(1)

$$f(x) = \begin{cases} x^2 & \text{se } x<1 \\ 0 & \text{se } x=1 \\ 1/x & \text{se } x>1 \end{cases}$$

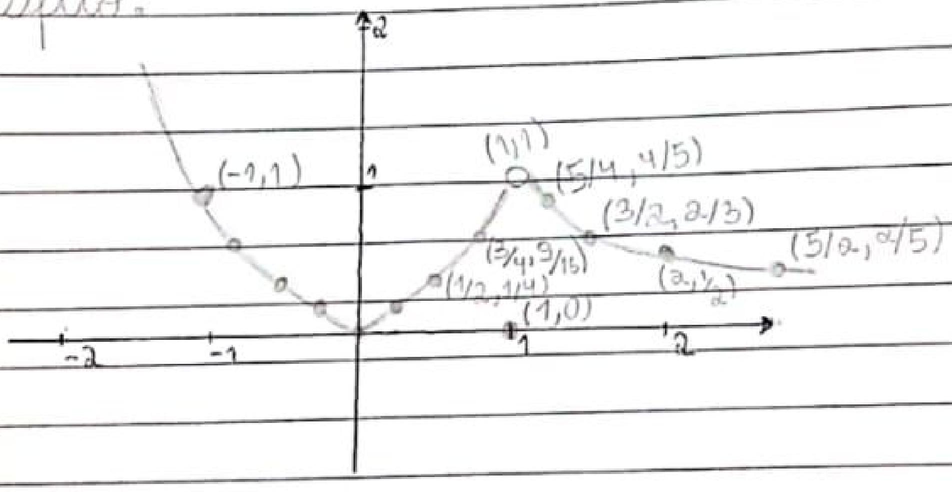
Gráfico

$f(x) = x^2$
- $\to x = 1/4 \to y = 1/16$
- $\to x = 1/2 \to y = 1/4$
- $\to x^2 = 0 \to y = 0$
- $\to x = 3/4 \to y = 9/16$

$f(x) = 0$
- $\to x = 1$

$f(x) = 1/x$
- $\to x = 3/2 \to y = 2/3$
- $\to y = 5/2 \to y = 2/5$
- $\to y = 2 \to y = 1/2$
- $\to x = 5/4 \to y = 4/5$

Gráfico:



a) $\lim\limits_{x\to 1^-} f(x) \to \lim\limits_{x\to 1^-} x^2 = 1$

b) $\lim\limits_{x\to 1^+} f(x) \to \lim\limits_{x\to 1^+} 1/x = 1$

c) $\lim\limits_{x\to 1} f(x) = \lim\limits_{x\to 1} f(x) = 1$

d) $\lim\limits_{x\to -1} f(x) = \lim\limits_{x\to -1} f(x) = 1$

e) $\lim\limits_{x\to +\infty} f(x) \to \lim\limits_{x\to +\infty} 1/x$

$\lim\limits_{x\to +\infty} 1/x = 0$

(2) (I) $\lim_{x \to 4} \frac{x^2-4x}{x^3-4x^2+x-4} \to \frac{(x^2-4x)}{(x^3-4x^2+x-4)} = \frac{x(x-4)}{x^2(x-4)+x-4}$

$\to \lim_{x \to 4} \frac{x}{x^2+1} \to \frac{4}{16+1} \to \frac{4}{17}$

(II) $\lim_{x \to 2} \frac{\sqrt{4x+1}-3}{x-2} \to \frac{\sqrt{4x+1}-3}{x-2} \cdot \frac{\sqrt{4x+1}+3}{\sqrt{4x+1}+3}$

$\to \frac{4x-8}{\sqrt{4x+1}+3 \cdot (x-2)} \to 4x-8 = 4(x-2) \to \frac{4(x-2)}{(x-2) \cdot \sqrt{4x+1}+3}$

$\lim_{x \to 2} \frac{4}{\sqrt{4x+1}+3} \to \frac{4}{\sqrt{9}+3} = \frac{4}{6} = \frac{2}{3}$

(III) $\lim_{x \to -\infty} \frac{3x^4-6x^2+1}{6x-x^3-2x^4} \to \frac{3x^4-6x^2+1}{6x-x^3-2x^4} = \frac{x^4(3-6/x^2+1/x^4)}{x^4(6/x^3-1/x-2)}$

$\to \frac{(3-6/x^2+1/x^4)}{(6/x^3-1/x-2)} \to \frac{(3-6+1)\,\cancel{x^3}\cdot\cancel{x}}{(6-1-2)\,\cancel{x^3}\cdot\cancel{x}} \to \frac{3-6\cdot0+0}{6\cdot0-0-2}$

$\lim_{x \to -\infty} \frac{(3-6/x^2+1/x^4)}{(6/x^3-1/x-2)} = -\frac{3}{2}$

(IV) $\lim_{x \to +\infty} (1+4/x)^{x+2} \to (1+4/x)^{x+2} = (1+4/x)^{x\left(\frac{x+2}{x}\right)}$

$\lim_{x \to +\infty} \left(1+\frac{4}{x}\right)^{x \lim_{x \to +\infty}\left(\frac{x+2}{x}\right)} \to \lim_{x \to +\infty}(x) = +\infty = 1$

$\to \lim_{x \to +\infty}\left(1+\frac{4}{x}\right)^x \to (1+t)^{4/t}$

$\to e^{4 \lim_{x \to +\infty}\left(\frac{x+2}{x}\right)^{=1}} \to e^4$

③ $f(x) = \frac{2x^2}{x^2-4}$ $\rightarrow$ x deve ser $\neq \pm 2$ $\rightarrow$ / /

$\lim\limits_{x \to -2} \left(\frac{2x^2}{x^2-4}\right) \rightarrow \nexists$ $\lim\limits_{x \to 2} \left(\frac{2x^2}{x^2-4}\right) \rightarrow \nexists$

• Assintonas verticais $\rightarrow$ 2 e -2

$f(x) = \frac{2x^2}{x^2-4}$

$\lim\limits_{x \to +\infty} \left(\frac{2x^2}{x^2-4}\right) \rightarrow \frac{\lim\limits_{x \to +\infty} (2x^2)}{\lim\limits_{x \to +\infty} (x^2-4)} = \frac{2x^2}{x^2\left(1-\frac{4}{x^2}\right)} \rightarrow \frac{2}{1-4/x^2}$

$\downarrow$

$2/_{1-4\cdot 0} = 2$

• Assintona horizontal da função $\rightarrow$ 2

④ $x^2 - e^x + 2 = 0$

$f(x) = 1,1 \Rightarrow (1,1)^2 - (e)^{1,1} + 2 = -0,23$

$f(x) = 1,5 \rightarrow (1,5)^2 - (e)^{1,5} + 2 = 0,21$

$\hookrightarrow$ Intervalo é $[1,1 \ ; \ 1,5]$

credeal

⑤ $f(x) = x^2 - 4$

derivada $\Rightarrow \frac{d}{dx}(x^2) - \frac{d}{dx}(4)$

$f'(x) = 2x$

$f'(1) = 2 \cdot 1 = 2$

$\rightarrow$ $f'(1)$ significa que é a tangente do ponto, quando o $x = 1$

⑥ (I) $y = \dfrac{x^2 - 1}{x^3 - sen(x)}$

$$y' = \frac{d}{dx}\left(\frac{x^2-1}{x^3 - sen(x)}\right) \rightarrow y' = \frac{\overbrace{d/dx(x^2-1)}^{2x} \cdot (x^3 - sen(x)) - (x^2-1) \cdot \frac{d}{dx}(x^3 - sen(x))}{((x^3 - sen(x))^2}$$

$$y' = \frac{2x(x^3 - sen(x)) - (x^2-1)(3x^2 - cos(x))}{(x^3 - sen(x))^2}$$

$$y' = \frac{-x^4 - 2x \cdot sen(x) + x^2 \cdot cos(x) + 3x^2 - cos(x)}{(x^3 - sen(x))^2}$$

$$y' = \frac{-x^4 - 2x \cdot sen(x) + x^2 \cdot cos(x) + 3x^2 - cos(x)}{(x^3 - sen(x))^2}$$

/ /

⑥ (II) $y = (x - \cos x^2)\ln(3x^4 - 2)$

$$y' = \frac{d}{dx}\left(x - \cos(x)^2 \cdot \ln(3x^4 - 2)\right)$$

$$\frac{d}{dx}\left(x - \cos(x)^2\right) = 1 - (-\operatorname{sen}(x^2) \cdot 2x)$$

$\downarrow$

$$y' = (1 - (-\operatorname{sen}(x)^2 \cdot 2x)\ln(3x^4 - 2) + (x - \cos(x)^2 \cdot \frac{d}{dx}(\ln(3x^4 - 2)$$

$$y' = (1 - (-\operatorname{sen}(x)^2 \cdot 2x)(\ln(3x^4 - 2) + (x - \cos(x)^2 \cdot \frac{1}{3x^4 - 2} \cdot 3 \cdot 4x^3$$

$$= \frac{12x^4 - 12x^3 \cdot \cos(x)^2}{3x^4 - 2}$$

$\downarrow$

$$y' = \frac{(1 + 2x \cdot \operatorname{sen}(x^2) \cdot \ln(3x^4 - 2) + 12x^4 - 12x^3 \cdot \cos(x)^2)}{3x^4 - 2}$$

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⑥ (III) $y = tg(x^3 - 7x) - 3 + cotg(5x)$

$y' = \frac{d}{dx} \cdot y \rightarrow y' = \frac{d}{dx}(tg(x^3-7x)) - \frac{d}{dx}(3) + \frac{d}{dx}(cotg(5x))$

$y' = sec(x^3-7x)^2 \cdot (3x^2-7) - cosec(5x)^2 \cdot 5$

$y' = sec(x^3-7x)^2 \cdot (3x^2-7) - 5cosec(5x)^2$

(IV) $y = 3(4x^3-5)^5 - \frac{3}{x^6} + e^{x-x^2}$

$y' = \frac{d}{dx} \cdot y \rightarrow y' = \frac{d}{dx}\left(tg(3)(4x^3-5)^5 - \frac{3}{x^6} + e^{x-x^2}\, 1-2x\right)$

$\frac{d}{dx}(tg(3)(4x^3-5)^5) - \frac{d}{dx}\left(\frac{3}{x^6}\right) + \frac{d}{dx}(e^{x-x^2})$

$\frac{d}{dx}(tg(3)(4x^3-5)^5) = 60\, tg(3)\, x^2(4x^3-5)^4 \rightarrow \frac{d}{dx}\left(\frac{3}{x^6}\right) = -18/x^7$

$\frac{d}{dx}(e^{x-x^2}) = e^{x-x^2}(1-2x)$

$\rightarrow 60\, tg(3) x^2 (4x^3-5)^4 + \frac{18}{x^7} + e^{x-x^2}(1-2x)$