ISE 3414 - POR: ICE 4 (Take-home) **NAMES:**

Dr. Bish, Fall 2018

Estimated work: 1-2 hours

- Take-home ICE. Solutions (one per group, with names of all group members included on the submission) to be submitted electronically via Canvas ICE 4 Assignment link by the deadline posted on Canvas. LATE submissions and submissions over e-mail receive ZERO credit.
- **Virginia Tech Honor Code** applies to this work, and will be strictly enforced. This work must be completed by the ICE 4 group only. You are NOT allowed to discuss these problems and your solutions with anyone, whether in person or over the internet (this includes the course instructor and the GTAs). All suspected violations will be promptly reported to the Honor Code System.
- Each group can have AT MOST four students.
- Open books and notes; calculators are allowed on all problems EXCEPT FOR PROBLEM 1.
- All solutions must represent the ENTIRE group's effort. After the submission of this assignment, each student is required to fill out **confidential** team evaluations over Canvas individually by the deadline posted on Canvas (these forms will be kept strictly confidential). The team evaluations will be in the form of a quiz, which will be open this Saturday 10am and to be closed next Monday 10pm. Differential grades among group members are possible.
- To obtain any credit you must clearly show all your work even if you used a calculator to actually compute it.
- Write legibly. If we cannot read your work, we will not grade it.

1. Random variables X and Y have the following joint probability density function (pdf):

$$f_{X,Y}(x,y) = \begin{cases} 2e^{-(x+y)}, & \text{if } 0 < x < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

Determine the conditional expectation $E[Y|X = x]$.

(Calculators are NOT allowed for this problem. <u>All</u> necessary integrations need to be solved.

2. A machine can produce three types of products. For product type i, the machine processing time is exponentially distributed with parameter:

$$\lambda_1 = 15 \text{ jobs per minute for } \; i = 1;$$
$$\lambda_2 = 8 \text{ jobs per minute for } i = 2;$$
$$\lambda_3 = 2 \text{ jobs per minute for } i = 3.$$

30% of the products produced by the machine are of type 1; 20% are of type 2; and 50% are of type 3. Let Y denote the process time of a random product; and let X denote the product type. Compute the following (numerical values are expected).

(a-i) $E[Y|X = 1]$

(a-ii) $E[Y|X = 2]$

(a-iii) $E[Y|X = 3]$

(b) $E[Y]$ (using the law of total expectation)

(c) Currently a type 2 product is being processed. Given that it has been processed for 30 seconds already, what is the probability that it will be processed for at least one more minute?

3. A component is purchased from 3 suppliers, A, B, and C, where the suppliers have respective defective rates of 2%, 6%, and 4%. Of all the components purchased, 20% comes from supplier A, 50% from supplier B, and 30% from supplier C, that is, each shipment comes from each of these suppliers with these probabilities. The company uses the following quality control policy. A sample of 15 units is randomly selected from each shipment of components. If at most 2 defective units are found in the sample, then the entire shipment is accepted; otherwise, the entire shipment is rejected.
Determine the following:

(a-1) probability that a shipment will be accepted given that it came from Supplier A.

(a-2) probability that a shipment will be accepted given that it came from Supplier B.

(a-3) probability that a shipment will be accepted given that it came from Supplier C.

(a-4) Using the law of total probability and your responses to parts (a-1)-(a-3) above, determine the probability that a shipment will be accepted.

(b-1) expected number of defectives in a random sample of 15 units, if the shipment came from Supplier A.

(b-2) expected number of defectives in a random sample of 15 units, if the shipment came from Supplier B.

(b-3) expected number of defectives in a random sample of 15 units, if the shipment came from Supplier C.

(b-4) Using the law of total expectation and your responses to parts (b-1)-(b-3) above, determine the expected number of defectives in a random sample of 15 units.

4. Consider the experiment of rolling a six-sided fair die. Let X denote the number of rolls it takes to obtain the first 5, Y denote the number of rolls until the first 2, and Z denote the number of rolls until the first 4. Numerical answers are needed only for parts (a) and (b). Expressions are sufficient for parts (c), (d), and (e).

(a) $E[X|Y = 1 \text{ or } Z = 1]$.

(b) $E[X|Y = 1 \text{ and } Z = 2]$.

(c) $E[X|Y = 1 \text{ and } Z = 3]$.

(d) $E[X|Y = 3 \text{ and } Z = 4]$.

(e) $E[X^2|Y = 1]$.