# Aula 11

9 Novembro

# Derivadas das funções trigonométricas inversas

$f: [0,\pi] \longrightarrow [-1,1]$ $\qquad$ $f^{-1}: [-1,1] \longrightarrow [0,\pi]$

$x \longmapsto \cos x$ $\qquad$ $y \longmapsto \arccos y$

$$y = \cos x\,,\ x \in [0,\pi] \Leftrightarrow x = \arccos y\,,\ y \in [-1,1]$$

Pelo Teorema da derivada da função inversa,

$$(f^{-1})'(y) = (f^{-1})'(f(x)) = \frac{1}{f'(x)} = \frac{1}{f'(f^{-1}(y))}$$

onde $f(x) = \cos x$ e, portanto, $f'(x) = -\operatorname{sen} x$, $x \in [0,\pi]$

Assim, $f'(x) \neq 0$ para todo o $x \in ]0,\pi[$ e

$f(0) = 1$, $f(\pi) = -1$

Consequentemente, o teorema é aplicável em $]-1,1[$ resultando

$$(f^{-1})'(y) = -\frac{1}{\operatorname{sen} x}$$

Resta escrever a derivada em termos da variável $y$.

Para tal basta atender a que

$y = f(x) = \cos x$ e a que $\operatorname{sen}^2 x + \cos^2 x = 1$,

pelo que $\operatorname{sen}^2 x = 1 - \cos^2 x$ e $\operatorname{sen} x = \sqrt{1-y^2}$

pois $x \in [0, \pi]$, onde o seno nunca é negativo

Concluindo

$$(f^{-1})'(y) = \frac{-1}{\sqrt{1-y^2}} \quad , \; y \in ]-1,1[$$

# Derivadas das funções hiperbólicas inversas

$$\operatorname{argsh} x = \ln\left(x + \sqrt{x^2+1}\right), \quad x \in \mathbb{R}$$

Assim,

$$\operatorname{argsh}' x = \frac{\left(x + \sqrt{x^2+1}\right)'}{x + \sqrt{x^2+1}} = \frac{1 + \frac{2x}{2\sqrt{x^2+1}}}{x + \sqrt{x^2+1}}$$

$$= \frac{1 + \frac{x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} = \frac{\frac{\sqrt{x^2+1} + x}{\sqrt{x^2+1}}}{x + \sqrt{x^2+1}} = \frac{1}{\sqrt{x^2+1}}$$

Observemos que.

$$\left(\sqrt{x^2+1}\right)' = \left(\left(x^2+1\right)^{1/2}\right)' = \frac{1}{2}\left(x^2+1\right)^{-1/2} \cdot 2x$$

$$= \frac{2x}{2\sqrt{x^2+1}} = \frac{x}{\sqrt{x^2+1}}$$

20. Calcule:

(a) $\cos(\arccos(1/8))$ (b) $\text{arctg}\left(\text{tg}\left(\frac{9\pi}{4}\right)\right)$ (c) $\text{arcsen}\left(\text{sen}\left(\frac{5\pi}{4}\right)\right)$

(d) $\text{sen}(\text{arcsen}(-1/2))$ (e) $\text{sen}(\text{arcsen}(1)+\pi)$ (f) $\text{arcsen}\left(\text{sen}\left(-\frac{\pi}{6}\right)\right)$

(g) $\text{arcsen}\left(\text{sen}\,\frac{23\pi}{6}\right)$ (h) $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$ (i) $\text{arctg}(\text{tg}\,\pi)$

(j) $\text{tg}\left(\arccos\left(\frac{2}{3}\right)\right)$ (k) $\cos\left(\text{arctg}\left(\frac{2}{3}\right)\right)$

a) $\cos : [0,\pi] \to [-1,1]$ $\qquad$ $\text{arc}\cos : [-1,1] \to [0,\pi]$

$x \mapsto \cos x$ $\qquad$ $x \mapsto \text{arc}\cos x$

Temos que:

$$\cos(\text{arc}\cos x) = x \quad \forall x \in [-1,1]$$

$$\text{arc}\cos(\cos x) = x \quad \forall x \in [0,\pi]$$

$\cos(\text{arc}\cos(1/8)) = 1/8$

b) $\text{arctg}\left(\text{tg}\left(\frac{9\pi}{4}\right)\right) = \text{arctg}\left(\text{tg}\left(2\pi + \frac{\pi}{4}\right)\right) = \text{arctg}\left(\text{tg}\,\frac{\pi}{4}\right) = \frac{\pi}{4}$

c) $\text{arc sen}\left(\text{sen}\left(5\frac{\pi}{4}\right)\right) = \text{arcsen}\left(\text{sen}\left(\pi + \frac{\pi}{4}\right)\right) =$

$= \text{arcsen}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

d) $\text{sen}\left(\text{arcsen}\left(-\frac{1}{2}\right)\right) = -\frac{1}{2}$

h) $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right) = \arccos\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

i) $\operatorname{arctg}\left(\operatorname{tg}\pi\right) = \operatorname{arctg}\left(\operatorname{tg}0\right) = 0$

24. Considere a função bijetiva $f : \mathbb{R}^+ \longrightarrow ]1, +\infty[$ tal que $f(x) = \operatorname{ch}\sqrt{x}$.

(a) Calcule a derivada de $f$.

(b) Mostre que $f^{-1}(x) = \ln^2\left(x + \sqrt{x^2 - 1}\right)$.

(c) Calcule a derivada da função inversa de $f$.

g) $\text{sen}\left(\text{arcsen}\left(-1/2\right)\right) = -1/2$

h) $\text{arc cos}\left(\cos\left(-\pi/3\right)\right) = \text{arc cos}\left(\cos\frac{\pi}{3}\right) = \frac{\pi}{3}$

$[0, \pi]$

i) $\text{arctg}\left(\text{tg}\ \pi\right) = \text{arctg}\left(\text{tg}\ 0\right) = 0$

$]-\pi/2, \pi/2[$