AVALIAÇÃO 2

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(1) $f(x,y)$ e $f(y,x)$ para $\ln\sqrt{x^2+y^2}$

$f(x,y) = \ln(x^2+y^2)^{1/2}$

- $\frac{\partial f}{\partial x} \to f(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot (x^2+y^2)^{1/2}$

$\to fx(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot \frac{1}{(x^2+y^2)^{1/2}}$

$$\boxed{fx(x,y) = x/(x^2+y^2)}$$

- $\frac{\partial f}{\partial y} \to fx(x,y) = \frac{x}{(x^2+y^2)}$

$\to fy(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot (x^2+y^2)^{1/2}$

$\to fy(x,y) = \frac{1}{(x^2+y^2)^{1/2}} \cdot y(x^2+y^2)^{1/2}$

$\to fy(x,y) = \frac{y}{(x^2+y^2)^{1/2+1/2}}$

$$\to \boxed{fy(x,y) = \frac{y}{(x^2+y^2)}}$$

2) a) $f(x,y) = x^2y^3 + x^3y^2$ $\begin{cases} x(t) = 1/t \\ y(t) = 1/t^2 \end{cases}$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial f}{\partial x} = 2xy^3 + 3x^2y^2$$

$$\frac{\partial f}{\partial y} = 3y^2x^2 + 2yx^3$$

$\hookrightarrow \frac{dx}{dt} = -1 \cdot t^{-1-1} = -t^{-2} = -1/t^2$

$\hookrightarrow \frac{dy}{dt} = -2 \cdot t^{-2-1} = -2t^{-3} = -2/t^3$

$\hookrightarrow \frac{dz}{dt} = (2xy^3 + 3x^2y^2)\left(\frac{-1}{t^2}\right) + 3y^2x^2 + 2yx^3 \cdot \left(\frac{-2}{t^3}\right)$

Agora, substituindo

$$\frac{dz}{dt} = \frac{2}{1} \cdot \left(\frac{1}{t}\right) \cdot \left(\frac{1}{t^2}\right) + 3\left(\frac{1}{t}\right)^2 \left(\frac{1}{t^2}\right) \cdot \left(\frac{-1}{t^2}\right) + 3\left(\frac{1}{t^2}\right)^2 \cdot \left(\frac{1}{t}\right)$$

$$+ 2\left(\frac{1}{t^2}\right)\left(\frac{1}{t}\right)^3 \cdot \left(\frac{-2}{t^3}\right)$$

continuando a 2...

$$\frac{dz}{dt} = \left(\frac{2}{t^2}\right) + \left(\frac{-3}{t^8}\right) + \frac{3}{t^6} + \left(\frac{-4}{t^8}\right)$$

$$\hookrightarrow \boxed{\frac{dz}{dt} = \left(\frac{-7}{t^8}\right) + \left(\frac{2}{t^7}\right) + \left(\frac{3}{t^6}\right)}$$

b) $f(x,y) = e^{x+y}$ $\rightarrow \begin{cases} x(t) = t^2 \\ y(t) = 2t^3 - 1 \end{cases}$

$$\frac{dz}{dt} = \frac{\partial f}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dt}$$

$$\hookrightarrow \frac{dz}{dt} = e^{x+y} \cdot 2t + e^{x+y} \cdot 6t^2$$

$$\hookrightarrow \frac{dz}{dt} = e^{x+y} \cdot (6t^2 + 2t)$$

- Agora, substituindo

$$\hookrightarrow \boxed{\frac{dz}{dt} = e^{t^2 + 2t^3 - 1} \cdot (6t^2 + 2t)}$$

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(3) $f(x,y) = \ln(e^{xy} - x^2y^3)$

a) $f_x(10, 15)$

$$\frac{\partial f}{\partial x} = \frac{1}{e^{xy} - x^2y^3} \cdot e^{xy} \cdot y - 2xy^3$$

$$\frac{\partial f}{\partial x} = \frac{e^{xy} \cdot y - 2xy^3}{e^{xy} - x^2y^3} \rightarrow \frac{y - 2x}{-x^2}$$

↳ Substituindo

↳ $\frac{\partial f}{\partial x}(10,15) \rightarrow \frac{15 - 2 \cdot 10}{-10^2} = \frac{-5}{100} = \boxed{-0,05}$

b) $f(11,15)$ e $f(10,15)$

$f(11,15) = \ln(e^{11 \cdot 15} - 11^2 \cdot 15^3)$

↳ $\ln(e^{165} - 408.375) = \boxed{165}$

$f(10,15) = \ln(e^{10 \cdot 15} - 10^2 \cdot 15^3)$

↳ $\ln(e^{150} - 337.500) = \boxed{150}$

c) $f_y(10,15)$

$$\frac{\partial f}{\partial y} = \frac{1}{e^{xy} - x^2y^3} \cdot e^{xy} \cdot x - 3x^2y^2 \Rightarrow \frac{x - 3}{-y}$$

↳ $\frac{\partial f}{\partial y}(10,15) = \frac{10 - 3}{-15} \cong \boxed{-0,47}$

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continuando a 3...

d) $f(10,16)$ e $f(10,15)$

$\hookrightarrow \ln(e^{10 \cdot 16} - 10^2 \cdot 16^3)$

$\hookrightarrow \ln(e^{160} - 409600) \rightarrow \boxed{160}$

$f(10,15) = \ln(e^{10 \cdot 15} - 10^2 \cdot 15^3)$

$\hookrightarrow \ln(e^{150} - 337500) \rightarrow \boxed{150}$

4) $fx = \frac{\partial f}{\partial x}$, $fy = \frac{\partial f}{\partial y}$ e $f(x,y) = \operatorname{sen}(xyz) + \frac{x^2 y^3}{z}$

a) $\frac{\partial f}{\partial x} = yz \cdot \cos(xyz) + 2x \cdot y^3/z \rightarrow 3y^2/z$

$2 \rightarrow \frac{\partial f}{\partial y} = 1 \cdot z \cdot -xz \cdot \operatorname{sen}(xyz) + 2x \cdot 3y^2/z$

$1 \rightarrow \frac{\partial f}{\partial x} = -z \cdot zy \cdot \cos(x,y,z) + 2 \cdot 3y^2/y$

$\hookrightarrow f121 = -z \cdot y \cdot \cos(x,y,z) + 6y^2$

continuando a 4...

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b) f_{221}

$2 \to \frac{\partial f}{\partial y} = xy \cdot \cos(x,y,z) + x^2 \cdot \frac{2y^2}{z}$

$2 \to \frac{\partial f}{\partial y} = xz - xz\,\text{sen}(x,y,z) + x^2 \cdot \frac{6y}{z}$

$1 \to \frac{\partial f}{\partial x} = -z^2 yz \cos(x,y,z) + 2x \cdot \frac{6y}{z}$

$$\boxed{f_{221} = -z^2 y \cos(x,yz) + 2x \cdot 6y}$$

c) f_{332}

$3 \to \frac{\partial f}{\partial z} = xy - x \cdot y \cdot \text{sen}(xyz) + x^2 \cdot \left(\frac{-y^3}{z^2}\right) = \frac{2y^3}{z^3}$

$3 \to \frac{\partial f}{\partial z} = xy - xy \cdot \text{sen}(xyz) + x^2 \left(\frac{2y^3}{3^3}\right)$

$2 \to \frac{\partial f}{\partial y} = -x^2 xz \cdot \cos(xyz) + x^2 \cdot \left(\frac{6y^2}{z^3}\right)$

$$2 \to \boxed{f_{332} = -x^3 \cdot z \cdot \cos(xyz) + x^2 \cdot \left(\frac{6y^2}{z^3}\right)}$$

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$$⑤\ f(x,y) = x^3 e^x + 10y$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x}(x^3 e^x) + \frac{\partial}{\partial x}(10y) = \boxed{3x^2 e^x + e^x x^3}$$

$$\frac{\partial f}{\partial y} = x^3 e^x + 10y = \boxed{10}$$

$$f(x,y) = 2y^2 \ln(x)$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} 2y^2 \ln(x) \rightarrow \boxed{\frac{2y^2}{x}}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} 2y^2 \ln(x) \rightarrow \boxed{4y . \ln(x)}$$

$$f(x,y) = 3y^2 \cos(x)$$

$$\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} 3y^2 \cos(x) \rightarrow \boxed{3y^2 . -\operatorname{sen}(x)}$$

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} 3y^2 \cos(x) \rightarrow \boxed{6y . \cos(x)}$$

$$f(x,y) = 4y^2 e^y + 6x^2 \rightarrow \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} = 4y^2 e^y + 6x$$

$$\downarrow$$

$$\frac{\partial f}{\partial y} = \boxed{8y . e^y} \qquad \boxed{12x}$$

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⑥ Achar $\frac{\partial z}{\partial x}$ e $\frac{\partial z}{\partial y}$ / /

$F(x,y) = 0 \rightarrow \frac{\partial F}{\partial x}(x,y,z) \cdot \frac{\partial y}{\partial x} + \frac{\partial F}{\partial y}(x,y,z)$

$\hookrightarrow \frac{\partial y}{\partial x} + \frac{\partial F}{\partial z}(x,y,z) \cdot \frac{\partial z}{\partial x}(x,y) = 0$

$\hookrightarrow \frac{\partial x}{\partial x} = 1$ e $\frac{\partial y}{\partial x} = 0$

$\rightarrow$ Assim $\rightarrow \frac{\partial F}{\partial x}(x,y,z) + \frac{\partial F}{\partial z}(x,y,z) \cdot \frac{\partial z}{\partial x}(x,y) = 0$

$\frac{\partial z}{\partial x}(x,y) = -\left(\frac{\partial F/\partial x\,(x,y,z)}{\partial F/\partial z\,(x,y,z)}\right) \rightarrow \frac{\partial z}{\partial y}(x,y) = -\left(\frac{\frac{\partial F}{\partial y}(x,y,z)}{\frac{\partial F}{\partial z}(x,y,z)}\right)$

• Substituindo:

$$\frac{\partial z}{\partial x}(x,y) = \left(\frac{3x^2 - y + 4z}{4x}\right) = \boxed{\frac{-3x^2 + y - 4z}{4x}}$$

$$\frac{\partial z}{\partial y}(x,y) = -\left(\frac{-x}{4x}\right) \rightarrow \boxed{\frac{1}{4}}$$

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7) Pontos de máximo e mínimo da função $f(x,y) = 3xy^2 + x^3 - 3x$

* Vetor gradiente: $\nabla f(x,y) = (f_x, f_y)$

$f_x = \frac{\partial}{\partial x}(3xy^2) + \frac{\partial}{\partial x}(x^3) - \frac{\partial}{\partial x}(3x) = 3y^2 + 3x^2 - 3$

$f_y = \frac{\partial}{\partial y}(3xy^2) + \frac{\partial}{\partial y}(x^3) - \frac{\partial}{\partial y}(3x) = 6xy$

$\hookrightarrow$ Gradiente $\rightarrow (3y^2 + 3x^2 - 3 \ , \ 6xy)$

$\hookrightarrow \begin{cases} 3y^2 + 3x^2 - 3 = 0 \\ 6xy = 0 \end{cases}$

$\hookrightarrow$ ou x ou y é zero

$\hookrightarrow 3x^2 = -3y^2 + 3 \rightarrow x = \sqrt{\frac{-3y^2 + 3}{3}}$

$\hookrightarrow$ Se o $y = 0 \rightarrow x = \sqrt{\frac{-3.0 + 3}{3}} \rightarrow x = \pm 1$

$\hookrightarrow$ Ponto crítico $\rightarrow P_1(1,0)$ e $P_2(-1,0)$

$\rightarrow$ Ponto de máximo para $(x,y) = (-1,0)$

$\rightarrow$ Ponto de mínimo para $(x,y) = (1,0)$

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