Diagnosis Test: Algebra

Try later:

8.9, 6.b, 9.b

R. J

github.com/royceanton

1. Evaluate each expression without using a calculator.

(a) $(-3)^4$ (b) -3^4 (c) 3^{-4}

(d) $\dfrac{5^{23}}{5^{21}}$ (e) $\left(\dfrac{2}{3}\right)^{-2}$ (f) $16^{-3/4}$

a) $(-3)^2 \cdot (-3)^2$
$= 9 \cdot 9 = 81$

b) $-3^4 = -1 \cdot 3^4$
$= -81$

c) $3^{-4} = \dfrac{1}{3^4} = \dfrac{1}{81}$

d) $\dfrac{5^{23}}{5^{21}} = 5^{23-21}$
$= 5^2 = 25$

e) $\left(\dfrac{2}{3}\right)^{-2} = \left(\dfrac{3}{2}\right)^2 = \dfrac{9}{4}$

f) $16^{-3/4} = \left(4^2\right)^{-3/4}$
$= (4)^{-3/2} = \left(2^2\right)^{-3/2}$
$= 2^{-3} = \dfrac{1}{8}$

2. Simplify each expression. Write your answer without negative exponents.

(a) $\sqrt{200} - \sqrt{32}$

(b) $(3a^3b^3)(4ab^2)^2$

(c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2}$

a) $\sqrt{200} = \begin{array}{r|l} 2 & 200 \\ 2 & 100 \\ 2 & 50 \\ 5 & 25 \\ & 5 \end{array} = 2 \cdot 5 \cdot \sqrt{2} = 10\sqrt{2}$

$\sqrt{32} = \begin{array}{r|l} 2 & 32 \\ 2 & 16 \\ 2 & 8 \\ 2 & 4 \\ & 2 \end{array} = 2 \cdot 2 \cdot \sqrt{2} = 4\sqrt{2}$

$10\sqrt{2} - 4\sqrt{2} = 6\sqrt{2}$

b) $(3a^3b^3)(4ab^2)^2$

$= 3a^3 \cdot b^3 \cdot 16a^2b^4$

$= 3a^5 \cdot b^7 \cdot 16 = 48a^5b^7$

c) $\left(\dfrac{3x^{3/2}y^3}{x^2y^{-1/2}}\right)^{-2} = \left(3x^{\frac{3}{2}-2}\, y^{3+\frac{1}{2}}\right)^{-2}$

$= 3x^{-\frac{1}{2}} \cdot y^{7/2})^{-2}$

$= \dfrac{1}{9} \cdot \dfrac{1}{(x^{-1/2})^2} \cdot \dfrac{1}{(y^{7/2})^2}$

$= \dfrac{1}{9} \cdot \dfrac{1}{x^{-1}} \cdot \dfrac{1}{y^7} = \dfrac{x}{9y^7}$

3. Expand and simplify.

(a) $3(x + 6) + 4(2x - 5)$

(b) $(x + 3)(4x - 5)$

(c) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b})$

(d) $(2x + 3)^2$

(e) $(x + 2)^3$

a) $3x + 18 + 8x - 20$

$= 11x - 2$

b) $(x+3)(4x-5)$

$= 4x^2 - 5x + 12x - 15$

$= 4x^2 - 7x - 15$

c) $a - b$

d) $(2x)^2 + (3)^2 + 12x$

$= 4x^2 + 9 + 12x$

$= 4x^2 + 12x + 9$

d) $(x+2)^3 = x^3 + 2^3 + 6x(x+2)$

$= x^3 + 2^3 + 6x^2 + 12x$

$= x^3 + 6x^2 + 12x + 8$

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* $(a+b)(a-b) = a^2 - b^2$
* $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
* $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

4. Factor each expression.

(a) $4x^2 - 25$

(b) $2x^2 + 5x - 12$

(c) $x^3 - 3x^2 - 4x + 12$

(d) $x^4 + 27x$

(e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

(f) $x^3y - 4xy$

a) $2^2x^2 - 5^2 = (2x)^2 - (5)^2 = (2x+5)(2x-5)$

b) $\left.\begin{array}{l} \underline{8} \times \underline{-3} = -24 \\ \underline{8} + \underline{-3} = 5 \end{array}\right\}$ $2x^2+8x-3x-12$

$= 2x(x+4)-3(x+4)$

$= (x+4)(2x-3)$

c) $x^3-3x^2-4x+12$

When $x=2 \Rightarrow 8-12-8+12=0$

$\therefore$ we know $(x-2)=0$ is a factor

$$\begin{array}{r|l} & x^2 - x - 6 \\ \hline x-2 & x^3 - 3x^2 - 4x + 12 \\ & x^3 - 2x^2 \quad (-) \\ \hline & -x^2 - 4x \\ & -x^2 + 2x \quad (-) \\ \hline & -6x + 12 \\ & -6x + 12 \\ \hline & 0 \end{array}$$

$x^2 - x - 6 =$

$x^2-3x+2x-6$

$x(x-3)+2(x-3)$

$(x-3)(x+2)$

$(x-2)(x-3)(x+2)$

d) $x^4+27x = x(x^3+27) = x(x^3+3^3)$

$(a+b)^3 = a^3+b^3+3ab(a+b)$

$\therefore \quad a^3+b^3 = (a+b)^3 - 3ab(a+b)$

$x((x+3)^3 - 9x(x+3))$

$x(x+3)((x+3)^2 - 9x)$

$x(x+3)(x^2+6x+9-9x)$

$x(x+3)(x^2-3x+9)$

e) $3x^{3/2} - 9x^{1/2} + 6x^{-1/2}$

$= 3(\sqrt{x})^3 - 9\sqrt{x} + \dfrac{6}{\sqrt{x}} = 3x\sqrt{x} - 9\sqrt{x} + \dfrac{6}{\sqrt{x}}$

$= \dfrac{3x^2-9x+6}{\sqrt{x}} = \dfrac{3(x^2-3x+2)}{\sqrt{x}}$

$\left.\begin{array}{l} \underline{-1} \times \underline{-2} = 2 \\ \underline{-1} + \underline{-2} = -3 \end{array}\right\} \Rightarrow \dfrac{3(x^2-x-2x+2)}{\sqrt{x}}$

$= \dfrac{3(x(x-1)-2(x-1))}{\sqrt{x}} = \dfrac{3(x-1)(x-2)}{\sqrt{x}}$

$= 3x^{-1/2}(x-1)(x-2)$

f) $x^3y - 4xy = xy(x^2 - 4)$

$= xy(x+2)(x-2)$

5. Simplify the rational expression.

(a) $\dfrac{x^2 + 3x + 2}{x^2 - x - 2}$

(b) $\dfrac{2x^2 - x - 1}{x^2 - 9} \cdot \dfrac{x+3}{2x+1}$

(c) $\dfrac{x^2}{x^2 - 4} - \dfrac{x+1}{x+2}$

(d) $\dfrac{\dfrac{y}{x} - \dfrac{x}{y}}{\dfrac{1}{y} - \dfrac{1}{x}}$

a) $\dfrac{x^2 + x + 2x + 2}{x^2 - 2x + x - 2} \Leftarrow \begin{cases} \underline{1} \times \underline{2} = 2 \\ \underline{1} + \underline{2} = 3 \\ \underline{-2} + \underline{1} = -1 \\ \underline{-2} \times \underline{1} = -2 \end{cases}$

$= \dfrac{x(x+1) + 2(x+1)}{x(x-2) + 1(x-2)}$

$= \dfrac{(x+1)(x+2)}{(x+1)(x-2)} = \dfrac{x+2}{x-2}$

b) $\dfrac{2x^2 - 2x + 1x - 1}{(x+3)(x-3)} \cdot \dfrac{(x+3)}{(2x+1)}$

$$= \frac{2x(x-1)+1(x-1)}{(x+3)(x-3)} \cdot \frac{(x+3)}{(2x+1)}$$

$$= \frac{(x-1)\cancel{(2x+1)}}{\cancel{(x+3)}(x-3)} \cdot \frac{\cancel{(x+3)}}{\cancel{(2x+1)}}$$

$$= \frac{x-1}{x-3}$$

c) $$\frac{x^2}{(x+2)(x-2)} - \frac{(x+1)}{(x+2)}$$

$$= \frac{x^2(x+2) - (x+1)(x+2)(x-2)}{(x+2)^2(x-2)}$$

$$= \frac{(x+2)\left[x^2 - (x+1)(x-2)\right]}{(x+2)^2(x-2)}$$

$$= \frac{x^2 - (x+1)(x-2)}{(x+2)(x-2)}$$

$$= \frac{x^2 - \left[x^2 - 2x + x - 2\right]}{(x+2)(x-2)}$$

$$= \frac{\cancel{x^2} - \cancel{x^2} + 2x - x + 2}{(x+2)(x-2)} = \frac{(x+2)}{(x+2)(x-2)} = \frac{1}{(x-2)}$$

d) $\dfrac{y^2 - x^2}{xy} \div \dfrac{x - y}{xy}$

$= \dfrac{y^2 - x^2}{x - y} = \dfrac{(y-x)(y+x)}{(x-y)}$

$= \dfrac{-1(x-y)(y+x)}{(x-y)}$

$= -1(y+x) = -x - y$

6. Rationalize the expression and simplify.

(a) $\dfrac{\sqrt{10}}{\sqrt{5} - 2}$

(b) $\dfrac{\sqrt{4 + h} - 2}{h}$

a) $\dfrac{\sqrt{10}(\sqrt{5} + 2)}{5 - 4} = \dfrac{\sqrt{50} + 2\sqrt{10}}{1}$

$= \sqrt{50} + 2\sqrt{10}$

$2|50$
$5|25$
5

$= 5\sqrt{2} + 2\sqrt{10}$

b) $\dfrac{(\sqrt{4+h} - 2)(\sqrt{4+h} + 2)}{h(\sqrt{4+h} + 2)}$

$= \dfrac{4 + h - 4}{h(\sqrt{4+h} + 2)} = \dfrac{1}{\sqrt{4+h} + 2}$

7. Rewrite by completing the square.

(a) $x^2 + x + 1$

(b) $2x^2 - 12x + 11$

a) $x^2 + x + 1$

$(x + a)^2 + b$

$x^2 + a^2 + \underline{2ax} + b$

extract 'a' and square it

$x^2 + x + 1$

$2ax = x$

$2a = 1$

$a = 1/2$

$\therefore \quad a^2 = 1/4$

$x^2 + x + \frac{1}{4} + 1 - \frac{1}{4}$

$\boxed{\left(x + \frac{1}{4}\right)^2 + \frac{3}{4}}$

b) $2x^2 - 12x + 11$

$\div$ by $2 \Rightarrow \quad x^2 - 6x + \frac{11}{2}$

we have $-6x = 2ax$

$$-6 = 2a$$

$$a = -3$$

$$\therefore \boxed{a^2 = 9}$$

$$x^2 - 6x + 9 - 9 + \frac{11}{2}$$

$$= (x-3)^2 - 9 + \frac{11}{2}$$

$$= (x-3)^2 - \frac{18 + 11}{2}$$

$$= (x-3)^2 - \frac{7}{2}$$

8. Solve the equation. (Find only the real solutions.)

(a) $x + 5 = 14 - \frac{1}{2}x$

(b) $\dfrac{2x}{x + 1} = \dfrac{2x - 1}{x}$

(c) $x^2 - x - 12 = 0$

(d) $2x^2 + 4x + 1 = 0$

(e) $x^4 - 3x^2 + 2 = 0$

(f) $3|x - 4| = 10$

(g) $2x(4 - x)^{-1/2} - 3\sqrt{4 - x} = 0$

a) $x + 5 = 14 - \frac{1}{2}x$

$x + \frac{1}{2}x = 9$

$3x = 18$

$x = 6$

b) $2x^2 = 2x^2 + 2x - x - 1$

$x = -1$

c) $\underline{-4} \times \underline{3} = -12$

$-\underline{4} + \underline{3} = -1$

$x^2 - 4x + 3x - 12 = 0$

$x(x-4) + 3(x-4) = 0$

$(x-4)(x+3) = 0$

$x_1 = 4 \quad x_2 = -3$

d) $2x^2 + 4x + 1 = 0$

$$\frac{-4 \pm \sqrt{16-8}}{2(2)} = \frac{-4 \pm \sqrt{8}}{4} = \frac{-4 \pm 2\sqrt{2}}{4} = -1 \pm \frac{1}{\sqrt{2}}$$

e) $x^4 - 3x^2 + 2 = 0$

replace $x^2 = m$

$\therefore \; m^2 - 3m + 2 = 0$

$m^2 - 2m - m + 2 = 0$

$m(m-2) - 1(m-2) = 0$

$(m-2)(m-1) = 0$

$m = 2 \quad m = 1$

$x^2 = 2 \quad x^2 = 1$

$x = \pm\sqrt{2} \quad x = \pm 1$

f) $3|x-4| = 10$

$|x-4| = \frac{10}{3}$

$x-4 = +\frac{10}{3}$ or $x-4 = -\frac{10}{3}$

$x = \frac{10}{3} + 4$ $\qquad$ $x = -\frac{10}{3} + 4$

$x = \frac{22}{3}$ $\qquad$ $x = \frac{2}{3}$

g) $2x(4-x)^{-1/2} - 3\sqrt{4-x} = 0$

$\frac{2x}{\sqrt{4-x}} = 3 \cdot \sqrt{4-x}$

$\frac{2x}{3} = 4-x$

$2x = 12 - 3x$

$5x = 12$

$x = \frac{12}{5}$

9. Solve each inequality. Write your answer using interval notation.

(a) $-4 < 5 - 3x \leqslant 17$ $\qquad$ (b) $x^2 < 2x + 8$

(c) $x(x-1)(x+2) > 0$ $\qquad$ (d) $|x-4| < 3$

(e) $\dfrac{2x-3}{x+1} \leqslant 1$

a) $-4-5 < 5-3x-5 \le 17-5$ $\quad (-5)$

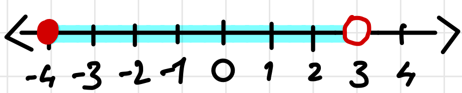
$-9 < -3x \le 12$ $\quad (\div -1)$

$\therefore$ sign change

$9 > 3x \ge -12$ $\quad (\div 3)$

$3 > x \ge -4$

$-4 \le x < 3$



$x \in [-4, 3)$

b) $x^2 \leq 2x + 8$

$x^2 - 2x - 8 \leq 0$

$x^2 - 4x + 2x - 8 \leq 0$

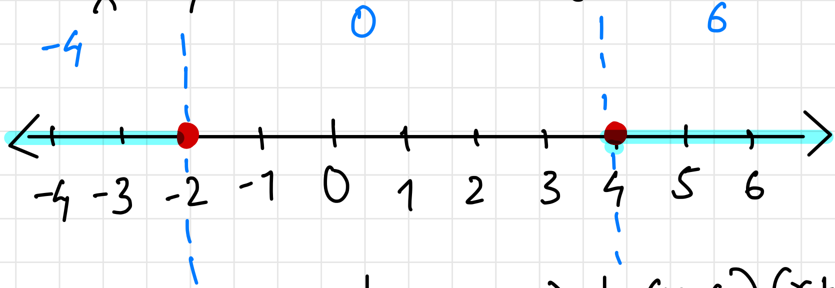
$x(x-4) + 2(x-4) \leq 0$

$(x-4)(x+2) \leq 0$

$x = 4$ & $x = -2$ (segments)

-8

$-4 \quad 2$



	$(x-4)$	$(x+2)$	$(x-4)(x+2)$
$x = -4$	$-$	$-$	$+$
$x = 0$	$-$	$+$	$-$
$x = 6$	$+$	$+$	$+$

given question inequality case $x < 0$

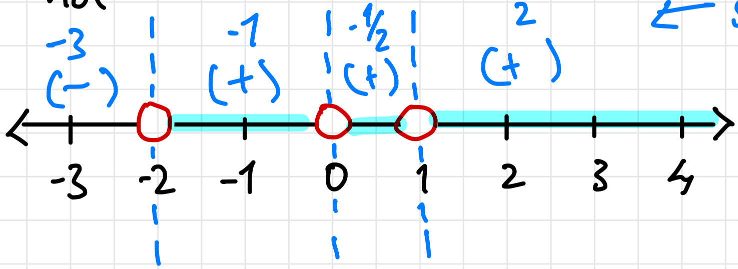
$\therefore$ we take the -ve region.

$[2, 3]$ where 2 & 3 are Included.

c) $x(x-1)(x+2) > 0$

→ regions $x=0$ $x=1$ and $x=-2$

→ not included as inequality is '>'



← selected values for regions.

	x	$x-1$	$x+2$	$x(x-1)(x+2)$
$x=-3$	−	−	−	−
$x=-1$	−	−	+	+
$x=-1/2$	−	−	+	+
$x=2$	+	+	+	+

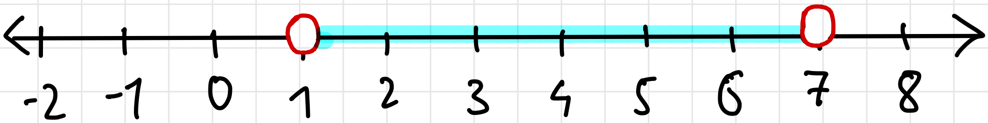
given question case ...... >0 ie +ve cases.

$(-2,0) \cup (0,1) \cup (1,\infty)$

d) $|x-4| < 3$

$x-4 > -3$ or $x-4 < +3$

$x > 1$ or $x < 7$



$x: (1, 7)$

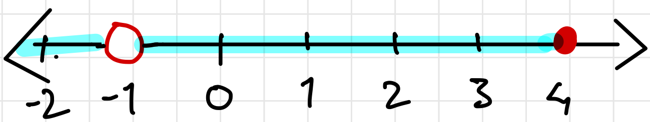
e) $\dfrac{2x-3}{x+1} \leq 1$

$\dfrac{2x-3-x-1}{x+1} \leq 0 \Rightarrow \dfrac{x-2}{x+1} \leq 0$

Here denominator cannot be equal to -1 as it gets $1/0$ form.

$x-4 \leq 0$

$x \leq 4$



$(-\infty, -1) \cup [-1, 4]$

10. State whether each equation is true or false.

(a) $(p+q)^2 = p^2 + q^2$

(b) $\sqrt{ab} = \sqrt{a}\,\sqrt{b}$

(c) $\sqrt{a^2+b^2} = a + b$

(d) $\dfrac{1+TC}{C} = 1 + T$

(e) $\dfrac{1}{x-y} = \dfrac{1}{x} - \dfrac{1}{y}$

(f) $\dfrac{1/x}{a/x - b/x} = \dfrac{1}{a-b}$

a) false

b) true

c) ~~true~~ false $\sqrt{a^2+b^2} \neq a+b$

d) false

e) false

f) true

$$(a+b)^n = {}^nC_0\, a^n b^0 + {}^nC_1\, a^{n-1} b^1 + {}^nC_2\, a^{n-2} b^2 + \dots {}^nC_{n=r}\, a^0 b^n$$

$a \rightarrow$ decreases

$b \rightarrow$ increases.

$${}^nC_r = \frac{n!}{r!(n-r)!}$$

pascals triangle

0						1					
1					1		1				
2				1		2		1			
3			1		3		3		1		
4		1		4		6		4		1	
5	1		5		10		10		5		1

$$(a+b)^3 = {}^3C_0\, a^3 b^0 + {}^3C_1\, a^2 b^1 + {}^3C_2\, a^1 b^2 + {}^3C_3\, a^0 b^3$$

$$= 1\, a^3 + 3\, a^2 b + 3\, a b^2 + 1\, b^3$$

$$= a^3 + 3a^2 b + 3ab^2 + b^3$$

55–60 Complete the square.

55. $x^2 + 2x + 5$

56. $x^2 - 16x + 80$

57. $x^2 - 5x + 10$

58. $x^2 + 3x + 1$

59. $4x^2 + 4x - 2$

60. $3x^2 - 24x + 50$

55) $2ax = 2x$

$2a = 2$

$a = 1$

$a^2 = 1$

$x^2 + 2x + 5 + 1 - 1$

$x^2 + 2x + 1 + 4$

$\boxed{(x+1)^2 + 4}$

56) $2ax = -16x$

$a = -8 \quad a^2 = 64$

$x^2 - 16x + 64 - 64 + 80 = 0$

$\boxed{(x-8)^2 + 16 = 0}$

57) $x^2 - 5x + 10$

$2ax = -5x$

$a = \frac{-5}{2} \quad a^2 = \frac{25}{4}$

$x^2 - 5x + \frac{25}{4} - \frac{25}{4} + 10$

$\boxed{= \left(x - \frac{5}{2}\right)^2 + \frac{15}{4}}$

58) $2ax = 3x$

$a = \frac{3}{2}$

$a^2 = \frac{9}{4}$

$x^2 + 3x + \frac{9}{4} - \frac{9}{4} + 1 = 0$

$\boxed{\left(x + \frac{3}{2}\right)^2 - \frac{5}{4} = 0}$

59) Make it into form $x^2 + 2ax + b^2$ by $\div 4$

$\frac{4x^2 + 4x - 2}{4} = x^2 + x - \frac{1}{2}$

Now $2ax = 1x \quad a^2 = \frac{1}{4}$

$a = 1/2$

$x^2 + x + \frac{1}{4} - \frac{1}{4} - \frac{1}{2}$

$\boxed{= \left(x + \frac{1}{2}\right)^2 - \frac{3}{4}}$

60) $3x^2 - 24x + 50$

$x^2 - 8x + \frac{50}{3}$

$2ax = -8x$

$a = -4$

$a^2 = 16$

$x^2 - 8x + 16 - 16 + \frac{50}{3}$

$\boxed{(x-4)^2 + \frac{2}{3}}$