The Dark Side of Decentralized Target Tracking

Unknown Correlations and Communication Constraints

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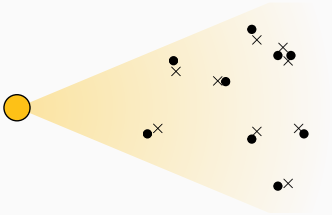
Speaker's Corner 2024-04-05

Introduction

Introduction

- Started at Saab 2016
- Industrial PhD student at Automatic Control, ISY, LiU between 2018–2023
- Main supervisor: Fredrik Gustafsson
- Research topic: Target tracking in decentralized sensor networks

A Target Tracking Problem



Multitarget tracking scene with measurements over multiple time instants

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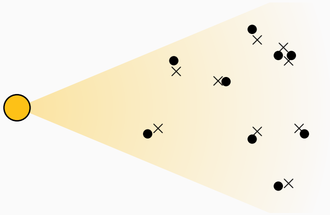
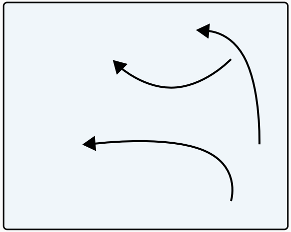
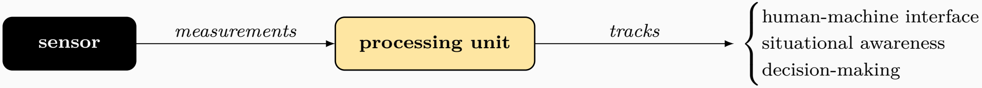


Multitarget tracking scene with measurements over multiple time instants



Refined picture using target tracking algorithms

Basic Target Tracking System

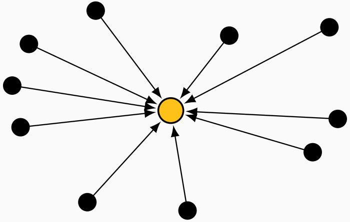


The Big Picture

- network-centric operations
- heterogenous agents: *ships*, *aircraft*, *ground vehicles* etc
- asymmetric capabilities
- distributed sensors
- information exchange

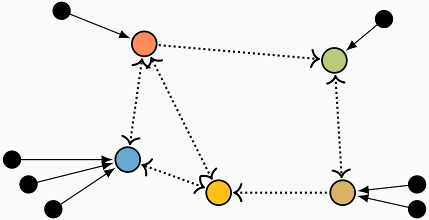
Sensor Network Architectures

Centralized sensor network



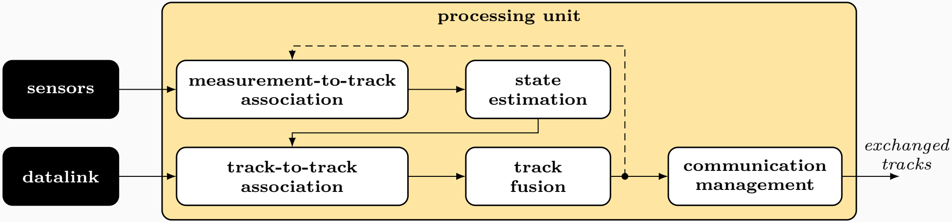
- possible optimal performance
- critical nodes, high complexity

Decentralized sensor network



- robust, modular, flexible
- dependencies (correlations)

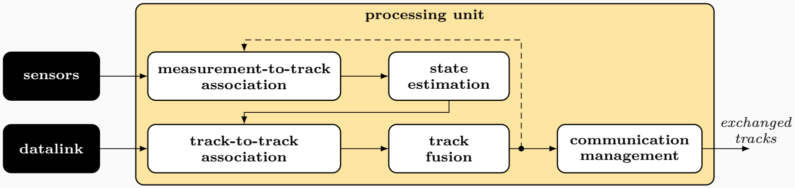
Decentralized Target Tracking: System Perspective



Research Problem

Two subproblems

1. robust track fusion under unknown correlations
2. efficient usage of the communication resource



Outline

Resources: https://github.com/robinforsling/dtt/

- MATLAB® source code and thesis summary

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Part I: Track fusion design and evaluation

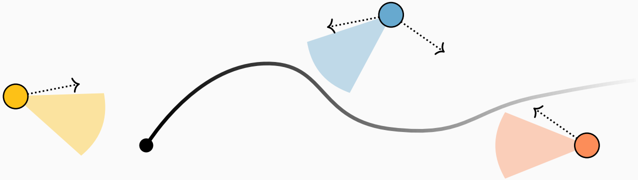
- track fusion methods
- evaluation measures and analysis

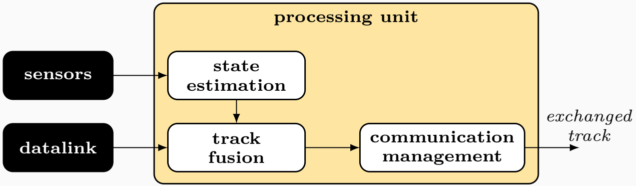
Part II: Communication management design and implementation

- two frameworks for reducing the communication load

Part I: Track Fusion Design and Evaluation

Decentralized Single-Target Tracking





Notation

- $x \in \mathbb{R}^{n_x}$: target state to be estimated
- I: identity matrix
- A^{T}: transpose of matrix (or vector) A
- A^{-1}: inverse of matrix A
- $A \succeq 0$: A is symmetric positive semidefinite
- $A \succ 0$: A is symmetric positive definite
- $\mathrm{E}(\boldsymbol{a})$: expected value of $\boldsymbol{a}$
- $\mathrm{cov}(\boldsymbol{a})$: covariance (matrix) of $\boldsymbol{a}$

Estimate Model

By (y_i, R_i) we denote the local estimate/track in ith agent, model as

$$y_i = H_i x + v_i \qquad\qquad R_i = \operatorname{cov}(\boldsymbol{v}_i)$$

where R_i is the covariance of the noise v_i

In this presentation $H_i = I$ is assumed for simplicity, i.e., $y_i \in \mathbb{R}^{n_x}$

linear model, but not necessarily Cartesian!

Track Fusion: Optimal Method

Consider:

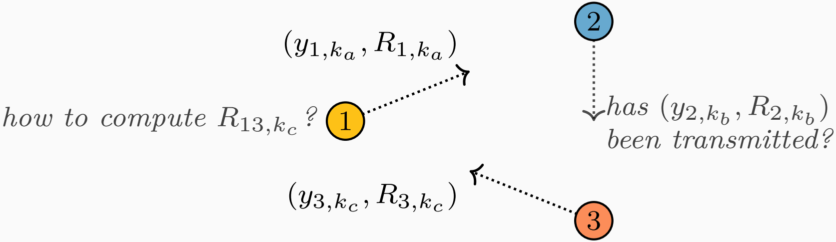
- (y_1, R_1) and (y_2, R_2) are to be fused
- $R_{12} = R_{21}^{\mathsf{T}} = \operatorname{cov}(\boldsymbol{y}_1, \boldsymbol{y}_2)$ is the cross-covariance between the estimates

An optimal fusion method is given by:

$$\hat{x} = K_1 y_1 + K_2 y_2 \qquad\qquad P = R_1 - K_2 S K_2^{\mathsf{T}}$$

where $K_1 = I - K_2$, $K_2 = (R_1 - R_{12})S^{-1}$, and $S = R_1 + R_2 - R_{12} - R_{12}^{\mathsf{T}}$

Why not use the optimal fusion method? R_{12} is unknown!



Track Fusion: Naive Solution

The naive solution: assume that $R_{12} = 0$

Optimal fusion given that $R_{12} = 0$:

$$\hat{x} = P\left(R_1^{-1} y_1 + R_2^{-1} y_2\right) \qquad P = \left(R_1^{-1} + R_2^{-1}\right)^{-1}$$

If $R_{12} \neq 0$, the uncertainty P is underestimated — double counting of information

Track Fusion: Conservative Estimators

Issues:

- unknown correlations
- if nonzero correlations are neglected the uncertainty P is underestimated

Possible solution: *conservative estimators*

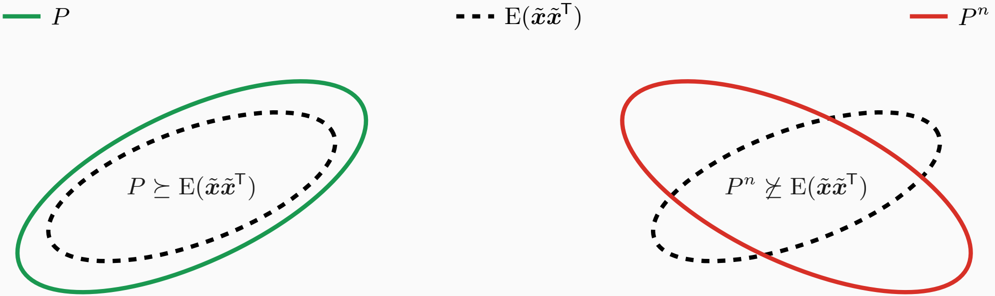
Conservative Estimate

An estimate $(\hat{x}, P)$ of x is *conservative* if

$$P - \mathrm{E}(\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^{\mathsf{T}}) \succeq 0$$

where $\tilde{\boldsymbol{x}} = \hat{\boldsymbol{x}} - x$ is the error

Conservative and Non-Conservative Estimates

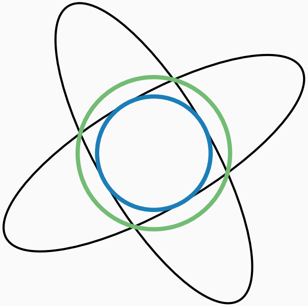


Track Fusion: Conservative Methods

Task: Fuse (y_1, R_1) and (y_2, R_2), where R_{12} is unknown

Conservative fusion methods:

- covarance intersection
- **largest ellipsoid method**
- inverse covariance intersection
- split covariance intersection
- $\cdots$



Covariance Intersection

The estimates are fused using *covariance intersection* (CI) according to

$$\hat{x} = P\left(\omega R_1^{-1} y_1 + (1-\omega) R_2^{-1} y_2\right) \qquad P = \left(\omega R_1^{-1} + (1-\omega) R_2^{-1}\right)^{-1}$$

where $\omega \in [0,1]$ is computed by solving

$$\underset{\omega}{\text{minimize}} \quad J(P)$$

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Similar in structure to the naive fusion method:

$$\hat{x} = P\left(R_1^{-1} y_1 + R_2^{-1} y_2\right) \qquad P = \left(R_1^{-1} + R_2^{-1}\right)^{-1}$$

Largest Ellipsoid Method

The estimates are fused using the *largest ellipsoid* (LE) method according to

1. Factorize $R_1 = U_1 \Sigma_1 U_1^\mathsf{T}$ and let $T_1 = \Sigma_1^{-\frac{1}{2}} U_1^\mathsf{T}$. Factorize $T_1 R_2 T_1^\mathsf{T} = U_2 \Sigma_2 U_2^\mathsf{T}$ and let $T_2 = U_2^\mathsf{T}$.
2. Transform using $T = T_2 T_1$ according to

$$z_1 = T y_1 \qquad C_1 = T R_1 T^\mathsf{T} = I \qquad z_2 = T y_2 \qquad C_2 = T R_2 T^\mathsf{T}$$

3. For each $i = 1, \ldots, n_x$, compute

$$([z]_i, [C]_{ii}) = \begin{cases} ([z_1]_i, 1)\,, & \text{if } 1 \leq [C_2]_{ii}, \\ ([z_2]_i, [C_2]_{ii})\,, & \text{if } 1 > [C_2]_{ii}. \end{cases}$$

4. Transform back:

$$\hat{x} = T^{-1} z \qquad P = T^{-1} C T^{-\mathsf{T}}$$

Monte Carlo Evaluation

Monte Carlo (MC) based approach for evaluation:

1. Specify local sensors, local state estimation filters, and communication pattern.
2. Specify the considered track fusion methods.
3. Define a metrics for tracking performance and conservativeness.
4. Define characteristic target trajectories.
5. Tune the local filters for the characteristic trajectories.
6. Using MC simulations, evaluate each fusion method with respect to performance and conservativeness.

An estimate at the ith MC run at time k is denoted $(\hat{x}_k^i, P_k^i)$

Performance Evaluation

Root mean squared error (RMSE) is a common measure for performance

- requires the true state to be known — cannot be computed online

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Root Mean Trace

The sampled *root mean trace* (RMT) at time k is defined as

$$\text{RMT}_k = \sqrt{\frac{1}{M}\sum_{i=1}^{M} \operatorname{tr}(P_k^i)}$$

Conservativeness Evaluation

Since $P = LL^\mathsf{T} \succ 0$, the Cholesky factor L is invertible such that

$$P \succeq \mathrm{E}(\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^\mathsf{T}) \iff I \succeq L^{-1}\,\mathrm{E}(\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^\mathsf{T})L^{-\mathsf{T}}$$

Hence $(\hat{x}, P)$ is conservative iff

$$\lambda_{\max}\left(L^{-1}\,\mathrm{E}(\tilde{\boldsymbol{x}}\tilde{\boldsymbol{x}}^\mathsf{T})L^{-\mathsf{T}}\right) \leq 1$$

$\lambda_{\max}(A)$ denotes the largest eigenvalue of a matrix A

Conservativeness Evaluation

Conservativeness Index

The sampled *conservativeness index* (COIN) at time k is defined as

$$\mathsf{COIN}_k = \lambda_{\max}\left(\underbrace{\frac{1}{M}\sum_{i=1}^{M}(L_k^i)^{-1}\tilde{x}_k^i(\tilde{x}_k^i)^{\mathsf{T}}(L_k^i)^{-\mathsf{T}}}_{\mathcal{C}_k}\right)$$

where $L_k^i(L_k^i)^{\mathsf{T}} = P_k^i$, $\tilde{x}_k^i$ is the error in the ith MC run, and $\mathcal{C}_k$ is the sampled normalized estimation error squared matrix

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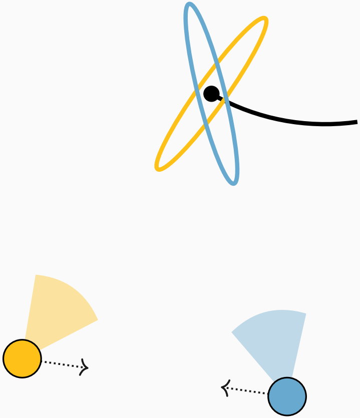
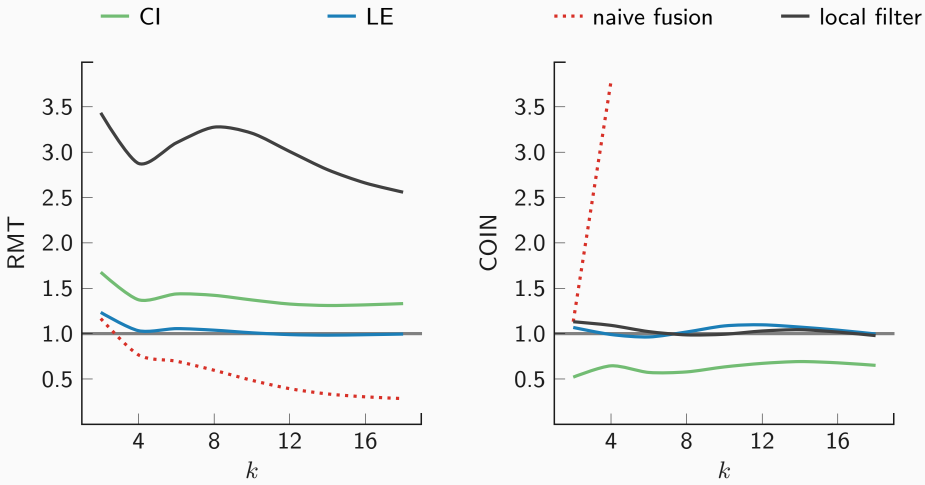
Want COIN_k to be smaller than or equal to 1

Evaluation Scenario

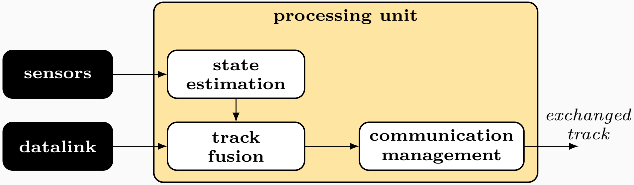


Evaluation Results



Part II: Communication Management Design and Implementation

Communication Management: Data Reduction



Two main approaches:

- diagonal covariance approximation (DCA)
- dimension-reduction (DR)

Diagonal Covariance Approximation

Problem:

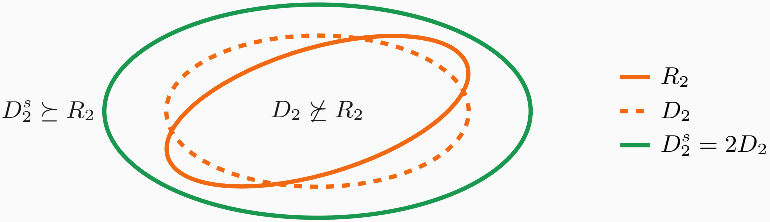
- Agent 2 is about to transmit (y_2, R_2) to Agent 1
- limited communication capacity: the data (y_2, R_2) must be reduced

Observation: y_i scales as n_x and R_i as n_x^2

Simple solution: exchange (y_2, D_2) where D_2 is diagonal — essentially an n_x-dimensional vector

Diagonal Covariance Approximation: Example

Let $R_2 = \left[\begin{smallmatrix} 4 & 1 \\ 1 & 1 \end{smallmatrix}\right]$ and $D_2 = \left[\begin{smallmatrix} 4 & 0 \\ 0 & 1 \end{smallmatrix}\right]$



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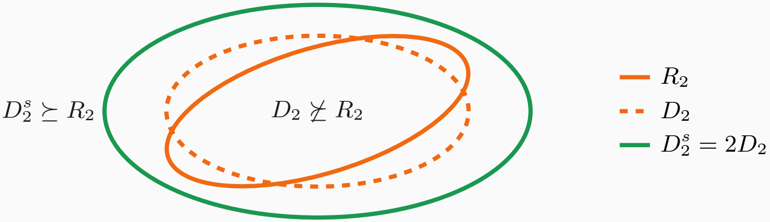


Agent 2 preserves conservativeness if (y_2, D_2^s) is exchanged

Diagonal Covariance Approximation: Two Options

Two options are considered:

- Agent 2 transmits (y_2, D_2^s) to Agent 1, where $D_2^s \succeq R_2$. In this case, Agent 2 has already preserved conservativeness, and hence, Agent 1 can use the received estimate directly without any additional action.
- Agent 2 transmits (y_2, D_2) to Agent 1. In this case, Agent 1 must explicitly handle that $D_2 \not\succeq R_2$ to ensure conservativeness after track fusion.

Diagonal Covariance Approximation: Eigenvalue Based Scaling

Eigenvalue Based Scaling

$$\begin{aligned} \underset{s}{\text{minimize}} \quad & s \\ \text{subject to} \quad & D_2^s = sD_2 \succeq R_2. \end{aligned}$$

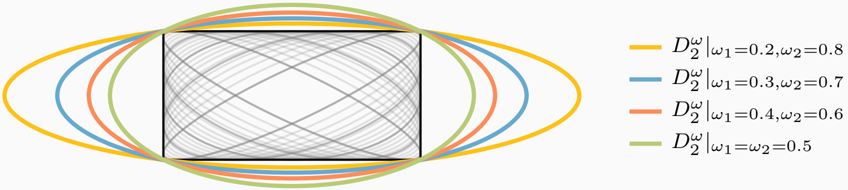
The solution is

$$s^\star = \lambda_{\max}(D_2^{-\frac{1}{2}} R_2 D_2^{-\frac{1}{2}})$$

Diagonal Covariance Approximation: Hyperrectangle Enclosing

Agent 1 receives (y_2, D_2) from Agent 2

- Assume $R_2 = \left[\begin{smallmatrix} 4 & 1 \\ 1 & 1 \end{smallmatrix}\right]$ such that $D_2 = \left[\begin{smallmatrix} 4 & 0 \\ 0 & 1 \end{smallmatrix}\right]$



The parametrization given by

$$D_2^\omega = \begin{bmatrix} \frac{4}{\omega_1} & 0 \\ 0 & \frac{1}{\omega_2} \end{bmatrix}$$

where $\omega_i > 0$ and $\sum_i \omega_i = 1$

Dimension Reduction: Basic Idea

Instead of transmitting (y_2, R_2) Agent 2 can transmit (y_Ψ, R_Ψ) where

$$y_\Psi = \Psi y_2 \qquad\qquad R_\Psi = \Psi R_2 \Psi^\mathsf{T}$$

and $\Psi \in \mathbb{R}^{m \times n_x}$ is a "wide matrix", i.e., $m < n_x$

This is a *dimension reduction* problem

Dimension Reduction: Designing Ψ

How to choose Ψ? Optimize for fusion performance!

Assume that (y_1, R_1) and (y_Ψ, R_Ψ) are fused according to

$$\hat{x} = P\left(R_1^{-1} y_1 + \Psi^\mathsf{T} R_\Psi^{-1} y_\Psi\right) \qquad P = \left(R_1^{-1} + \Psi^\mathsf{T} R_\Psi^{-1} \Psi\right)^{-1}$$

which is optimal given that the estimates are uncorrelated

Dimension Reduction: Designing Ψ

Fusion Optimal Dimension Reduction

A fusion optimal $\Psi^\star$ is computed by solving

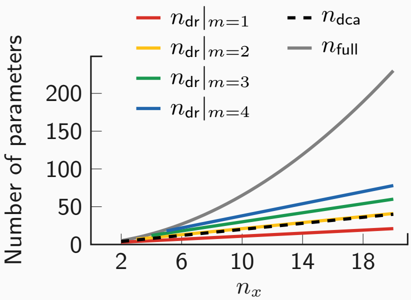
$$\underset{\Psi}{\text{minimize}} \quad \mathrm{tr}(P).$$

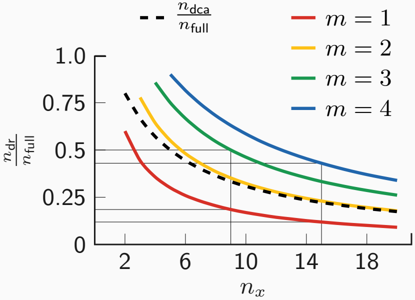
where $P = \left(R_1^{-1} + \Psi^\mathsf{T} R_\Psi^{-1} \Psi\right)^{-1}$

The solution is given by an eigenvalue problem!

Communication Reduction

Let n_{dca}, n_{dr}, and n_{full} denote the number of parameters to be transmitted using DCA, DR, and full estimates, respectively





Summary

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Excluded material:

- the CLUE framework
- common information estimate — keeping track of network common information
- practical and theoretical aspects related to the data reduction techniques

Related resources: `https://github.com/robinforsling/dtt/`

- MATLAB® library source code for all examples and simulation
- thesis summary
- posters, papers, bibliography, figures