$$L_1(w,q|u) = \frac{P(w)P(q)S_1(u|w,q)}{\sum_{q'}\int_{w'} P(w')P(q')S_1(u|w',q')} \tag{1}$$

$$= \frac{P(w)P(q)S_1(u|w,q)}{K} \tag{2}$$

In the equation above, we define $K = \sum_{q'}\int_{w'} P(w')P(q')S_1(u|w',q')$.
To compute the marginal distribution over QUDs:

$$L_1(q|u) = \int_w L_1(w,q|u) \tag{3}$$

$$= \frac{1}{K}P(q)\int_w P(w)S_1(u|w,q) \tag{4}$$

$$= \frac{1}{K}P(q)\int_w P(w)S_1(u|w_q,q) \tag{5}$$

$$= \frac{1}{K}P(q)\int_w P(w_q,w^\perp)S_1(u|w_q,q) \tag{6}$$

$$= \frac{1}{K}P(q)\int_w P(w_q)P(w^\perp)S_1(u|w_q,q) \tag{7}$$

$$= \frac{1}{K}P(q)\int_{w^\perp\in Q^\perp} P(w^\perp)\int_{w_q\in Q} P(w_q)S_1(u|w_q,q) \tag{8}$$

$$= \frac{1}{K}P(q)\int_{w_q\in Q} P(w_q)S_1(u|w_q,q) \tag{9}$$

Here $w, q \in \mathbb{R}^n$, and w_q is the projection of w onto the vector q. In addition, Q is the subspace of $\mathbb{R}^n$ spanned by the vector q, and $Q^\perp$ is the orthogonal complement of Q. The vector $w^\perp$ is the projection of vector w onto the subspace $Q^\perp$.

The final equation is a one-dimensional integral, and can be computed using a discrete approximation to the integral. The constant K can be found from the constraint $\sum_q L_1(q|u) = 1$.