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Recuperação 1

① Matrícula: 201110005

$$S = 10 \quad \alpha = 4$$

$$T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$$

$$N(T) = [(4, 5, 4, 6)]$$

$$\mathcal{B} = \{(4, 5, 4, 6); (1, 0, 0, 0); (0, 1, 0, 0); (0, 0, 1, 0)\} \subset \mathbb{R}^4$$

$$\mathcal{B}' = \{(1, 0, 0, 0); (0, 1, 0, 0); (0, 0, 1, 0); (0, 0, 0, 1)\} \subset \mathbb{R}^4$$

$$\begin{cases} T(4, 5, 4, 6) = (0, 0, 0, 0) \\ T(1, 0, 0, 0) = (1, 0, 0, 0) \\ T(0, 1, 0, 0) = (0, 1, 0, 0) \\ T(0, 0, 1, 0) = (0, 0, 1, 0) \end{cases}$$

$$(x, y, z, t) = a(4, 5, 4, 6) + b(1, 0, 0, 0) + c(0, 1, 0, 0) + d(0, 0, 1, 0)$$

$$(x, y, z, t) = (4a + b, 5a + c, 4a + d, 6a)$$

$$\begin{cases} 4a + b = x \rightarrow b = x - 4a \\ 5a + c = y \rightarrow c = y - 5a \\ 4a + d = z \rightarrow d = z - 4a \\ 6a = t \rightarrow a = t/6 \end{cases}$$

$$\hookrightarrow (x, y, z, t) = \left(\frac{t}{6}\right)(4, 5, 4, 6) + \left(\frac{x - 4t}{6}\right)(1, 0, 0, 0) + \left(\frac{y - 5t}{6}\right)(0, 1, 0, 0)$$

$$\hookrightarrow + \left(\frac{z - 4t}{6}\right)(0, 0, 1, 0)$$

$$\begin{aligned} \rightarrow T(x, y, z, t) &= \left(\frac{t}{6}\right)(0, 0, 0, 0) + \left(\frac{x-2t}{3}\right)(1, 0, 0, 0) \\ &\quad + \left(\frac{y-5t}{6}\right)(0, 1, 0, 0) + \left(\frac{z-2t}{3}\right)(0, 0, 1, 0) \end{aligned}$$

$$\rightarrow T(x, y, z, t) = \left(\frac{t}{6}, \frac{x-2t}{3}, \frac{y-5t}{6}, \frac{z-2t}{3}\right)$$

$$T: \mathbb{R}^4 \rightarrow \mathbb{R}^4$$

$$T(x, y, z, t) = \left(\frac{t}{6}, \frac{x-2t}{3}, \frac{y-5t}{6}, \frac{z-2t}{3}\right)$$

\rightarrow Agora, achar uma base para a imagem

$$\rightarrow \{(a, b, c, d) \in \mathbb{R}^4, T(x, y, z, t) = (a, b, c, d)\}$$

$$\rightarrow \{(a, b, c, d) \in \mathbb{R}^4; \frac{t}{6} = a, \frac{x-2t}{3} = b, \frac{y-5t}{6} = c, \frac{z-2t}{3} = d\}$$

$$\rightarrow \{(a, b, c, d) \in \mathbb{R}^4; d = 0\}$$

$$\rightarrow \{a(1, 0, 0, 0) + b(0, 1, 0, 0) + c(0, 0, 1, 0); a, b, c \in \mathbb{R}\}$$

\rightarrow base para a imagem ~~$B = \{(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0)\}$~~

$$\rightarrow \{(1, 0, 0, 0); (0, 1, 0, 0); (0, 0, 1, 0)\}$$

$$\textcircled{2} \alpha = 4$$

$$(\alpha, \alpha, 2\alpha+1) = (4, 4, 9)$$

$$(2\alpha+1, \alpha, \alpha) = (9, 4, 4)$$

$$\hookrightarrow \beta(1, 0, 0, 0), (0, 1, 0, 0), (0, 0, 1, 0), (0, 0, 0, 1)$$

$$\begin{cases} T(1, 0, 0, 0) = (4, 4, 9) \\ T(0, 1, 0, 0) = (9, 4, 4) \\ T(0, 0, 1, 0) = (0, 0, 0) \\ T(0, 0, 0, 1) = (0, 0, 0) \end{cases}$$

$$T(x, y, z, t) = x \cdot T(1, 0, 0, 0) + y \cdot T(0, 1, 0, 0) + z \cdot T(0, 0, 1, 0) + t \cdot T(0, 0, 0, 1)$$

$$\hookrightarrow x = (4, 4, 9) + y(9, 4, 4) + z(0, 0, 0) + t(0, 0, 0)$$

$$\hookrightarrow T(x, y, z, t) = (4x + 9y), (4x + 4y), (9x + 4y)$$

Agora, achar uma base para o núcleo

$$\hookrightarrow \{(x, y, z, t) \in \mathbb{R}^4; T(x, y, z, t) = (0, 0, 0)\}$$

$$\hookrightarrow \{(x, y, z, t) \in \mathbb{R}^4; (x, y, 0) = (0, 0, 0)\}$$

$$\hookrightarrow \{(x, y, z, t) \in \mathbb{R}^4; x = 0, y = 0\}$$

$$\hookrightarrow \{(0, 0, z, t); z, t \in \mathbb{R}\}$$

$$\hookrightarrow \{(0, 0, 1, 0)z + (0, 0, 0, 1)t; z, t \in \mathbb{R}\}$$

$$\hookrightarrow \text{base} \rightarrow \beta' = \{(0, 0, 1, 0); (0, 0, 0, 1)\}$$

$$\textcircled{3} T: P_3(\mathbb{R}) \rightarrow P_3(\mathbb{R})$$

$$\alpha = 4$$

/ /

$$T(4x) = x$$

$$T = ?$$

$$T(4x^2 + 1) = x^2$$

$$T(2) = 1$$

$$T(x^3) = x^3$$

$$\hookrightarrow B = \{4x, 4x^2 + 1, 1, x^3\}$$

$$\begin{cases} T(4x) = x \\ T(4x^2 + 1) = x^2 \\ T(2) = 1 \\ T(x^3) = x^3 \end{cases}$$

$$\hookrightarrow Ax^3 + Bx^2 + Cx + D //$$

$$\hookrightarrow a(4x) + b(4x^2 + 1) + c(2) + d(x^3)$$

$$\hookrightarrow Ax^3 + Bx^2 + Cx + D = 4ax + 4bx^2 + b + 2c + dx^3$$

$$\begin{cases} d = A \end{cases}$$

$$\begin{cases} 4b = B \rightarrow b = B/4 \end{cases}$$

$$\begin{cases} 4a = C \rightarrow a = C/4 \end{cases}$$

$$\begin{cases} b + 2c = D \rightarrow c = \left(\frac{D - B}{4} \right) / 2 \rightarrow c = \frac{D - B}{8} \end{cases}$$

$$\hookrightarrow Ax^3 + Bx^2 + Cx + D = \left(4x \cdot \frac{C}{4} \right) + \left(4x^2 \cdot \frac{B}{4} \right) + \left(\frac{B}{4} \right) + 2 \left(\frac{D - B}{8} \right) + Ax^3$$

$$T(Ax^3 + Bx^2 + Cx + D) = \left(4x \cdot \frac{C}{4} \right) + \left(4x^2 \cdot \frac{B}{4} \right) + \frac{B}{4} + 2 \cdot \left(\frac{D - B}{8} \right) + Ax^3$$

$$(4) T: \mathbb{R}^3 \rightarrow \mathbb{R}^3 \quad \alpha = 4$$

$$T(x, y, z) = (x, x + 4y + z, z)$$

$$T^{-1} = ?$$

$$\hookrightarrow (x, y, z)$$

$$\hookrightarrow \text{Seja } \begin{cases} T(1, 0, 0) = (1, 1, 1) \\ T(0, 1, 0) = (1, 1, 0) \rightarrow \text{valor} \\ T(0, 0, 1) = (1, -1, 0) \rightarrow \text{aleatório} \end{cases}$$

$$\hookrightarrow (x, y, z) = (1, 0, 0) + 4(0, 1, 0) + (0, 0, 1)$$

$$\hookrightarrow T^{-1} \hookrightarrow \begin{cases} T^{-1}(1, 1, 1) = (1, 0, 0) \\ T^{-1}(1, 1, 0) = (0, 1, 0) \\ T^{-1}(1, -1, 0) = (0, 0, 1) \end{cases}$$

$$\hookrightarrow T^{-1}(x, y, z) = (1, 1, 1) + 4(1, 1, 0) + (1, -1, 0)$$

$$\hookrightarrow T^{-1}(x, y, z)$$