AVALIAÇÃO 2

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① $T(x,y,z) = (x+2y+3z, 4y+6z, -y-z)$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

↳ base canônica $\beta = \{(1,0,0), (0,1,0), (0,0,1)\}$

$$[T]_\beta = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix} \qquad \begin{bmatrix} x & y & z \\ 0 & y & z \\ 0 & 0 & z \end{bmatrix}$$

→ p/ ser triangular superior

det T é inversível

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1/2 \end{bmatrix}$$

↳ Multiplicar por $-1/2$
↳ Subtrair 1

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & 0 & 1/2 \end{bmatrix} \longrightarrow \text{Det} = 2$$

↳ Matriz triangular Superior

↳ base → $\{(2,0,0), (0,2,0), (0,0,2)\}$

credeal

② a) Determine $T(x,y,z)$

$$[T] = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \boxed{T(x,y,z) = (x+y, -x+z, -y-z)}$$

b) Matriz de T em relação a base

$\beta' = \{(-1,1,0), (1,-1,1), (0,1,-1)\}$

$\rightarrow T(-1,1,0) = (-1+1, 1+0, -1-0) = (0,1,-1)$

$\rightarrow T(1,-1,1) = (1-1, -1+1, 1-1) = (0,0,0)$

$\rightarrow T(0,1,-1) = (0+1, 0-1, -1+1) = (1,-1,0)$

$$\begin{cases} x_1(-1,1,0) + y_1(1,-1,1) + z_1(0,1,-1) = (0,1,-1) \\ x_2(-1,1,0) + y_2(1,-1,1) + z_2(0,1,-1) = (0,0,0) \\ x_3(-1,1,0) + y_3(1,-1,1) + z_3(0,1,-1) = (1,-1,0) \end{cases}$$

$$\rightarrow (x_1, y_1, z_1) = \begin{cases} -x + y = 0 \\ x - y + z = 1 \\ y - z = -1 \end{cases} \rightarrow \begin{matrix} x_1 = 0 \\ y_1 = 0 \\ z_1 = 1 \end{matrix}$$

$$\rightarrow (x_2, y_2, z_2) = \begin{cases} -x + y = 0 \\ x - y + z = 0 \\ y - z = 0 \end{cases} \rightarrow \begin{matrix} x_2 = 0 \\ y_2 = 0 \\ z_2 = 0 \end{matrix}$$

credeal

$$\Rightarrow (x_3, y_3, z_3) = \begin{cases} -x+y=1 \\ x-y+z=-1 \\ y-z=0 \end{cases}$$

$$\Rightarrow \begin{array}{l} x_3=-1 \\ y_3=0 \\ z_3=0 \end{array}$$

A matriz é $\rightarrow \begin{bmatrix} x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \\ z_1 & z_2 & z_3 \end{bmatrix}$

Portanto, a matriz é $\begin{bmatrix} 0 & 0 & -1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

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③ Autovalores e autovetores do operador $T(x,y,z) = (x+2y+3z, 4y+6z, -y-z)$

$$\begin{bmatrix} x & 2y & 3z \\ 0 & 4y & 6z \\ 0 & -y & -z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix} \rightarrow \begin{bmatrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 6 \\ 0 & -1 & -1-\lambda \end{bmatrix}$$

$$\rightarrow \left|\begin{matrix} 1-\lambda & 2 & 3 \\ 0 & 4-\lambda & 6 \\ 0 & -1 & -1-\lambda \end{matrix}\right|\begin{matrix} 1-\lambda & 2 \\ 0 & 4-\lambda \\ 0 & -1 \end{matrix}$$

$\rightarrow \text{Det} = (-\lambda+1)(\lambda^2-3\lambda+2)$

$\rightarrow$ Igualar a zero e achar os valores p/ λ

$\rightarrow (-\lambda+1)(\lambda^2-3\lambda+2) = 0$

$\rightarrow \lambda_1 = 1$ e $\lambda_2 = 2$

- Autovalores são $\lambda = 1$ e $\lambda = 2$

$\rightarrow$ Agora, calcular os autovetores

$\rightarrow \boxed{Ax = \lambda x}$

- para $\lambda = 1$ $\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 1.\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

- para $\lambda = 2$ $\rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 6 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = 2\begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow \begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix}$

- Autovetores para $\lambda = 1$ são $\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ e para $\lambda = 2$ são $\begin{pmatrix} -3 \\ -3 \\ 1 \end{pmatrix}$

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4 $[T]_{\beta'} = \begin{bmatrix} 1 & -4 & -2 & -2 \\ -4 & -1 & -2 & -2 \\ 2 & 2 & 1 & 4 \\ 2 & 2 & 4 & 1 \end{bmatrix}$

↳ Autovalores → $\lambda = -3$
$\lambda = 3$
$\lambda = 1 + 2\sqrt{2}$
$\lambda = 1 - 2\sqrt{2}$

↳ Autovetores de

$\lambda = 3 \Rightarrow \begin{pmatrix} 0 \\ -1 \\ 1 \\ 1 \end{pmatrix}$

$\lambda = -3 \Rightarrow \begin{pmatrix} 0 \\ 0 \\ -1 \\ 1 \end{pmatrix}$

$\lambda = 1 + 2\sqrt{2} \Rightarrow \begin{pmatrix} \sqrt{2} \\ -2 \\ 1 \\ 1 \end{pmatrix}$

$\lambda = 1 - 2\sqrt{2} \Rightarrow \begin{pmatrix} -\sqrt{2} \\ -2 \\ 1 \\ 1 \end{pmatrix}$

↳ Matriz M → $\begin{bmatrix} 0 & 0 & \sqrt{2} & -\sqrt{2} \\ 0 & -1 & -2 & -2 \\ -1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$