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HW-1

Problem 1. $a = 5$

→ Hidden layer :- $n1, n2, n3$
→ Input nodes :- $x1, x2$
→ Output :- $\hat{y}$
→ Learning rate $\alpha = 0.1$
→ Activation Function = Sigmoid.
→ loss function, $MSE = \frac{1}{2}(y - \hat{y})^2$

→ Target output $y = 1$

[a] $x1 = a = 5$ $\quad w1 = w2 = 1 \quad b1 = 1$

$x2 = 1$

$z_1 = w1 * x1 + w2 * x2 + b1$

$= 1 \times 5 + 1 \times 1 + 1$

$= 5 + 1 + 1 = 7$

$a1 = \sigma(7) = \frac{1}{1+e^{7}} = \boxed{0.99908}$

$z_2 = w3 * x1 + w4 * x2 + b2$

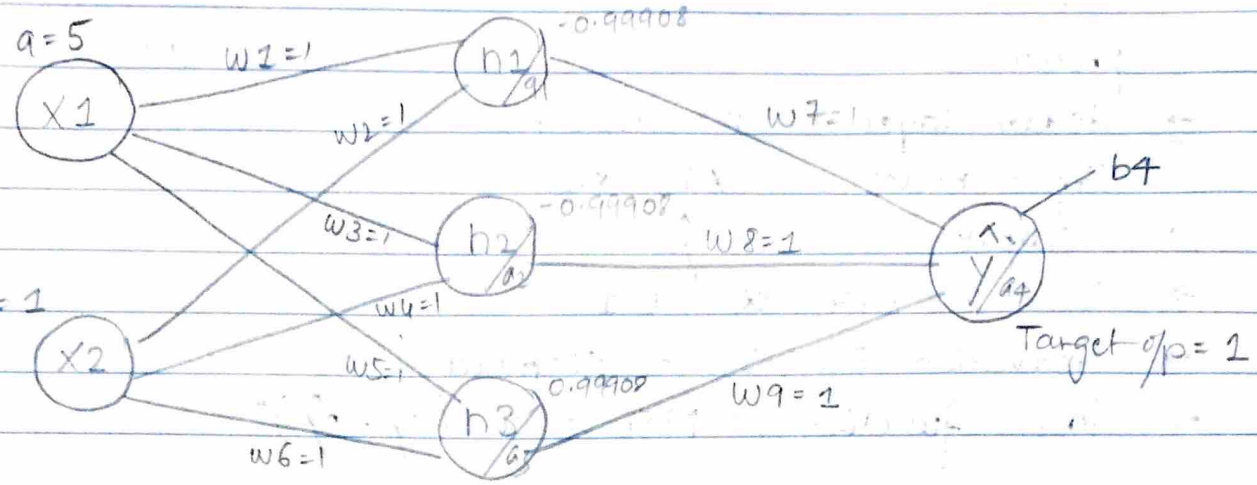
$= 1 * 5 + 1 * 1 + 1 = 7$

$a2 = \sigma(7) = \boxed{0.99908}$

$z_3 = w5 * x_1 + w6 * x_2 + b3$

$= 1 * 5 + 1 * 1 + 1 = 7$

$a3 = \sigma(7) = \boxed{0.99908}$

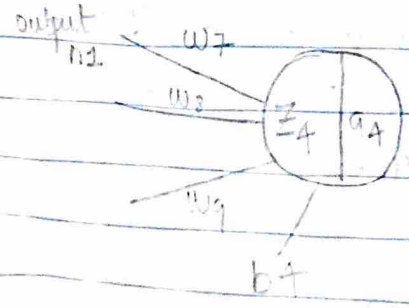


$$Z_4 = 0.99908 \times w_7 + 0.99908 \times w_8 + 0.99908 \times w_9 + b4$$
$$= 0.99908 + 0.99908 + 0.99908 + 1$$
$$= 3.9972$$
$$\sigma(3.9972)$$
$$a_4 = \frac{1}{1+e^{-3.99}} = \underline{\underline{0.9818}}$$

MSE

$$\text{Loss}(L) = (y-\hat{y})^2/2$$
$$= \frac{(1-0.9818)^2}{2}$$
$$= \underline{\underline{0.00016562}}$$

Back Propagation.



$$\frac{\partial E_{total}}{\partial w_5} =$$

$$\frac{\partial Z_4}{\partial w_7} \cdot \frac{\partial a_4}{\partial Z_4} \cdot \frac{\partial E}{\partial a_4}$$

$$E_{total} = E_1 + E_2 = \frac{1}{2}(y_1 - \hat{y}_1)^2 + \frac{1}{2}(y_2 - \hat{y}_2)^2$$

$$\frac{\partial E_{total}}{\partial a_4} = 2 * \frac{1}{2}(y_1 - \hat{y}_1)^{2-1} * -1 + 0$$

$$= -(y_1 - \hat{y}_1) = -(1 - 0.9818) = \underline{-0.0182}$$

$$a_4 = \frac{1}{1 + e^{-z_4}}$$

$$\frac{\partial a_4}{\partial z_4} = a_4(1 - a_4) = 0.9818(1 - 0.9818) = \underline{0.01786876}$$

$$\frac{\partial z_4}{\partial w_7} = \frac{\partial}{\partial w_7}(w_7 a_1 + w_8 a_2 + w_9 a_3)$$

$$= \frac{\partial(w_7 a_1)}{\partial w_7} = a_1 * 1 = \underline{0.99908}$$

$$\frac{\partial z_4}{\partial w_7} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial E}{\partial a_4} = \frac{\partial E_{total}}{\partial w_7}$$

$$= 0.99908 * 0.01786876 * (-0.0182)$$

$$= \underline{-0.00032491}$$

$$w_7' = w_7 - \eta * \frac{\partial E_{total}}{\partial w_7} = 1 - 0.1 * (-0.00032491)$$

$$= 1 - (-0.00003249) = \underline{1.00003249}$$

(a). Updated weights:

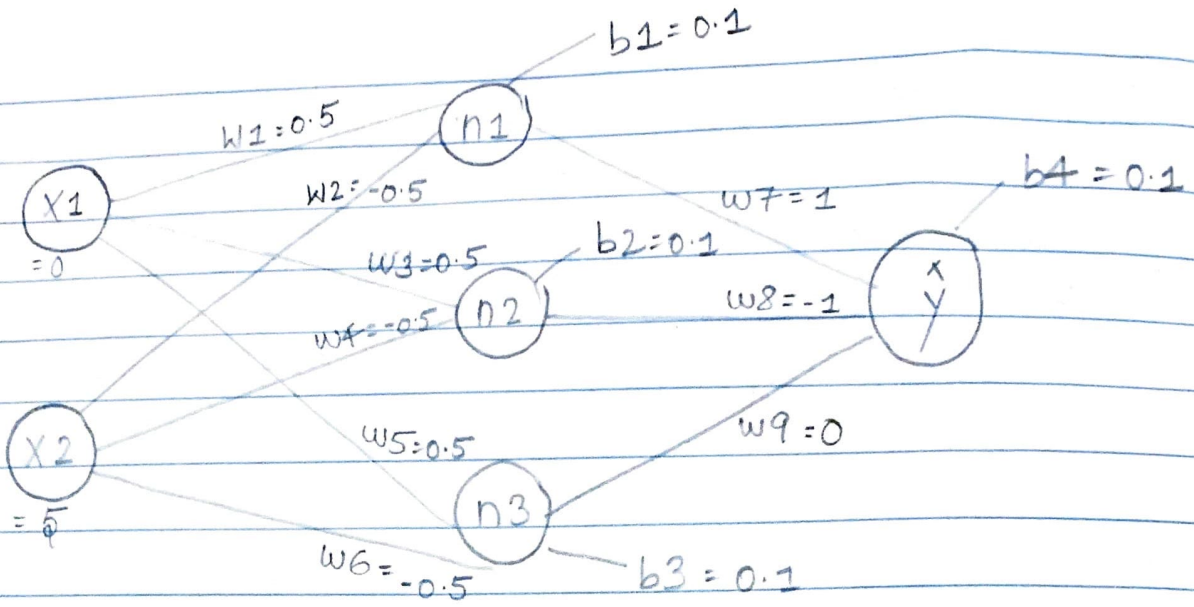
$$w_7' = w_7 - \eta * \frac{\partial E_{total}}{\partial w_7} = 1 - 0.1 * (-0.00032491) = 1.00003249//$$

$$w_8' = w_8 - \eta * \frac{\partial E_{total}}{\partial w_8} = 1 - 0.1 * (-0.00032491) = 1.00003249//$$

$$w_9' = w_9 - \eta * \frac{\partial E_{total}}{\partial w_9} = 1 - 0.1 * (-0.00032491) = 1.00003249$$

It is same as weights & hidden layer o/p are same.

[b.]



$$Z_1 = w_1 * X_1 + w_2 * X_2 + b1$$
$$= 0.5 * 0 + (-0.5) * 5 + 0.1$$
$$= -2.40$$

$$a_1 = \sigma(-2.40) = \frac{1}{1+e^{-(-2.4)}} = \frac{1}{1+e^{2.4}} = 0.0831$$

$$Z_2 = w_3 * X_1 + w_4 \cdot X_2 + b2$$
$$= 0.5 * 0 + (-0.5) \cdot 5 + 0.1$$
$$= -2.4$$

$$a_2 = \sigma(-2.4) = \frac{1}{1+e^{-(-2.4)}} = \frac{1}{1+e^{2.4}} = 0.0831$$

$$Z_3 = w_5 \cdot X_1 + w_6 \cdot X_2 + b3$$
$$= 0.5 \cdot 0 + (-0.5) \cdot 5 + 0.1$$
$$= -2.4$$

$$a_3 = \sigma(-2.4) = \frac{1}{1+e^{-(-2.4)}} = \frac{1}{1+e^{2.4}} = 0.0831$$

$z_4 = a_1 \cdot w_7 + a_2 \cdot w_8 + a_3 \cdot w_9 + b_4$

$= 0.0831 \cdot 1 + -1 \cdot 0.0831 + 0 \times 0.0831 + 0.1$

$= 0.0831 + (-0.0831) + 0 + 0.1$

$= 0.1$

$a_4 = \frac{1}{1+e^{-(0.1)}} = \underline{0.5249}$

$E = 1/2 (y - \hat{y})^2 = 1/2 (1 - 0.5249)^2 = \underline{0.11286}$

Back Propagation

$E_{total} = E_1 + E_2 = \frac{1}{2}(y_1 - \hat{y}_1)^2 + \frac{1}{2}(y_2 - \hat{y}_2)^2$

$\frac{\partial E_{total}}{\partial a_4} = \frac{1}{2} * 2 (y_1 - \hat{y}_1)^{2-1} * -1 + 0$

$= -1 (y_1 - \hat{y}_1)$

$= -1 (1 - 0.5249)$

$= \underline{-0.4751}$

$a_4 = 1/1+e^{-z_4}$

$\frac{\partial a_4}{\partial z_4} = a_4 (1 - a_4) = 0.5249 (1 - 0.5249)$

$= \underline{0.2493}$

$\frac{\partial z_4}{\partial w_7} = \frac{\partial}{\partial w_7} (w_7 a_1 + w_8 a_2 + w_9 a_3 + b_4)$

$= \frac{\partial (w_7 a_1)}{\partial w_7} = a_1 \cdot 1 = 0.0831 //$

$\frac{\partial z_4}{\partial w_7} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial E_{total}}{\partial a_4} = \frac{\partial E_{total}}{\partial w_7} =$

$0.0831 \cdot 0.2493 \cdot (-0.4751) = \underline{-0.00984257}$

$$\frac{\partial E_{total}}{\partial w_8} = \frac{\partial E}{\partial a_4} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial w_8}$$

$$\frac{\partial E}{\partial a_4} = -0.4751$$

$$\frac{\partial a_4}{\partial z_4} = 0.2493$$

$$\frac{\partial z_4}{\partial w_8} = \frac{\partial (w_8 \cdot a_2 + w_7 \cdot a_1 + w_9 a_3)}{\partial w_8}$$

$$= \frac{\partial (w_8 \cdot a_2)}{\partial w_8} = a_2 \cdot (-1) = 0.0831 (-1)$$

$$= -0.0831$$

$$\frac{\partial E_{total}}{\partial w_8} = -0.4751 \cdot 0.2493 \cdot (-0.0831)$$

$$= \underline{\underline{0.00098426}}$$

$$\frac{\partial E_{total}}{\partial w_9} = \frac{\partial E}{\partial a_4} \cdot \frac{\partial a_4}{\partial z_4} \cdot \frac{\partial z_4}{\partial w_9}$$

$$\frac{\partial E}{\partial a_4} = -0.4751 \qquad \frac{\partial a_4}{\partial z_4} = 0.2493$$

$$\frac{\partial z_4}{\partial w_9} = \frac{\partial (w_9 \cdot a_3)}{\partial w_9} = a_3 \cdot (0) = 0 //$$

$$\frac{\partial E_{total}}{\partial w_9} = 0 //$$

Updated weights.

$w_7 = 1$ $\quad w_8 = -1$ $\quad w_9 = 0$

$$w_7' = w_7 - \eta \frac{\partial E_{total}}{\partial w_7}$$

$$= 1 - 0.1(-0.00984257)$$

$$= \underline{1.00098426}$$

$$w_8' = w_8 - \eta \frac{\partial E_{total}}{\partial w_8}$$

$$= -1 - 0.1(0.00098426)$$

$$= \underline{-1.00009843}$$

$$w_9' = w_9 - \eta \frac{\partial E_{total}}{\partial w_9}$$

$$= 0 - 0.1(0)$$

$$= 0 //$$