Universidade Federal da Fronteira Sul
Ciência da Computação
Cálculo 1 - Professor Milton Kist
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Exercícios → Lista 4 - Integrais

(1) (I) $\int \left(9t^2 + \frac{1}{\sqrt{t^3}}\right) dt \rightarrow \int 9t^2 + \frac{1}{t\sqrt{t}}\, dt \rightarrow \int 9t^2 + \frac{1}{t^{3/2}}\, dt$

$\hookrightarrow \int 9t^2\, dt + \int \frac{1}{t^{3/2}}\, dt \hookrightarrow 3t^3 - \frac{2}{\sqrt{t}} \rightarrow \boxed{3t^3 - \frac{2}{\sqrt{t}} + C,\ C \in \mathbb{R}}$

(II) $\int (ax^4 + bx^3 + 3c)\, dx \rightarrow \int ax^4\, dx + \int bx^3\, dx + \int 3c\, dx$

$\hookrightarrow \frac{ax^5}{5} + \frac{bx^4}{4} + 3cx \rightarrow \boxed{\frac{ax^5}{5} + \frac{bx^4}{4} + 3cx + C,\ C \in \mathbb{R}}$

(III) $\int \left(\sqrt{2y} - \frac{1}{\sqrt{2y}}\right) dy \rightarrow \int (2y)^{1/2} - \frac{1}{(2y)^{1/2}}\, dy$

$\hookrightarrow \int 2^{1/2} \cdot y^{1/2}\, dy - \int \frac{1}{2^{1/2} \cdot y^{1/2}}\, dy \hookrightarrow \frac{2y\sqrt{2y}}{3} - \sqrt{2y}$

$\hookrightarrow \boxed{\frac{2y\sqrt{2y}}{3} - 2y + C,\ C \in \mathbb{R}}$

(IV) $\int \frac{x^5 + 2x^2 - 1}{x^4}\, dx \rightarrow \int \frac{x^5}{x^4} + \frac{2x^2}{x^4} - \frac{1}{x^4}\, dx \rightarrow \int x + \frac{2}{x^2} - \frac{1}{x^4}\, dx$

$\hookrightarrow \int x\, dx + \int \frac{2}{x^2}\, dx - \int \frac{1}{x^4}\, dx \hookrightarrow \frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3}$

$\hookrightarrow \boxed{\frac{x^2}{2} - \frac{2}{x} + \frac{1}{3x^3} + C,\ C \in \mathbb{R}}$

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$$(V)\ \int \left(\frac{e^t}{2} + \sqrt{t} + \frac{1}{t}\right) dt \rightarrow \int \frac{e^t}{2} dt + \int t^{1/2} dt + \int \frac{1}{t} dt$$

$$\hookrightarrow \frac{e^t}{2} + \frac{2t\sqrt{t}}{3} + \ln(|t|) \rightarrow \boxed{\frac{e^t}{2} + \frac{2t\sqrt{t}}{3} + \ln(|t|) + C, C \in \mathbb{R}}$$

$$(VI)\ \int \frac{8x^4 - 9x^3 + 6x^2 - 2x + 1}{x^2} dx$$

$$\hookrightarrow \int \frac{8x^4}{x^2} - \frac{9x^3}{x^2} + \frac{6x^2}{x^2} - \frac{2x}{x^2} + \frac{1}{x^2} dx \rightarrow \int 8x^2 - 9x + 6 - \frac{2}{x} + \frac{1}{x^2} dx$$

$$\hookrightarrow \int 8x^2 dx - \int 9x dx + \int 6 dx - \int \frac{2}{x} dx + \int \frac{1}{x^2} dx$$

$$\hookrightarrow \boxed{\frac{8x^3}{3} - \frac{9x^2}{2} + 6x - 2\ln(|x|) - \frac{1}{x} + C, C \in \mathbb{R}}$$

$$(VII)\ \int \cos\theta \cdot tg\theta\, d\theta \rightarrow \int \cancel{\cos\theta} \cdot \frac{sen\theta}{\cancel{\cos\theta}} d\theta$$

$$\hookrightarrow \int sen\theta\, d\theta \rightarrow -\cos\theta$$

$$\hookrightarrow \boxed{-\cos\theta + C, C \in \mathbb{R}}$$

$$(VIII)\ \int tg^2 x \operatorname{cossec}^2 x\, dx \rightarrow \int (tg x \operatorname{cossec} x)^2 dx$$

$$\hookrightarrow \int \left(\frac{sen x}{\cos x} \cdot \frac{1}{sen x}\right)^2 dx \rightarrow \int \frac{1}{\cos x^2} dx \rightarrow tg x$$

$$\hookrightarrow \boxed{tg x + C, C \in \mathbb{R}}$$

② $f(x) = x^{2/3} + x$

$F(x) = \int x^{2/3} + x \, dx = \frac{x^{2/3+1}}{2/3+1} + \frac{x^{1+1}}{1+1}$

$\to \frac{x^{5/3}}{5/3} + \frac{x^2}{2} \Rightarrow \frac{3x^{5/3}}{5} + \frac{x^2}{2} \to F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} + C$

$F(1) = 1 \to \frac{3\sqrt[3]{1^5}}{5} + \frac{1^2}{2} + C = 1 \to \frac{3}{5} + \frac{1}{2} + C = 1 \to \frac{11}{10} + C = 1$

$\to C = -1/10 \to \boxed{F(x) = \frac{3\sqrt[3]{x^5}}{5} + \frac{x^2}{2} - \frac{1}{10}}$

③ $\int f(x)\,dx = x^2 + \frac{1}{2}\cos 2x + C$

$\to f(x) = x^2 + \frac{1}{2}\cos 2x + C \to y' = 2x + \frac{1}{2}(-\operatorname{sen} 2x \cdot 2)$

$\to \boxed{y' = 2x - \operatorname{sen} 2x}$

④ (I) $\int (2x^2 + 2x - 3)^{10}(2x+1)\,dx$

$u = 2x^2 + 2x - 3 \to \int \frac{u^{10}}{2} du \to \frac{1}{2}\int u^{10} du = \frac{1}{2} \frac{u^{10+1}}{10+1}$

$\to \frac{1}{2} \frac{(2x^2+2x-3)^{10+1}}{10+1} \to \frac{1}{22}(2x^2+2x-3)^{11}$

$\to \boxed{\frac{1}{22}(2x^2+2x-3)^{11} + C, \ C \in \mathbb{R}}$

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(4) II) $\int 5x\sqrt{4-3x^2}\,dx \rightarrow 5.\int x\sqrt{4-3x^2}\,dx \rightarrow 5.\int -\frac{1}{6}\sqrt{t}\,dt$

$\hookrightarrow 5.\left(-\frac{1}{6}\right).\int \sqrt{t}\,dt \rightarrow -\frac{5}{6}\int t^{1/2}\,dt \rightarrow -\frac{5}{6}\cdot 2t\sqrt{t}/3$

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$\hookrightarrow -\frac{5}{6}\cdot\frac{2(4-3x^2)\sqrt{4-3x^2}}{3} \rightarrow -\frac{5\sqrt{4-3x^2}(4-3x^2)}{9}$

$\hookrightarrow \boxed{-\frac{5\sqrt{4-3x^2}(4-3x^2)}{9}+C,\ C\in\mathbb{R}}$

(III) $\int (e^{2t}+2)^{1/3}\,e^{2t}\,dt \rightarrow \int \frac{u^{1/3}}{2}\,du \rightarrow \frac{1}{2}\int u^{1/3}\,du$

$\hookrightarrow \frac{1}{2}\cdot\frac{3u\sqrt[3]{u}}{4} \rightarrow \frac{1}{2}\cdot\frac{3(e^{2t}+2)\sqrt[3]{e^{2t}+2}}{4}$

$\hookrightarrow \frac{3\sqrt[3]{e^{2t}+2}\,(e^{2t}+2)}{8} \rightarrow \boxed{\frac{3\sqrt[3]{e^{2t}+2}\,(e^{2t}+2)}{8}+C,\ C\in\mathbb{R}}$

(IV) $\int \operatorname{sen}^4 x \cos x\,dx \rightarrow \int t^4\,dt \rightarrow t^5/5$

$\hookrightarrow \frac{\operatorname{sen} x^5}{5} \hookrightarrow \boxed{\frac{\operatorname{sen}(x)^5}{5}+C,\ C\in\mathbb{R}}$

(V) $\int \frac{\operatorname{sen} x}{\cos^5 x}\,dx \hookrightarrow \int \frac{-1}{t^5}\,dt \rightarrow -\int\frac{1}{t^5}\,dt \rightarrow -\left(\frac{-1}{4t^4}\right)$

$\hookrightarrow \frac{1}{4\cos(x)^4} \hookrightarrow \boxed{\frac{1}{4\cos(x)^4}+C,\ C\in\mathbb{R}}$

(VI) $\int e^x \cos 2\, e^x\, dx \rightarrow \frac{(e \cdot \cos 2e)^x}{\ln(e \cos 2e)} \hookrightarrow \frac{e^x \cdot \cos(2e)^x}{1 + \ln(\cos(2e))}$

$\hookrightarrow \boxed{\frac{e^x \cdot \cos(2e)^x}{1 + \ln(\cos(2e))} + C,\ C \in \mathbb{R}}$

5 (I) $\int x \operatorname{sen} 5x\, dx \rightarrow u = x \rightarrow dv = \operatorname{sen} 5x\, dx$

$\hookrightarrow du = dx \rightarrow v = -\frac{\cos 5x}{5} \hookrightarrow x\left(\frac{-\cos 5x}{5}\right) - \int \frac{-\cos 5x}{5} dx$

$\hookrightarrow x\left(\frac{-\cos 5x}{5}\right) + \frac{1}{5}\int \frac{\cos t}{5} dt$

$\hookrightarrow x\left(\frac{-\cos 5x}{5}\right) + \frac{1}{25} \operatorname{sen} 5x \rightarrow -\frac{x \cdot \cos 5x}{5} + \frac{\operatorname{sen} 5x}{25}$

$\hookrightarrow \boxed{-\frac{x \cdot \cos 5x}{5} + \frac{\operatorname{sen} 5x}{25} + C,\ C \in \mathbb{R}}$

(II) $\int (x+1) \cos 2x\, dx \rightarrow \int x \cdot \cos(2x) + \int \cos(2x)\, dx$

$\hookrightarrow \frac{x \cdot \operatorname{sen} 2x}{2} + \frac{\cos 2x}{4} + \frac{\operatorname{sen} 2x}{2} \rightarrow \frac{x \operatorname{sen} 2x + \operatorname{sen} 2x}{2} + \frac{\cos 2x}{4}$

$\hookrightarrow \boxed{\frac{x \operatorname{sen} 2x + \operatorname{sen} 2x}{2} + \frac{\cos 2x}{4} + C,\ C \in \mathbb{R}}$

(III) $\int x \ln 3x\, dx$

$\hookrightarrow 3 \ln \int x^2 \cdot dx \rightarrow 3 \ln \frac{x^{2+1}}{2+1} = \ln x^3 \rightarrow \boxed{\ln x^3 + C}$

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(IV) $\int x^2 \cos x \, dx \rightarrow \int \frac{t^2 \cdot \cos t}{a^3} dt \rightarrow \frac{1}{a^3} \cdot \int t^2 \cos t \, dt$

$\hookrightarrow \frac{1}{a^3} \cdot \left(t^2 \cdot \operatorname{sen}(t) - 2 \int \operatorname{sen}(t) \cdot t \, dt \right)$

$\hookrightarrow \frac{1}{a^3} \left(t^2 \cdot \operatorname{sen}(t) - 2 \left(t \left(-\cos(t) \right) - \int -\cos(t) \, dt \right) \right.$

$\hookrightarrow \frac{1}{a^3} \left((ax)^2 \cdot \operatorname{sen}(ax) - 2 (ax \cdot (-\cos(ax)) + \operatorname{sen}(ax) \right)$

$\hookrightarrow \boxed{\frac{a^2 x^2 \cdot \operatorname{sen}(ax) + 2ax \cdot \cos(ax) - 2\operatorname{sen}(ax)}{a^3} + C, \; C \in \mathbb{R}}$

(V) $\int x^2 e^x \, dx \rightarrow u = x^2 \rightarrow dv = e^x dx$

$du = 2x \, dx \rightarrow v = e^x$

$\hookrightarrow x^2 e^x - \int e^x \cdot 2x \, dx \rightarrow x^2 e^x - 2 \cdot \int e^x x \, dx$

$\hookrightarrow x^2 e^x - 2 \int x e^x \, dx \rightarrow x^2 e^x - 2(xe^x - e^x)$

$\hookrightarrow x^2 e^x - 2xe^x + 2e^x$

$\hookrightarrow \boxed{x^2 e^x - 2xe^x + 2e^x + C, \; C \in \mathbb{R}}$

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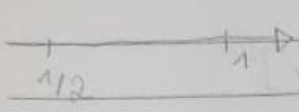
⑥ I) $x = 1/2$, $x = \sqrt{y}$ e $y = -x + 2$

$\sqrt{y} = x \to x^2 = y \to x^2 + x - 2 = 0 \to x' = 1$



$n = 4 \to \frac{1}{2} : 4 = \frac{1}{8}$

p/ $x = 5/8 \to y = -5/8 + 2 = 11/8 \to \frac{5}{8} \cdot \frac{11}{8} = \boxed{\frac{55}{64}}$ ua

p/ $x = 3/4 \to y = -3/4 + 2 = 5/4 \to \frac{3}{4} \cdot \frac{5}{4} = \boxed{\frac{15}{16}}$ ua

p/ $x = 7/8 \to y = -7/8 + 2 = 9/8 \to \frac{7}{8} \cdot \frac{9}{8} = \boxed{\frac{63}{64}}$ ua

p/ $x = 1 \to y = -1 + 2 = 1 \to 1 \cdot 1 = \boxed{1 \text{ ua}}$

$\hookrightarrow$ Área $\to$ $\boxed{A < 3{,}78125}$

(II) $y^2 = 2x$ e $x^2 = 2y$

$y = x^2/2$

$\hookrightarrow y^2 = 2x \to \left(\frac{x^2}{2}\right)^2 = 2x \to \frac{x^4}{4} = 2x$

$\downarrow$

$x^4 - 8x = 0$

$\hookrightarrow x = 0$ e 2

$\hookrightarrow A = \int_0^2 [f(x) - g(x)]\, dx$

$\hookrightarrow A = \int_0^2 \sqrt{2x} \cdot x^2/2 \; dx$

$\hookrightarrow A = ((2\sqrt{2})/3 \cdot 2^{3/2} - 1/6 \cdot 8 - ((2\sqrt{2})/3) \cdot 0 - 1/6 \cdot 0)$

$\hookrightarrow \frac{2\sqrt{2} \cdot 2\sqrt{2}}{3} - \frac{4}{3} \to \frac{8}{3} - \frac{4}{3} \hookrightarrow \boxed{A = \frac{4}{3} \text{ ua}}$

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(III) $y = \operatorname{sen} x$ e $y = -\operatorname{sen} x$, $x \in [0, 2\pi]$

$-\operatorname{sen} x \leq y \leq \operatorname{sen} x$

$0 \leq x \leq 2\pi \rightarrow \int_0^{2\pi} [y]_{-\operatorname{sen} x}^{\operatorname{sen}} dx$

$\hookrightarrow \int_0^{2\pi} 2\operatorname{sen} x \, dx = [-2\cos x]^{2\pi}$

$\hookrightarrow -2\cos 2\pi + 2\cos 0 = \boxed{4ua}$