

Álgebra linear - Avaliação 1

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$$\textcircled{1} S = \{ \alpha = 1$$

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \rightarrow N(T) = \{(1, 0, 2)\}$$

$$\bullet B = \{(1, 0, 2), (1, 0, 0), (0, 1, 0)\} \subset \mathbb{R}^3$$

$$\bullet B' = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\} \subset \mathbb{R}^3$$

$$\begin{cases} T(1, 0, 2) = (0, 0, 0) \\ T(1, 0, 0) = (1, 0, 0) \\ T(0, 1, 0) = (0, 1, 0) \end{cases}$$

$$(x, y, z) = a(1, 0, 2) + b(1, 0, 0) + c(0, 1, 0)$$

$$(x, y, z) = (a+b, a+c, a)$$

$$\begin{cases} a+b = x \\ a+c = y \\ a = z \end{cases} \rightarrow \begin{cases} b = x-z \\ c = y-x \\ a = z \end{cases}$$

$$(x, y, z) = z(1, 0, 2) + (x-z)(1, 0, 0) + (y-x)(0, 1, 0)$$

$$T(x, y, z) = z(0, 0, 0) + (x-z)(1, 0, 0) + (y-x)(0, 1, 0)$$

$$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$$

$$T(x, y, z) = (x-z, y-x, 0) \rightarrow N(T) = \{(1, 0, 2)\}$$

2)

$$T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$$

$$\text{Im}(T) = [(1, 0, 2); (1, -1, 1)]$$

I) base β para $\text{Im}(T) = \mathbb{R}^3$

$$\beta = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)\}$$

$$\begin{cases} T(1, 0, 0, 0) = (1, 0, 2) \\ T(0, 1, 0, 0) = (1, -1, 1) \\ T(0, 0, 1, 0) = (0, 0, 0) \\ T(0, 0, 0, 1) = (0, 0, 0) \end{cases}$$

$$(x, y, z, t) = x(1, 0, 0, 0) + y(0, 1, 0, 0) + z(0, 0, 1, 0) + t(0, 0, 0, 1)$$

$$T(x, y, z, t) = x(1, 0, 2) + y(1, -1, 1) + 0 + 0$$

$$T(x, y, z, t) = x(1, 0, 2) + y(1, -1, 1)$$

$$T(x, y, z, t) = (x+y, -y, 3z)$$

$$T: \mathbb{R}^4 \rightarrow \mathbb{R}^3$$

$$T(x, y, z, t) = (x+y, -y, 3z)$$

$$\text{Im}(T) = [(1, 0, 2); (1, -1, 1)]$$

$$\textcircled{3} T: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$$

$$\begin{cases} T(1+x) = 2 \\ T(x^3+x^2) = x^2+2x \end{cases}$$

$$\begin{cases} T(1+x) = 2 \\ x^2+2x = 0 \end{cases} \quad B = \{1, x, x^2\} \subset P_3(\mathbb{R})$$

$$x + yt + zt^2 = x^2 + 2x$$

$$\hookrightarrow \boxed{x^3 + 2x^2 + 2x + 2 = 0}$$

$$\textcircled{4} T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \quad T(x, y, z) = (x, -y, z)$$

$$\begin{cases} T(1, 0, 0) = (1, -1, 1) \\ T(0, 0, 1) = (1, -1, 0) \\ T(0, 1, 0) = (1, -1, 1) \end{cases}$$

$$\hookrightarrow \begin{cases} T^{-1}(1, -1, 1) = (1, 0, 0) \\ T^{-1}(1, -1, 0) = (0, 0, 1) \\ T^{-1}(1, -1, 1) = (0, 1, 0) \end{cases}$$