Diagnostic Test: Functions

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1. The graph of a function f is given at the left.

(a) State the value of $f(-1)$.

(b) Estimate the value of $f(2)$.

(c) For what values of x is $f(x) = 2$?

(d) Estimate the values of x such that $f(x) = 0$.

(e) State the domain and range of f.

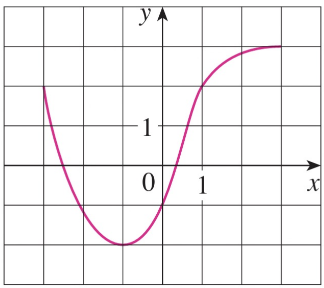


FIGURE FOR PROBLEM 1

a) $f(-1)$; when $x = -1$, $y = -2$

b) $f(2) = 2.8$

c) $f(x) = 2$, the value of x when y is 2 is horizontal line drawn at $y = 2$ and points it touches the curve

$\therefore$ here at $x = 1, -3$

d) $f(x) = 0$, when $y = 0$, x are -2.5 & ~ 0.4

e) Domain x : $[-3, 3]$ $-3 \leq x \leq 3$

Range y : $[-2, 3]$ $-2 \leq y \leq 3$

2. If $f(x) = x^3$, evaluate the difference quotient $\dfrac{f(2+h) - f(2)}{h}$ and simplify your answer.

3. Find the domain of the function.

(a) $f(x) = \dfrac{2x+1}{x^2 + x - 2}$ (b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2+1}$ (c) $h(x) = \sqrt{4-x} + \sqrt{x^2 - 1}$

2. given $f(x) = x^3$

ie when $y = x^3$

$f(2+h) = (2+h)^3$

$f(2) = 2^3$

$$\frac{f(2+h) - f(2)}{h}$$

$$= \frac{(2+h)^3 - 8}{h}$$

$$= \frac{2^3 + h^3 + 6h(2+h) - 8}{h}$$

$$= \frac{8 + h^3 + 12h + 6h^2 - 8}{h}$$

$$= \frac{h^3 + 12h + 6h^2}{h}$$

$$= h^2 + 6h + 12$$

$$\boxed{(a+b)^3 = a^3 + b^3 + 3ab(a+b)}$$

0				1				
1			1		1			
2		1		2		1		
3	1		3		3		1	

$^3C_0 \quad ^3C_1 \quad ^3C_2 \quad ^3C_3$

Deriving using Pascals triangle as practice

$^3C_0 a^n b^0 + {}^3C_1 a^{n-1} b^1 + {}^3C_2 a^{n-2} b^2 + {}^3C_3 a^{n-3} b^3$

$1 \cdot a^3 b^0 + 3 \cdot a^2 b^1 + 3a^1 b^2 + 1a^0 b^3$

$= a^3 + 3a^2 b + 3ab^2 + b^3$

3.a) $f(x) = \dfrac{2x+1}{x^2+x-2}$

Denominator: $x^2 + x - 2 = 0$

-2

$-1 \quad 2$

$$x^2 - x + 2x - 2 = 0$$
$$x(x-1) + 2(x-1) = 0$$
$$(x-1)(x+2) = 0$$
$$x = 1 \qquad x = -2$$

$\therefore$ Domain $x \in R$ where $x \neq 1, 2$

$$(-\infty, -2) \cup (-2, 1) \cup (1, \infty)$$

b) $g(x) = \dfrac{\sqrt[3]{x}}{x^2+1}$

denominator $x^2 + 1 = 0$

D: $(-\infty, \infty)$

$x\left(x + \frac{1}{x}\right) = 0$

$x = 0 \qquad x + \frac{1}{x} = 0$

$x = -\frac{1}{x}$

$x^2 = -1 \qquad x = i$

c) $h(x) = \sqrt{4-x} + \sqrt{x^2-1}$

Root of a -ve number is imaginary ie, it is not defined as real number. The domain consists of all values such that the value inside root equated to ≥ 0.

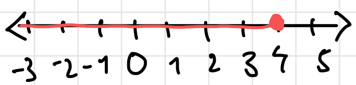
$$4 - x \geq 0 \quad \& \quad x^2 - 1 \geq 0$$

$$4 \geq x$$

Here we have the domain as

$$x \leq 4$$

ie all the vallues less than or equal to 4



Domain for $\sqrt{4-x}$

$(-\infty, 4]$

$$x^2 - 1 \geq 0$$

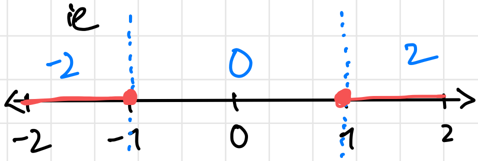
$$(x+1)(x-1) \geq 0$$

when equating to 0 we have

$$(x+1)(x-1) = 0$$

$$x = -1 \quad \& \quad x = +1$$

here $f(x) \geq 0$ ∴ we look for +ve values ie



when	$x+1$	$x-1$	$(x+1)(x-1)$
-2	−	−	+
0	+	−	−
2	+	+	+

we take the +ve regions

$(-\infty, -1] \cup [1, \infty)$

is the domain for $\sqrt{x^2-1}$

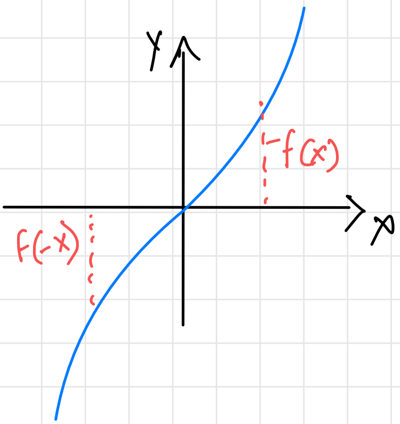
∴ Domain of complete function $\sqrt{4-x} + \sqrt{x^2-1}$ is

$$(-\infty, -1] \cup [1, 4]$$

4. How are graphs of the functions obtained from the graph of f?

(a) $y = -f(x)$ (b) $y = 2f(x) - 1$ (c) $y = f(x - 3) + 2$

a)
$y = -f(x)$
$f(x) = -f(x)$
∴ it this is an odd function
the graph would be on opposite sides of the x-axis, top & bottom



b) $2f(x)$ means the function is stretched twice ie mutiplied/magnified and shifted by -1 ie all data points are shifted 1 unit down

c) $f(x-3)$ means the original function is shifted 3 units to the right
+2 would indicate the whole function is shifted upwards by 2 units.

5. Without using a calculator, make a rough sketch of the graph.

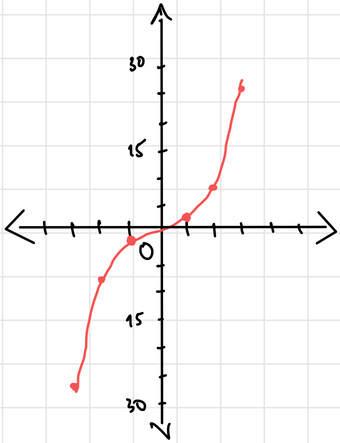
(a) $y = x^3$ (b) $y = (x+1)^3$ (c) $y = (x-2)^3 + 3$

(d) $y = 4 - x^2$ (e) $y = \sqrt{x}$ (f) $y = 2\sqrt{x}$

(g) $y = -2^x$ (h) $y = 1 + x^{-1}$

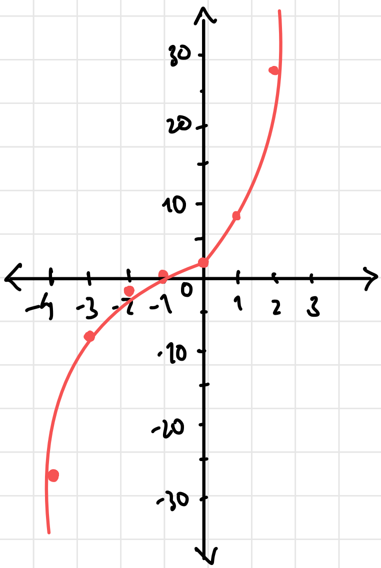
a) $y = x^3$

x	-3	-2	-1	0	1	2	3
y	-27	-8	-1	0	1	8	27



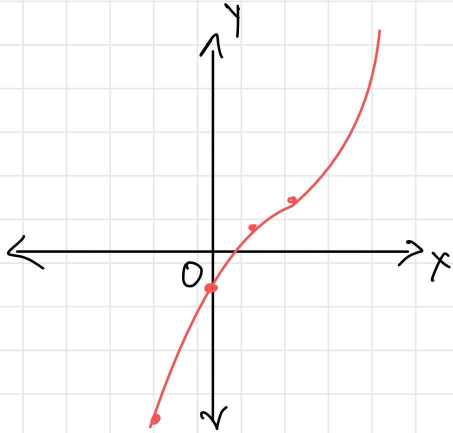
b) $y = (x+1)^3$

x	y
-3	-8
-2	-1
-1	0
0	1
1	8
2	27
3	64



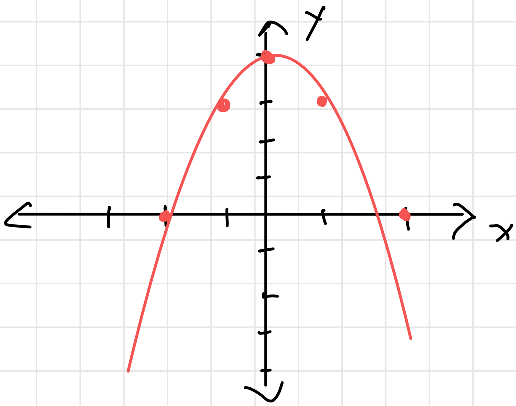
c) $y = (x-2)^3 + 3$

x	y
-3	-122
-2	-61
-1	-24
0	-5
1	2
2	3
3	4
5	30



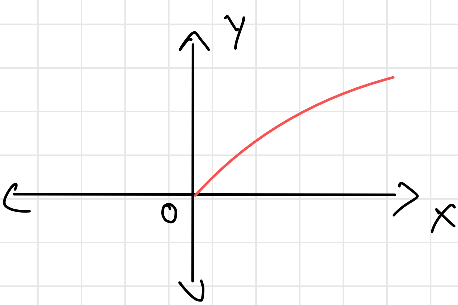
d) $y = 4 - x^2$

x	y
-2	0
-1	3
0	4
1	3
2	0



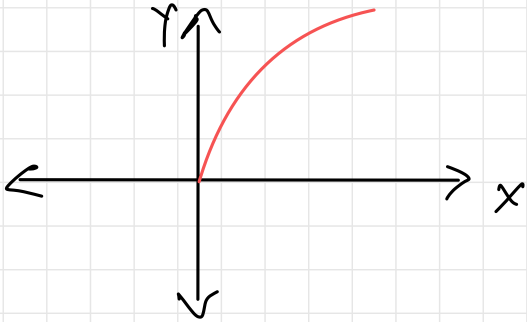
e) $y = \sqrt{x}$

x	y
0	0
1	1
2	$\sqrt{2}$
3	$\sqrt{3}$



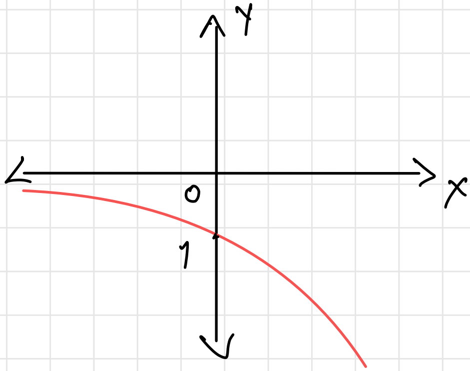
f) $y = 2\sqrt{x}$

stretched & multiplied graph of $\sqrt{x}$



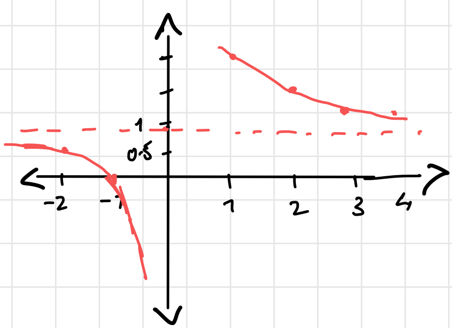
g) $y = -2^{x}$

x	y
-2	$-1/4$
-1	$-1/2$
0	1
1	-2
2	-4
3	$-1 \cdot 8 = -8$
4	$-1 \cdot 16 = -16$



h) $y = 1 + x^{-1}$

x	y
-2	$1 - 1/2 = 0.5$
-1	$1 - 1 = 0$
0	1
1	$1 + 1^{-1} = 2$
2	$1 + \frac{1}{2} = 1.5$
3	1.33
4	1.25



6. Let $f(x) = \begin{cases} 1 - x^2 & \text{if } x \leqslant 0 \\ 2x + 1 & \text{if } x > 0 \end{cases}$

(a) Evaluate $f(-2)$ and $f(1)$. (b) Sketch the graph of f.

7. If $f(x) = x^2 + 2x - 1$ and $g(x) = 2x - 3$, find each of the following functions.

(a) $f \circ g$ (b) $g \circ f$ (c) $g \circ g \circ g$

6. a) $f(-2)$ ie when $x = -2$

here $x < 0$ $\therefore$ $f(x) = 1 - x^2$

$f(-2) = 1 - (-2)^2 = 1 - 4 = -3$

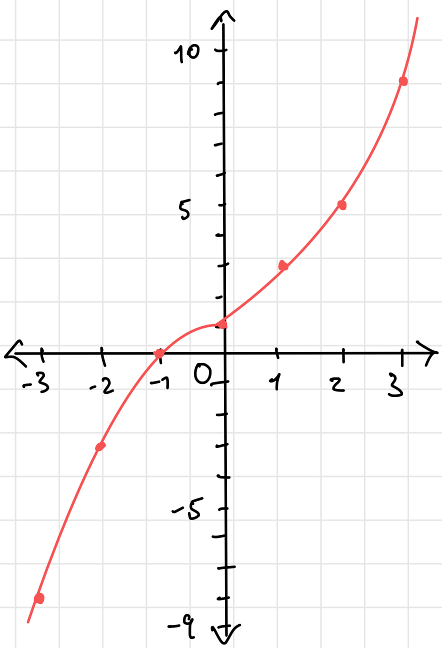
$f(1)$ ie when $x = 1$

$x > 0$ $\therefore$ $f(x) = 2x + 1$

$f(1) = 2(1) + 1 = 3$

b)

x	-3	-2	-1	0
$1-x^2$	-8	-3	0	1
x	1	2	3	4
$2x+1$	3	5	7	9



7.a) $f \circ g \Rightarrow f(g(x))$

$f(x) = x^2 + 2x + 1$

$f(g(x)) = f(2x-3)$

$= (2x-3)^2 + 2(2x-3) + 1$

$= 4x^2 + 9 - 12x + 4x - 6 + 1$

$= 4x^2 + 3 - 8x + 1$

$= 4x^2 - 8x + 4$

7b) $g \circ f \quad g(f(x))$

$g(x^2 + 2x - 1)$

$= 2(x^2 + 2x - 1) - 3$

$= 2x^2 + 4x - 2 - 3$

$= 2x^2 + 4x - 5$

7c) $g \circ g \circ g \Rightarrow g(g(g(x)))$

$g(x) = 2x - 3$

$g(g(x)) = 2(2x-3) - 3$

$g(g(g(x))) = 2[2(2x-3) - 3] - 3$

$= 2[4x - 6 - 3] - 3$

$= 2[4x - 9] - 3 = 8x - 18 - 3$

$= 8x - 21$