

UPF Team Notebook: The Binary Three

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1 Basic

1.1 Auxiliar Comparer

```
// returns true if the first argument goes before the second argument
// in the strict weak ordering it defines, and false otherwise.
struct classcomp {
    bool operator() (const int& lhs, const int& rhs) const
    {return lhs > rhs;}
};

int main() {
    set<int> set1;
    set<int, classcomp> set2;
    set1.insert(26); set1.insert(93); set1.insert(42); // 26, 42, 93
    set2.insert(26); set2.insert(93); set2.insert(42); // 93, 42, 26

    for (auto it=set1.begin(); it!=set1.end(); ++it) cout << *it << " ";
    cout << "\n";
    for (auto it=set2.begin(); it!=set2.end(); ++it) cout << *it << " ";
}
```

1.2 Libraries

algorithm	heap, sort	map	map<S, T>
cfloat	DBL_MAX	queue	priority_queue
cmath	pow, sqrt	set	set<S>
cstdlib	abs, rand	sstream	istringstream, ostringstream
iostream	cin, cout	string	string
io manip	setprecision	utility	pair<S, T>
list	list<T>	vector	vector<T>

1.3 Macros

```
#define X first
#define Y second
#define LI long long
#define MP make_pair
#define PB push_back
```

```
#define SZ size()
#define SQ(a) ((a)*(a))
#define MAX(a,b) ((a)>(b)?(a):(b))
#define MIN(a,b) ((a)<(b)?(a):(b))
#define FOR(i,x,y) for(int i=(int)x; i<(int)y; i++)
#define RFOR(i,x,y) for(int i=(int)x; i>(int)y; i--)
#define SORT(a) sort(a.begin(), a.end())
#define RSORT(a) sort(a.rbegin(), a.rend())
#define IN(a,pos,c) insert(a.begin()+pos,1,c)
#define DEL(a,pos,cant) erase(a.begin()+pos,cant)
```

1.4 Permutations

```
int N = 3;
int a[] = {1,2,3};
do {
    for (int i = 0; i < N; ++i) cout << a[i] << " ";
    cout << "\n";
}
while (next_permutation(a, a + N));
```

1.5 Precision cout

```
cout.setf(ios::fixed);
cout.precision(8);
```

2 Data Structures

2.1 Big Numbers

```
#include <cassert>
#define BASE 1000000000

struct big {
    vector<int> V;
    big(): V(1, 0) {}
    big(int n): V(1, n) {} // supone n < 1000000000 !!!
    big(const big &b): V(b.V) {}
```

```

bool operator==(const big &b) const { return V==b.V; }
int &operator[](int i) { return V[i]; }
int operator[](int i) const { return V[i]; }
int size() const { return V.SZ; }
void resize(int i) { V.resize(i); }

bool operator<(const big &b) const {
    for (int i = b.SZ-1; SZ == b.SZ && i >= 0; i--)
        if (V[i] == b[i]) continue;
        else return (V[i] < b[i]);
    return (SZ < b.SZ);
}

void add_digit(int l) {
    if (l > 0) V.PB(l);
}

};

inline big suma(const big &a, const big &b, int k) {
    LI l = 0;
    int size = MAX(a.SZ, b.SZ+k);
    big c; c.resize(size);
    for (int i = 0; i < size; ++i) {
        l += i < a.SZ ? a[i] : 0;
        l += (k <= i && i < k + b.SZ) ? b[i-k] : 0;
        c[i] = l%BASE;
        l /= BASE;
    }
    c.add_digit(int(l));
    return c;
}

inline big operator+(const big &a, const big &b) {
    return suma(a, b, 0);
}

inline big operator+(const big &a, int b) {return a+big(b);}
inline big operator+(int b, const big &a) {return a+big(b);}

inline big operator-(const big &a, const big &b) {
    assert(b < a || a == b);
    LI l = 0, m = 0;
    big c; c.resize(a.SZ);
    for (int i = 0; i < a.SZ; ++i) {
        l += a[i];

```

```

        l -= i < b.SZ ? b[i] + m : m;
        if (l < 0) { l += BASE; m = 1; }
        else m = 0;
        c[i] = l%BASE;
        l /= BASE;
    }
    if (c[c.SZ-1] == 0 && c.SZ > 1) c.resize(c.SZ-1);
    return c;
}

inline big operator-(const big &a, int b) {return a-big(b);}

inline big operator*(const big &a, int b) {
    if (b == 0) return big(0);
    big c; c.resize(a.SZ);
    LI l = 0;
    for (int i = 0; i < a.SZ; ++i) {
        l += (LI)b*a[i];
        c[i] = l%BASE;
        l /= BASE;
    }
    c.add_digit(int(l));
    return c;
}

inline big operator*(int b, const big &a) {return a*b;}
inline big operator*(const big &a, const big &b) {
    big res;
    for (int i = 0; i < b.SZ; ++i)
        res = suma(res, a*b[i], i);
    return res;
}

inline void divmod(const big &a, int b, big &div, int &mod) {
    div.resize(a.SZ);
    LI l = 0;
    for (int i = a.SZ-1; i >= 0; --i) {
        l *= BASE;
        l += a[i];
        div[i] = l/b;
        l %= b;
    }
    if (div[div.SZ-1] == 0 && div.SZ > 1) div.resize(div.SZ-1);
    mod=int(l);
}

inline big operator/(const big &a, int b) {

```

```

    big div; int mod;
    divmod(a, b, div, mod);
    return div;
}

inline int operator%(const big &a, int b) {
    big div; int mod;
    divmod(a, b, div, mod);
    return mod;
}

inline istream &operator>>(istream &is, big &b) {
    string s;
    if (is >> s) {
        b.resize((s.SZ - 1)/9 + 1);
        for (int n = s.SZ, k = 0; n > 0; n -= 9, k++) {
            b[k] = 0;
            for (int i = MAX(n-9, 0); i < n; i++)
                b[k] = 10*b[k] + s[i]-'0';
        }
    }
    return is;
}

inline ostream &operator<<(ostream &os, const big &b) {
    os << b[b.SZ - 1];
    for (int k = b.SZ-2; k >= 0; k--)
        os << setw(9) << setfill('0') << b[k];
    return os;
}

void p10519() { //10519: calcula 2+2+4+6+8+10+...+2*n
    for (big n; cin >> n; ) {
        if (n == big(0)) cout << 1 << endl;
        else cout << 2 + n*(n-1) << endl;
    }
}

int main(){
    p10519();
}

```

2.2 Binary Indexed Tree

```

/* Binary indexed tree. Supports cumulative sum queries in O(log n) */
#define N (1<<18)
LL bit[N];

void add(LL* bit,int x,int val) {
    for(; x<N; x+=x&-x)
        bit[x]+=val;
}

LL query(LL* bit,int x) {
    LL res=0;
    for(;x;x-=x&-x)
        res+=bit[x];
    return res;
}

```

2.3 Square Root Trick

```

/* Partitions an array in sqrt(n) blocks of size sqrt(n) to support
 * O(sqrt(n)) range sum queries, O(sqrt(n)) range sum updates, and O(1)
 * point updates */
void update(LL *S, LL *A, int i, int k, int x) {
    S[i/k] = S[i/k] - A[i] + x;
    A[i] = x;
}

LL query(LL *S, LL *A, int lo, int hi, int k) {
    int sum=0, i=lo;
    while((i+1)%k != 0 && i <= hi)
        sum += A[i++];
    while(i+k <= hi)
        sum += S[i/k], i += k;
    while(i <= hi)
        sum += A[i++];
    return sum;
}

```

3 Dynamic Programming

3.1 Change Making Problem

```
int N = 8; // numero de monedas
int m[] = {1,2,5,10,20,50,100,200}; // monedas
int A[100001]; // vector de resultados

int main() {
    int C; // monto C <= 100000
    cin >> C;
    A[0] = 0;
    for (int i = 1; i <= C; i++) {
        A[i] = 1000000;
        for (int j = 0; j < N && m[j] <= i; j++)
            A[i] = MIN(A[i], A[i-m[j]] + 1);
    }
    cout << A[C] << endl;
}
```

3.2 Cocke-Younger-Kasami (Context-free parsing)

```
// O(n^3 |G|) worst case, bigger constant factor
int rules[3][MAX_RULES], nrules;

struct {
    char t;
    int nt;
} nonterminals[MAX_RULES];
int n_nt;

int len;
bool parsed[N_CHARS][MAX_LEN][MAX_LEN];

bool mark(int c, int e, int d) {
    if(parsed[c][e][d])
        return false;
    if(c == ROOT_NONTERMINAL && e == 0 && d == len-1)
        return true;
    parsed[c][e][d] = true;
    int i;
    for(i=0; i<nrules; i++) {
```

```
        if (c == rules[1][i]) {
            int k, j = rules[2][i], d_1 = d+1;
            for(k = d_1; k < len; k++)
                if(parsed[j][d_1][k] && mark(rules[0][i], e, k))
                    return true;
        }
        if (c == rules[2][i]) {
            int k, j = rules[1][i], e_1 = e-1;
            for(k = e_1; k >= 0; k--)
                if(parsed[j][k][e_1] && mark(rules[0][i], k, d))
                    return true;
        }
    }
    return false;
}

scanf("%s", str);
// scan rules
// rules[0][i] := rules[1][i] rules[2][i]

len = strlen(str);
memset(parsed, 0, sizeof(parsed));
int j;
for(j=0; j<n_nonterminals; j++) {
    int i;
    for(i=0; i<len; i++) {
        if(str[i] == nonterminals[j].t && mark(nonterminals[j].nt, i, i)) {
            putchar('1');
            goto finish;
        }
    }
}
putchar('0');
finish: putchar('\n');

// O(n^3 |G|) worst case, smaller constant factor. Can parse n=1000 with
// about
// 20 rules in less than 5s
for(i=0; i<len; i++) {
    int a = str[i] - 'a';
    int b;
    for(b=0; b<N_CHARS; b++)
        parsed[b][i][i] = nonterminals[b][a];
    int j;
    for(j=i-1; j >= 0; j--) {
```

```

int l;
for(l=0; l<N_CHARS; l++)
    parsed[l][i][j] = false;
int k;
for(k=j; k<i; k++) {
    int r;
    for(r=0; r<nrules; r++) {
        if(parsed[rules[1][r]][k][j] &&
            parsed[rules[2][r]][i][k+1])
            parsed[rules[0][r]][i][j] = true;
    }
}
}

if(parsed['S'-'A'][len-1][0])
    putchar('1');
else
    putchar('0');
putchar('\n');

```

3.3 Edit Distance (Damerau-Levenshtein)

$m[0][_] = m[_][0] = 0$. $m[i][j]$ is the edit distance of $s1[0..i]$ and $s2[0..j]$. $m[i][j] = \min(\text{operations})$, where operations are:

- Add character, subtract character $m[i-1][j] + 1$, $m[i][j-1] + 1$ respectively
- Change character $m[i-1][j-1] + (\text{if } s1[i] == s2[j] \text{ then } 1 \text{ else } 0)$
- Swap last two characters $m[i-2][j-2] + (\text{if } s1[i-1] == s2[j] \text{ \&\& } s1[i] == s2[j-1] \text{ then } 1)$.

3.4 Knapsack Problem

```

int N = 8; // numero de objetos N <= 1000
int v[] = {1,6,7,1,8,3,7,5}; // valor de objetos
int p[] = {5,3,7,1,8,2,7,3}; // peso de objetos
int A[1001][1001]; // matriz de resultados

int main() {
    int C = 7; // capacidad C <= 1000

```

```

for (int j = 0; j <= C; j++)
    A[0][j] = 0;
for (int i = 1; i <= N; i++) {
    A[i][0] = 0;
    for (int j = 1; j <= C; j++) {
        A[i][j] = A[i-1][j];
        if (p[i-1] <= j) {
            int r = A[i-1][j-p[i-1]] + v[i-1];
            A[i][j] = MAX(A[i][j], r);
        }
    }
}
cout << A[N][C] << endl; // output: 12
}

```

3.5 Longest Common Subsequence

```

table[_][0] = 0;
for(int i=1; i<n+1; i++) {
    table[i][0] = 0;
    for(int j=1; j<n+1; j++) {
        if(x[i-1] == y[j-1])
            table[i][j] = table[i-1][j-1] + 1;
        else
            table[i][j] = max(table[i-1][j], table[i][j-1]);
    }
}

```

3.6 Longest Increasing Subsequence

```

// O(n^2)

for(int i=0; i<N; i++) {
    inc[i] = 1;
    for(int j=0; j<N; j++) {
        if(seq[j] < seq[i]) {
            int v = inc[j] + 1;
            if(v > inc[i])
                inc[i] = v;
        }
    }
}

```

```

    }
    if(inc[i] > max)
        max = inc[i];
}

// O(n log n)

ind[0] = 0;
ind_sz = 1;
while(scanf("%d", &seq[seq_sz++]) == 1) {
    /* Add next element if it's bigger than the current last */
    int i = seq_sz-1;
    if (seq[ind[ind_sz-1]] < seq[i]) {
        predecessor[i] = ind[ind_sz-1];
        ind[ind_sz++] = i;
        continue;
    }
    /* bsearch to find element immediately bigger */
    int u = 0, v = ind_sz-1;
    while(u < v) {
        int c = (u + v) / 2;
        if (seq[ind[c]] < seq[i])
            u = c+1;
        else
            v = c;
    }
    /* Update b if new value is smaller then previously referenced value */
    if (seq[i] < seq[ind[u]]) {
        if (u > 0)
            predecessor[i] = ind[u-1];
        ind[u] = i;
    }
}

```

3.7 Maximum Subarray Sum (Kadane)

/* We show the 2D version here. the 1D version is the code block separated by a newline. You can keep track of where the sequence starts and ends by messing with the max_here and max assignments respectively. Use > max_here to keep longer subsequences, >= max_here to keep shorter ones. Take into account circular arrays by adding the sum of all elements and the max of the array with sign changed. */

```

max = mat[0][0];
for(i=0; i<N; i++) {
    memset(aux, 0, sizeof(aux));
    for(k=i; k<N; k++) {
        for(j=0; j<N; j++)
            aux[j] += mat[k][j];

        max_here = aux[0];
        if(max_here > max)
            max = max_here;
        for(j=1; j<N; j++) {
            max_here += aux[j];
            if(aux[j] > max_here)
                max_here = aux[j];
            if(max_here > max)
                max = max_here;
        }
    }
}

```

3.8 Traveling Salesman Problem

// TSP in $O(n^2 * 2^n)$. Subset is bitmask, Cost is cost.
// tsp_memoize[subset][j] stores the shortest path starting at node -1, including the nodes in the subset and finishing at node j.
// This is for the TSP with N+1 nodes. We pick the first one arbitrarily.
Cost distances[N][N], tsp_memoize[1 << (N+1)][N];
const Cost sentinel=-0x3f3f3f3f;
#define TSP(subset, i) (tsp_memoize[subset][i] == sentinel ? \
tsp(subset, i) : \
tsp_memoize[subset][i])

```

Cost tsp(const Subset subset, const int i) {
    Subset without = subset ^ (1 << i);
    Cost minimum = numeric_limits<Cost>::max();
    for(int j=0; j<n_nodes; j++) {
        if(j==i || (without & (1 << j)) == 0)
            continue;
        Cost v = TSP(without, j);
        v += distances[i][j];
        if(v < minimum)
            minimum = v;
    }
}

```

```

        return tsp_memoize[subset][i] = minimum;
    }

    /* fill tsp_memoize with sentinel */
    tsp_memoize[1<<i][i] = distance /* from -1 to i */
    for(int i=0; i<n_nodes; i++)
        tsp(0xffff >> (16 - n_nodes), i) /* + distance from i to -1 */;

```

4 Geometry

4.1 Convex Hull

```

typedef int T;    // posiblemente cambiar a double
typedef pair<T,T> P;
T xp(P p, P q, P r) {
    return (q.X-p.X)*(r.Y-p.Y) - (r.X-p.X)*(q.Y-p.Y);
}

struct Vect {
    P p, q; T dist;
    Vect(P &a, P &b) {
        p = a; q = b;
        dist = SQ(a.X - b.X) + SQ(a.Y - b.Y);
    }
    bool operator<(const Vect &v) const {
        T t = xp(p, q, v.p);
        return t < 0 || t == 0 && dist < v.dist;
    }
};

vector<P> convexhull(vector<P> v) { // v.SZ >= 2
    sort(v.begin(), v.end());
    vector<Vect> u;
    for (int i = 1; i < (int)v.SZ; i++)
        u.PB(Vect(v[i], v[0]));
    sort(u.begin(), u.end());
    vector<P> w(v.SZ, v[0]);
    int j = 1; w[1] = u[0].p;
    for (int i = 1; i < (int)u.SZ; i++) {
        T t = xp(w[j-1], w[j], u[i].p);
        for (j--; t < 0 && j > 0; j--)
            t = xp(w[j-1], w[j], u[i].p);
        j += t > 0 ? 2 : 1;
    }
}

```

```

        w[j] = u[i].p;
    }
    w.resize(j+1);
    return w;
}

int main() {
    vector<P> v;
    v.PB(MP(0, 1));
    v.PB(MP(1, 2));
    v.PB(MP(3, 2));
    v.PB(MP(2, 1));
    v.PB(MP(3, 1));
    v.PB(MP(6, 3));
    v.PB(MP(7, 0));
    vector<P> w = convexhull(v);
} // resultado: (0,1) (7,0) (6,3) (1,2)

```

4.2 Line Intersection

Intersection between two lines: here is the system solved. Swap all x s and y s to avoid dividing by zero if $p_x = 0$.

$$s = \frac{P_y - Q_y + \frac{p_y}{p_x}(Q_x - P_x)}{q_y - \frac{p_y}{p_x}q_x}$$

$$x = Q_x + q_x s; \quad y = Q_y + q_y s$$

$$t = \frac{Q_x - P_x + q_x s}{p_x}$$

4.3 Routines

```

double INF = 1e100;
double EPS = 1e-12;

struct PT {
    double x, y;
    PT() {}
    PT(double x, double y) : x(x), y(y) {}
    PT(const PT &p) : x(p.x), y(p.y) {}
    PT operator + (const PT &p) const { return PT(x+p.x, y+p.y); }
    PT operator - (const PT &p) const { return PT(x-p.x, y-p.y); }
}

```



```

PT operator * (double c) const { return PT(x*c, y*c ); }
PT operator / (double c) const { return PT(x/c, y/c ); }
};

double dot(PT p, PT q) { return p.x*q.x+p.y*q.y; }
double dist2(PT p, PT q) { return dot(p-q,p-q); }
double cross(PT p, PT q) { return p.x*q.y-p.y*q.x; }
ostream &operator<<(ostream &os, const PT &p) {
    os << "(" << p.x << "," << p.y << ")";
}

// rotate a point CCW or CW around the origin
PT RotateCCW90(PT p) { return PT(-p.y,p.x); }
PT RotateCW90(PT p) { return PT(p.y,-p.x); }
PT RotateCCW(PT p, double t) {
    return PT(p.x*cos(t)-p.y*sin(t), p.x*sin(t)+p.y*cos(t));
}

// project point c onto line through a and b
// assuming a != b
PT ProjectPointLine(PT a, PT b, PT c) {
    return a + (b-a)*dot(c-a, b-a)/dot(b-a, b-a);
}

// project point c onto line segment through a and b
PT ProjectPointSegment(PT a, PT b, PT c) {
    double r = dot(b-a,b-a);
    if (fabs(r) < EPS) return a;
    r = dot(c-a, b-a)/r;
    if (r < 0) return a;
    if (r > 1) return b;
    return a + (b-a)*r;
}

// compute distance from c to segment between a and b
double DistancePointSegment(PT a, PT b, PT c) {
    return sqrt(dist2(c, ProjectPointSegment(a, b, c)));
}

// compute distance between point (x,y,z) and plane ax+by+cz=d
double DistancePointPlane(double x, double y, double z,
                           double a, double b, double c, double d)
{
    return fabs(a*x+b*y+c*z-d)/sqrt(a*a+b*b+c*c);
}

```

```

// determine if lines from a to b and c to d are parallel or collinear
bool LinesParallel(PT a, PT b, PT c, PT d) {
    return fabs(cross(b-a, c-d)) < EPS;
}

bool LinesCollinear(PT a, PT b, PT c, PT d) {
    return LinesParallel(a, b, c, d)
        && fabs(cross(a-b, a-c)) < EPS
        && fabs(cross(c-d, c-a)) < EPS;
}

// determine if line segment from a to b intersects with
// line segment from c to d
bool SegmentsIntersect(PT a, PT b, PT c, PT d) {
    if (LinesCollinear(a, b, c, d)) {
        if (dist2(a, c) < EPS || dist2(a, d) < EPS ||
            dist2(b, c) < EPS || dist2(b, d) < EPS) return true;
        if (dot(c-a, c-b) > 0 && dot(d-a, d-b) > 0 && dot(c-b, d-b) > 0)
            return false;
        return true;
    }
    if (cross(d-a, b-a) * cross(c-a, b-a) > 0) return false;
    if (cross(a-c, d-c) * cross(b-c, d-c) > 0) return false;
    return true;
}

// compute intersection of line passing through a and b
// with line passing through c and d, assuming that unique
// intersection exists; for segment intersection, check if
// segments intersect first
PT ComputeLineIntersection(PT a, PT b, PT c, PT d) {
    b=b-a; d=c-d; c=c-a;
    assert(dot(b, b) > EPS && dot(d, d) > EPS);
    return a + b*cross(c, d)/cross(b, d);
}

// compute center of circle given three points
PT ComputeCircleCenter(PT a, PT b, PT c) {
    b=(a+b)/2;
    c=(a+c)/2;
    return ComputeLineIntersection(b, b+RotateCW90(a-b), c,
                                    c+RotateCW90(a-c));
}

```

```

// determine if point is in a possibly non-convex polygon (by William
// Randolph Franklin); returns 1 for strictly interior points, 0 for
// strictly exterior points, and 0 or 1 for the remaining points.
// Note that it is possible to convert this into an *exact* test using
// integer arithmetic by taking care of the division appropriately
// (making sure to deal with signs properly) and then by writing exact
// tests for checking point on polygon boundary
bool PointInPolygon(const vector<PT> &p, PT q) {
    bool c = 0;
    for (int i = 0; i < p.size(); i++){
        int j = (i+1)%p.size();
        if ((p[i].y <= q.y && q.y < p[j].y ||
            p[j].y <= q.y && q.y < p[i].y) &&
            q.x < p[i].x + (p[j].x - p[i].x) * (q.y - p[i].y) / (p[j].y -
                p[i].y))
            c = !c;
    }
    return c;
}

// determine if point is on the boundary of a polygon
bool PointOnPolygon(const vector<PT> &p, PT q) {
    for (int i = 0; i < p.size(); i++)
        if (dist2(ProjectPointSegment(p[i], p[(i+1)%p.size()], q), q) < EPS)
            return true;
    return false;
}

// compute intersection of line through points a and b with
// circle centered at c with radius r > 0
vector<PT> CircleLineIntersection(PT a, PT b, PT c, double r) {
    vector<PT> ret;
    b = b-a;
    a = a-c;
    double A = dot(b, b);
    double B = dot(a, b);
    double C = dot(a, a) - r*r;
    double D = B*B - A*C;
    if (D < -EPS) return ret;
    ret.push_back(c+a+b*(-B+sqrt(D+EPS))/A);
    if (D > EPS)
        ret.push_back(c+a+b*(-B-sqrt(D))/A);
    return ret;
}

```

```

// compute intersection of circle centered at a with radius r
// with circle centered at b with radius R
vector<PT> CircleCircleIntersection(PT a, PT b, double r, double R) {
    vector<PT> ret;
    double d = sqrt(dist2(a, b));
    if (d > r+R || d+min(r, R) < max(r, R)) return ret;
    double x = (d*d-R*R+r*r)/(2*d);
    double y = sqrt(r*r-x*x);
    PT v = (b-a)/d;
    ret.push_back(a+v*x + RotateCCW90(v)*y);
    if (y > 0)
        ret.push_back(a+v*x - RotateCCW90(v)*y);
    return ret;
}

// This code computes the area or centroid of a (possibly nonconvex)
// polygon, assuming that the coordinates are listed in a clockwise or
// counterclockwise fashion. Note that the centroid is often known as
// the "center of gravity" or "center of mass".
double ComputeSignedArea(const vector<PT> &p) {
    double area = 0;
    for(int i = 0; i < p.size(); i++) {
        int j = (i+1) % p.size();
        area += p[i].x*p[j].y - p[j].x*p[i].y;
    }
    return area / 2.0;
}

double ComputeArea(const vector<PT> &p) {
    return fabs(ComputeSignedArea(p));
}

PT ComputeCentroid(const vector<PT> &p) {
    PT c(0,0);
    double scale = 6.0 * ComputeSignedArea(p);
    for (int i = 0; i < p.size(); i++){
        int j = (i+1) % p.size();
        c = c + (p[i]+p[j])*(p[i].x*p[j].y - p[j].x*p[i].y);
    }
    return c / scale;
}

```

5 Graphs

5.1 Bellman-Ford (Shortest Path with Negative Weights)

```
// Complexity: E * V - Input: directed graph
typedef pair<pair<int,int>,int> P; // par de nodos + coste
int N; // numero de nodos
vector<P> v; // representacion aristas

int bellmanford(int a, int b) {
    vector<int> d(N, 1000000000);
    d[a] = 0;
    for (int i = 1; i < N; i++)
        for (int j = 0; j < (int)v.SZ; j++)
            if (d[v[j].X.X] < 1000000000 && d[v[j].X.X] + v[j].Y <
                d[v[j].X.Y])
                d[v[j].X.Y] = d[v[j].X.X] + v[j].Y;
    for (int j = 0; j < (int)v.SZ; j++)
        if (d[v[j].X.X] < 1000000000 && d[v[j].X.X] + v[j].Y < d[v[j].X.Y])
            return -1000000000; // existe ciclo negativo
    return d[b];
}

int main(){
    N=8;
    v.PB(MP(MP(0, 1), +2)); v.PB(MP(MP(1, 2), -1)); v.PB(MP(MP(1, 3),
        +1));
    v.PB(MP(MP(2, 3), +1)); v.PB(MP(MP(6, 4), -1)); v.PB(MP(MP(4, 5),
        -1));
    v.PB(MP(MP(5, 6), -1));

    // min distance, negative cycle, unreachable
    cout << bellmanford(0, 3) << " " << bellmanford(4, 6) << " "
        << bellmanford(0, 7) << endl;
}
```

5.2 Bron-Kerbosch (Maximum Clique in Undirected Graph)

```
#define U unsigned int
typedef vector<short int> V;
```

```
vector<vector<U> > graf; // vertices/aristas del grafo
U numv, kmax; // # conjuntos/tamano grupo independiente
```

```
int evalua(V &vec) {
    for (int n = 0; n < vec.size(); n++)
        if (vec[n] == 1) return n;
    return -1;
}

void Bron_i_Kerbosch() {
    vector<U> v;
    U i, j, aux, k = 0, bandera = 2;
    vector<V> I, Ve, Va;
    I.PB(V()); Ve.PB(V()); Va.PB(V());
    for (i = 0; i < numv; i++) {
        I[0].PB(0); // conjunto vacio
        Ve[0].PB(0); // conjunto vacio
        Va[0].PB(1); // contiene todos
    }
    while(true) {
        switch(bandera) {
            case 2: // paso 2
                v.PB(evalua(Va[k]));
                I.PB(V(I[k].begin(), I[k].end()));
                Va.PB(V(Va[k].begin(), Va[k].end()));
                Ve.PB(V(Ve[k].begin(), Ve[k].end()));
                aux = graf[v[k]].size();
                I[k+1][v[k]] = 1; Va[k+1][v[k]] = 0;
                for (i = 0; i < aux; i++) {
                    j = graf[v[k]][i]; Ve[k+1][j] = Va[k+1][j] = 0;
                }
                k = k + 1; bandera = 3;
                break;
            /*****/
            case 3: // paso 3
                for (i = 0, bandera = 4; i < numv; i++) {
                    if (Ve[k][i] == 1) {
                        aux = graf[i].size();
                        for (j = 0; j < aux; j++)
                            if (Va[k][graf[i][j]] == 1)
                                break;
                        if (j == aux) { i = numv; bandera = 5; }
                    }
                }
                break;
        }
    }
}
```

```

/*****/
case 4: // paso 4
    if (evalua(Ve[k]) == -1 && evalua(Va[k]) == -1) {
        for (int n = 0; n < numv; n++)
            if (I[k][n] == 1) cout << n << " ";
        cout << endl;
        if (k > kmax) kmax = k;
        bandera = 5;
    }
    else bandera = 2; // ir a paso 2
break;
/*****/
case 5: // paso 5
    k = k - 1; v.pop_back(); I[k].clear();
    I[k].assign(I[k+1].begin(), I[k+1].end());
    I[k][v[k]] = 0; I.pop_back(); Ve.pop_back();
    Va.pop_back(); Ve[k][v[k]] = 1; Va[k][v[k]] = 0;
    if (k == 0) {
        if (evalua(Va[0]) == -1) return;
        bandera = 2; // ir a paso 2
    }
    else bandera = 3; // ir a paso 3
break;
}
}

int main() {
    U idx, i; stringstream ss; string linea;
    while (cin >> numv) {
        getline(cin, linea);
        for (i = 0; i < numv; i++) { // Lectura del grafo
            // vertices adyacentes al i-esimo vertice
            vector<U> bb; graf.PB(bb);
            getline(cin, linea);
            ss << linea;
            while (ss >> idx) graf[i].PB(idx);
            ss.clear();
        }
        // Llamada al algoritmo
        kmax = 0;
        cout << "Conjuntos independientes: " << endl;
        if (numv > 0)
            Bron_i_Kerbosch();
        cout << "kmax: " << kmax << endl;
    }
}

```

```

// Limpieza variables
for (i = 0; i < numv; i++) graf[i].clear();
graf.clear();
}
}

```

5.3 Dijkstra (Shortest Path)

```

// Complexity: ElogV - Input: undirected graph
typedef int V; // tipo de costes
typedef pair<V,int> P; // par de (coste,nodo)
typedef set<P> S; // conjunto de pares

int N; // numero de nodos
vector<P> A[10001]; // listas adyacencia (coste,nodo)

// int prec[201]; // predecesores (nodes from s to t)
// another way to obtain a path (above all, if there is
// more than one, consists in using BFS from the target
// and add to the queue those nodes that lead to the
// minimum cost in the preceeding node)

V dijkstra(int s, int t) {
    S m; // cola de prioridad
    vector<V> z(N, 1000000000); // distancias iniciales
    z[s] = 0; // distancia a s es 0
    m.insert(MP(0, s)); // insertar (0,s) en m
    while (m.SZ > 0) {
        P p = *m.begin(); // p=(coste,nodo) con menor coste
        m.erase(m.begin()); // elimina este par de m
        if (p.Y == t) return p.X; // cuando nodo es t, acaba
        // para cada nodo adyacente al nodo p.Y
        for (int i = 0; i < (int)A[p.Y].SZ; i++) {
            // q = (coste hasta nodo adyacente, nodo adyacente)
            P q(p.X + A[p.Y][i].X, A[p.Y][i].Y);
            // si q.X es la menor distancia hasta q.Y
            if (q.X < z[q.Y]) {
                m.erase(MP(z[q.Y], q.Y)); // borrar anterior
                m.insert(q); // insertar q
                z[q.Y] = q.X; // actualizar distancia
                // prec[q.Y] = p.Y; // actualizar
                // predecesores
            }
        }
    }
}

```

```

    }
}
return -1;
}

int main() {
    N = 6;          // solucion 0-1-2-4-3-5, coste 11
    A[0].PB(MP(2, 1)); // arista (0, 1) con coste 2
    A[0].PB(MP(5, 2)); // arista (0, 2) con coste 5
    A[1].PB(MP(2, 2)); // arista (1, 2) con coste 2
    A[1].PB(MP(7, 3)); // arista (1, 3) con coste 7
    A[2].PB(MP(2, 4)); // arista (2, 4) con coste 2
    A[3].PB(MP(3, 5)); // arista (3, 5) con coste 3
    A[4].PB(MP(2, 3)); // arista (4, 3) con coste 2
    A[4].PB(MP(8, 5)); // arista (4, 5) con coste 8
    cout << dijkstra(0, 5) << endl;
}

```

5.4 Floyd-Warshall (All Pairs Shortest Path)

```

// Complexity: n^3
// A: matriz n*n de adyacencia con costes
// ausencia de arista representada por un numero grande
for (int k = 0; k < n; k++)
    for (int i = 0; i < n; i++)
        for (int j = 0; j < n; j++)
            A[i][j] = MIN(A[i][j], A[i][k] + A[k][j]);

```

5.5 Kruskal (Minimum Spanning Tree)

```

// Complexity: ElogV - Input: undirected graph
typedef vector<pair<int, pair<int, int>>> V;

int N, mf[2000]; // numero de nodos N <= 2000
V v;             // vector de aristas
                // (coste, (nodo1, nodo2))

// vector< pair<long, int> > K[3001]; // kruskal tree

int set(int n) { // conjunto conexo de n
    if (mf[n] == n) return n;
}

```

```

    else mf[n] = set(mf[n]); return mf[n];
}

int kruskal() {
    int a, b, sum = 0;
    sort(v.begin(), v.end());
    for (int i = 0; i < N; i++)
        mf[i] = i; // inicializar conjuntos conexos
    for (int i = 0; i < (int)v.SZ; i++) {
        a = set(v[i].Y.X), b = set(v[i].Y.Y);
        if (a != b) { // si conjuntos son diferentes
            mf[b] = a; // unificar los conjuntos
            sum += v[i].X; // agregar coste de arista
                        // K[v[i].Y.X].PB(MP(v[i].X, v[i].Y.Y));
                        // K[v[i].Y.Y].PB(MP(v[i].X, v[i].Y.X));
        }
    }
    return sum;
}

int main() {
    N = 5; // solucion 13 (0,3),(1,2),(2,3),(3,4)
    v.PB(MP(4, MP(0, 1))); // arista (0,1) coste 4
    v.PB(MP(4, MP(0, 2))); // arista (0,2) coste 4
    v.PB(MP(3, MP(0, 3))); // arista (0,3) coste 3
    v.PB(MP(6, MP(0, 4))); // arista (0,4) coste 6
    v.PB(MP(3, MP(1, 2))); // arista (1,2) coste 3
    v.PB(MP(7, MP(1, 4))); // arista (1,4) coste 7
    v.PB(MP(2, MP(2, 3))); // arista (2,3) coste 2
    v.PB(MP(5, MP(3, 4))); // arista (3,4) coste 5
    cout << kruskal() << endl;
}

```

5.6 Maximum Bipartite Matching

```

/* Input: VVI with 1 if connected, 0 if not. mr and mc have the matches
 * for each side. Complexity: E * V.
 * From Stanford University's notebook. */
typedef vector<int> VI;
typedef vector<VI> VVI;

bool FindMatch(int i, const VVI &w, VI &mr, VI &mc, VI &seen) {
    for (int j = 0; j < w[i].size(); j++) {

```

```

    if (w[i][j] && !seen[j]) {
        seen[j] = true;
        if (mc[j] < 0 || FindMatch(mc[j], w, mr, mc, seen)) {
            mr[i] = j;
            mc[j] = i;
            return true;
        }
    }
}
return false;
}

int BipartiteMatching(const VVI &w, VI &mr, VI &mc) {
    mr = VI(w.size(), -1);
    mc = VI(w[0].size(), -1);

    int ct = 0;

    for (int i = 0; i < w.size(); i++) {
        VI seen(w[0].size());
        if (FindMatch(i, w, mr, mc, seen)) ct++;
    }
    return ct;
}

```

5.7 Maximum Cut

- $A = \emptyset$
- $B = \emptyset$
- For every vertex v
 - Place v in A or B with probability $1/2$
- Return A, B

El tall que retorna esperat és almenys la meitat de l'òptim

5.8 Min Cost Max Flow

```

/* From Stanford University's notebook.
 * To perform minimum weighted bipartite matching:
 * - Capacity between nodes = 1 (cost whatever given by the problem)

```

```

 * - Capacity from source = 1 and cost = 0
 * - Capacity to sink = 1 and cost = 0
 * Output: <maximum flow value - minimum cost value>
 * Complexity:  $O(|V|^2)$  per augmentation
 *           max flow:  $O(|V|^3)$  augmentations
 *           min cost max flow:  $O(|V|^4 * \text{MAX\_EDGE\_COST})$  augmentations
 */
typedef vector<int> VI;
typedef vector<VI> VVI;
typedef long long L;
typedef vector<L> VL;
typedef vector<VL> VVL;
typedef pair<int, int> PII;
typedef vector<PII> VPII;

const L INF = numeric_limits<L>::max() / 4;

struct MinCostMaxFlow {
    int N;
    VVL cap, flow, cost;
    VI found;
    VL dist, pi, width;
    VPII dad;

    MinCostMaxFlow(int N) :
        N(N), cap(N, VL(N)), flow(N, VL(N)), cost(N, VL(N)),
        found(N), dist(N), pi(N), width(N), dad(N) {}

    void AddEdge(int from, int to, L cap, L cost) {
        this->cap[from][to] = cap;
        this->cost[from][to] = cost;
    }

    void Relax(int s, int k, L cap, L cost, int dir) {
        L val = dist[s] + pi[s] - pi[k] + cost;
        if (cap && val < dist[k]) {
            dist[k] = val;
            dad[k] = make_pair(s, dir);
            width[k] = min(cap, width[s]);
        }
    }

    L Dijkstra(int s, int t) {
        fill(found.begin(), found.end(), false);
        fill(dist.begin(), dist.end(), INF);

```

```

fill(width.begin(), width.end(), 0);
dist[s] = 0;
width[s] = INF;

while (s != -1) {
    int best = -1;
    found[s] = true;
    for (int k = 0; k < N; k++) {
        if (found[k]) continue;
        Relax(s, k, cap[s][k] - flow[s][k], cost[s][k], 1);
        Relax(s, k, flow[k][s], -cost[k][s], -1);
        if (best == -1 || dist[k] < dist[best]) best = k;
    }
    s = best;
}

for (int k = 0; k < N; k++)
    pi[k] = min(pi[k] + dist[k], INF);
return width[t];
}

pair<L, L> GetMaxFlow(int s, int t) {
    L totflow = 0, totcost = 0;
    while (L amt = Dijkstra(s, t)) {
        totflow += amt;
        for (int x = t; x != s; x = dad[x].first) {
            if (dad[x].second == 1) {
                flow[dad[x].first][x] += amt;
                totcost += amt * cost[dad[x].first][x];
            }
            else {
                flow[x][dad[x].first] -= amt;
                totcost -= amt * cost[x][dad[x].first];
            }
        }
    }
    return make_pair(totflow, totcost);
}
};

```

5.9 Minimum Cut

Mientras queden > 2 vértices

- Selecciona arista al azar
- Fusiona u y v en un único vértice
- Retorna el coste representado por los dos vértices finales

Si repetimos $N = c \cdot n^2 \cdot \log n$ veces —, $P(\text{fracas}) \leq \frac{1}{n^c}$

5.10 Stable Marriage Problem

```

short W[1000][1000], M[1000][1000];
short wP[1001], mP[1001], coW[1001];
short listing[50000];
int N;

void stableMarriage(){
    int ac = 0, total = 0, ws, woman;
    for( int i = 0; i < N; i++){
        listing[total++] = i;
        coW[i] = 0;
    }

    while( ac < total ){
        ws = listing[ac++];
        if( wP[ws] != -1 )
            continue;

        for( ; coW[ws] < N; coW[ws]++){
            if (mP[ W[ws][ coW[ws] ] ] == -1 ) {
                wP[ws] = W[ws][ coW[ws] ];
                mP[ W[ws][ coW[ws] ] ] = ws;
                break;
            }
            else {
                woman = W[ws][ coW[ws] ];
                if( M[woman][ mP[woman] ] > M[woman][ws] ){
                    listing[total++] = mP[woman];
                    wP[ mP[woman] ] = -1;
                    mP[woman] = ws;
                    wP[ws] = woman;
                    break;
                }
            }
        }
    }
}

```

```
}
```

5.11 Tarjan (Strongly Connected Components)

```
// Complexity |V| + |E|
int index, ct;
vector<bool> I;
// L indica el indice del conjunto fuertemente conexo al que pertenece
// cada nodo
vector<int> D, L, S;
vector<vector<int>> V; // listas de adyacencia (grafo dirigido)

void tarjan (unsigned n) {
    D[n] = L[n] = index++;
    S.push_back(n);
    I[n] = true;
    for (unsigned i = 0; i < V[n].size(); ++i) {
        if (D[V[n][i]] < 0) {
            tarjan(V[n][i]);
            L[n] = MIN(L[n], L[V[n][i]]);
        }
        else if (I[V[n][i]])
            L[n] = MIN(L[n], D[V[n][i]]);
    }
    if (D[n] == L[n]) {
        ++ct;
        // todos los nodos eliminados de S pertenecen al mismo scc
        while (S[S.size() - 1] != n) {
            I[S.back()] = false;
            S.pop_back();
        }
        I[n] = false;
        S.pop_back();
    }
}

void scc() {
    index = ct = 0;
    I = vector<bool>(V.size(), false);
    D = vector<int>(V.size(), -1);
    L = vector<int>(V.size());
    S.clear();
    for (unsigned n = 0; n < V.size(); ++n)
```

```
        if (D[n] < 0)
            tarjan(n);
        // ct = numero total de scc
    }
}
```

5.12 Topological Sort

```
vector<int> A[101]; // adjacency list (directed graph without cycles)
int inbound[101]; // number of nodes that point to each node
vector<int> fo; // final order

// M = number of nodes (there might be 'lonely' nodes)
void toposort(int M) {
    stack<int> order;
    int current;

    // Search for roots (identifiers might change between
    // problems (e.g. 1 to M))
    for(int m = 0; m < M; m++){
        if(inbound[m] == 0)
            order.push(m);
    }

    // Start topsort from roots
    while(!order.empty()){
        // Pop from stack
        current = order.top();
        order.pop();
        // Save order in fo
        fo.push_back(current);
        // Add childs only if inbound is 0
        for (int i = 0; i < A[current].size(); ++i)
        {
            inbound[A[current][i]]--;
            if (inbound[A[current][i]] == 0)
                order.push(A[current][i]);
        }
    }
}

int main() {
    A[0].push_back(1); A[0].push_back(2); A[2].push_back(1);
    inbound[0] = 0; inbound[1] = 2; inbound[2] = 1;
```



```

toposort(3);
for (int i = 0; i < fo.size(); ++i) cout << fo[i] << " ";
// 0 2 1
}

```

6 Math

6.1 Catalan Numbers

```

unsigned long long v[34]; // 1, 1, 2, 5, 14, 42, 132, 429, 1430, ...
// Cn = number of strings of n*2 consistent parentheses.
// ((( ))) ()( ) ()( ) ()( ) ()( )
// Cn = number of non-isomorphic ordered trees with n vertices.
// Cn = number of full binary trees with n + 1 leaves, and n internal
//      nodes
// Cn = number of ways to tile a staircase shape of height n with n
//      rectangles
/* Cn = number of monotonic lattice paths along the edges of a grid with
//      n n
//      square cells, which do not pass above the diagonal */
void catalan(){
    v[0] = 1;
    for (int i = 1; i < 34; ++i){
        unsigned long long sum = 0;
        for (int j = 0; j < i; ++j){
            sum += v[j] * v[i-j-1];
        }
        v[i] = sum;
    }
}

```

6.2 Complex Numbers

```

// Complex number class, from Stanford's Notebook. Required for FFT
struct cpx {
    cpx(){}
    cpx(double aa):a(aa){}
    cpx(double aa, double bb):a(aa),b(bb){}
    double a, b;
    double modsq(void) const { return a * a + b * b; }
}

```

```

    cpx bar(void) const { return cpx(a, -b); }
};
cpx operator +(cpx a, cpx b) { return cpx(a.a + b.a, a.b + b.b); }
cpx operator *(cpx a, cpx b) {
    return cpx(a.a * b.a - a.b * b.b, a.a * b.b + a.b * b.a);
}
cpx operator /(cpx a, cpx b) {
    cpx r = a * b.bar();
    return cpx(r.a / b.modsq(), r.b / b.modsq());
}
cpx EXP(double theta) { return cpx(cos(theta), sin(theta)); }

```

6.3 Exponent

```

template <typename T, typename U> T expo(T &t, U n) {
    if (n == U(0)) return T(1);
    else {
        T u = expo(t, n/2);
        if (n%2 > 0) return u*u*t;
        else return u*u;
    }
}

```

6.4 Fast Fourier Transform

```

// from Stanford's notebook:
// https://web.stanford.edu/~liszt90/acm/notebook.html
// in:    input array
// out:    output array
// step:   {SET TO 1} (used internally)
// size:   length of the input/output {MUST BE A POWER OF 2}
// dir:    either plus or minus one (direction of the FFT)
// RESULT: out[k] = \sum_{j=0}^{size - 1} in[j] * exp(dir * 2pi * i * j *
//           k / size)
const double two_pi = 4 * acos(0);
void FFT(cpx *in, cpx *out, int step, int size, int dir)
{
    if (size < 1) return;
    if (size == 1)
    {
        out[0] = in[0];
    }
}

```

```

    return;
}
FFT(in, out, step * 2, size / 2, dir);
FFT(in + step, out + size / 2, step * 2, size / 2, dir);
for(int i = 0 ; i < size / 2 ; i++)
{
    cpx even = out[i];
    cpx odd = out[i + size / 2];
    out[i] = even + EXP(dir * two_pi * i / size) * odd;
    out[i + size / 2] = even + EXP(dir * two_pi * (i + size / 2) /
        size) * odd;
}
}
}

```

6.5 Greatest Common Divisor

```

// in algorithm library: __gcd(a, b)
int gcd(int a, int b) {
    if (a < b) return gcd(b, a);
    else if (a%b == 0) return b;
    else return gcd(b, a%b);
}

gcd(a,b)*lcm(a,b) = a*b

```

6.6 Matrix Multiplication

```

#define SIZE 15 // tamaño de matriz cuadrado
#define MOD 10007 // modulo de la multiplicacion
struct matriz {
    int v[SIZE][SIZE];
    matriz() { init(); } // matriz de 0's
    matriz(int x) { // matriz con x's en la diagonal
        init();
        for (int i = 0; i < SIZE; i++) v[i][i] = x;
    }
    void init() {
        for (int i = 0; i < SIZE; i++)
            for (int j = 0; j < SIZE; j++) v[i][j] = 0;
    }
}
// multiplicacion de matrices modulo MOD

```

```

matriz operator*(matriz &m) {
    matriz n;
    for (int i = 0; i < SIZE; i++)
        for (int j = 0; j < SIZE; j++)
            for (int k = 0; k < SIZE; k++)
                n.v[i][j] = (n.v[i][j] + v[i][k]*m.v[k][j])%MOD;
    return n;
}
};

```

6.7 Modular Linear Equations

```

// returns d = gcd(a,b); finds x,y such that d = ax + by
int extended_euclid(int a, int b, int &x, int &y) {
    int xx = y = 0;
    int yy = x = 1;
    while (b) {
        int q = a/b;
        int t = b; b = a%b; a = t;
        t = xx; xx = x-q*xx; x = t;
        t = yy; yy = y-q*yy; y = t;
    }
    return a;
}

```

```

// finds all solutions to ax = b (mod n)
VI modular_linear_equation_solver(int a, int b, int n) {
    int x, y;
    VI solutions;
    int d = extended_euclid(a, n, x, y);
    if (!(b%d)) {
        x = mod (x*(b/d), n);
        for (int i = 0; i < d; i++)
            solutions.push_back(mod(x + i*(n/d), n));
    }
    return solutions;
}

// computes b such that ab = 1 (mod n), returns -1 on failure
int mod_inverse(int a, int n) {
    int x, y;
    int d = extended_euclid(a, n, x, y);
    if (d > 1) return -1;
}

```

```

    return mod(x,n);
}

// Chinese remainder theorem (special case): find z such that
// z % x = a, z % y = b. Here, z is unique modulo M = lcm(x,y).
// Return (z,M). On failure, M = -1.
PII chinese_remainder_theorem(int x, int a, int y, int b) {
    int s, t;
    int d = extended_euclid(x, y, s, t);
    if (a%d != b%d) return make_pair(0, -1);
    return make_pair(mod(s*b*x+t*a*y,x*y)/d, x*y/d);
}

// Chinese remainder theorem: find z such that
// z % x[i] = a[i] for all i. Note that the solution is
// unique modulo M = lcm_i (x[i]). Return (z,M). On
// failure, M = -1. Note that we do not require the a[i]'s
// to be relatively prime.
PII chinese_remainder_theorem(const VI &x, const VI &a) {
    PII ret = make_pair(a[0], x[0]);
    for (int i = 1; i < x.size(); i++) {
        ret = chinese_remainder_theorem(ret.second, ret.first, x[i], a[i]);
        if (ret.second == -1) break;
    }
    return ret;
}

// computes x and y such that ax + by = c; on failure, x = y == -1
void linear_diophantine(int a, int b, int c, int &x, int &y) {
    int d = gcd(a,b);
    if (c%d) {
        x = y = -1;
    } else {
        x = c/d * mod_inverse(a/d, b/d);
        y = (c-a*x)/b;
    }
}

```

6.8 Newton Method

```

long double tolerance = 1E-6;
long double c0 = 1.0;
long double c1 = 1.0;

```

```

bool solutionFound = false;

// find the value of 'c' that makes the function equal to = 0
// might also be used in optimization problems setting y as
// the first derivative and yprime as the second
while (true)
{
    long double y = /* formula of the original function */;
    long double yprime = /* formula of the first derivative respect to
                           c */;
    c1 = c0 - y / yprime;
    if ((fabs(c1 - c0) / fabs(c1)) < tolerance)
    {
        solutionFound = true;
        break;
    }
    c0 = c1;
}

```

6.9 Polynomial Multiplication

```

const int MAX_LEN = 262144 * 2;
cpx A[MAX_LEN], B[MAX_LEN], C[MAX_LEN];
int A_len, B_len, C_len;

/* set the appropriate coefficients in the inputs A and B's real-valued
   part,
   * and their length in A_len and B_len. */

for(C_len = 1; !(C_len > A_len + B_len - 1); C_len *= 2);
assert(C_len < MAX_LEN);
memset(A + A_len, 0, (C_len - A_len) * sizeof(cpx));
memset(B + B_len, 0, (C_len - B_len) * sizeof(cpx));
FFT(A, C, 1, C_len, 1);
FFT(B, A, 1, C_len, 1);
for(int i=0; i<C_len; i++)
    A[i] = A[i] * C[i];
FFT(C, A, 1, C_len, -1);
for(int i=0; i<C_len; i++)
    C[i].a /= C_len;
// now C[i].a (the real-valued parts) contain the result

```

6.10 Primes

```
int v[10000]; // primes

void savePrimes()
{
    int k = 0;
    v[k++] = 2;
    for (int i = 3; i <= 10010; i += 2) {
        bool b = true;
        for (int j = 0; b && v[j] * v[j] <= i; j++)
            b = i%v[j] > 0;
        if (b)
            v[k++] = i;
    }

    bool isPrime(int x){
        bool prime = true;
        for (int j = 0; prime && v[j] * v[j] <= x; j++)
            prime = x%v[j] > 0;
        return prime;
    }

    // probar si un numero x <= 100000000 es primo
    int main()
    {
        savePrimes();
        cout << isPrime(4);
    }
```

7 Sequences

7.1 Binary Search

```
// binary_search function can be found at algorithm library
// devuelve el i mas pequeno tal que t <= v[i]
// si no existe tal i, devuelve v.SZ
template<typename T> int bb(T t, vector<T> &v) {
    int a = 0, b = v.SZ;
    while (a < b) {
        int m = (a + b)/2;
```

```
        if (v[m] < t) a = m+1; else b = m;
    }
    return a;
}
```

7.2 Ternary Search

```
double E = 0.0000001; // tolerance
double L = 200000; // R and L are extreme possible values...
double R = -200000; // ... for the optimized parameter
while (1) {
    double dist = R - L;
    if (fabs(dist) < E) break;
    double leftThird = L + dist / 3;
    double rightThird = R - dist / 3;
    // f is the function which we are optimizing
    if (f(leftThird) < f(rightThird))
        R = rightThird;
    else
        L = leftThird;
}
```

7.3 Vector Partition

```
bidirectional_iterator partition(bidirectional_iterator start,
                                bidirectional_iterator end,
                                Predicate p);
```

```
bool IsOdd(int i) {return (i%2==1);}
```

```
int main () {
    vector<int> myvector;
    vector<int>::iterator it, bound;

    // set some values:
    for (int i=1; i<10; ++i)
        myvector.push_back(i); // 1 2 3 4 5 6 7 8 9

    bound = partition(myvector.begin(), myvector.end(), IsOdd);

    // print out content:
```

```

cout << "odd members:";
for (it=myvector.begin(); it!=bound; ++it)
    cout << " " << *it;
cout << "\neven members:";
for (it=bound; it!=myvector.end(); ++it)
    cout << " " << *it;
cout << endl;
}

```

8 Strings

8.1 Knuth-Morris-Pratt

```

/*Search of substring in O(n+k)*/
void TablaKMP(string T,vector<int> &F)
{
    int pos = 2; // posicion actual en F
    int cnd = 0; // ndice en T del siguiente carcter del actual candidato
                    en la subcadena

    F[0] = -1;
    while(pos <= T.size())
    {
        if(T[pos - 1] == T[cnd] )
        { //siguiente candidato coincidente en la cadena
            cnd++;
            F[pos] = cnd;
            pos++;
        } else if(cnd > 0)
        { //si fallan coincidencias consecutivas entonces asignamos valor
            conocido la primera vez
            cnd = F[cnd];
        } else{
            F[pos] = 0 ;
            pos++;
        }
    }
}

vector<int> KMPSearch(string T, string P)//T: texto donde se busca ,P:
    palabra a buscar ,salida: vector de posiciones match
{
    int k = 0 ; //puntero de T

```

```

int i = 0 ; //avance en P

vector<int> F(T.size(),0),sol;

if(T.size() >= P.size())
{
    TablaKMP(T,F); //optimizacin para no repetir busquedas de
                    subcadenas que no hacen match
    while(k+i < T.size())
    {
        if(P[i] == T[k+i])
        {
            if(i == P.size()-1)
            {
                sol.push_back(k); //modificando el return podemos
                    devolver todos los matches
            }
            i++;
        } else{
            k += i-F[i];
            if(i > 0)
            {
                i = F[i];
            }
        }
    }
}

return sol;
}

int main(){
    string T = "PARTICIPARIA CON MI PARACAIDAS PARTICULAR";
    string P = "A";
    vector<int> founds = KMPSearch(T,P);
    for(int i = 0 ; i < founds.size();++i)
    {
        cout<<founds[i]<<endl;
    }
}

```

8.2 Regular Expressions

```

String regex = BuildRegex();

```

```

Pattern pattern = Pattern.compile (regex);

Scanner s = new Scanner(System.in);

pattern.matcher(removed_period).find() // Boolean

// Matcher documentation
/* Matcher has an internal index, and find() finds the next instance
of the pattern. */
// int start(): Returns the start index of the previous match.
// int end(): Returns the offset after the last character matched.
/* boolean find(int start): Resets this matcher and then attempts to
find the next subsequence of the input sequence that matches the
pattern, starting at the specified index. */
/* boolean matches(): Attempts to match the entire region against the
pattern. */
// String replaceAll(String replacement)
// String replaceFirst(String replacement)

```

8.3 Suffix Arrays

```

// Suffix array construction in  $O(L \log^2 L)$  time. Routine for
// computing the length of the longest common prefix of any two
// suffixes in  $O(\log L)$  time.
//
// INPUT: string s
//
// OUTPUT: array suffix[] such that suffix[i] = index (from 0 to L-1)
//         of substring s[i...L-1] in the list of sorted suffixes.
//         That is, if we take the inverse of the permutation suffix[],
//         we get the actual suffix array.

```

```

#include <vector>
#include <iostream>
#include <string>

using namespace std;

struct SuffixArray {
    const int L;
    string s;
    vector<vector<int>> > P;
    vector<pair<pair<int,int>,int> > M;

```

```

SuffixArray(const string &s) : L(s.length()), s(s), P(1,
    vector<int>(L, 0)), M(L) {
    for (int i = 0; i < L; i++) P[0][i] = int(s[i]);
    for (int skip = 1, level = 1; skip < L; skip *= 2, level++) {
        P.push_back(vector<int>(L, 0));
        for (int i = 0; i < L; i++)
            M[i] = make_pair(make_pair(P[level-1][i], i + skip < L ?
                P[level-1][i + skip] : -1000), i);
        sort(M.begin(), M.end());
        for (int i = 0; i < L; i++)
            P[level][M[i].second] = (i > 0 && M[i].first ==
                M[i-1].first) ? P[level][M[i-1].second] : i;
    }
}

vector<int> GetSuffixArray() { return P.back(); }

// returns the length of the longest common prefix of s[i...L-1] and
// s[j...L-1]
int LongestCommonPrefix(int i, int j) {
    int len = 0;
    if (i == j) return L - i;
    for (int k = P.size() - 1; k >= 0 && i < L && j < L; k--) {
        if (P[k][i] == P[k][j]) {
            i += 1 << k;
            j += 1 << k;
            len += 1 << k;
        }
    }
    return len;
}

};

int main() {
    // bobocel is the 0'th suffix
    // obocel is the 5'th suffix
    // bocel is the 1'st suffix
    // ocel is the 6'th suffix
    // cel is the 2'nd suffix
    // el is the 3'rd suffix
    // l is the 4'th suffix
    SuffixArray suffix("bobocel");
    vector<int> v = suffix.GetSuffixArray();

```

```

// indices of the first character in the ith suffix
// 0th suffix (bobocel) -> 0
// 1st suffix (bocel) -> 2
// 2nd suffix (cel) -> 4
vector<int> s(v.size());
for (int i = 0; i < v.size(); ++i)
{
    s[v[i]] = i;
}

// with the 's' vector we would compare whether suffix i
// has a common prefix with all suffixes from i + 1 to
// i + M by doing the LCP between just i and i + M.
// for (int i = 0; i <= N - M; ++i)
// {
//     int s1 = S[i];
//     int s2 = S[i + M - 1];
//     int length = suffix.LongestCommonPrefix(s1, s2);
// }

// Expected output: 0 5 1 6 2 3 4
//                2
for (int i = 0; i < v.size(); i++) cout << v[i] << " ";
cout << endl;
cout << suffix.LongestCommonPrefix(0, 2) << endl;
}

```
