LCC 2015/2016

Análise

Teste 1 modelo

1.

a) $f(2,1) = 2^2 + (1-1)^2 + 1 = 5$

$f(-2,1) = (-2)^2 + (1-1)^2 + 1 = 5$

b)

$C_1 = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 1\} = \{(x,y) \in \mathbb{R}^2 : x^2 + (y-1)^2 = 0\}$

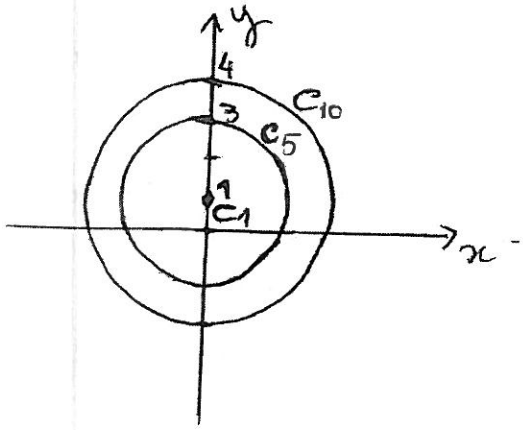
$= \{(0,1)\}$

$C_5 = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 5\} = \{(x,y) \in \mathbb{R}^2 : x^2 + (y-1)^2 = 4\}$

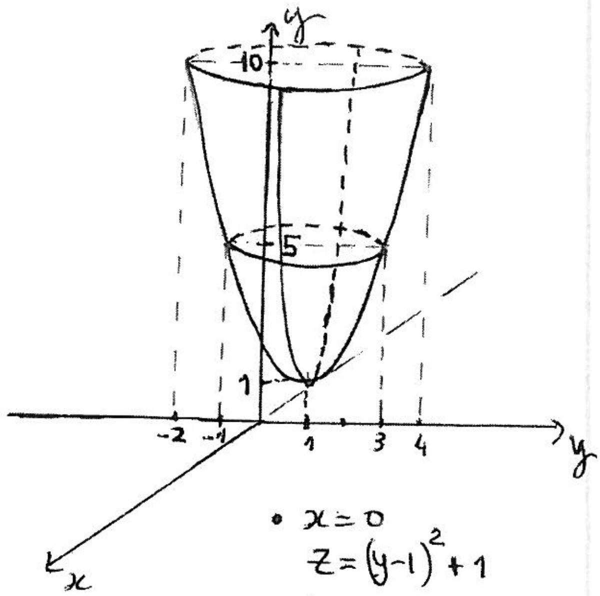
Circunferência de centro em $(0,1)$ e raio 2

$C_{10} = \{(x,y) \in \mathbb{R}^2 : f(x,y) = 10\} = \{(x,y) \in \mathbb{R}^2 : x^2 + (y-1)^2 = 9\}$

Circunferência de centro em $(0,1)$ e raio 3



c)



- $x = 0$
  $z = (y-1)^2 + 1$
- $y = 1$
  $z = x^2 + 1$

2.

a)

- $$\lim_{\substack{(x,y)\to(0,0)\\ y=0}} \frac{x^3-y^3}{y^3+x^2} = \lim_{x\to 0} \frac{x^3}{x^2} = \lim_{x\to 0} x = 0$$

- $$\lim_{\substack{(x,y)\to(0,0)\\ x=0}} \frac{x^3-y^3}{y^3+x^2} = \lim_{y\to 0} \frac{-y^3}{y^3} = \lim_{y\to 0} (-1) = -1$$

Como temos dois limites trajetoriais distintos, concluímos que não existe $\lim_{(x,y)\to(0,0)} \frac{x^3-y^3}{y^3+x^2}$.

b)

- Se $(x,y) \neq (0,0)$, f é contínua por ser o quociente de duas funções contínuas (funções polinomiais);

- $$\lim_{(x,y)\to(0,0)} (x+y)\cdot\frac{y^2}{2x^2+y^2} = 0, \text{ uma vez que}$$

$$\lim_{(x,y)\to(0,0)} (x+y) = 0 \quad \text{e} \quad \left|\frac{y^2}{2x^2+y^2}\right| \leq \frac{y^2}{y^2} = 1$$

(função limitada)

Como $f(0,0)=0$, concluímos que f é também contínua em $(0,0)$.

Logo, f é contínua em $\mathbb{R}^2$.

c) $\begin{cases} z = 2y^2 + x \\ x = 1 \end{cases}$

$$\frac{\partial z}{\partial y} = 4y$$

Declive da reta tangente à parábola $z = 2y^2 + 1$, no plano $x = 1$, em $(1, -1, 3)$ é igual a

$$\frac{\partial z}{\partial y}(1, -1) = -4 < 0$$

d) Taxa de variação de $z = x^2 y + 2y^2 x$ na direção do eixo dos xx:

$$\frac{\partial z}{\partial x} = 2xy + 2y^2$$

Se $x = -y$, vem $\frac{\partial z}{\partial x} = 2(-y)y + 2y^2 = -2y^2 + 2y^2 = 0$.

3. $z(x, t) = x + at + e^{x-at}$, $a \in \mathbb{R}$

$$\frac{\partial z}{\partial x} = 1 + e^{x-at} \qquad \frac{\partial z}{\partial t} = a - a\,e^{x-at}$$

$$\frac{\partial^2 z}{\partial x^2} = e^{x-at} \qquad \frac{\partial^2 z}{\partial t^2} = (-a)\cdot(-a)\,e^{x-at} = a^2 e^{x-at}$$

Logo,

$$\frac{\partial^2 z}{\partial t^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

4. $z = g(x, y)$, $x = s + t$, $y = s - t$

$$\begin{tikzcd} & & s \\ & x \arrow[ur] \arrow[r] & t \\ z \arrow[ur] \arrow[dr] & & \\ & y \arrow[r] \arrow[dr] & s \\ & & t \end{tikzcd}$$

$$\frac{\partial z}{\partial s} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial s} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}$$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x}\cdot\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\cdot\frac{\partial y}{\partial t} = \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}$$

Assim,

$$\frac{\partial z}{\partial s}\cdot\frac{\partial z}{\partial t} = \left(\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}\right)\cdot\left(\frac{\partial z}{\partial x}-\frac{\partial z}{\partial y}\right)$$
$$= \left(\frac{\partial z}{\partial x}\right)^2-\left(\frac{\partial z}{\partial y}\right)^2$$

5. $f(x,y,z) = x^2y^3(z+1)^4$

$(x,y,z) = (1,1,0)$

$(x+\Delta x,\ y+\Delta y,\ z+\Delta z) = (1.05,\ 0.9,\ 0.01) \Rightarrow$ $\Delta x = 0.05$, $\Delta y = -0.1$, $\Delta z = 0.01$

$$\Delta f \simeq df = \frac{\partial f}{\partial x}\cdot\Delta x + \frac{\partial f}{\partial y}\cdot\Delta y + \frac{\partial f}{\partial z}\cdot\Delta z$$
$$= 2xy^3(z+1)^4\cdot\Delta x + 3x^2y^2(z+1)^4\cdot\Delta y + 4x^2y^3(z+1)^3\cdot\Delta z$$

Para $(x,y,z)=(1,1,0)$ e $(\Delta x,\Delta y,\Delta z) = (0.05,\ -0.1,\ 0.01)$, temos

$$\Delta f \simeq 2\times 0.05 + 3\times(-0.1) + 4\times 0.01 = 0.1 - 0.3 + 0.04 = -0.16$$

6. $V(x,y,z) = 2x^2 - 3xy + xyz$

a) $\vec{E}(x,y,z) = -\vec{\nabla}V(x,y,z) = -\left(\frac{\partial V}{\partial x},\frac{\partial V}{\partial y},\frac{\partial V}{\partial z}\right)$

$$= -\left(4x-3y+yz,\ -3x+xz,\ xy\right)$$

b) $P = (2,1,0)$

$\vec{v} = (1,1,-1)$ $\qquad \vec{u} = \frac{\vec{v}}{\|\vec{v}\|} = \frac{1}{\sqrt{3}}(1,1,-1) = \left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},-\frac{\sqrt{3}}{3}\right)$

$\vec{\nabla}f(2,1,0) = (5,-6,2)$

$D_{\vec{u}}f(2,1,0) = (5,-6,2)\cdot\left(\frac{\sqrt{3}}{3},\frac{\sqrt{3}}{3},-\frac{\sqrt{3}}{3}\right) = -\sqrt{3}$

e) Variação mínima ocorre na direção de

$$-\vec{\nabla} V(P) = (-5, 6, -2)$$

Taxa mínima: $-\|\vec{\nabla} V(P)\| = -\sqrt{65}$

7. $y^2 z e^x - \text{sen}(xyz) = 1$

a) Seja $g(x,y,z) = y^2 z e^x - \text{sen}(xyz)$.

$$\vec{\nabla} g = \left(y^2 z e^x - yz\cos(xyz),\ 2yze^x - xz\cos(xyz),\ y^2 e^x - xy\cos(xyz) \right)$$

$$\vec{\nabla} g(0,1,1) = (1-1,\ 2-0,\ 1-0) = (0, 2, 1)$$

Plano tangente à superfície no ponto $(0,1,1)$:

$$\vec{\nabla} g(0,1,1) \cdot (x-0, y-1, z-1) = 0$$

$$\Leftrightarrow (0,2,1) \cdot (x, y-1, z-1) = 0$$

$$\Leftrightarrow 2(y-1) + z - 1 = 0$$

$$\Leftrightarrow 2y - 2 + z - 1 = 0$$

$$\Leftrightarrow 2y + z - 3 = 0$$

b)

$$\frac{\partial z}{\partial y} = -\frac{\frac{\partial g}{\partial y}}{\frac{\partial g}{\partial z}} = -\frac{2yze^x - xz\cos(xyz)}{y^2 e^x - xy\cos(xyz)}$$

$$\underset{(z=1)}{\frac{\partial z}{\partial y}(0,1)} = -\frac{2-0}{1-0} = -2$$