Diagnosis Test: Analytical Geometry

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1. Find an equation for the line that passes through the point $(2, -5)$ and

(a) has slope -3

(b) is parallel to the x-axis

(c) is parallel to the y-axis

(d) is parallel to the line $2x - 4y = 3$

$y = mx + c$

where m is the slope

c is the intercept

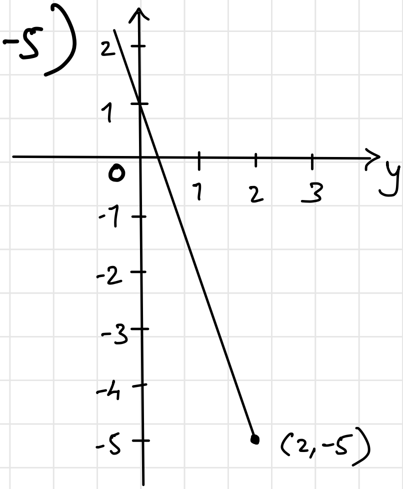
a) given $m = -3$ & point $(2, -5)$

$-5 = -3(2) + C$

$-5 = -6 + C$

$C = 1$

$\therefore y = -3x + 1$

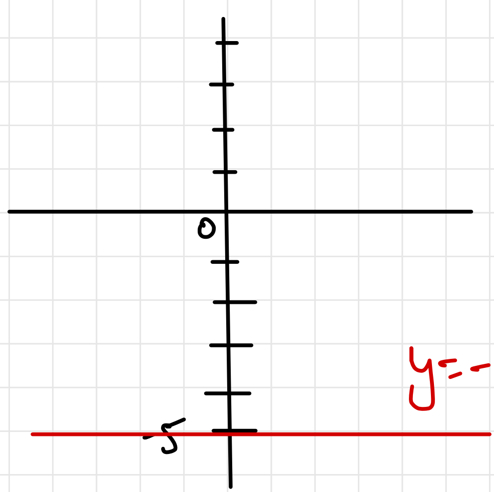


b) Parallel to x axis

is the y value

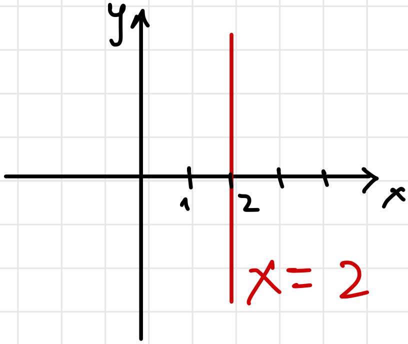
of the point away

from x axis



$y = -5$

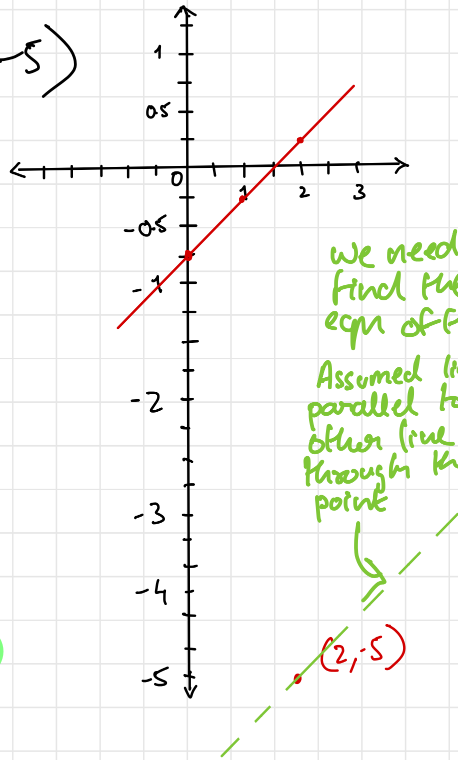
c) parallel to y axis is the x value of the point away from y axis



d) parallel to the line $2x - 4y = 3$ & point $(2, -5)$

$$4y = 2x - 3$$

$$y = \frac{1}{2}x - \frac{3}{4}$$



we need to find the eqn of the line

Assumed line parallel to the other line & through the point

The intercept is at $-\frac{3}{4} = -0.75$ &

$m = \frac{1}{2}$

when $x = 1$; $y = 0.5 - 0.75 = -0.25$

$x = 2$; $y = 1 - 0.75 = 0.25$

Given our line is parallel to the line $2x - 4y = 3$

This means that all parallel lines have the same slope

∴ To construct a new line we need that slope and new data point ie here $(2, -5)$

$$y = \frac{1}{2}x + c$$

$$-5 = \frac{1}{2}(2) + c$$

$$-6 = c$$

$$\therefore \quad y = \frac{1}{2}x - 6$$

2. Find an equation for the circle that has center $(-1, 4)$ and passes through the point $(3, -2)$.

3. Find the center and radius of the circle with equation $x^2 + y^2 - 6x + 10y + 9 = 0$.

2. center $(-1, 4)$ $\quad C(h, k)$
point $(3, -2)$ $\quad P(x, y)$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(3+1)^2 + (-2-4)^2 = r^2$$
$$16 + 36 = r^2$$
$$r = \sqrt{52} \qquad r^2 = 52$$

eqn: $(x+1)^2 + (y-4)^2 = 52$

3. $x^2 + y^2 - 6x + 10y + 9 = 0$

$(x-h)^2 + (y-k)^2 = r^2$ ⇐ This is the form we need

∴ we need to complete the squares on the question eqn to get that form after grouping

$$(x^2 - 6x) + (y^2 + 10y) = -9$$

$-6x = 2ax$
$2a = -6$
$a = -3$
$a^2 = 9$

$10y = 2ay$
$2a = 10$
$a = 5$
$a^2 = 25$

$x^2 - 6x + 9 - 9 + y^2 + 10y + 25 - 25 = -9$

$(x-3)^2 - 9 + (y+5)^2 - 25 = -9$

$(x-3)^2 + (y+5)^2 = 25$ $\quad \therefore r = 5$ as $r^2 = 25$ here

center $(h,k) \Rightarrow (-3, 5)$

$r \Rightarrow 5$

4. Let $A(-7, 4)$ and $B(5, -12)$ be points in the plane.

(a) Find the slope of the line that contains A and B.

(b) Find an equation of the line that passes through A and B. What are the intercepts?

(c) Find the midpoint of the segment AB.

(d) Find the length of the segment AB.

(e) Find an equation of the perpendicular bisector of AB.

(f) Find an equation of the circle for which AB is a diameter.

4.a) $A(-7, 4)$ $\quad B(5, -12)$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-12 - 4}{5 + 7} = \frac{-16}{12} = \frac{-4}{3}$$

b) $(y - y_1) = m(x - x_1)$ is the eqn for slope & a point we have slope $= -\frac{4}{3}$ and a point $(-7, 4)$

$$y - 4 = -\frac{4}{3}(x + 7)$$

$$3y - 12 = -4x - 28$$

$$3y + 4x = -16 \longrightarrow (1)$$

$$y = -\frac{4}{3}x - \frac{16}{3} \qquad m = -\frac{4}{3} \qquad b = -\frac{16}{3}$$ which is the y intercept

For x intercept we take $y=0$ in (1)

$$0 + 4x = -16$$

$$x = -4$$

c) Mid point of line segment

$A(-7, 4)$ $B(5, -12)$

$$\left(\frac{-7+5}{2}, \frac{4+-12}{2}\right) = (-1, -4)$$

d) length $AB = \sqrt{(5+7)^2 + (-12-4)^2}$

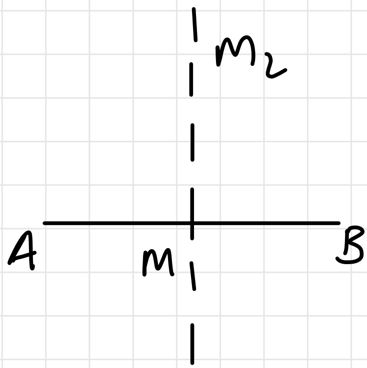
$$= \sqrt{12^2 + 16^2} = \sqrt{400} = 20$$

e) Find the eqn of $\perp$r bisector of AB

$\perp$r bisector means line perpendicular to AB $\therefore$ it has opposite slope sign

we initially found the slope as $-\frac{4}{3}$

$\therefore$ $\perp$r bisector line slope is



$$m_1 m_2 = -1$$

$$-\frac{4}{3} \cdot m_2 = -1$$

$$m_2 = \frac{3}{4}$$

midpoint $M(-1, -4)$

eqn of $\perp$r is:

$$y - -4 = \frac{3}{4}(x - -1)$$

$$4(y+4) = 3(x+1)$$

$$4y + 16 = 3x + 3$$

$$4y - 3x = -13$$

$$y = \frac{3x}{4} - \frac{13}{4}$$

f) ie from the midpoint we can draw a $\perp r$ line that bisects AB $\therefore$ it can form a circle or radius 'r'

$\therefore$ we use circle eqn:

$C(h,k) \quad P(x,y)$

$$(x-h)^2 + (y-k)^2 = r^2$$

here C is the midpoint $(-1,-4)$

a point is either A or B, we take $A(-7,4)$

$$\therefore r = \sqrt{(-7+1)^2 + (4+4)^2}$$

$$= \sqrt{36+64} = \sqrt{100} \quad \therefore r^2 = 100$$

eqn: $(x+1)^2 + (y+4)^2 = 100$

5. Sketch the region in the xy-plane defined by the equation or inequalities.

(a) $-1 \leqslant y \leqslant 3$

(b) $|x| < 4$ and $|y| < 2$

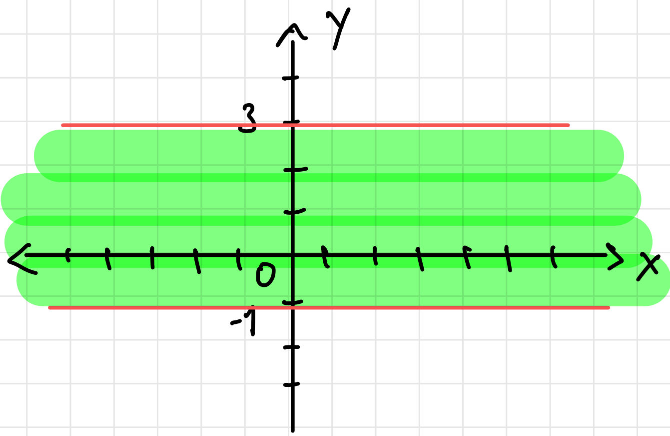
(c) $y < 1 - \frac{1}{2}x$

(d) $y \geqslant x^2 - 1$

(e) $x^2 + y^2 < 4$

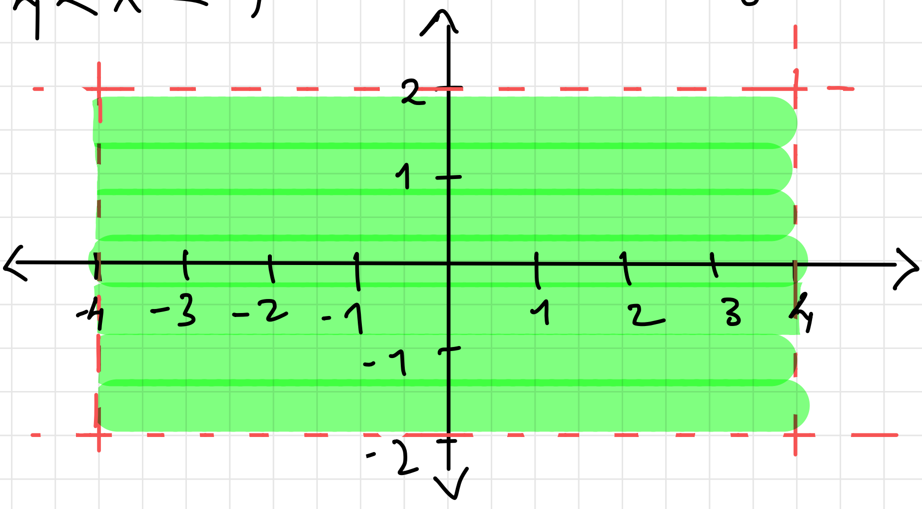
(f) $9x^2 + 16y^2 = 144$

a)



b) $|x| < 4$ & $|y| < 2$

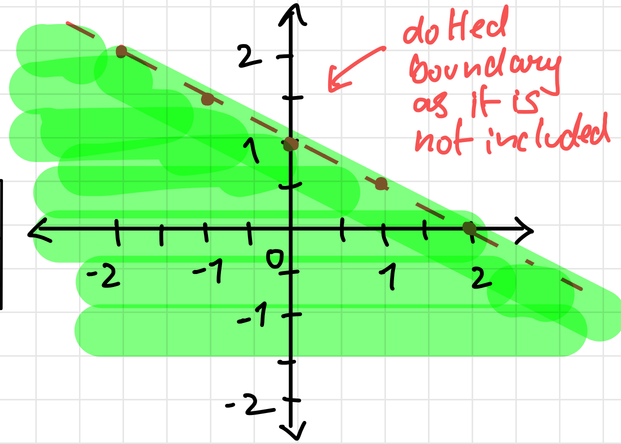
$-4 < x < 4$ $-2 < y < 2$



Dotted lines as they are not included

c) $y < 1 - \frac{1}{2}x$

x	-2	-1	0	1	2
y	2	1.5	1	0.5	0

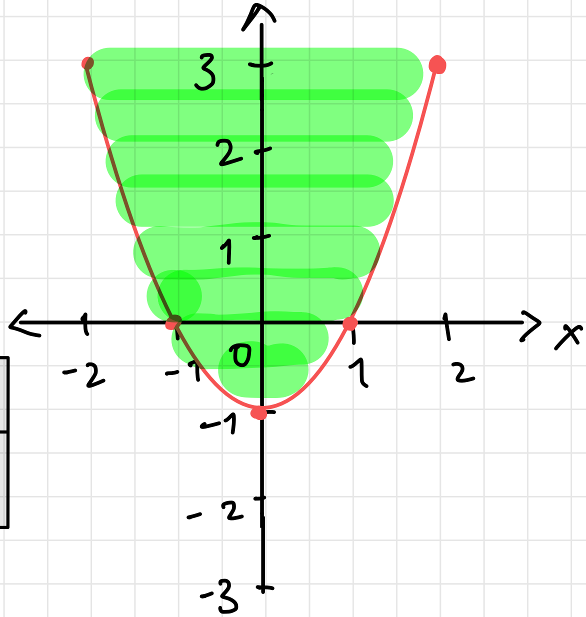


The corresponding y values when we solve $y = 1 - \frac{1}{2}x$ and all the values of the function on the left side as we have '<'

d) $y \geq x^2 - 1$

$y = x^2 - 1$

x	-2	-1	0	1	2
y	3	0	-1	0	3

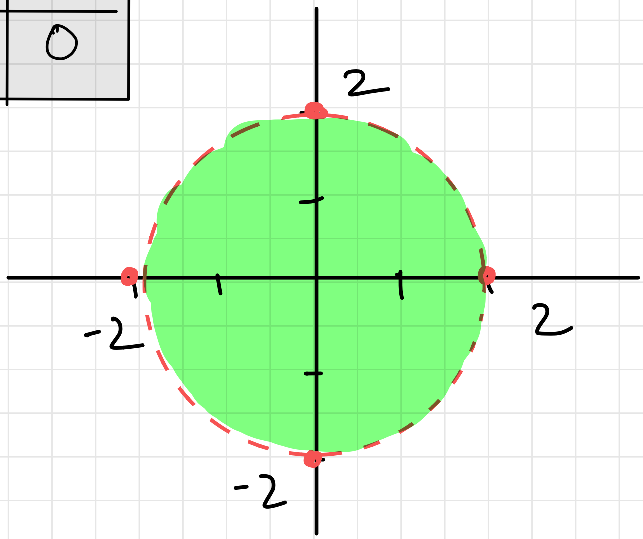


e) $x^2 + y^2 < 4$

$y^2 = 4 - x^2$

Notice this is an eqn of a circle.

x	-2	-1	0	1	2
y	0	$\pm\sqrt{3}$	± 2	$\pm\sqrt{3}$	0



f) $9x^2 + 16y^2 = 144$

eqn of ellipse with centre $c(0,0)$

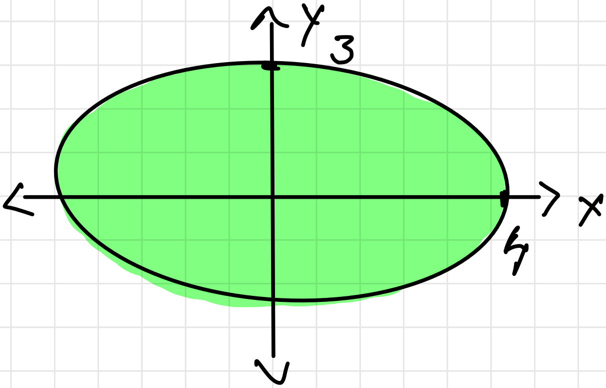
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \qquad \text{where } a > b$$

$$\therefore \quad \frac{9x^2}{144} + \frac{16y^2}{144} = 1$$

$$\frac{x^2}{144/9} + \frac{y^2}{144/16} = 1$$

here $\frac{\cancel{144}\ \cancel{48}\ 16}{\cancel{9}\ 3\ 1}$ $\qquad$ $\frac{\cancel{144}\ \cancel{72}\ \cancel{36}\ 9}{\cancel{16}\ 8\ 4\ 1}$

$$\frac{x^2}{16} + \frac{y^2}{9} = 1$$



a is under x

$\therefore$ horizontal

major axis is x

$a^2 = 16 \quad a = 4$

$b^2 = 9 \quad b = 3$