

Systems of Linear Equations

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System of Linear Equations

$$\begin{array}{rcl} x - 2y + 5z & = 3 \\ 3x + y + z & = 2 \\ x + z & = 1 \\ -4x + y - z & = 10 \end{array}$$

* We do not need a square ($n \times n$)

* Line of zeros means infinitely many solus
no meaning (wrong)

* We cannot find solus by looking

Gaussian Elimination

$$\left(\begin{array}{ccc|c} 1 & -2 & 5 & 3 \\ 3 & 1 & 1 & 2 \\ 1 & 0 & 1 & 1 \\ -4 & 1 & -1 & 10 \end{array} \right) \quad \begin{array}{l} \text{II} \rightarrow \text{II} - \frac{3}{1} \text{I} \\ \text{III} \rightarrow \text{III} - \frac{1}{1} \text{I} \\ \text{IV} \rightarrow \text{IV} - \frac{-4}{1} \text{I} \end{array}$$

Algorithm:

- ① Go to row I, get zeros below the pivot
(subtract rows from each other.)

$$\underline{\text{II}} - 3\underline{\text{I}}$$

$$\begin{aligned} 3 - 3(1) &= 0 \\ 1 - 3(-2) &= 7 \\ 1 - 3(5) &= -14 \\ 2 - 3(3) &= -7 \end{aligned}$$

$$\underline{\text{III}} - 1\underline{\text{I}}$$

$$\begin{aligned} 1 - 1 &= 0 \\ 0 - (-2) &= 2 \\ 1 - (5) &= -4 \\ 1 - (3) &= -2 \end{aligned}$$

$$\underline{\text{IV}} + 4\underline{\text{I}}$$

$$\begin{aligned} -4 + 4(1) &= 0 \\ 1 + 4(-2) &= -7 \\ -1 + 4(5) &= 19 \\ 10 + 4(3) &= 22 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 5 & 3 \\ 0 & 7 & -14 & -7 \\ 0 & 2 & -4 & -2 \\ 0 & -7 & 19 & 22 \end{array} \right) \quad \begin{array}{l} \text{III} - \frac{2}{7}\text{II} \\ \text{IV} - \frac{-7}{7}\text{II} \end{array}$$

$$\underline{\text{III}} - \frac{2}{7}\underline{\text{II}}$$

$$\begin{aligned} 0 - \frac{2}{7}(0) &= 0 \\ 2 - \frac{2}{7}(7) &= 0 \\ -4 - \frac{2}{7}(-14) &= -4 + \frac{28}{7} = 0 \\ -2 - \frac{2}{7}(-7) &= -2 + 2 = 0 \end{aligned}$$

$$\underline{\text{IV}} + \underline{\text{II}}$$

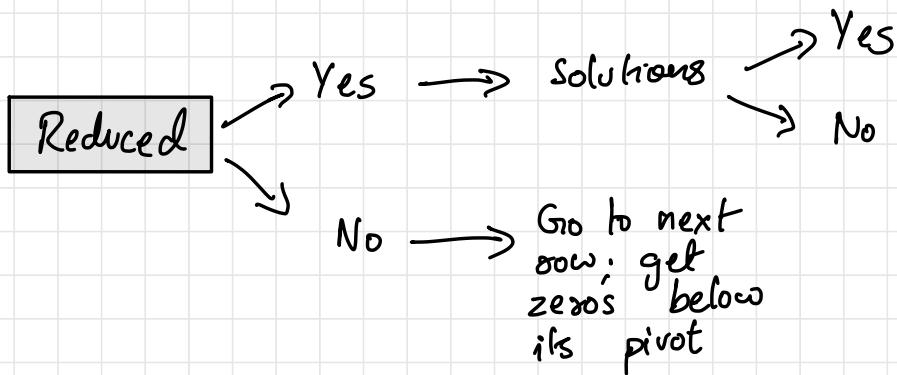
$$\begin{aligned} 0 + 0 &= 0 \\ -7 + 7 &= 0 \\ 19 + (-14) &= 5 \\ 22 + (-7) &= 15 \end{aligned}$$

$$\left(\begin{array}{cccc|c} 1 & -2 & 5 & 3 \\ 0 & 7 & -14 & -7 \\ 0 & 0 & 5 & 15 \\ 0 & 0 & 0 & 0 \end{array} \right)$$

The system is now reduced
∴ we stop & look for solutions

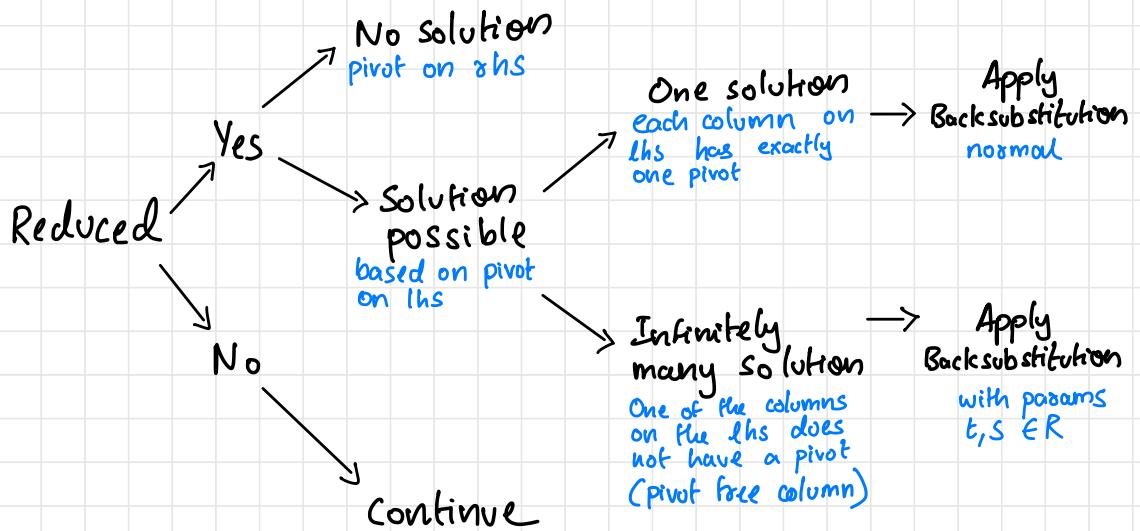
How do we know the system is reduced?

- Systems reduced, meaning at most 1 pivot in each column.
- just counting the already defined pivots



A no solution case : When pivot on RHS

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{array} \right)$$



3. Solve the following SLE using back-substitution and row-reduction echelon form:[unique solution? infinitely many solutions? no solution?]

$$x_1 - 2x_2 + x_3 = 0$$

$$4x_1 + 5x_2 - 6x_3 = 10$$

$$-2x_1 - 7x_2 + 2x_3 = -4$$

\$x_1\$	\$x_2\$	\$x_3\$	
1	-2	1	0
4	5	-6	10
-2	-7	2	-4

$\text{II} - \frac{1}{4}\text{I}$

$\text{III} - \frac{-2}{1}\text{I}$

$$\text{II} - 4\text{I}$$

$$4 - 4(1) = 0$$

$$8 - 4(-2) = 8 + 8 = 16$$

$$-6 - 4(1) = -6 - 4 = -10$$

$$10 - 4(0) = 10 - 0 = 0$$

$$\text{III} + 2\text{I}$$

$$-2 + 2(1) = 0$$

$$-7 + 2(-2) = -7 - 4 = -11$$

$$2 + 2(1) = 2 + 2 = 4$$

$$-4 + 2(0) = -4 + 0 = -4$$

$$\begin{array}{cccc|c} 1 & -2 & 1 & 0 \\ 0 & 13 & -10 & 0 \\ 0 & -11 & 4 & -4 \end{array} \xrightarrow{\text{III} - \frac{-11}{13}\text{II}}$$

$$\text{III} + \frac{11}{13}\text{II}$$

$$0 + \frac{11}{13}(0) = 0$$

$$-11 + \frac{11}{13}(13) = 0$$

$$4 + \frac{11}{13}(-10) = 4 - \frac{110}{13} = \frac{52 - 110}{13} = \frac{-58}{13}$$

$$4 + \frac{11}{13}(0) = 4$$

$$\left| \begin{array}{ccc|c} 1 & -2 & 1 & 0 \\ 0 & 13 & -10 & 0 \\ 0 & 0 & -\frac{58}{13} & 4 \end{array} \right.$$

1 pivot
in each column
 \therefore unique soln.

$$\begin{array}{r} -58 \\ \hline 13 \\ \times 3 = 4 \end{array}$$

$$x_3 = -\frac{52}{58} = -\frac{26}{29}$$

$$13x_2 + 10 \cdot \frac{26}{29} = 0$$

$$377x_2 = -260$$

$$x_2 = \frac{-260}{377} = -\frac{20}{29}$$

$$\begin{array}{r} 2 \\ 29 \\ 13 \\ \hline 187 \\ 29 \\ \hline 377 \end{array}$$

$$1x_1 - 2\left(\frac{-20}{29}\right) + \left(\frac{-26}{29}\right) = 0$$

$$x_1 = \frac{26}{29} - \frac{40}{29} = -\frac{14}{29}$$

$$x_1 = -\frac{14}{29} \quad x_2 = -\frac{20}{29} \quad x_3 = -\frac{26}{29}$$

Checking:

$$-\frac{14}{29} - 2\left(-\frac{20}{29}\right) + \left(-\frac{26}{29}\right) = 0$$

4. Solve the following SLE:[unique solution? infinitely many solutions? no solution?]

$$x_2 - 4x_3 = 8$$

$$2x_1 - 3x_2 + 2x_3 = 1$$

$$4x_1 - 8x_2 + 12x_3 = 1$$

Hint: draw the planes on <https://www.geogebra.org/3d?lang=en>

$$\begin{aligned} 0x_1 + 1x_2 - 4x_3 &= 8 \\ 2x_1 - 3x_2 + 2x_3 &= 1 \\ 4x_1 - 8x_2 + 12x_3 &= 1 \end{aligned}$$

$$\left| \begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 2 & -3 & 2 & 1 \\ 0 & 1 & -4 & 1 \end{array} \right|$$

$\text{II} - \frac{2}{4} \text{I}$

$$\left| \begin{array}{ccc|c} 0 & 1 & -4 & 8 \\ 0 & 1 & -4 & 1 \\ 0 & 1 & -4 & 1 \end{array} \right|$$

$\text{III} - \frac{1}{1} \text{II}$

$$\begin{aligned} \text{II} - \frac{1}{2} \text{I} \\ 2 - \frac{1}{2}(4) &= 0 \\ -3 - \frac{1}{2}(-8) &= -3 + 4 = 1 \\ 2 - \frac{1}{2}(12) &= 2 - 6 = -4 \\ 1 - \frac{1}{2}(1) &= 1 - \frac{1}{2} = \frac{1}{2} \end{aligned}$$

$$\left| \begin{array}{ccc|c} 4 & -8 & 12 & 1 \\ 0 & 1 & -4 & \frac{1}{2} \\ 0 & 0 & 0 & \frac{15}{2} \end{array} \right.$$

$\underline{\text{III}} - \underline{\text{II}}$
 $0 - (0) = 0$
 $1 - (1) = 0$
 $-4 - (-4) = 0$
 $8 - \left(\frac{1}{2}\right) = \frac{15}{2}$

Pivot on rhs

$0 = \frac{15}{2}$: No solution

5. Solve the following SLE:[unique solution? infinitely many solutions? no solution?]

$$-8x_1 + 16x_2 - 24x_3 = -2$$

$$6x_1 - 9x_2 + 6x_3 = 3$$

$$4x_1 - 8x_2 + 12x_3 = 1$$

$$\left| \begin{array}{ccc|c} -8 & 16 & -24 & -2 \\ 6 & -9 & 6 & 3 \\ 4 & -8 & 12 & 1 \end{array} \right.$$

$\underline{\text{II}} - \frac{6}{-8} \underline{\text{I}}$
 $\underline{\text{III}} - \frac{4}{-8} \underline{\text{I}}$

$$\left| \begin{array}{ccc|c} -8 & 16 & -24 & -2 \\ 0 & 3 & 0 & \frac{3}{2} \\ 0 & 0 & 0 & 0 \end{array} \right.$$

↑
pivot free
column

$$III + \frac{1}{2}I$$

$$9 + \frac{1}{2}(-8) = 0$$

$$-8 + \frac{1}{2}(16) = 0$$

$$12 + \frac{1}{2}(-24) = 0$$

$$1 + \frac{1}{2}(-2) = 0$$

$$II + \frac{3}{4}I$$

$$6 + \frac{3}{4}(-8) = 6 - 3(-2) \\ = 0$$

$$-9 + \frac{3}{4}(16) = -9 + 3(4) \\ = 3$$

$$6 + \frac{3}{4}(-24) = 6 - 3(6) \\ = 0$$

$$3 + \frac{3}{4}(-2) = 3 - \frac{3}{2} \\ = \frac{3}{2}$$

$$3x_2 = \frac{3}{2}$$

$$x_2 = \frac{1}{2} .$$

pivot free column infinitely
 $\therefore x_3 = t ; t \in R$ many
 solns.

$$-8x_1 + 16\left(\frac{1}{2}\right) - 24t = -2$$

$$-8x_1 + 8 - 24t = -2$$

$$x_1 = \frac{10 - 24t}{8}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} \frac{90t - 24}{8} \\ \frac{1}{2} \\ t \end{pmatrix}$$

6. a)

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ -4 & 5 & 6 & 0 \\ 7 & 8 & 9 & 10 \end{array} \right] \quad \begin{matrix} \text{I} \\ \text{II} - \frac{-4}{1} \text{ I} \\ \text{III} - \frac{7}{1} \text{ I} \end{matrix} \quad 19:40$$

$$\text{II} + 4\text{I}$$

$$-7 + 4(1) = 0$$

$$5 + 4(2) = 13$$

$$6 + 4(3) = 18$$

$$0 + 4(4) = 16$$

$$\text{III} - 7\text{I}$$

$$7 - 7(1) = 0$$

$$8 - 7(2) = -6$$

$$9 - 7(3) = -12$$

$$10 - 7(4) = -18$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 13 & 18 & 16 \\ 0 & -6 & -12 & -18 \end{array} \right] \xrightarrow{\text{III} - \frac{-6}{13} \text{II}}$$

$$\text{III} + \frac{6}{13} \text{II}$$

$$0 + \frac{6}{13}(0) = 0$$

$$-6 + \frac{6}{13}(13) = 0$$

$$-12 + \frac{6}{13}(18) = \frac{-156 + 108}{13} = \frac{-48}{13}$$

$$-18 + \frac{6}{13}(16) = \frac{-234 + 96}{13} = \frac{-138}{13}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 3 & 4 \\ 0 & 13 & 18 & 16 \\ 0 & 0 & \frac{-48}{13} & \frac{-138}{13} \end{array} \right]$$

System is Reduced

One pivot in each column.
Unique solution.

$$\frac{-48}{13} z = -\frac{138}{13}$$

$$z = \frac{138}{48} = \frac{23}{8}$$

$$z = \frac{23}{8}$$

$$23 \cdot \frac{9}{4}$$

$$13y + 18 \cdot \frac{23}{8} = 16$$

$$\begin{array}{r} 2 \\ 2 \\ 9 \\ \hline 207 \end{array}$$

$$13y + \frac{207}{4} = 16$$

$$13y = 16 - \frac{207}{4}$$

$$13y = \frac{64 - 207}{4}$$

$$52y = 143$$

$$y = \frac{143}{52}$$

$$\begin{array}{r} 2 \\ 16 \\ 4 \\ \hline 64 \\ 110 \\ 807 \\ 64 \\ \hline 143 \end{array}$$

$$1x + 2 \cdot \frac{143}{52} + 3 \cdot \frac{23}{8} = 4$$

$$x = 4 - \frac{143}{26} - \frac{69}{8}$$

$$x = \frac{104 - 143}{26} - \frac{69}{8}$$

$$= -\frac{39}{26} - \frac{69}{8}$$

$$= -\frac{312 - 1794}{208}$$

$$= \frac{2106}{208} = \frac{1053}{104}$$

$$\begin{array}{r} 2 \\ 26 \\ \hline 4 \\ \hline 104 \end{array}$$

$$\begin{array}{r} 7 \\ 39 \\ \hline 8 \\ \hline 312 \end{array}$$

$$\begin{array}{r} 1 \\ 5 \\ 69 \\ \hline 26 \\ \hline 414 \end{array}$$

$$\begin{array}{r} 1 \\ 38 \\ \hline 1794 \end{array}$$

$$\begin{array}{r} 1 \\ 1 \\ 1794 \\ \hline 312 \\ \hline 2106 \end{array}$$

$$\begin{array}{r} 4 \\ 26 \\ \hline 8 \\ \hline 208 \end{array}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \frac{1053}{104} \\ \frac{143}{52} \\ \frac{23}{8} \end{pmatrix}$$

12:33

$$b) \left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 1 & 4 & 7 & 13 \\ 4 & 8 & 9 & 24 \end{array} \right] \xrightarrow{\text{I} - \frac{1}{2}\text{II}} \left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 1 & 4 & 7 & 13 \\ 0 & 0 & 0 & 0 \end{array} \right] \xrightarrow{\text{III} - \frac{1}{2}\text{II}} \left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 1 & 4 & 7 & 13 \\ 0 & 0 & 0 & 0 \end{array} \right]$$

$$\text{II} - \frac{1}{2}\text{I}$$

$$4 - \frac{1}{2}(2) = 0$$

$$4 - \frac{1}{2}(3) = \frac{5}{2}$$

$$7 - \frac{1}{2}(6) = 4$$

$$13 - \frac{1}{2}(9) = \frac{7}{2}$$

$$\text{III} - 2\text{I}$$

$$4 - 2(2) = 0$$

$$8 - 2(3) = 2$$

$$9 - 2(6) = -3$$

$$24 - 2(9) = 6$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 0 & \frac{5}{2} & 4 & 7/2 \\ 0 & 2 & -3 & 6 \end{array} \right] \xrightarrow{\text{III} - \frac{2}{5}\text{II}} \left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 0 & \frac{5}{2} & 4 & 7/2 \\ 0 & 0 & -1 & -\frac{1}{5} \end{array} \right]$$

$$\text{III} - \frac{4}{5} \quad \text{II}$$

$$0 - \frac{4}{5}(0) = 0$$

$$2 - \frac{4}{5}\left(\frac{5}{2}\right) = 0$$

$$-3 - \frac{4}{5}(4) = -\frac{31}{5}$$

$$6 - \frac{4}{5}\left(\frac{7}{2}\right) = \frac{16}{5}$$

$$-3 - \frac{16}{5} = \frac{-15-16}{5} = -\frac{31}{5}$$

$$6 - \frac{14}{5} = \frac{30-14}{5} = \frac{16}{5}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 9 \\ 0 & \frac{5}{2} & 4 & 3 \\ 0 & 0 & -\frac{31}{5} & \frac{16}{5} \end{array} \right]$$

The system is reduced
 One pivot in each column
 \therefore 1 - unique solution

Back sub:

$$-\frac{31}{5} z = \frac{16}{5}$$

$$z = \frac{16}{-31}$$

$$\begin{array}{r} 2 \\ 16 \\ 4 \\ \hline 64 \end{array}$$

$$\frac{5}{2} y + 4 \cdot \frac{16}{-31} = 3$$

$$\frac{5}{2} y = 3 + \frac{64}{31}$$

$$\begin{array}{r} 31 \\ 3 \\ \hline 93 \\ 64 \\ \hline \end{array}$$

$$\frac{5}{2} y = \frac{157}{31}$$

$$y = \frac{157}{31} \cdot \frac{2}{5} = \frac{314}{155}$$

$$\begin{array}{r} 1 \\ 15 \\ 7 \\ \hline 2 \\ \hline 314 \\ 3 \\ \hline \end{array}$$

$$2x + 3\left(\frac{314}{155}\right) + 6\left(\frac{16}{-31}\right) = 9$$

$$\begin{array}{r} 9 \\ 4 \\ 2 \\ \hline 316 \\ 6 \\ \hline 9 \\ 6 \end{array}$$

$$2x + \frac{942}{155} - \frac{96}{31} = 9$$

$$x = \frac{9 + \frac{96}{31} - \frac{942}{755}}{2}$$

c)

$$\left[\begin{array}{ccc|c} -2 & 3 & 6 & 7 \\ 1 & 4 & -7 & -2 \\ -3 & -8 & 22 & 11 \end{array} \right] \quad \begin{matrix} \text{II} - \frac{1}{-2} \text{ I} \\ \text{III} - \frac{-3}{-2} \text{ I} \end{matrix} \quad :19$$

$$\text{II} + \frac{1}{2} \text{ I} \quad \cancel{\times 2} \quad \text{III} - \frac{3}{2} \text{ I} \quad \cancel{\times 2}$$

$$1 + \frac{1}{2}(-2) = 0 = 0 \quad -3 - \frac{3}{2}(-2) = 0 = 6$$

$$4 + \frac{1}{2}(3) = \frac{11}{2} = 11 \quad -8 - \frac{3}{2}(3) = -\frac{25}{2} = -25$$

$$-7 + \frac{1}{2}(6) = -4 = -8 \quad 22 - \frac{3}{2}(6) = 13 = 26$$

$$-2 + \frac{1}{2}(-7) = \frac{3}{2} = 3 \quad 11 - \frac{3}{2}(-7) = -10 = -20$$

$$\left[\begin{array}{ccc|c} -2 & 3 & 6 & 7 \\ 0 & 11 & -8 & 3 \\ 0 & -25 & 26 & -20 \end{array} \right] \xrightarrow{\text{III} - \frac{-25}{11} \text{II}}$$

$$\text{III} + \frac{25}{11} \text{II}$$

$$0 + \frac{25}{11}(0) = 0$$

$$-25 + \frac{25}{11}(11) = 0$$

$$26 + \frac{25}{11}(-8) = \frac{26(11) - 25(8)}{11} = 86$$

$$-20 + \frac{25}{11}(3) = \frac{-20(11) + 25(3)}{11} = 795$$

multiplied by
 $\underline{x 11}$

$\frac{26}{11}$

$\frac{26}{26}$

286

$\frac{4}{25}$
 $\frac{8}{200}$

$$\left[\begin{array}{ccc|c} -2 & 3 & 6 & 7 \\ 0 & 11 & -8 & 3 \\ 0 & 0 & 86 & 145 \end{array} \right]$$

System is reduced
 1 pivot in each column
 1 unique soln.

$\frac{112^0}{880}$
 $\frac{75}{145}$

$$\left[\begin{matrix} 26 & 11 \\ 8 & 25 \end{matrix} \right] C = -20(11) + 25(8)$$

$$\left[\begin{matrix} 286 & -200 \end{matrix} \right] C = -220 + 75$$

$$86 C = 145$$

$$C = \frac{145}{86}$$

$$11 \cdot b - 8 \cdot \frac{145}{86} = 3$$

$$11b = 3 + \frac{145 \cdot 8}{86} =$$

$$= \frac{258 + 1160}{86} : \frac{1918}{86}$$

$$b = \frac{1918}{86 \cdot 11}$$

$$-2a + 3 \cdot \left(\frac{1918}{86 \cdot 11} \right) + \frac{145}{86} \cdot 6 = 7$$

$$\begin{array}{r}
 3 \quad 4 \\
 145 \\
 \times 8 \\
 \hline
 1160 \\
 186 \\
 \hline
 3 \\
 258 \\
 \hline
 1418
 \end{array}$$

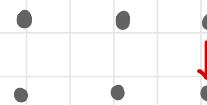
$$22 \times 32 = 604$$

$$51 \times 63 = 3213$$

15 + 6 = 21 carry to left

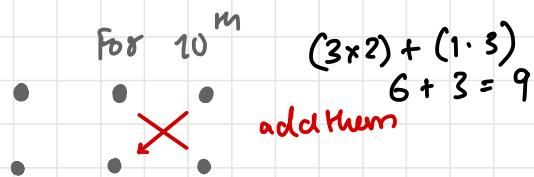
$$\begin{array}{r}
 213 \\
 \times 323 \\
 \hline
 68799
 \end{array}$$

For unit



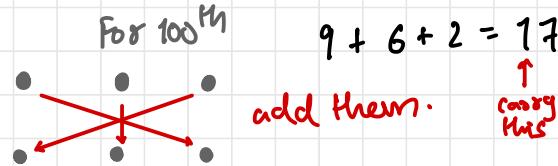
$$3 \times 3 = 9$$

For 10^m



$$(3 \times 2) + (1 \cdot 3) \\ 6 + 3 = 9$$

For 100^m



$$9 + 6 + 2 = 17$$

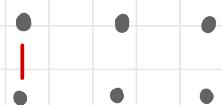
carry this

For 1000^m



$$4 + 3 + 1 \\ = 8$$

For last part



$$2 \cdot 3 = 6$$

Exam q1:

$$4a - 6b + c = 1$$

$$2a - b + c = 5$$

$$-6a + 3b - 2c = -3$$

13:27

$$\left[\begin{array}{ccc|c} a & b & c & \\ \textcircled{1} & -6 & 1 & 1 \\ \textcircled{2} & -1 & 1 & 5 \\ \textcircled{-6} & 3 & -2 & -3 \end{array} \right] \xrightarrow{\text{II} - \frac{2}{4}} \left[\begin{array}{ccc|c} a & b & c & \\ \textcircled{1} & -6 & 1 & 1 \\ \textcircled{2} & -1 & 1 & 5 \\ \textcircled{-6} & 3 & -2 & -3 \end{array} \right] \xrightarrow{\text{III} - \frac{-6}{4}} \left[\begin{array}{ccc|c} a & b & c & \\ \textcircled{1} & -6 & 1 & 1 \\ \textcircled{2} & -1 & 1 & 5 \\ \textcircled{3} & 0 & -1 & 0 \end{array} \right]$$

$$\text{II} - \frac{1}{2} \text{I}$$

$$2 - \frac{1}{2}(1) = 0$$

$$-1 - \frac{1}{2}(-6) = 2$$

$$1 - \frac{1}{2}(1) = \frac{1}{2}$$

$$5 - \frac{1}{2}(1) = \frac{9}{2}$$

$$\text{II} + \frac{3}{2} \text{I}$$

$$-6 + \frac{3}{2}(1) = 0$$

$$3 + \frac{3}{2}(-6) = -6$$

$$-2 + \frac{3}{2}(1) = -\frac{1}{2}$$

$$-3 + \frac{3}{2}(1) = -\frac{3}{2}$$

$$\left[\begin{array}{ccc|c} a & b & c & \\ 4 & -6 & 1 & 1 \\ 0 & 2 & \frac{1}{2} & \frac{9}{2} \\ 0 & -6 & -\frac{1}{2} & -\frac{3}{2} \end{array} \right] \xrightarrow{\text{III} - 3\text{II}} \left[\begin{array}{ccc|c} & & & \\ & & & \\ & & & \\ & & & \end{array} \right]$$

$$\text{III} + 3\text{II}$$

$$0 + 3(0) = 0$$

$$-6 + 3(2) = -6 + 6 = 0$$

$$-\frac{1}{2} + 3\left(\frac{1}{2}\right) = -\frac{1}{2} + \frac{3}{2} = \frac{2}{2} = 1$$

$$-\frac{3}{2} + 3\left(\frac{9}{2}\right) = -\frac{3}{2} + \frac{27}{2} = \frac{24}{2} = 12$$

$$\left[\begin{array}{ccc|c} a & b & c & \\ 4 & -6 & 1 & 1 \\ 0 & 2 & \frac{1}{2} & \frac{9}{2} \\ 0 & 0 & 1 & 12 \end{array} \right]$$

System is reduced
1 pivot in each column

1- unique solution

$$7c = 12$$

$c = \frac{12}{7}$

$$2b + \frac{1}{2}c = \frac{9}{2}$$

$$4b + c = 9$$

$$4b = 9 - 12 = -3$$

$b = \frac{-3}{4}$

$$4a - 6\left(-\frac{3}{4}\right) + 12 = 1$$

$$4a + \frac{18}{4} = -11$$

$$16a + 18 = -44$$

$$16a = -62$$

$$a = \frac{-62}{16} = -\frac{31}{8}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 12 \\ -\frac{3}{4} \\ -\frac{31}{8} \end{pmatrix}$$

$$16 : 36$$

$$d) \left[\begin{array}{ccc|c} 2 & 3 & 6 & 7 \\ 1 & 9 & 7 & 6 \\ 4 & 8 & 9 & 16 \end{array} \right] \xrightarrow{\text{II} - \frac{1}{2}\text{I}} \left[\begin{array}{ccc|c} 2 & 3 & 6 & 7 \\ 0 & 8 & 5 & 6 \\ 4 & 8 & 9 & 16 \end{array} \right] \xrightarrow{\text{III} - \frac{1}{2}\text{I}} \left[\begin{array}{ccc|c} 2 & 3 & 6 & 7 \\ 0 & 8 & 5 & 6 \\ 0 & 0 & 4 & 10 \end{array} \right]$$

$$\text{II} - \frac{1}{2}\text{I}$$

$$1 - \frac{1}{2}(2) = 0$$

$$4 - \frac{1}{2}(3) = \frac{5}{2}$$

$$7 - \frac{1}{2}(6) = 4$$

$$6 - \frac{1}{2}(7) = \frac{5}{2}$$

$$\times 2 \\ \Downarrow$$

$$0$$

$$5$$

$$8$$

$$5$$

$$\text{III} - 2\text{I}$$

$$4 - 2(2) = 0$$

$$8 - 2(3) = 2$$

$$9 - 2(6) = -3$$

$$16 - 2(7) = 2$$

$$4 - \frac{3}{2} = \frac{8-3}{2} = \frac{5}{2} \quad 9 - 12 = -3$$

$$\cancel{12 - 7} = \frac{5}{2} \quad 16 - 19 = 2$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 7 \\ 0 & 5 & 8 & 5 \\ 0 & 2 & -3 & 2 \end{array} \right] \text{III} - \frac{2}{5} \text{II}$$

$$\text{III} - \frac{2}{5} \text{II}$$

$$0 - \frac{2}{5}(0) = 0$$

$$2 - \frac{2}{5}(5) = 0$$

$$-3 - \frac{2}{5}(8) = -3 - \frac{16}{5} = \frac{-25-16}{5} = \frac{-31}{5}$$

$$2 - \frac{2}{5}(5) = 0$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 7 \\ 0 & 5 & 8 & 5 \\ 0 & 0 & \frac{-31}{5} & 0 \end{array} \right]$$

reduced ; 1 pivot in each column

Back sub:

$$\begin{array}{r} -31 \\ \hline 5 \\ C = 0 \end{array}$$

$$5b + 8(0) = 5$$

$$b = 1$$

$$2a + 3 + 6(0) = 7$$

$$2a + 3 = 7$$

$$2a = 4$$

$$a = 2$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$$

16:46

$$\text{c) } \left[\begin{array}{ccc|c} 2 & 3 & 6 & 11 \\ 1 & -9 & 7 & 15 \\ 9 & 8 & 9 & 11 \end{array} \right] \quad \begin{matrix} \text{II} - \frac{1}{2} \text{I} \\ \text{III} - \frac{9}{2} \text{I} \end{matrix}$$

$$\text{II} - \frac{1}{2} \text{I}$$

$$\text{III} - 2 \text{I}$$

$$1 - \frac{1}{2}(2) = 0 \quad \begin{matrix} \cancel{x^2} \\ \cancel{y} \\ 0 \end{matrix} \quad 9 - 2(2) = 0$$

$$-4 - \frac{1}{2}(3) = -\frac{11}{2} = -11 \quad 8 - 2(3) = 2$$

$$7 - \frac{1}{2}(6) = 4 = 8 \quad 9 - 2(6) = -3$$

$$15 - \frac{1}{2}(11) = \frac{19}{2} = 9.5 \quad 11 - 2(11) = -11$$

$$-4 - \frac{3}{2} = -\frac{8-3}{2} = -\frac{11}{2}$$

$$15 - \frac{11}{2} = \frac{30-11}{2} = \frac{19}{2}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 11 \\ 0 & -11 & 8 & 38 \\ 0 & 2 & -3 & -11 \end{array} \right] \xrightarrow{\text{III} - \frac{2}{11} \text{II}}$$

$$\text{III} + \frac{2}{11} \text{II}$$

$$0 + \frac{2}{11}(0) = 0$$

$$2 + \frac{2}{11}(-11) = 0$$

$$-3 + \frac{2}{11}(8) = -3 + \frac{16}{11} = \frac{-33+16}{11} = \frac{-17}{11}$$

$$-11 + \frac{2}{11}(38) = \frac{-121+36}{11} =$$

$$\left[\begin{array}{ccc|c} 2 & 3 & 6 & 11 \\ 0 & -11 & 8 & 38 \\ 0 & 0 & -\frac{17}{11} & -\frac{83}{11} \end{array} \right]$$

reduced, solvable
one pivot in each column
one unique soln.

$$-17c = -83$$

$c = \frac{-83}{17}$

$$-11b + 8 \cdot \frac{-83}{17} = 38$$

$$\frac{-181b + 360}{17} = 38$$

$$-181b + 360 = 646$$

$$360 - 646 = 181b$$

$$b = \frac{360 - 646}{181}$$

$b = \frac{-286}{181}$

$$\begin{array}{r} 17.11 \\ = 181 \\ \hline 45 \\ 8 \\ \hline 360 \end{array}$$

$$\begin{array}{r} 38 \\ 17 \\ \hline 35 \\ 6 \end{array}$$

$$2a + 3\left(\frac{-286}{181}\right) + 6\left(\frac{45}{17}\right) = 11$$

$$2a = 11 + \frac{286 \times 3}{181} - \frac{6 \times 45}{17}$$

$$2a = 11 + \frac{856}{181} - \frac{270}{17}$$

$$a = \frac{1}{2} \left(11 + \frac{856 \times 17 - 270 \times 181}{181 \times 17} \right)$$

$$a = \frac{1}{2} \left(\frac{11 + 14952 - 48870}{3077} \right)$$

$$= \frac{-34907}{6154}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{34907}{6154} \\ -\frac{286}{181} \\ \frac{45}{17} \end{pmatrix}$$

5:29

$$f) \left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 7 & 4 & 1 & 13 \\ 9 & 8 & 4 & 24 \end{array} \right] \xrightarrow{\text{II} - \frac{7}{6}\text{I}} \left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{13}{6} \\ 9 & 8 & 4 & 24 \end{array} \right] \xrightarrow{\text{III} - \frac{9}{6}\text{I}} \left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{13}{6} \\ 0 & \frac{7}{2} & 1 & \frac{21}{2} \end{array} \right]$$

$$\text{II} - \frac{7}{6}\text{I}$$

$$\text{III} - \frac{3}{2}\text{I}$$

$$7 - \frac{7}{6}(6) = 0$$

$$9 - \frac{3}{2}(6) = 0$$

$$4 - \frac{7}{6}(3) = \frac{1}{3}$$

$$8 - \frac{3}{2}(3) = \frac{7}{2}$$

$$1 - \frac{7}{6}(2) = \frac{4}{3}$$

$$4 - \frac{3}{2}(2) = 1$$

$$13 - \frac{7}{6}(9) = \frac{5}{2}$$

$$24 - \frac{3}{2}(9) = \frac{21}{2}$$

$$\left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & \frac{1}{3} & \frac{1}{6} & \frac{13}{6} \\ 0 & \frac{7}{2} & 1 & \frac{21}{2} \end{array} \right] \xrightarrow{\text{II} \times 3 \Rightarrow} \left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & 1 & \frac{1}{2} & \frac{13}{2} \\ 0 & \frac{7}{2} & 1 & \frac{21}{2} \end{array} \right] \xrightarrow{\text{III} \times 2 \Rightarrow}$$

$$\left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & 1 & 4 & 15/2 \\ 0 & 7 & 2 & 21 \end{array} \right] \xrightarrow{\text{III} - \frac{7}{1} \text{II}} \left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & 1 & 4 & 15/2 \\ 0 & 0 & -26 & -7 \end{array} \right]$$

$$\text{IV} - 7 \text{II}$$

$$0 - 7(0) = 0$$

$$7 - 7(1) = 0$$

$$2 - 7(4) = 2 - 28 = -26$$

$$21 - 7\left(\frac{15}{2}\right) = \frac{42 - 105}{2} = \frac{63}{2}$$

$$\begin{matrix} 63 \\ 42 \\ 105 \end{matrix}$$

$$\left[\begin{array}{ccc|c} 6 & 3 & 2 & 9 \\ 0 & 1 & 4 & 15/2 \\ 0 & 0 & -26 & 63/2 \end{array} \right]$$

= System is reduced
 Solvable ; One pivot in each column
 1 unique soln

Basis Sub:

$$-26z = \frac{63}{2}$$

$$\boxed{z = -\frac{63}{52}}$$

$$16 - 4 \left(-\frac{63}{52} \right) = \frac{15}{2}$$

$$\begin{array}{r} 240 + \\ 12 \\ \hline 252 \end{array}$$

$$52b - 252 = 26 \cdot 15$$

$$52b = 390 + 252$$

$$\boxed{b = \frac{642}{52} = \frac{321}{26}}$$

$$\begin{array}{r} 26 \\ 15 \\ \hline 13 \\ 390 \\ 252 \\ \hline 642 \end{array}$$

$$6a + 3 \cdot \frac{321}{26} + 2 \left(-\frac{63}{52} \right) = 9$$

321

$$\frac{156a + 963}{26} - \frac{63}{26} = 9$$

$$\frac{3}{963}$$

$$156a + 900 = 26 \cdot 9$$

$$156a = 234 - 900$$

$$\begin{array}{r} 1 \\ 180 \\ \hline 59 \end{array}$$

$$\frac{59}{34}$$

$$a = \frac{666}{156} = \frac{333}{78} = \frac{111}{26}$$

$$\frac{666}{234} \overline{)00}$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} -\frac{63}{52} \\ 321/26 \\ 111/26 \end{pmatrix}$$

g)

$$\left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 1 & -4 & 7 & -13 \\ -4 & 8 & 9 & 29 \end{array} \right] \xrightarrow{\text{I} \rightarrow -\frac{1}{2}\text{I}} \left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 1 & -4 & 7 & -13 \\ -4 & 8 & 9 & 29 \end{array} \right] \xrightarrow{\text{II} \rightarrow -\frac{1}{4}\text{II}} \left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 1 & -4 & 7 & -13 \\ 1 & -2 & \frac{9}{4} & \frac{29}{4} \end{array} \right]$$

$$\text{II} - \frac{1}{2}\text{I}$$

$$\text{III} + 2\text{I}$$

$$1 - \frac{1}{2}(2) = 0$$

$$-4 + 2(2) = 0$$

$$-4 - \frac{1}{2}(3) = -\frac{11}{2}$$

$$8 + 2(3) = 14$$

$$7 - \frac{1}{2}(-6) = 10$$

$$9 + 2(-6) = -3$$

$$-13 - \frac{1}{2}(9) = -\frac{35}{2}$$

$$29 + 2(9) = 42$$

$$x2 \Rightarrow \left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 0 & -\frac{11}{2} & 20 & -\frac{35}{2} \\ 0 & 14 & -3 & 42 \end{array} \right]$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 0 & -11 & 20 & -35 \\ 0 & 14 & -3 & 42 \end{array} \right] \xrightarrow{\text{III} - \frac{14}{11} \text{II}}$$

$$\text{III} + \frac{14}{11} \text{II}$$

$$0 + \frac{14}{11}(0) = 0$$

$$14 + \frac{14}{11}(-11) = 0$$

$$-3 + \frac{14}{11}(20) = \frac{-33 + 280}{11} = \frac{247}{11}$$

$$42 + \frac{14}{11}(-35) = \frac{462 - 490}{11} = -\frac{28}{11}$$

$$\left[\begin{array}{ccc|c} 2 & 3 & -6 & 9 \\ 0 & -11 & 20 & -35 \\ 0 & 0 & \frac{247}{11} & -\frac{28}{11} \end{array} \right]$$

System is reduced
 Solvable - One pivot in each column
 1 - unique soln

$$247z = -28$$

$$z = \frac{-28}{247}$$

$$-11y + 20 \left(\frac{-28}{247} \right) = -35$$

$$(-11)(247)y + (20)(-28) = (-35)(247)$$

$$-2717y - 560 = -8645$$

$$-2717y = -8645 + 560$$

$$y = \frac{8085}{2717}$$

$$2a + 3 \left(\frac{8085}{2717} \right) - 6 \left(\frac{-28}{247} \right) = -9$$

week1 week2

x student job	\rightarrow	2	1	4
y office job	\rightarrow	4	3	4
z math job	\rightarrow	<u>3</u>	<u>8</u>	<u>17</u>
		16 €	17.5	17

$$\begin{array}{ccc|c}
 x & y & z & \\
 \textcircled{2} & 4 & 3 & 16 \\
 \textcircled{1} & 3 & 8 & 17.5 \text{ II} - \frac{1}{2} \text{ I} \\
 \textcircled{4} & 4 & 1 & 17 \text{ III} - \frac{4}{2} \text{ I} \\
 \hline
 \end{array}$$

$$\text{II} - \frac{1}{2} \text{I}$$

$$\text{III} - 2 \text{I}$$

$$1 - \frac{1}{2}(2) = 0 \quad 4 - 2(2) = 0$$

$$3 - \frac{1}{2}(4) = 1 \quad 4 - 2(4) = -4$$

$$8 - \frac{1}{2}(3) = \frac{13}{2} \quad 1K-2(3) = -5 \quad \text{K} = 6$$

$$17.5 - \frac{1}{2}(16) = 9.5 \quad 17 - 2(16) = -15$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 3 & 16 \\ 0 & 1 & \frac{13}{2} & 9.5 \\ 0 & -4 & -5 & -15 \end{array} \right] \xrightarrow{\text{I} - \frac{-4}{1} \text{II}} \left[\begin{array}{ccc|c} 2 & 4 & 3 & 16 \\ 0 & 1 & \frac{13}{2} & 9.5 \\ 0 & 0 & 21 & -15 \end{array} \right]$$

$$= \text{III} + 4 \text{II}$$

$$0 + 4(0) = 0$$

$$-4 + 4(-1) = 0$$

$$-5 + 4(\frac{13}{2}) = -5 + 26 = 21$$

$$-15 + 4(9.5) = -15 + 38 = 23$$

$$\text{K} - 6 + 26 = 21$$

$$\text{K} + 20 = 21$$

$$\text{K} = 1$$

$$\left[\begin{array}{ccc|c} 2 & 4 & 3 & 16 \\ 0 & 1 & \frac{13}{2} & 9.5 \\ 0 & 0 & 21 & 23 \end{array} \right]$$

$$21 z = 23$$

$$z = \frac{23}{21} \approx 1.09 \text{ hrs}$$

