Aula 13

17 de Novembro

Exemplos de Primitivas Imediatas

Exercício 3 da Folha de Exercícios nº 6

(2) $\int (3x^2 - 2x^5)\, dx = \int 3x^2\, dx - \int 2x^5\, dx =$

$= 3\int x^2\, dx - 2\int x^5\, dx = 3\,\frac{x^3}{3} - 2\,\frac{x^6}{6} + C$

$= x^3 - \frac{x^6}{3} + C, \quad C \in \mathbb{R}$

Observação:

$$\int x^5\, dx = \int \underbrace{1}_{f'}\, \underbrace{x^5}_{f^5}\, dx \overset{R2}{=} \frac{x^6}{6} + C, \quad C \in \mathbb{R}$$

$f(x) = x$

$\alpha = 5$

$f'(x) = 1$

③ $\int (2x+10)^{20}\,dx = \int \frac{1}{2}\, 2\,(2x+10)^{20}\,dx =$

$f(x) = 2x+10$

$\alpha = 20$

$f'(x) = 2$

$= \frac{1}{2}\int \underbrace{2}_{f'}\,\underbrace{(2x+10)^{20}}_{f^{20}}\,dx \underset{R2}{=} \frac{1}{2}\,\frac{(2x+10)^{21}}{21} + C,\quad C\in\mathbb{R}$

$= \frac{(2x+10)^{21}}{42} + C,\quad C\in\mathbb{R}$

⑦ $\int \sqrt{2x+1}\,dx = \int (2x+1)^{1/2}\,dx = \frac{1}{2}\int \underbrace{2}_{f'}\,\underbrace{(2x+1)^{1/2}}_{f^{1/2}}\,dx$

$f(x) = 2x+1$

$\alpha = 1/2$

$f'(x) = 2$

$\underset{R2}{=} \frac{1}{2}\,\frac{(2x+1)^{3/2}}{\frac{3}{2}} + C = \frac{1}{3}(2x+1)^{3/2} + C,\quad C\in\mathbb{R}$

⑨ $$\int \frac{1}{4-3x}\,dx = -\frac{1}{3}\int \frac{\overset{f'}{-3}}{\underset{f}{4-3x}}\,dx \underset{R3}{=} -\frac{1}{3}\ln|4-3x| + C, \quad C \in \mathbb{R}$$

$f(x) = 4-3x$
$f'(x) = -3$

⑩ $$\int \frac{1}{e^{3x}}\,dx = \int e^{-3x}\,dx = -\frac{1}{3}\int \underbrace{-3}_{f'}\,\underbrace{e^{-3x}}_{e^{f}}\,dx \underset{R4}{=} -\frac{1}{3}e^{-3x} + C, \quad C \in \mathbb{R}$$

$f(x) = -3x$
$a = e$
$f'(x) = -3$

⑭ $$\int \frac{1}{x(\ln^2 x+1)}\,dx = \int \frac{\frac{1}{x}}{\ln^2 x + 1}\,dx = \int \frac{\overset{f'}{\frac{1}{x}}}{\underset{1+f^2}{1+\ln^2 x}}\,dx$$

$f(x) = \ln x$
$f'(x) = \frac{1}{x}$

$$\underset{R15}{=} \operatorname{arctg}(\ln x) + C, \quad C \in \mathbb{R}$$

㉔ $$\int \frac{x}{\sqrt{x^2-1}}\,dx = \int x\,(x^2-1)^{-1/2}\,dx = \frac{1}{2}\int \underbrace{2x}_{f'}\,\underbrace{(x^2-1)^{-1/2}}_{f^{-1/2}}\,dx$$

$f(x) = x^2-1$
$\alpha = -1/2$
$f'(x) = 2x$

$$\underset{R2}{=} \frac{1}{2}\,\frac{(x^2-1)^{1/2}}{\frac{1}{2}} + C = \sqrt{x^2-1} + C, \quad C \in \mathbb{R}$$

34)

$$\int \frac{\operatorname{sen} x}{\sqrt{1+\cos x}}\,dx = \int \operatorname{sen} x\,(1+\cos x)^{-1/2}\,dx =$$

$$f(x) = 1+\cos x$$
$$\alpha = -1/2$$
$$f'(x) = -\operatorname{sen} x$$

$$= -\int \underbrace{(-\operatorname{sen} x)}_{f'}\,\underbrace{(1+\cos x)^{-1/2}}_{f^{-1/2}}\,dx \underset{R_2}{=} -\frac{(1+\cos x)^{1/2}}{\frac{1}{2}} + C$$

$$= -2\sqrt{1+\cos x} + C, \qquad C\in\mathbb{R}$$

33)

$$\int \frac{e^{\operatorname{arctg} x}}{1+x^2}\,dx = \int \underbrace{\frac{1}{1+x^2}}_{f'}\,\underbrace{e^{\operatorname{arctg} x}}_{e^f}\,dx \underset{R_4}{=} e^{\operatorname{arctg} x} + C, \quad C\in\mathbb{R}$$

$$f(x) = \operatorname{arctg} x$$
$$a = e$$
$$f'(x) = \frac{1}{1+x^2}$$

(21) $$\int \operatorname{sen}^2 x \, dx = \int \frac{1-\cos(2x)}{2}\, dx = \int \frac{1}{2}\, dx - \int \frac{\cos(2x)}{2}\, dx$$

$$= \int \frac{1}{2}\, dx - \frac{1}{2}\int \cos(2x)\, dx = \int \frac{1}{2}\, dx - \frac{1}{2}\,\frac{1}{2}\int \underbrace{2}_{f'}\underbrace{\cos(2x)}_{\cos(f)}dx$$

$$f(x) = 2x$$
$$f'(x) = 2$$

$$= \frac{1}{2}x - \frac{1}{4}\operatorname{sen}(2x) + C, \quad C \in \mathbb{R}$$

$R_1 + R_5$