

Program Structures and Algorithms  
Spring 2023(SEC –8)

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**Task:**

- Implement height-weighted Quick Union with Path Compression
- Using your implementation of UF\_HWQUPC, develop a UF ("union-find") client that takes an integer value  $n$  from the command line to determine the number of "sites." Then generates random pairs of integers between 0 and  $n-1$ , calling `connected()` to determine if they are connected and `union()` if not. Loop until all sites are connected then print the number of connections generated.
- Determine the relationship between the number of objects ( $n$ ) and the number of pairs ( $m$ ) generated to accomplish this (i.e. to reduce the number of components from  $n$  to 1). Justify your conclusion in terms of your observations and what you think might be going on.

**Relationship Conclusion:** It should be kind of linear or little more than just linear.  $M = NC$ . Where  $C$  is some constant. First let's start with how many edges do we need to create a single component. It should be  $N - 1$ . This is the minimum number of edges or Unions we need to call, to get all sites connected. Proof - Suppose there are 3 nodes to connect all three of them the minimum we will require is 2. And similarly for 4 nodes the minimum is 3.

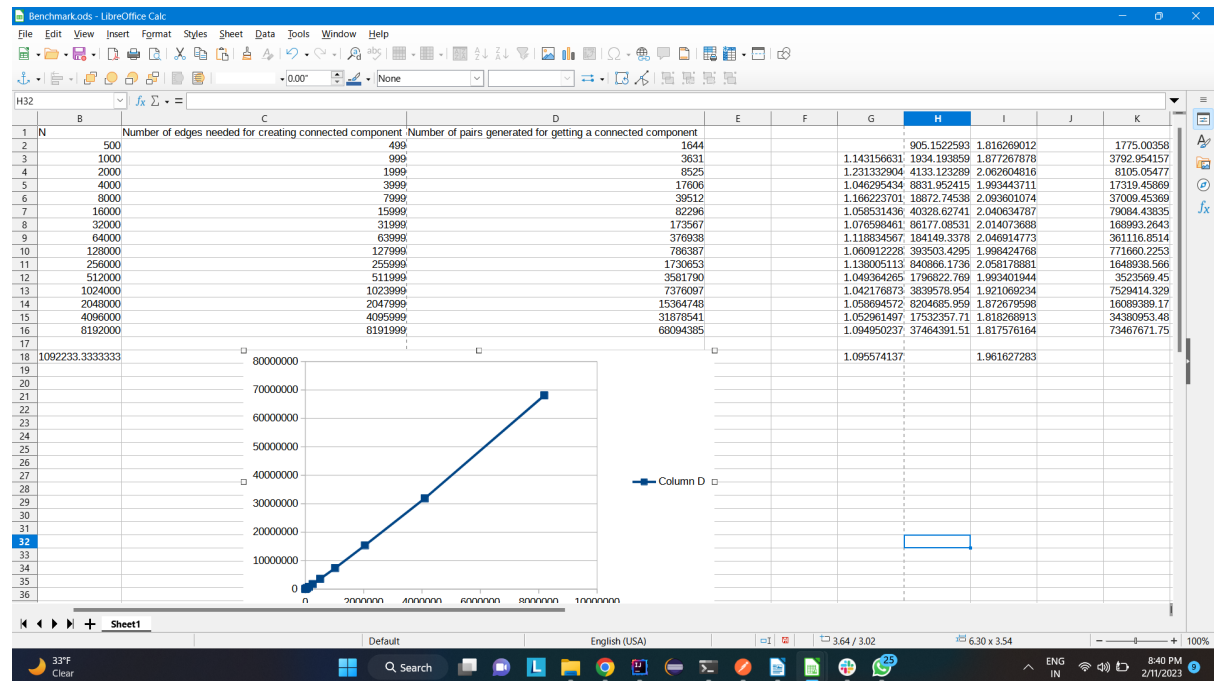
When we start generating two random numbers between 0 and  $n - 1$  to union. then when there exist no edges or in the starting it will have a higher probability of generating edges in the starting the probability would be:  $(n^2 - n)/n^2$ . This is because there could be  $n^2$  possible type of pair which we could generate for example in case of 3 nodes we can randomly generate 9 edges like – (0, 0), (0, 1), (0, 2), (1, 1), (1, 0), (1, 2), (2, 2), (2, 0), (2, 1), and “-n” because we cannot create self edges and there could be  $n$  self edges. And probability is number of edges which we can create to get the edges which we need for connecting nodes –  $(n^2 - n)$  divided by total number of probable outcomes. But as the graphs gets connected the probability start decreasing – because now the number of edges increases and edges which we want to connect decreases and probability becomes -  $(n^2 - n - 2x)/n^2$ , here  $x$  is the number of edges which we have already created. And 2 is because if we have already created an edge between 0 and 1 then if our random number generator generated (1, 0) or (0, 1) it will not create any edge as it is already present. So the probability start decreasing and since in the starting the probability is fairly high as compared to at the end. Because of this interplay it could be that the  $M$  is linearly proportional to  $N$ . Since more the Number of  $N$  the more edges we need to create. Thus increasing kind of linearly.

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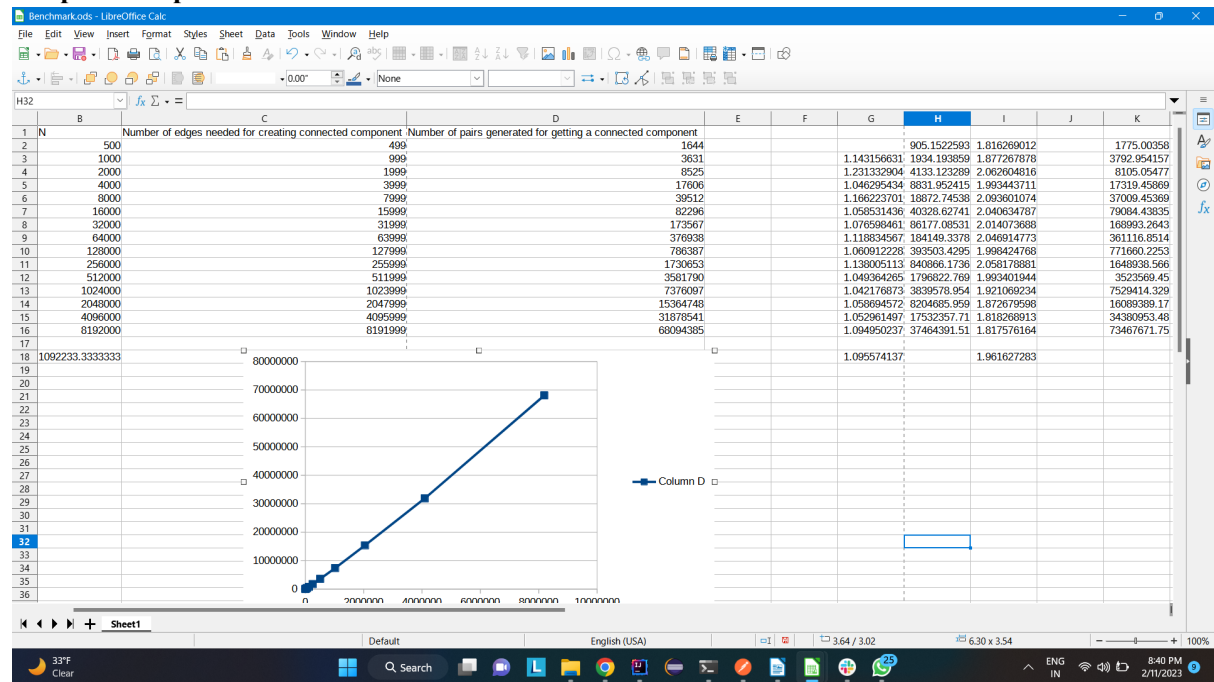
**Evidence to support that conclusion:**

In the below screenshot we can see by the slope of the log of  $M$  and log  $N$  values in the column G and averaging it out so we get slope – 1.0955. thus the relationship turn out to be like  $M = N^{1.09}C$ . And now  $C$  is calculated by  $C = M/N^{1.09}$ . Here we are inserting the actual values of  $M$  which we got from the experiment which is present in the column D,  $N^{1.09}$  here we are inserting the original values of  $N$  present in the column B. The result of this ratio is present in I. And averaging it out we get  $C$  as 1.9. This the relationship turn out to be around  $M = 1.9N^{1.09}$ . And plotting the graph between column B(experimental values of  $N$ ) and column D(experimental values of  $M$ ) we nearly get a straight line. Which means it should be

kind of linear. To get these values the code is written in the file named UF\_BenchMark. Also this xlsx file is named Benchmark.xlsxs.



## Graphical Representation:



## Unit Test Screenshots:

First Screenshot of the UF\_BenchMark.java running.



