves, depicted in Fig. 1, show that that the proposed detector (9) outperforms the GLRT (3) and the cross-correlation based detector T_{cc}. This shows the advantage of integrating out the unknown parameters instead of replacing them by their maximum likelihood estimates.

The second experiment compares the probability of missed detection (p_m) for a probability of false alarm $p_{fa} = 10^{-3}$

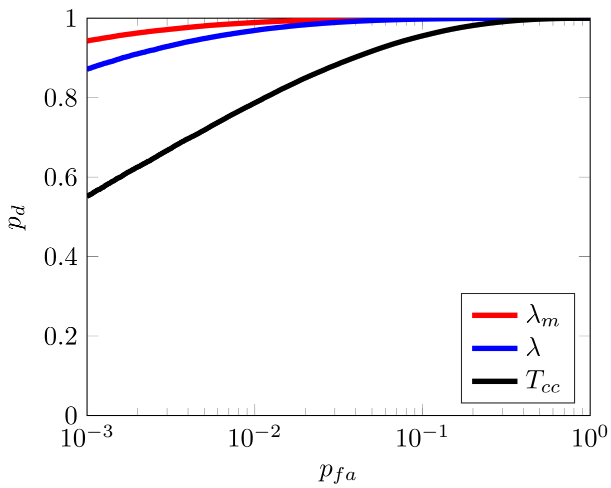

Fig. 1. ROC curves in a scenario with $N = 50, L_s = L_r = p = 10, \text{SNR}_s = -7$ and $\text{SNR}_r = 8$ dBs

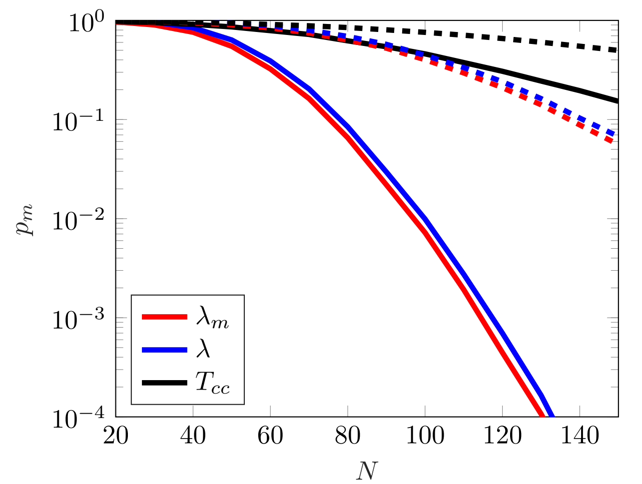

Fig. 3. Probability of missed detection vs. N for $p_{fa} = 10^{-3}$ in a scenario with $\text{SNR}_s = -10, L_s = L_r = p = 10$, and two different cases for $\text{SNR}_r = \text{SNR}_s + \Delta\text{SNR}$: 1) Solid line: $\Delta\text{SNR} = 20$; 2) Dashed line: $\Delta\text{SNR} = 10$

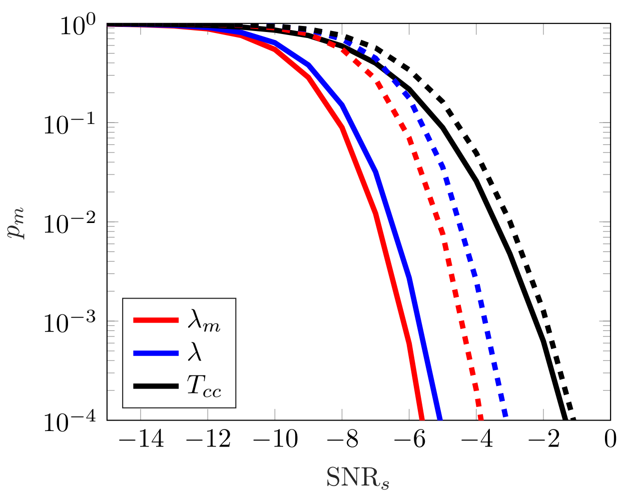

Fig. 2. Probability of missed detection vs. SNR_s for $p_{fa} = 10^{-3}$ in a scenario with $N = 50, L_s = L_r = p = 10$, and two different cases for $\text{SNR}_r = \text{SNR}_s + \Delta\text{SNR}$: 1) Solid line: $\Delta\text{SNR} = 20$; 2) Dashed line: $\Delta\text{SNR} = 10$

vs. the SNR_s. The experiment considers $N = 50$, $L_s = L_r = p = 10$, and $\text{SNR}_r = \text{SNR}_s + \Delta\text{SNR}$, with $\Delta\text{SNR} = 10$ and $\Delta\text{SNR} = 20$ dBs. The results for this experiment are shown in Fig. 2, where the marginalized detector (9) outperforms (3) and T_{cc} over the whole range of SNR_s and SNR_r values. Interestingly, this figure also shows that increasing the SNR_r by 10 dBs can be compensated by much smaller increases of SNR_s in this particular scenario.

The third experiment evaluates the probability of missed detection vs. the number of snapshots N for $p_{fa} = 10^{-3}$ in a scenario with $\text{SNR}_s = -10$ dBs, $L_s = L_r = p = 10$, and the same two different cases for ΔSNR as the previous experiment. Again, the proposed detector outperforms the GLR and the cross-correlation detector, as Fig. 3 shows. This figure shows that compensating the loss of SNR_r requires a large increase in the number of samples.

V. Conclusions

One of the most common detection techniques for passive radar is the generalized likelihood ratio test (GLRT), which substitutes the unknown parameters by their maximum likelihood estimates in the likelihood ratio. As an alternative to the GLRT, in this work, we have derived the marginalized likelihood ratio test, which integrates out the unknown parameters by assigning them a prior distribution. In particular, we have considered the detection of a Gaussian transmitted signal in noises with positive-definite, but otherwise unstructured covariance matrices. The formulation of this problem boils down to a test for the covariance structure of the observations, in particular, whether the covariance matrix is block diagonal or unstructured. Hence, by placing complex inverse-Wishart priors on these covariance matrices, we are able to solve the integrals and obtained a closed-form detector, which is a function of regularized canonical correlations for certain prior hyper-parameters. Interestingly, the regularization term appeared naturally in the derivation and automatically adapts to the number of samples. Finally, simulation results showed that the proposed detector slightly outperforms the second-order GLRT and the well-known cross-correlation detector.

References

[1] H. D. Griffiths and C. J. Baker, "Passive coherent location radar systems. Part 1: Performance prediction," *IEE Proc. Radar, Sonar and Navigation*, vol. 152, no. 3, p. 153, 2005.

[2] C. J. Baker, H. D. Griffiths, and I. Papoutsis, "Passive coherent location radar systems. Part 2: Waveform properties," *IEE Proc. Radar, Sonar and Navigation*, vol. 152, no. 3, p. 160, 2005.

[3] Y. Liu, G. Liao, J. Xu, Z. Yang, and Y. Yin, "Improving detection performance of passive MIMO radar by exploiting the preamble information of communications signal," *IEEE Systems Journal*, vol. 15, no. 3, pp. 4391–4402, 2021.

[4] D. E. Hack, L. K. Patton, B. Himed, and M. A. Saville, "Centralized passive MIMO radar detection without direct-path reference signals," *IEEE Trans. Signal Process.*, vol. 62, no. 11, pp. 3013–3023, 2014.

[5] ——, "Detection in passive MIMO radar networks," *IEEE Trans. Signal Process.*, vol. 62, no. 11, pp. 2999–3012, 2014.

[6] N. J. Willis and H. D. Griffiths, *Advances in Bistatic Radar*. SciTech Publishing, 2007.

[7] M. Baczyk and M. Malanowski, "Reconstruction of the reference signal in DVB-T-based passive radar," *Int. J. of Electronics and Telecommunications*, vol. 57, no. 1, pp. 43–48, 2011.

[8] P. Howland, D. Maksimiuk, and G. Reitsma, "FM radio based bistatic radar," *IEE Proc. Radar, Sonar and Navigation*, vol. 152, no. 3, pp. 107–115, 2005.

[9] L. T. McWhorter, L. Scharf, C. Moore, and M. Cheney, "Passive multichannel detection: A general first-order statistical theory," *IEEE Open J. Signal Process.*, vol. 4, pp. 437–451, 2023.

[10] F. Colone, D. W. O'Hagan, P. Lombardo, and C. J. Baker, "A multistage processing algorithm for disturbance removal and target detection in passive bistatic radar," *IEEE Trans. Aero. Electr. Syst.*, vol. 45, no. 2, pp. 698–722, 2009.

[11] J. Liu, H. Li, and B. Himed, "On the performance of the cross-correlation detector for passive radar applications," *Signal Process.*, vol. 113, pp. 32–37, 2015.

[12] K. S. Bialkowski, I. V. L. Clarkson, and S. D. Howard, "Generalized canonical correlation for passive multistatic radar detection," in *IEEE Work. Stat. Signal Process.*, Jul. 2011, pp. 417–420.

[13] S. Gogineni, P. Setlur, M. Rangaswamy, and R. R. Nadakuditi, "Passive radar detection with noisy reference channel using principal subspace similarity," *IEEE Trans. Aero. Electr. Syst.*, vol. 54, no. 1, pp. 18–36, 2018.

[14] M. Fazlollahpoor, M. Derakhtian, and S. Khorshidi, "Rao retector for passive MIMO radar with direct-path interference," *IEEE Trans. Aero. Electr. Syst.*, vol. 56, no. 4, pp. 2999–3009, Aug. 2020.

[15] X. Zhang, J. Sward, H. Li, A. Jakobsson, and B. Himed, "A sparsity-based passive multistatic detector," *IEEE Trans. Aero. Electr. Syst.*, vol. 55, no. 6, pp. 3658–3666, Dec. 2019.

[16] X. Zhang, H. Li, and B. Himed, "Multistatic detection for passive radar with direct-path interference," *IEEE Trans. Aero. Electr. Syst.*, vol. 53, no. 2, pp. 915–925, Apr. 2017.

[17] Y. Liu, R. S. Blum, G. Liao, and S. Zhu, "Passive MIMO radar detection exploiting known format of the communication signal observed in colored noise with unknown covariance matrix," *Signal Process.*, vol. 174, p. 107611, Sep. 2020.

[18] I. Santamaría, L. Scharf, J. Via, Y. Wang, and H. Wang, "Passive detection of correlated subspace signals in two MIMO channels," *IEEE Trans. Signal Process.*, vol. 65, no. 7, pp. 1752–1764, Mar. 2017.

[19] D. Ramírez, I. Santamaría, and L. Scharf, *Coherence: In Signal Processing and Machine Learning*. Springer Nature, 2023.

[20] J. Manco-Vásquez, M. Lázaro-Gredilla, D. Ramírez, J. Vía, and I. Santamaría, "A Bayesian approach for adaptive multiantenna sensing in cognitive radio networks," *Signal Process.*, vol. 96, Part B, pp. 228–240, Mar. 2014.

[21] J. Liu, J. Han, Z.-J. Zhang, and J. Li, "Bayesian detection for MIMO radar in Gaussian clutter," *IEEE Trans. Signal Process.*, vol. 66, no. 24, pp. 6549–6559, 2018.

[22] N. R. Goodman, "Statistical analysis based on a certain multivariate complex Gaussian distribution (An introduction)," *Ann. Math. Statist.*, vol. 34, no. 1, pp. 152–177, 1963.

[23] K. V. Mardia, J. T. Kent, and J. M. Bibby, *Multivariate Analysis*. New York: Academic, 1979.

[24] A. T. James, "Distributions of matrix variates and latent roots derived from Normal samples," *Ann. Math. Statist.*, vol. 35, no. 2, pp. 475–501, 1964.