Aula 22

7 Janeiro

1) Calcule $\displaystyle\int \frac{x + (\arcsen(2x))^2}{\sqrt{1-4x^2}}\,dx$

$$\int \frac{x + (\arcsen(2x))^2}{\sqrt{1-4x^2}}\,dx = \int \frac{x}{\sqrt{1-4x^2}}\,dx + \int \frac{(\arcsen(2x))^2}{\sqrt{1-4x^2}}\,dx$$

$$= \int x\,(1-4x^2)^{-1/2}\,dx + \int \frac{1}{\sqrt{1-(2x)^2}}\,(\arcsen(2x))^2\,dx$$

$f(x) = 1-4x^2$
$\alpha = -1/2$
$f'(x) = -8x$

$f(x) = \arcsen(2x)$
$\alpha = 2$
$f'(x) = \dfrac{2}{\sqrt{1-(2x)^2}}$

$$= -\frac{1}{8}\int -8x\,(1-4x^2)^{-1/2}\,dx + \frac{1}{2}\int \frac{2}{\sqrt{1-(2x)^2}}\,(\arcsen(2x))^2\,dx$$

$$= -\frac{1}{8}\,\frac{(1-4x^2)^{1/2}}{\frac{1}{2}} + \frac{1}{2}\,\frac{(\arcsen(2x))^3}{3} + C$$

$$= -\frac{1}{4}\sqrt{1-4x^2} + \frac{1}{6}(\arcsen(2x))^3 + C,\ C\in\mathbb{R}$$

2) Calcule $\int_0^{\sqrt{3}/2} \operatorname{arcsen} x \, dx =$

$f'(x) = 1 \qquad f(x) = x$

$g(x) = \operatorname{arcsen} x \qquad g'(x) = \dfrac{1}{\sqrt{1-x^2}}$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} - \int_0^{\frac{\sqrt{3}}{2}} \frac{x}{\sqrt{1-x^2}}\, dx$$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} + \int_0^{\frac{\sqrt{3}}{2}} \frac{-x}{\sqrt{1-x^2}}\, dx$$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} + \int_0^{\frac{\sqrt{3}}{2}} -x\,(1-x^2)^{-1/2}\, dx$$

$$\boxed{\begin{array}{l} f(x) = 1-x^2 \\ \alpha = -1/2 \\ f'(x) = -2x \end{array}}$$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} + \frac{1}{2}\int_0^{\frac{\sqrt{3}}{2}} \underbrace{-2x}_{f'}\, \underbrace{(1-x^2)^{-1/2}}_{f^{-1/2}}\, dx$$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} + \cancel{\frac{1}{2}}\left[\frac{(1-x^2)^{1/2}}{\cancel{\frac{1}{2}}}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \left[x \operatorname{arcsen} x\right]_0^{\frac{\sqrt{3}}{2}} + \left[\sqrt{1-x^2}\right]_0^{\frac{\sqrt{3}}{2}}$$

$$= \left(\frac{\sqrt{3}}{2} \underbrace{\operatorname{arcsen} \frac{\sqrt{3}}{2}}_{\pi/3} - 0\right) + \left(\sqrt{1-\frac{3}{4}} - \sqrt{1}\right) = \frac{\sqrt{3}\,\pi}{6} + \frac{1}{2} - 1 = \frac{\sqrt{3}\pi}{6} - \frac{1}{2}$$

3) Calcule $\int \frac{3x^2 - 5x + 1}{(x-1)^2(x+1)}\,dx$

(i) Zeros de $D(x) = (x-1)^2(x+1)$

$\boxed{x = 1}$ real de multiplicidade 2

→ contribui com 2 frações simples

$\boxed{x = -1}$ real de multiplicidade 1

→ contribui com 1 fração simples

(ii) Decomposição em frações simples

$$\frac{3x^2 - 5x + 1}{(x-1)^2(x+1)} = \frac{A}{(x-1)^2} + \frac{B}{x-1} + \frac{C}{x+1}$$

Reduzindo ao mesmo denominador, da última equação sai

$$3x^2 - 5x + 1 = A(x+1) + B(x-1)(x+1) + C(x-1)^2$$

$x = 1 \rightarrow -1 = 2A \iff A = -1/2$

$x = -1 \rightarrow 9 = 4A \iff A = 9/4$

$x = 0 \rightarrow 1 = -1/2 - B + \frac{9}{4} \iff B = \frac{3}{4}$

(iii) Cálculo das primitivas

$$\int \frac{3x^2-5x+1}{(x-1)^2(x+1)}\,dx = \int \frac{-1/2}{(x-1)^2}\,dx + \int \frac{3/4}{x-1}\,dx + \int \frac{9/4}{x+1}\,dx$$

$$= -\frac{1}{2}\int (x-1)^{-2}\,dx + \frac{3}{4}\int \frac{1}{x-1}\,dx + \frac{9}{4}\int \frac{1}{x+1}\,dx$$

$$= -\frac{1}{2}\,\frac{(x-1)^{-1}}{-1} + \frac{3}{4}\ln|x-1| + \frac{9}{4}\ln|x+1| + C$$

$$= \frac{1}{2(x-1)} + \frac{3}{4}\ln|x-1| + \frac{9}{4}\ln|x+1| + C,\quad C\in\mathbb{R}$$