Statistical Signal and Data Processing through Applications

Mini-Project

MUSIC for DOA

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MUSIC (Multiple Signal Classification)

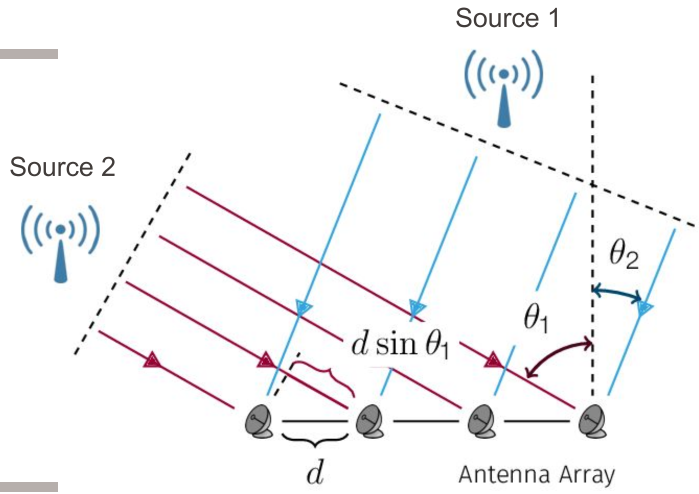
- Developed during the *60s - 80s*
- Usages
  - Frequency estimation
  - DOA
    - Communication, Sonar, Radar, Speech processing, ...

Summary

- MUSIC for DOA
- Generated Data
  - MUSIC
  - Improved MUSIC
- Real data
  - MUSIC
  - Improved MUSIC
- Conclusion

Assumptions on the system

- D narrowband source signals with identical center frequency
- Uniform Linear Array of M (M > D) isotropic microphones
- Far-field
- Presence of Gaussian White Noise
- Array element interval not larger than half the wavelength of the highest signal frequency

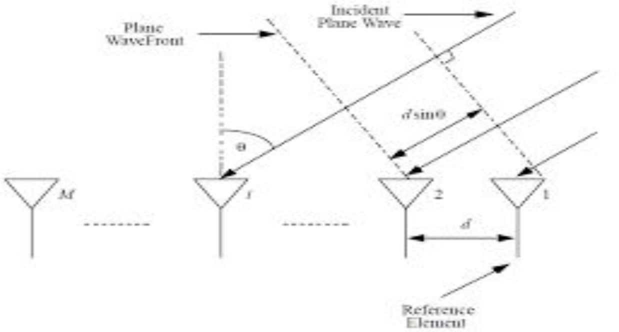


Data model

- Array steering vector a(θ): M x 1 vector with $a_m(\theta_k) = \exp[-j(m-1)\frac{2\pi \, \mathrm{d} \sin\theta_k}{\lambda}]$
- Direction matrix of the incident signals $A=[a(\theta_1), a(\theta_2), \ldots, a(\theta_D)]^T$
- Output signal of each array element $x_m(t) = \sum_{k=1}^{D} a_m(\theta_k) s_k(t) + n_m(t)$
- Matrices formulation: $X=AS+N$

$$N=[n_1(t), n_2(t), \ldots, n_M(t)]^T$$

$$X=[x_1(t), x_2(t), \ldots, x_M(t)]^T$$

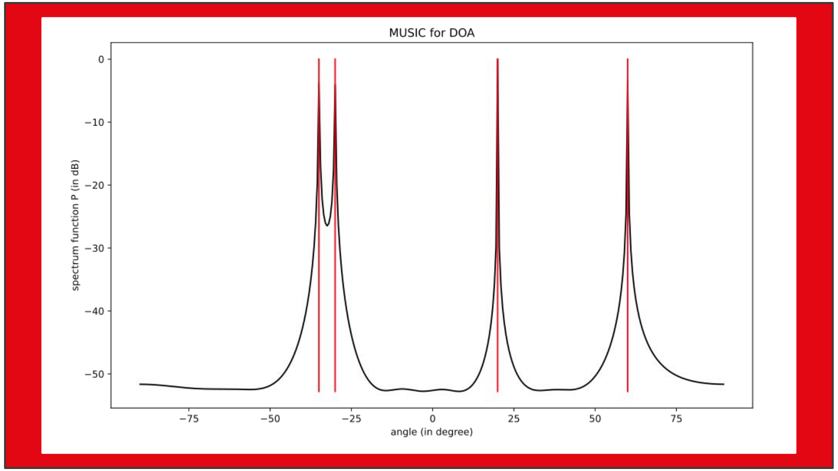


Basic algorithm

1) $R_x = \frac{1}{N}\sum_{i=1}^{N} X(i)X^H(i)$ $\longrightarrow$ $R_x = AR_sA^H + \sigma^2 I$

2) $A^H v_i=0 \quad i=D+1, D+2,\ldots,M$ $\longrightarrow$ $E_n=[V_{D+1},V_{D+2},\ldots,V_M]$

3) $\boxed{P_{mu}(\theta) = \frac{1}{a^H(\theta)E_nE_n^H a(\theta)}}$



DEMO 1 :

Improved algorithm

- Conjugate reconstruction of the data matrix of the MUSIC algorithm
    - Improve precision for noisy and correlated signals

New correlation matrix for the algorithm: $R = R_x + R_y = AR_sA^H + J[AR_sA^H]^*J + 2\sigma^2 I$

$$J = \begin{bmatrix} 0 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 0 \end{bmatrix}$$

Transition matrix J

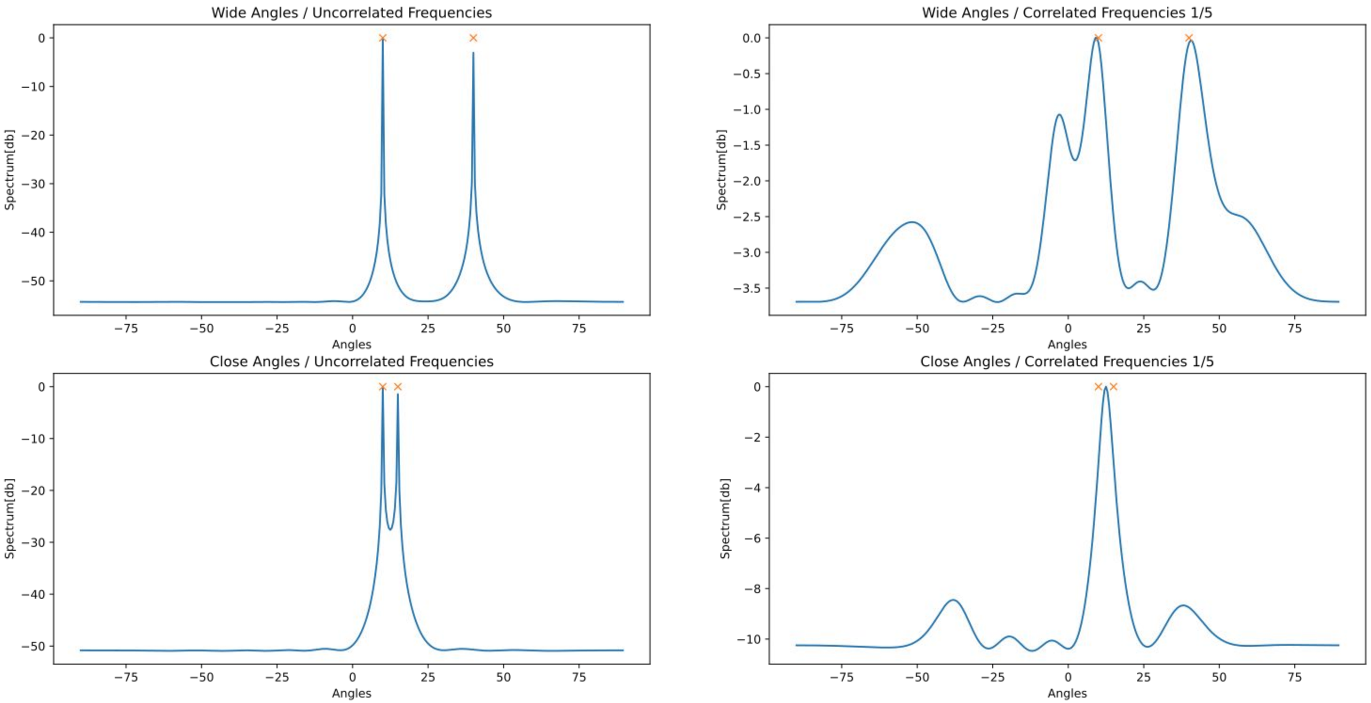
- Based on the pre-processing of the array steering vector
    - Error estimation and correction of the sensors

$$a(\theta) = \begin{bmatrix} a_1(\theta) & 0_{M\times L} \\ 0_{L\times 1} & a_2(\theta) \end{bmatrix} \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix}_{(L+1)\times 1} = \tilde{a}(\theta)\Gamma$$

$$\hat{\theta} = \arg\max_{\theta} \frac{1}{\det\left[\tilde{a}^H(\theta)U_N U_N^H \tilde{a}(\theta)\right]}$$

DEMO 2 :

Generated data results for the Basic Algorithm

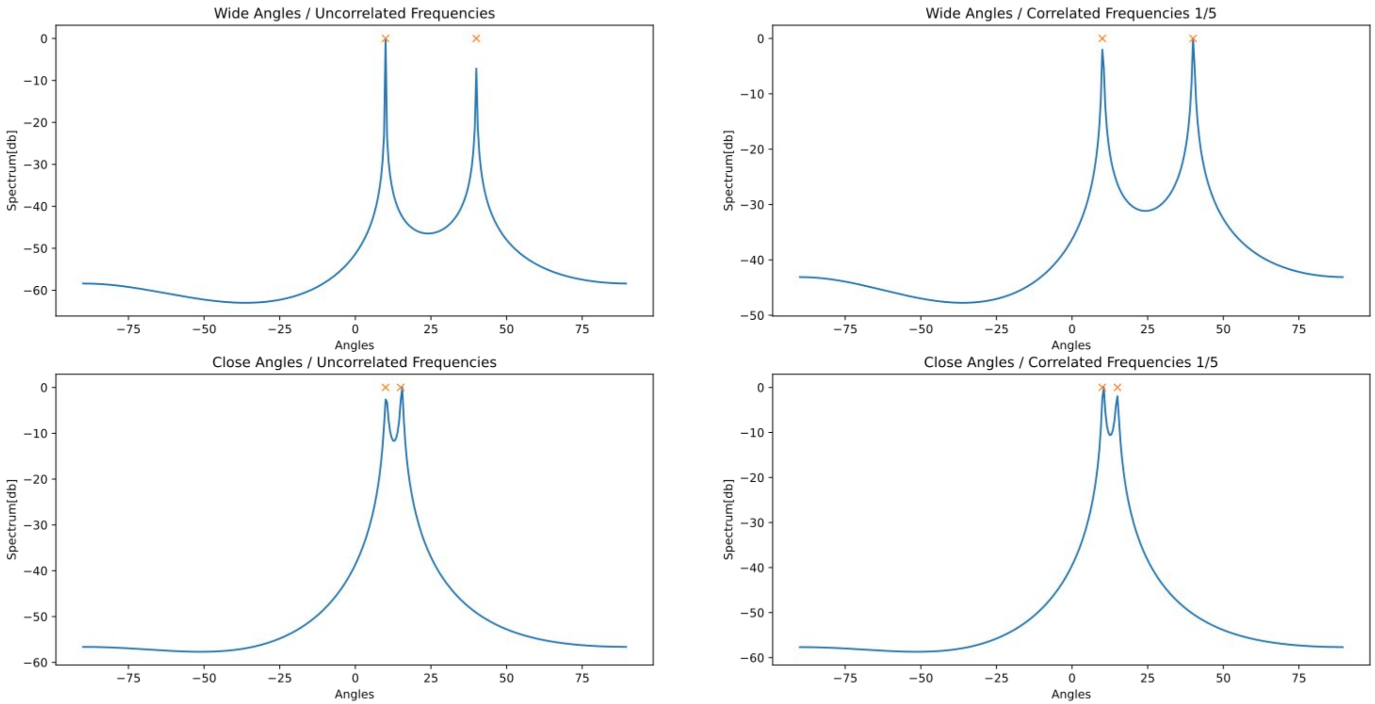


Thoughts

- Basic algo works well for simple and generated signals (even with relatively close angles).
- Huge drop of accuracy when used on generated and correlated signals.

EPFL

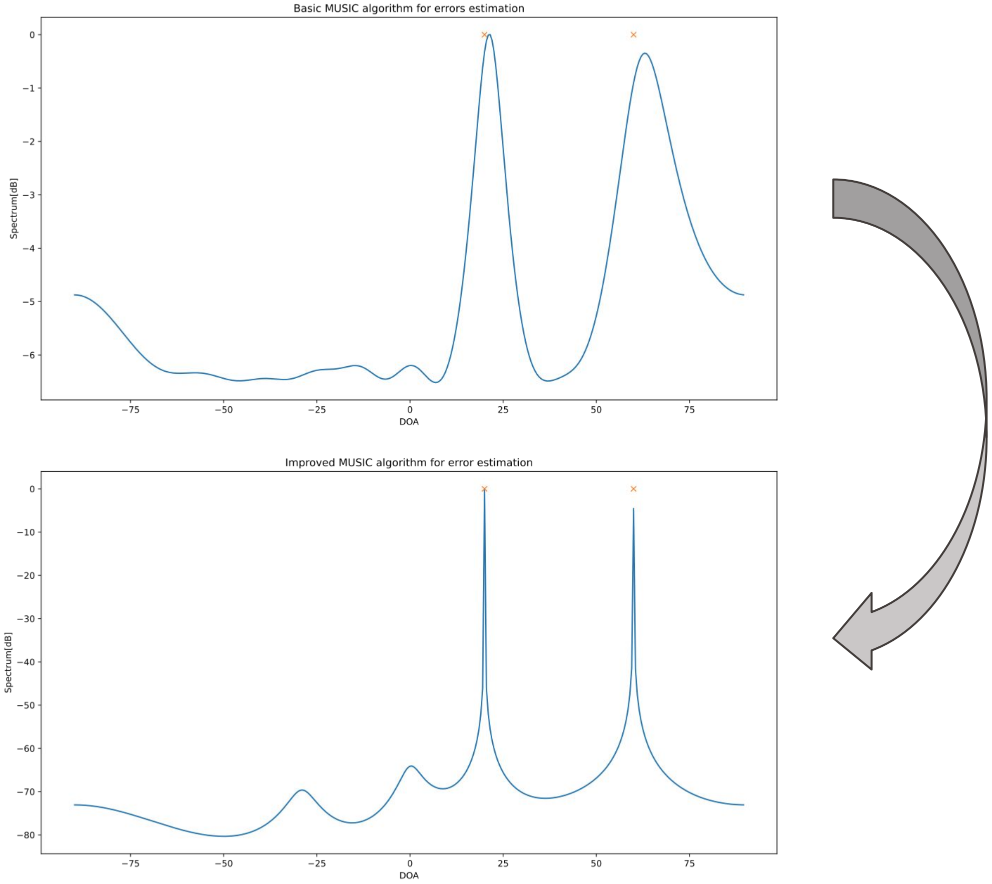
Generated data results for the Improved Algorithm



Thoughts

- Improved algo induces noise in the spatial spectrum function.
- Less precise for <u>uncorrelated</u> <u>close angles</u>.
- Great improvements for <u>correlated</u> signals.

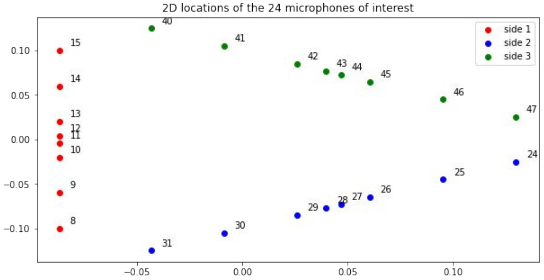
Generated data results with error estimation and correction



Thoughts

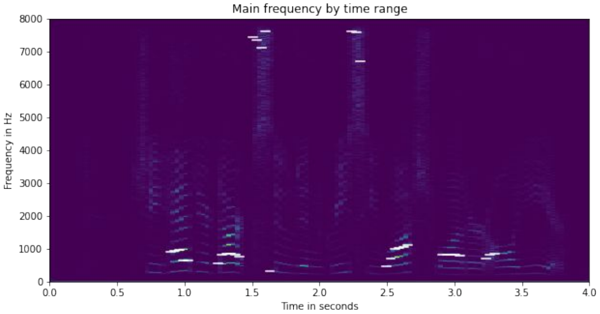
- Accurate estimation and correction of microphones induced errors.

Real data

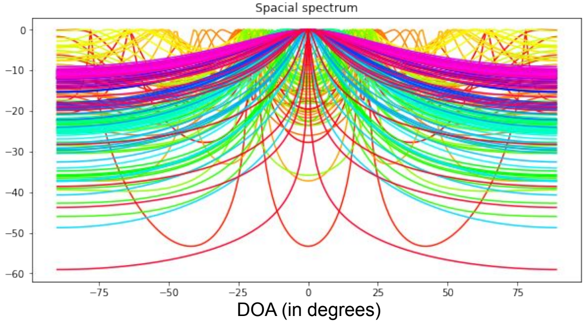


- Frame based technique
- Finding the n peakiest peaks

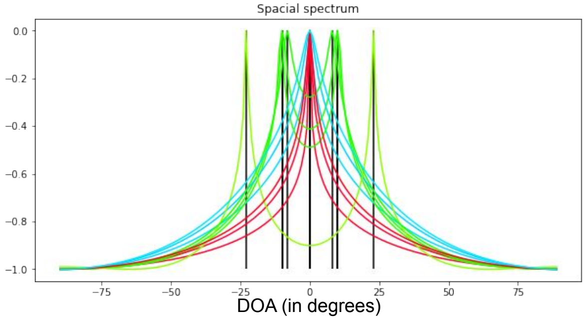
DEMO 3



Main frequencies by time range



MUSIC output for each frame

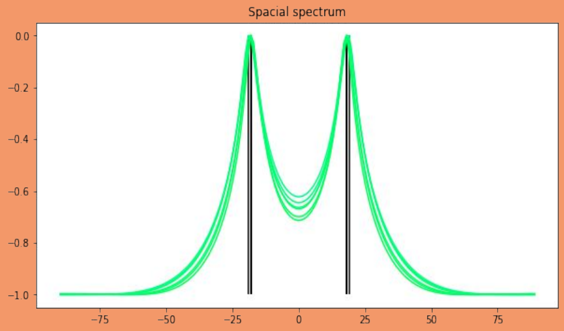


Keep only the n “peakiest” peaks

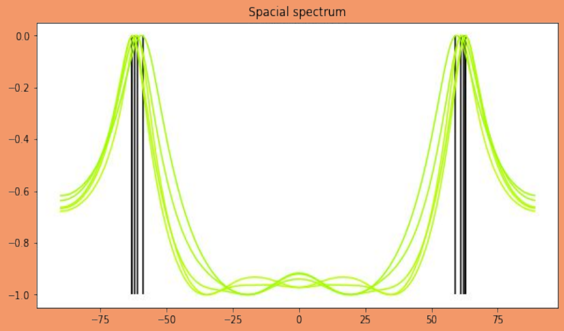
Real data results

MUSIC

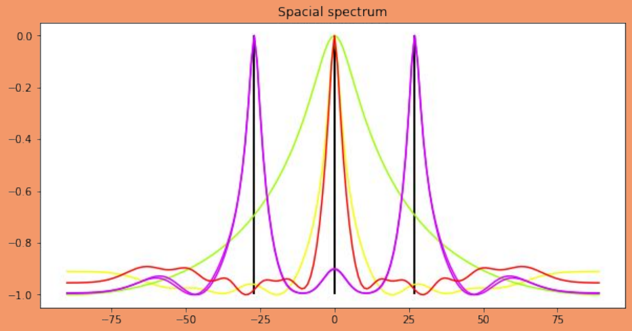
Single sweep source at 20 degrees



Two corr. speech sources at 32 & 66 degrees

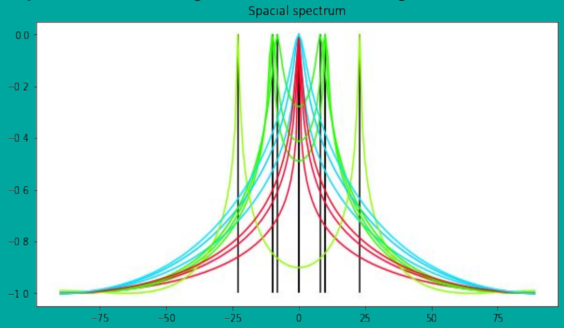


Two uncorr. sweep sources at 30 & 60 degrees

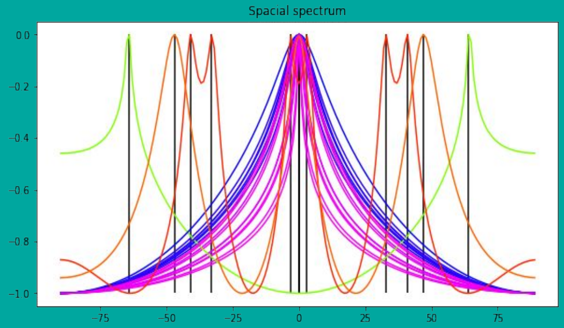


Improved

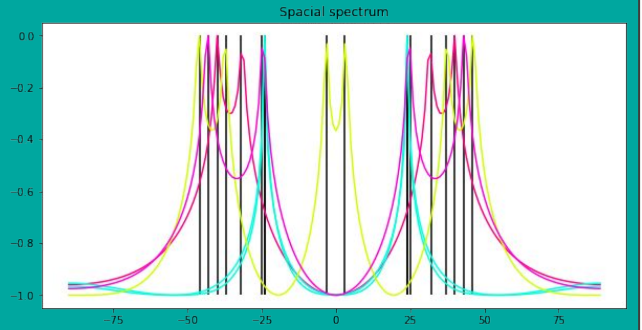
Single source at 20 degrees



Two speech sources at 32 & 66 degrees



Two uncorr. sweep sources at 30 & 60 degrees



Conclusion

- Generated data
- Real data
- Github link

Further Work

- Data preprocessing
- Take advantage of the 3D structure of the Pyramic device
- Clustering on the output array of DOAs
- Machine Learning

Thank you for your time!