Performance of Music Algorithm for DOA Estimation

Conference Paper · November 2016

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ICAYS Engineering Mechanics and Interdisciplinary
12-13 November 2016, Beijing, P.R.China
1st International Conference in Aerospace for Young Scientists

Performance of Music Algorithm for DOA Estimation

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Abstract: Direction of Arrival (DOA) plays an important role in increasing the throughput and capacity of network by focusing the reception and transmission of signal in the desired direction.
In practical scenario multiple signals arrive from multiple sources towards the antenna. The capacity of the system can be increased if multiple signals are focused in desired direction. This paper shows the performance analysis of MUSIC algorithm using uniform linear array. The MUSIC algorithm performs well if the signals are incoherent but if the signal has coherent nature the performance of MUSIC algorithm degrades. The MUSIC algorithm is improved in this paper which performs better if the signals fare coherent.

Keywords: Antenna array, Direction of Arrival Estimation, Array signal processing, MUSIC Algorithm.

1. Introduction

DOA estimation is an important technique in the field of signal processing in which the exact location of the source is find form where the signals are arriving [1]. This technique is used in the array signal processing for the purpose of wireless communication and audio/ speech processing systems [2-3].

One technique used for DOA estimation is using fixed antenna in which there is one disadvantage of limiting the antennas main lobe beam width which reduces the efficiency and capacity of the antenna.

There is an inverse relationship between antennas main lobe and physical size. So by increasing the

Physical size for improving the accuracy of angle measurement is not a practical solution [4-5].

The smart antenna uses array of antenna instead of using a single antenna improves the efficiency and resolution of the DOA estimation. Many methods are proposed for estimating the direction and number of signals. The DOA estimation algorithm proposed in past are Capon, Bartlett, ESPRIT and MUSIC. The MUSIC algorithm is accurate method and has high resolution which is used in design of smart antennas.[6] In this paper MUSIC DOA estimator is simulated in the MATLAB and then the improved MUSIC algorithm is presented in the case of coherent signal and its results are evaluated.

2. Data Model

The model used considers that the sources of signal are narrow bands; the center frequency of each source is same. D is the number of testing signal source. The antenna array are uniformly spaced and consists of M (M>D) elements of array. The element of array has similar characteristics and is isotropic in nature. The spacing between the elements is Δ and the interval between array element interval is not larger the half wavelength of the largest frequency. The antenna element is in the far field source. The antenna array receiving the signals from source is plane wave. The test signals and array elements are uncorrelated and the variance is zero mean Gaussian noise.

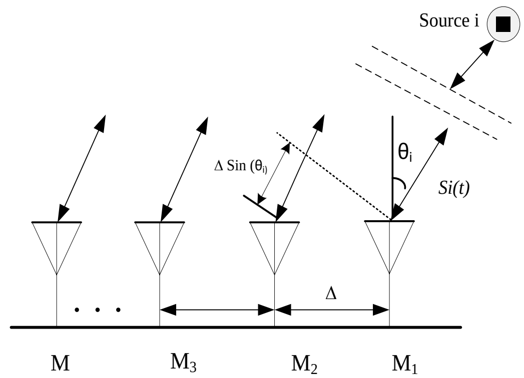

Fig. A, Physical model of the system

By considering the aforementioned assumptions if the signal generated by the remote source "i" impinging on the array at an angle θi at the rightmost element is $s_i(t)$, then the signal received by the M element of the array can be written

$$s_{im}(t) = s_i(t) e^{jm\mu_i} \qquad m = 0 \ldots M-1 \quad (1)$$

Where $\mu_i = -2\pi \frac{\Delta}{\lambda} \sin(\theta_i)$ is called additional phase difference caused by the path difference between M array element and the first array element The total signal at the M receiving element is

$$x_m(t) = \sum_{i=0}^{d-1} s_{im}(t) + n_m(t)$$

The total received signal can be written as

$$\mathbf{x}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{n}(t) \quad (2)$$

Where;

$$\mathbf{x}(t) = \left[x_1(t) \quad x_2(t) \quad \ldots \quad x_{M-1}(t) \right]^T \quad (3)$$

is the data column vector received by the array.

$$\mathbf{A} = \begin{bmatrix} 1 & 1 & \ldots & 1 \\ e^{j\mu_1} & e^{j\mu_2} & \ldots & e^{j\mu_d} \\ \ldots & \ldots & \ldots & \ldots \\ e^{j(M-1)\mu_1} & e^{j(M-1)\mu_2} & \ldots & e^{j(M-1)\mu_d} \end{bmatrix} \quad (4)$$

Is the vandermonde structure of the array steering matrix A.

$$s(t) = \left[s_1(t) \quad s_2(t) \quad \ldots \quad s_{M-1}(t) \right]^T \quad (5)$$

is the signal Column vector generated by the source.

$$\mathbf{n}(t) = \left[n_1(t) \quad n_2(t) \quad \ldots \quad n_{M-1}(t) \right]^T \quad (7)$$

is the noise vector generated by the source.

The data covariance matrix can be written as

$$\mathbf{R}_{xx} = E\left\{ \mathbf{x}(t) \mathbf{x}^H(t) \right\} \quad (6)$$

Where $E\{\ldots\}$ denotes expected value and $\mathbf{x}^H(t)$ is the complex-conjugated transpose of $\mathbf{x}(t)$. And The Rxx is called as data covariance matrix.

3. Music Algorithm

Music stands for Multiple Signal Classification which is the most classic DOA techniques based on eigen-decomposition.[7] This algorithm was proposed by Schmidt in 1979. In this algorithm first characteristic decomposition is performed for the covariance matrix for any output data of the array. The characteristic decomposition results in signal subspace which is orthogonal containing a noise subspace in the signal component. The two orthogonal subspaces are used to construct a spectrum function. In the spectrum function the peaks of the signals are searched and DOA signals are detected. [8-10]

The implementation of the MUSIC algorithm is as follow.

- In the first step the estimation of the input covariance matrix is based on N received signal vector or data estimation is done by collecting the data which is in below equation.

$$R_{X=E\left|XX^H\right|}$$

- The second important step is to find the eigenvalue which is find by decomposing the covariance matrix from the aforementioned Equation

$$Rx = ARsA^H + \partial^2$$

- The eigenvalue is calculated from above Equation 2 and then according to order of Eigenvalues take that eigenvector and eigenvalue which are equal to number of signal D as and take that as a part of space.
- The remaining M-D eigenvectors and eigenvalues are taken as a part of noise.
- After this the noise matrix is obtained is through following equation $E_n = A^H vi = 0$

- Compute the MUSIC spectrum According to formulae

  $$P_{mu}(\varnothing) = \frac{1}{a^H(\varnothing)E_nE_n^Ha(\varnothing)}$$

  Vary the value of $\varnothing$
- In the last step spectrum function is calculated and the estimated value of DOA is find by searching the peak values [7].

4. Improved Music Algorithm

The MUSIC algorithm is limited to uncorrelated signal. If the signals are correlated the performance of the MUSIC algorithm is not good. The improvement done in MUSIC algorithm is that the conjugate reconstruction of the data matrix is done of the MUSIC algorithm. The results are analyzed in next section.

5. Simulation Results

The simulations were performed in MATLAB. The simulation parameters were defined as the incident angles are 10°, 30° and 50 ° respectively. The three signals are independent narrow band signals which are not correlated. The model of the noise is taken as additive white Gaussian noise (AWGN). The SNR is considered as 20dB. The number of array element is 10. The spacing of the array elements is half of the wavelength of input signal. The number of snapshot taken is 200 in this simulation. The following simulation result shows the relationship between the key factors effecting the DOA estimation and MUSIC algorithm.

5.1 Basic DOA Estimation for MUSIC Algorithm.

Figure 1 shows the basic simulation of the MUSIC algorithm in which two independent signals are taken for the hypothetical situation. By using MUSIC algorithm a spectrum peak algorithm can easily be constructed. It can estimate the direction and number of incident signal accurately. If accurate model is used DOA estimation is very accurate in MUSIC algorithm and it performs very well with high resolution in the multiple signal environments.

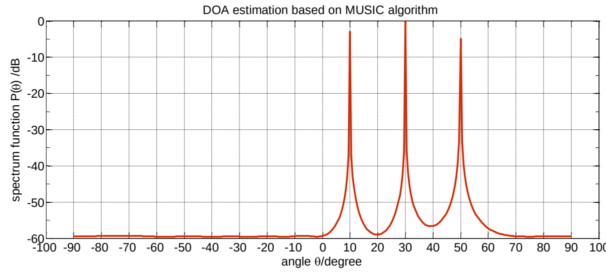

Figure 1 MUSIC ALGORITHM

5.2 Music Algorithm and Number of Array Elements

In the figure 2 the relationship between the number of array elements and DOA estimation is shown. With above simulation parameters the array element number is 10, 50 and 100 and the results are shown.

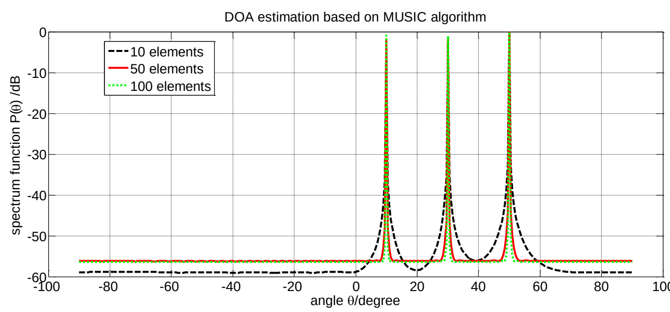

Figure 2 Relationship between number of elements and MUSIC algorithm.

Keeping simulation parameters same and increasing the number of arrays elements is shown that spectral beam width becomes narrow, directivity increases which enhance the ability to distinguish spatial signals. By increasing the number of array elements the DOA estimation is improved but the data processing speed decreases.

5.3 Music Algorithm and the Element Spacing.

In figure 3 relationship between element spacing and DOA estimation is shown. The simulation parameters are same, number of array element is 10 and the array spacing is λ, $\lambda/6$ and $\lambda/2$.

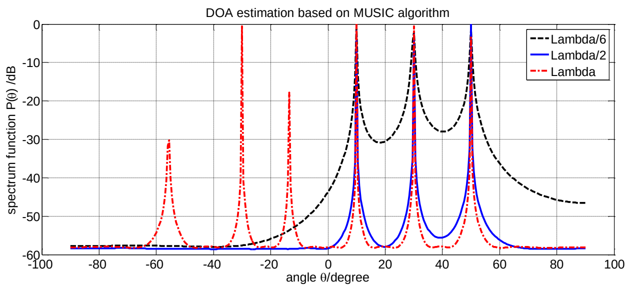

Figure 3 Effect of spacing of array of elements.

In the above figure it can be seen from the results that as the spacing between array elements is increasing beam width of DOA estimation is becoming narrower. The resolution of MUSIC algorithm increases with increase is spacing, however, when the spacing is greater than λ/2, except for the signal source direction, other directions appeared as false peaks.

5.4 Music Algorithm and Number of Snapshot.

The relationship between snapshot and MUSIC algorithm is shown is Figure 4. The parameters are the same just the snap shot is varied to 5, 50 and 200.

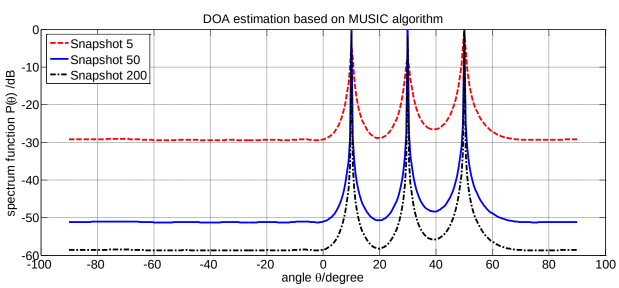

Figure 4 Relationship between snapshots on MUSIC algorithm

The result shows that as the number of snapshots is increased the spectrum of DOA estimation increase and accuracy of MUSIC algorithm is also increased but complexity of processing of more data is in there.

5.5 Music Algorithm and SNR.

The figure 5 shows the relationship of SNR and DOA estimation. All other simulation parameters are kept constant, snapshot is kept 200 and SNR is varied -20dB, 0dB and 20dB.

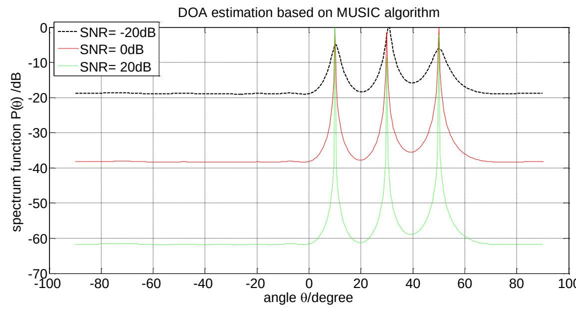

Figure5 Effect of SNR on MUSIC algorithm

All other simulation parameters are kept constant and if the SNR is increased the direction of signal becomes clears and accuracy of MUSIC algorithm improves. The spectrum of DOA estimation becomes narrow.

5.6 Music Algorithm and Incident Angle.

Figure 6 shows the effect of signal incident angle difference. All simulation parameters are kept constant, SNR is 20dB and the incident angle is 5, 10 and 20 degree.

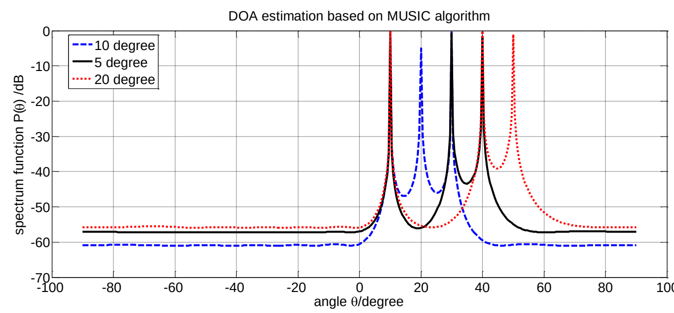

Figure 6 Effect of incident angle on MUSIC algorithm

The incident angle shows how MUSIC algorithm recognizes three signals. The results shows that if incident angle in increased the resolution of signal is increased and direction of signal is improved.

5.7 Music Algorithm for Coherent Signal.

Figure 7 shows the effect of improved MUSIC algorithm on coherent signals.

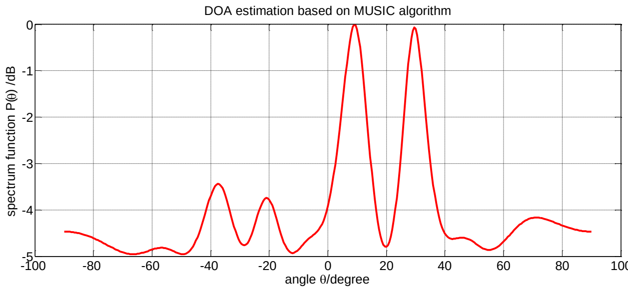

Figure 7 MUSIC algorithm when signals are coherent.

5.8 Improved Music Algorithm for Coherent Signal.

Figure 7 shows the old MUSIC algorithms when the signal is coherent and Figure 8 shows the improved MUSIC algorithm when signal is coherent.

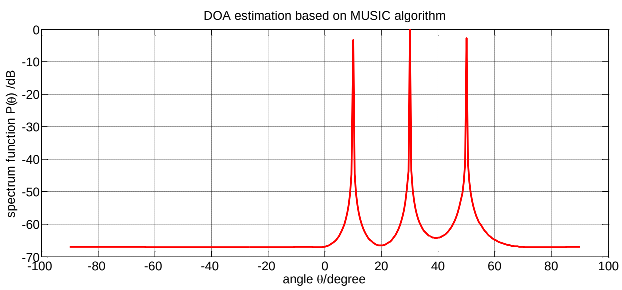

Figure 8 Improved MUSIC when signals are coherent

The comparison of MUSIC and improved MUSIC is done is figure 7 and 8 and it is clearly shown in results that improved MUSIC performs well when the signals are coherent. The improved MUSIC clearly distinguished the coherent signals and angle of estimation was accurate.

5.9 Spectrum comparison of music.

Keeping all the parameters same, figure 9 shows that MUSIC has superior performance compared to Conventional methods.

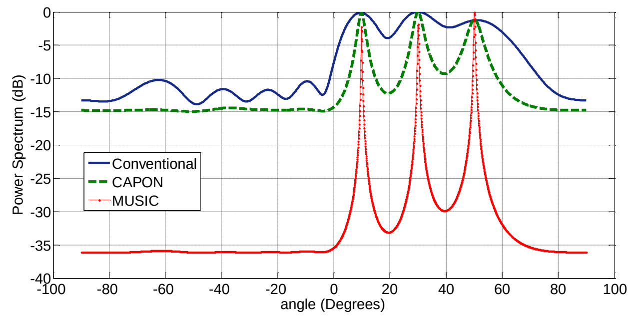

Figure 9, spectrum comparison of MUSIC with Conventional Beam-former.

The figure 9 shows that the width of the beam and height of the side lobe effect the resolution of the conventional method while capon overcome these limitations but it fails if the signals are highly correlated. MUSIC algorithm has better separated and sharper peaks and the side lobes at other angles are reduced.

Conclusion & Discussion

In this paper DOA Estimation for uniform linear array using MUSIC algorithm investigated in detail. The simulation results shows that the MUSIC algorithm provide superior resolution due to its inherent eigen-structure. It can be seen from the simulation that the more the number of element and snapshot, the larger the difference between the incident angles, the greater the array elements spacing, the MUSIC algorithm has a high resolution.

The improvements in MUSIC are analyzed in the form of sharper peaks and smaller errors in angle detection. The MUSIC algorithm performed well in the case of incoherent signals but degrade its performance in the presence of coherent signals. MUSIC algorithm is improved and it is seen from the results that it's DOA and resolution is clear if the signals are coherent.

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