EML Assignment 4 Problem 3

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Question 1

For $\hat{f}_1$,

$$\hat{\beta}_1 = \arg\min_{\beta} \sum_i \left(y_i - \alpha - \hat{f}_2(x_{2i}) - \beta x_{1i}\right)^2 + \lambda\beta^2$$

We differentiate with respect to β and equate to 0.

$$-2\sum_i x_{1i}\left(y_i - \alpha - \hat{f}_2(x_{2i}) - \hat{\beta}_1 x_{1i}\right) + 2\lambda\hat{\beta}_1 = 0$$

$$2(\lambda + \sum_i x_{1i}^2)\hat{\beta}_1 = 2\sum_i x_{1i}\left(y_i - \alpha - \hat{f}_2(x_{2i})\right)$$

$$\hat{\beta}_1 = \frac{1}{\lambda + \sum_i x_{1i}^2}\sum_i x_{1i}\left(y_i - \alpha - \hat{f}_2(x_{2i})\right)$$

For the next step, we will use the new value for $\hat{\beta}_1$ when evaluating $\hat{f}_1(X_1) = \hat{\beta}_1 X_1$.

Similar to the previous analysis, we get the following for $\hat{f}_2$,

$$\hat{\beta}_2 = \arg\min_{\beta} \sum_i \left(y_i - \alpha - \hat{f}_1(x_{1i}) - \beta x_{2i}\right)^2 + \lambda\beta^2$$

$$\hat{\beta}_2 = \frac{1}{\lambda + \sum_i x_{2i}^2}\sum_i x_{2i}\left(y_i - \alpha - \hat{f}_1(x_{1i})\right)$$

Question 2

The case of the squared error we first compute the derivative with respect to all parameters in the model and then we modify all the parameters at the same time. In the case of GAM compute the error with respect to each parameter alone and then performing backfit iteratively change its value in the function and in this case if some of the variables are correlated then when it comes to the time of modifying the coefficeint related to that predictor its update will not be the same as with the case global older method discussed in the clases.

Question 3

When X1 and X2 are correlated, the order in which fj is updated first will affect the outcomes of the back fitting process with the smoothing operator Sλ. This is due to the fact that updating f2 using f1's value from the previous iteration will differ from updating f2 using f2's value from f1's value.

Question 4

Write down the smoothing operator based on cubic smoothing splines.

$$\sum_{i=1}^{n}(y_i - g(x_i))^2 + \lambda \int g^{''}(t)^2 \, dt \tag{1}$$