Universidade Federal da Fronteira Sul / /
Curso: Ciência da Computação
Disciplina: Cálculo 1
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1) a) $\lim_{x \to 2} \frac{x^2+x-6}{x-2}$

$x^2+(a+b)x+ab \to (x+a).(x+b) \to \frac{(x-2).(x+3)}{(x-2)}$

$\to \lim_{x \to 2} (x+3) \to 2+3 = \boxed{5}$

b) $\lim_{x \to -4} \frac{x^2+5x+4}{x^2+3x-4} \to \frac{(-4)^2+5.(-4)+4}{(-4)^2+3.(-4)-4} = \frac{16-20+4}{16-12-4}$

$\to \frac{0}{0} \to \frac{(x+1)(x+4)}{(x-1)(x+4)} = \frac{x+1}{x-1} \to \frac{-4+1}{-4-1} = \boxed{\frac{3}{5}}$

c) $\lim_{x \to 2} \frac{x^2-x+6}{x-2}$

$\lim_{x \to a} -f(x) \neq \lim_{x \to a} +f(x) \to$ não existe limite

$\lim_{x \to 2} + \left(\frac{x^2-x+6}{x-2}\right) = \infty$ $\boxed{\lim =]-\infty, +\infty[}$

$\lim_{x \to 2} - \left(\frac{x^2-x+6}{x-2}\right) = -\infty$

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d) $\lim_{x \to 4} \frac{x^2-4x}{x^2-3x-4} \rightarrow \frac{x(x-4)}{x^2-3x-4} \rightarrow \frac{x(x-4)}{(x+1)(x-4)}$

$\frac{x}{(x+1)} \rightarrow \frac{4}{4+1} = \boxed{\frac{4}{5}}$

e) $\lim_{t \to 3} \frac{t^2-9}{2t^2+7t+3} \rightarrow \frac{(t+3)(t-3)}{2t^2+7t+3} \rightarrow \frac{(t+3)(t-3)}{(2x+1)(t+3)}$

$\frac{(t-3)}{(2x+1)} \rightarrow \frac{-3-3}{2.-3+1} = \boxed{\frac{6}{5}}$

f) $\lim_{x \to 1} \frac{x^2-4x}{x^2-3x-4}$

$\lim_{x \to a^-} f(x) \neq \lim_{x \to a^+} f(x) \rightarrow$ não existe limite

$\boxed{\lim =]-\infty, +\infty[}$

② $\lim_{x \to 3} \frac{x^4-8x^3+18x^2-27}{x^4-10x^3+36x^2-54x+27}$

$\frac{x^4-8x^3+18x^2-27}{x^4-10x^3+36x^2-54x+27} \rightarrow$ teorema das raízes racionais $\rightarrow \lim_{x \to 3} \frac{(x+1)}{(x-1)}$

$\lim_{x \to} \frac{(x+1)}{(x-1)} \rightarrow \frac{3+1}{3-1} = \frac{4}{2} = \boxed{2}$

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b) $\lim\limits_{x \to 2} \dfrac{x-2}{\sqrt{2x-4}}$

$\dfrac{(x-2)}{\sqrt{2x-4}} \cdot \dfrac{(\sqrt{2x-4})}{(\sqrt{2x-4})} \to \dfrac{(x-2).(\sqrt{2x-4})}{2x-4} \to \dfrac{(x-2).(\sqrt{2x-4})}{2(x-2)}$

$\lim\limits_{x \to 2} \dfrac{\sqrt{2x-4}}{2} \to \dfrac{\sqrt{4-4}}{2} \to \boxed{0}$

c) $\lim\limits_{x \to 4} \dfrac{x-4}{\sqrt{x}-2}$

$\dfrac{(x-4)(\sqrt{2}+2)}{(\sqrt{x}-2)(\sqrt{2}+2)} \to \dfrac{(x-4)(\sqrt{x}+2)}{x-4} \to \sqrt{x}+2 \to \sqrt{4}+2 = \boxed{4}$

d) $\lim\limits_{x \to 0} \dfrac{x}{2-\sqrt{4-x}} \to \dfrac{x(2-\sqrt{4-x})}{(2-\sqrt{4-x})(2-\sqrt{4-x})} \to \dfrac{x(2-\sqrt{4-x})}{x}$

$2+\sqrt{-x+4} \to 2+\sqrt{4} \to \boxed{4}$

3 a) $\lim\limits_{x \to \infty} \dfrac{1}{2x+3} \to \dfrac{\lim_{x \to \infty}(1)}{\lim_{x \to \infty}(2x+3)} \to \dfrac{\lim_{x \to \infty}(1)=1}{\lim_{x \to \infty}(2x+3)=}$

$\boxed{1/\infty}$

b) $\lim\limits_{x \to \infty} \dfrac{3x+5}{x-4} \to \lim\limits_{x \to \infty}\left(\dfrac{3+5/x}{1-4/x}\right) \to \dfrac{\lim_{x \to \infty}(3+5/x)}{\lim_{x \to \infty}(1-4/x)}$

$\lim\limits_{x \to \infty}(3+5/x) = 3$

$\to \dfrac{3}{1} = \boxed{3}$

$\lim\limits_{x \to \infty}(1-4/x) = 1$

c) $\lim_{t \to -\infty} \frac{1-x-x^2}{2x^2-7}$

$\lim_{t \to -\infty} \left(\frac{1}{t^2} - \frac{1}{t} - 1\right) = -1 \to \frac{-1}{2} \Rightarrow \boxed{-1/2}$

$\lim_{t \to -\infty} (2-7/x^2) = 2$

d) $\lim_{x \to \infty} \frac{2-3x^2}{5x^2+4x} \to \lim_{x \to \infty} \left(\frac{2}{x^2} - 3\right) = -3 \to \boxed{-3/5}$

$\lim_{x \to \infty} \left(5 + \frac{4}{x}\right) = 5$

e) $\lim_{x \to \infty} \frac{x^3+5x}{2x^3-x^2+4} \to \frac{\lim_{x \to \infty} (1+5/x^2) \to 1}{\lim_{x \to \infty} (2-1/x+4/x^3) \to 2} \to \boxed{\frac{1}{2}}$

f) $\lim_{t \to -\infty} \frac{t^2+2}{t^3+t^2-1} \to \frac{\lim_{x \to -\infty} (1+2/x^3) = 1}{\lim_{x \to -\infty} \left(1+\frac{1}{x} - \frac{1}{x^3}\right) = 1} \to 1/1 = \boxed{1}$

④ a) $\lim_{x \to 5^+} \frac{6}{x-5}$

$\lim_{x \to 5^+} \frac{6}{x-5} \to x > 5 \to x - 5 > 0 \to \boxed{+\infty}$ ▷ valor positivo próx do 0

b) $\lim_{x \to 5^-} \frac{6}{x-5}$

▷ valor negativo que se aproxima de 0

$\lim_{x - 5^-} \frac{6}{x-5} \to x < 5 \to x - 5 < 0 \to \boxed{-\infty}$

c) $\lim_{x \to 3} \frac{1}{(x-3)^8}$

$\lim_{x \to a} - f(x) = \lim_{x \to a} + f(x) = L \to$ então $\lim_{x \to a} f(x) = L$

$\lim_{x \to 3} 3^- \left(\frac{1}{(x-3)^8}\right) = \infty$

$\lim_{x \to 3} 3^+ \left(\frac{1}{(x-3)^8}\right) = \infty$

$\to \boxed{\infty}$

d) $\lim_{x \to 0} \frac{x-1}{x^2(x+2)}$

$\lim_{x \to a} f(x) = L \to \lim_{x \to 0^-} \left(\frac{x-1}{x^2(x+2)}\right) = -\infty$

$\boxed{-\infty}$

$\lim_{x \to 0^+} \left(\frac{x-1}{x^2(x+2)}\right) = -\infty$

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e) $\lim\limits_{x \to -2^+} \dfrac{x-1}{x^2(x+2)} \to (x-1) \cdot \dfrac{1}{x^2(x+2)}$

$\lim\limits_{x \to -2^+} \left((x-1) \cdot \dfrac{1}{x^2(x+2)} \right) \to \lim\limits_{x \to -2^+} (x-1) \cdot \lim\limits_{x \to -2^+} \left(\dfrac{1}{x^2(x+2)} \right)$

$\lim\limits_{x \to -2^+} (x-1) = -3$

$-3 \cdot \infty = \boxed{-\infty}$

$\lim\limits_{x \to -2^+} \left(\dfrac{1}{x^2(x+2)} \right) = \infty$

f) $\lim\limits_{x \to 5^+} \ln(x+5) \to \lim\limits_{x \to 5^+} (\ln(x-5)) \to \ln(5-5) \to \boxed{0 \text{ (zero)}}$

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5 c) $\lim_{x \to 0} \left(1+\frac{x}{3}\right)^{1/x}$

$\lim_{x \to 0} (1+\frac{1}{z})^{3z}$ $\quad z = x/3 \to x = 3z$

$\lim_{x \to 0} \left[\left[1+\frac{1}{z}\right]^{z}\right]^{3} \to \lim_{x \to 0} e^{1/3} \to \boxed{\lim_{x \to 0} \sqrt[3]{e}}$

5 d) $\lim_{x \to \infty} (1+2/x)^{x} \to (1+1/x/2) \to t = x/2$

$\left(1+\frac{1}{t}\right)^{2t} \to \left(\left(1+\frac{1}{t}\right)^{t}\right)^{2} = \boxed{e^{2}}$

6) a) $f(x) = \frac{3x}{x-1}$

$\lim\limits_{x \to +\infty} f(x) = \lim\limits_{x \to \infty} \frac{3x}{x-1} \to \frac{1}{x}\left(\frac{3x}{x-1}\right) \to \lim\limits_{x \to 1} \to \left(\frac{3x}{x-1}\right)$

Assintona vertical = 1
Assintona horizontal = 3

b) $f(x) = \frac{2x}{\sqrt{x^2+4}} \to \lim\limits_{x \to +\infty} \left(\frac{2x}{\sqrt{x^2+4}}\right)$

Assintona horizontal $\begin{cases} y = 2 \\ y = -2 \end{cases}$

$\lim\limits_{x \to -\infty} \left(\frac{2x}{\sqrt{x^2+4}}\right)$

$\lim\limits_{x \to +\infty} \left(\frac{2x}{\frac{\sqrt{x^2+4}}{x}}\right)$

c) $f(x) = \frac{2x^2+1}{2x^2-3x}$

$\lim\limits_{x \to 0} \left(\frac{2x^2+1}{2x^2-3x}\right)$

$f(x) = \frac{2x^2+1}{2x^2-3x}$

$\lim\limits_{x \to 3/2} \left(\frac{2x^2+1}{2x^2-3x}\right)$

VERTICAL $\begin{cases} x = 0 \\ x = 3/2 \end{cases}$

$\lim\limits_{x \to +\infty} \left(\frac{2x^2+1}{2x^2-3x}\right) \to$ HORIZONTAL $\to \{ y = 1$

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d) $f(x) = \frac{x}{\sqrt{x^2-4}}$

$f(x) = \frac{x}{\sqrt{x^2-4}}$ $\begin{cases} \lim\limits_{x \to -2} \left(x / \sqrt{x^2-4} \right) \\ \lim\limits_{x \to 2} \left(x / \sqrt{x^2-4} \right) \end{cases}$ VERTICAL $\begin{cases} x = -2 \\ x = 2 \end{cases}$

$\lim\limits_{x \to \infty} \left(\frac{x}{\sqrt{x^2-4}} \right)$

$\lim\limits_{x \to -\infty} \left(x / \sqrt{x^2-4} \right)$ HORIZONTAL $\begin{cases} y = 1 \\ y = -1 \end{cases}$

e) $f(x) = \frac{x^3+1}{x^2+4}$

$\lim\limits_{x \to \infty} \left(\frac{10^3+1}{10^2+4} \right) \to \infty$ $\lim\limits_{x \to -\infty} \left(\frac{10^3+1}{10^2+4} \right) \to -\infty$

NÃO TEM ASSINTONAS

f) $f(x) = \frac{x}{\sqrt[4]{x^4+1}}$

$\lim\limits_{x \to \infty} \left(\frac{x}{\sqrt[4]{x^4+1}} \right) \to 1$ $\lim\limits_{x \to -\infty} \left(\frac{x}{\sqrt[4]{x^4+1}} \right) \to -1$

VERTICAIS $\begin{cases} y = 1 \\ y = -1 \end{cases}$

(7) a) $f(x) = \begin{cases} x^2 - 4 / x+2, & x \neq -2 \\ 1 & x = -2 \end{cases}$

$f(-2) = 1$

$\lim_{x \to -2} \frac{x^2-4}{x+2} \to \lim_{x \to -2} (x-2) = -4$

$f(-2) \neq \lim_{x \to -2} f(x) \to$ NÃO É CONTÍNUA

b) $f(x) = x^3 - 2x + 3$

$f(1) = 1^3 - 2 \cdot 1 + 3$

$\to f(x)$ é contínua se x for igual a 1

c) $f(x) = \frac{x}{x^2-1} \to \frac{-2}{-2^2-1} \to \frac{-2}{5}$

$\to$ NÃO É CONTÍNUA

⑧ $f(x) = \begin{cases} 1+ax, & \text{se } x \leq 0 \\ x^2+2a, & \text{se } x > 0 \end{cases}$

$$\lim_{x \to p} f(x) = f(p) \to \begin{cases} \lim\limits_{x \to p^+} f(x) = f(p) \\ \lim\limits_{x \to p^-} f(x) = f(p) \end{cases}$$

$$\lim_{x \to p^-} f(x) = f(p)$$

$$\lim_{x \to 0^-} 1+ax = f(0) \to f(0) = 1$$

$$\lim_{x \to p^+} f(x) = f(p)$$

$$\lim_{x \to 0^+} x^4+2a = f(0) \to 2a = 1 \to a = 1/2$$

$$\boxed{f(x) = \begin{cases} 1+x/2, & \text{se } x \leq 0 \\ x^4+1, & \text{se } x > 0 \end{cases}}$$

9 a) f é contínua no intervalo [0,1]
↳ função polinomial

$f(0) = -1$

$f(1) = 1$ O zero está entre $f(0)$ e $f(1) = f(x_0) = 0$

b) f é contínua no intervalo [1,0]

$f(1) = -1$ ↳ função polinomial

$f(2) = 9$

O zero está entre $f(1)$ e $f(2) = f(x_0) = 0$

c) f é contínua (função polinomial)

$f\left(\frac{1}{2}\right) = 1 + \frac{\sqrt{2}}{4} > 0$ $f\left(\frac{3}{2}\right) = 1 - \frac{3\sqrt{2}}{4} < 0$

Zero está entre $f(1/2)$ e $f(3/2) = f(x_0) = 0$