2 In the simple linear regression model $y = \beta_0 + \beta_1 x + u$, suppose that $\mathrm{E}(u) \neq 0$. Letting $\alpha_0 = \mathrm{E}(u)$, show that the model can always be rewritten with the same slope, but a new intercept and error, where the new error has a zero expected value.

3 The following table contains the *ACT* scores and the *GPA* (grade point average) for eight college students. Grade point average is based on a four-point scale and has been rounded to one digit after the decimal.

Student	*GPA*	*ACT*
1	2.8	21
2	3.4	24
3	3.0	26
4	3.5	27
5	3.6	29
6	3.0	25
7	2.7	25
8	3.7	30

(i) Estimate the relationship between *GPA* and *ACT* using OLS; that is, obtain the intercept and slope estimates in the equation

$$\widehat{GPA} = \hat{\beta}_0 + \hat{\beta}_1 ACT.$$

Comment on the direction of the relationship. Does the intercept have a useful interpretation here? Explain. How much higher is the *GPA* predicted to be if the *ACT* score is increased by five points?

(ii) Compute the fitted values and residuals for each observation, and verify that the residuals (approximately) sum to zero.

(iii) What is the predicted value of *GPA* when $ACT = 20$?

(iv) How much of the variation in *GPA* for these eight students is explained by *ACT*? Explain.

10 Let $\hat{\beta}_0$ and $\hat{\beta}_1$ be the OLS intercept and slope estimators, respectively, and let $\bar{u}$ be the sample average of the errors (not the residuals!).

(i) Show that $\hat{\beta}_1$ can be written as $\hat{\beta}_1 = \beta_1 + \sum_{i=1}^{n} w_i u_i$, where $w_i = d_i/\mathrm{SST}_x$ and $d_i = x_i - \bar{x}$.

(ii) Use part (i), along with $\sum_{i=1}^{n} w_i = 0$, to show that $\hat{\beta}_1$ and $\bar{u}$ are uncorrelated. [*Hint*: You are being asked to show that $\mathrm{E}[(\hat{\beta}_1 - \beta_1) \cdot \bar{u}] = 0$.]

(iii) Show that $\hat{\beta}_0$ can be written as $\hat{\beta}_0 = \beta_0 + \bar{u} - (\hat{\beta}_1 - \beta_1)\bar{x}$.

(iv) Use parts (ii) and (iii) to show that $\mathrm{Var}(\hat{\beta}_0) = \sigma^2/n + \sigma^2(\bar{x})^2/\mathrm{SST}_x$.

(v) Do the algebra to simplify the expression in part (iv) to equation (2.58). [*Hint*: $\mathrm{SST}_x/n = n^{-1}\sum_{i=1}^{n} x_i^2 - (\bar{x})^2$.]

3 The following model is a simplified version of the multiple regression model used by Biddle and Hamermesh (1990) to study the tradeoff between time spent sleeping and working and to look at other factors affecting sleep:

$$sleep = \beta_0 + \beta_1 totwrk + \beta_2 educ + \beta_3 age + u,$$

where *sleep* and *totwrk* (total work) are measured in minutes per week and *educ* and *age* are measured in years. (See also Computer Exercise C3 in Chapter 2.)

(i) If adults trade off sleep for work, what is the sign of β_1?

(ii) What signs do you think β_2 and β_3 will have?

(iii) Using the data in SLEEP75, the estimated equation is

$$\widehat{sleep} = 3{,}638.25 - .148\ totwrk - 11.13\ educ + 2.20\ age$$
$$n = 706,\ R^2 = .113.$$

If someone works five more hours per week, by how many minutes is *sleep* predicted to fall? Is this a large tradeoff?

(iv) Discuss the sign and magnitude of the estimated coefficient on *educ*.

(v) Would you say *totwrk*, *educ*, and *age* explain much of the variation in *sleep*? What other factors might affect the time spent sleeping? Are these likely to be correlated with *totwrk*?

9 The following equation describes the median housing price in a community in terms of amount of pollution (*nox* for nitrous oxide) and the average number of rooms in houses in the community (*rooms*):

$$\log(price) = \beta_0 + \beta_1 \log(nox) + \beta_2 rooms + u.$$

(i) What are the probable signs of β_1 and β_2? What is the interpretation of β_1? Explain.

(ii) Why might *nox* [or more precisely, log(*nox*)] and *rooms* be negatively correlated? If this is the case, does the simple regression of log(*price*) on log(*nox*) produce an upward or a downward biased estimator of β_1?

(iii) Using the data in HPRICE2, the following equations were estimated:

$$\widehat{\log(price)} = 11.71 - 1.043\ \log(nox),\ n = 506,\ R^2 = .264.$$
$$\widehat{\log(price)} = 9.23 - .718\ \log(nox) + .306\ rooms,\ n = 506,\ R^2 = .514.$$

Is the relationship between the simple and multiple regression estimates of the elasticity of *price* with respect to *nox* what you would have predicted, given your answer in part (ii)? Does this mean that −.718 is definitely closer to the true elasticity than −1.043?

16 The following equations were estimated using the data in LAWSCH85:

$$\underset{}{\widehat{lsalary}} = \underset{(.24)}{9.90} - \underset{(.0003)}{.0041}\ rank + \underset{(.069)}{.294}\ GPA$$
$$n = 142,\ R^2 = .8238$$

$$\widehat{lsalary} = \underset{(.29)}{9.86} - \underset{(.0004)}{.0038}\ rank + \underset{(.083)}{.295}\ GPA + \underset{(.00036)}{.00017}\ age$$
$$n = 99,\ R^2 = .8036$$

How can it be that the R-squared is smaller when the variable *age* is added to the equatic