Diagnostic Test: Trigonometry

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1. Convert from degrees to radians.

(a) 300°　　(b) −18°

2. Convert from radians to degrees.

(a) $5\pi/6$　　(b) 2

3. Find the length of an arc of a circle with radius 12 cm if the arc subtends a central angle of 30°.

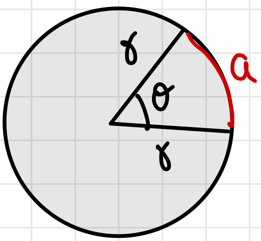
1) a) $\cancel{300} \cdot \frac{\pi}{18\cancel{0}} = \frac{10\pi}{6} = \frac{5\pi}{3}$

b) $-18^\circ = -18 \cdot \frac{\pi}{180} = -\frac{\pi}{10}$

2) a) $\frac{5\cancel{\pi}}{\cancel{6}_{1}} \cdot \frac{\cancel{180}^{30}}{\cancel{\pi}} = 150^\circ$

b) $2 \cdot \frac{180}{\pi} = \frac{360^\circ}{\pi}$

3)



$\frac{\theta}{2\pi} = \frac{a}{2\pi r}$　$\frac{\text{arc length}}{\text{circumference}}$

$a = r\theta$

$\cancel{30} \cdot \frac{\pi}{\cancel{180}_{6}}$

given $r = 12\text{cm}$　$\theta = 30^\circ$

$a = 12\text{ cm} \cdot \frac{\pi}{6} = 2\pi \text{ cm}$

4. Find the exact values.

(a) $\tan(\pi/3)$ (b) $\sin(7\pi/6)$ (c) $\sec(5\pi/3)$

5. Express the lengths a and b in the figure in terms of θ.

6. If $\sin x = \frac{1}{3}$ and $\sec y = \frac{5}{4}$, where x and y lie between 0 and $\pi/2$, evaluate $\sin(x + y)$.

4. a) $\tan\left(\frac{\pi}{3}\right) = \tan\left(\frac{180}{3}\right) = \tan 60^\circ$

$= \frac{\sin 60^\circ}{\cos 60^\circ} = \frac{\sqrt{3}/2}{1/2} = \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

b) $\sin\left(\frac{7\pi}{6}\right) = \sin\left(\frac{7 \cdot \cancel{180}^{\,3}}{\cancel{6}_{\,1}}\right) = \sin 210^\circ$

$= \sin(90 + 120^\circ)$

$= \cos 120^\circ$

$= \cos(90^\circ + 30^\circ)$

$= -\sin 30$

$= -\frac{1}{2}$

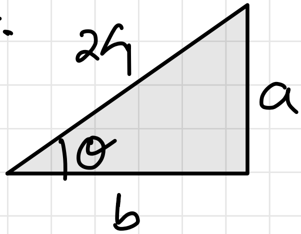
$\sin 90 \pm \theta = \cos\theta$

$\cos 90 \pm \theta = \mp \sin\theta$

$\tan 90 \pm \theta = \mp \cot\theta$

$\cot 90 \pm \theta = \mp \tan\theta$

5.

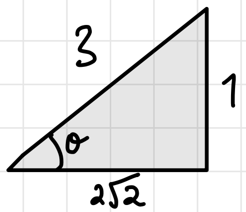


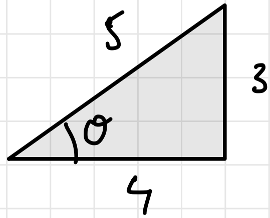
$\sin\theta = \frac{a}{24} \therefore a = 24\sin\theta$

$\cos\theta = \frac{b}{24} \therefore b = 24\cos\theta$

6. $\sin x = \frac{1}{3}$ $\quad \sec y = \frac{5}{4}$ $\quad x, y \Rightarrow [0, \pi/2)$

$$\sin(a+b) = \sin a \cos b + \cos a \sin b$$





$\sqrt{3^2 - 1^2}$
$= \sqrt{8}$
$= 2\sqrt{2}$

$\sin a = \frac{1}{3}$

$\cos a = \frac{2\sqrt{2}}{3}$

$\cos b = \frac{4}{5}$

$\sin b = \frac{3}{5}$

$\sqrt{25-16}$
$= \sqrt{9} = 3$

$$\therefore \sin(x+y) = \frac{1}{3} \cdot \frac{4}{5} + \frac{2\sqrt{2}}{3} \cdot \frac{3}{5}$$

$$= \frac{4}{15} + \frac{6\sqrt{2}}{15} = \frac{1}{15}(4 + 6\sqrt{2})$$

7. Prove the identities.

(a) $\tan\theta \sin\theta + \cos\theta = \sec\theta$ (b) $\dfrac{2\tan x}{1+\tan^2 x} = \sin 2x$

8. Find all values of x such that $\sin 2x = \sin x$ and $0 \leqslant x \leqslant 2\pi$.

9. Sketch the graph of the function $y = 1 + \sin 2x$ without using a calculator.

7. a) $\tan\theta \sin\theta + \cos\theta$

$$\frac{\sin^2\theta + \cos^2\theta}{\cos\theta} = \frac{1}{\cos\theta} = \sec\theta = RHS$$

b) $$\frac{2\tan x}{1+\tan^2 x} = \frac{2\tan x}{\frac{\cos^2 x + \sin^2 x}{\cos^2 x}}$$

$$= 2\tan x \cos^2 x = 2\frac{\sin x}{\cos x}\cos^2 x$$

$$= 2\sin x \cos x$$

$$= \sin 2x$$

$$= RHS$$

8. given $0 \le x \le 2\pi$

we have to find all values such that $\sin 2x = \sin x$

$\sin 0 = 0 \qquad \sin 2 \cdot 0 = 0$

$\sin 2x = \sin x$

$\sin 2x - \sin x = 0$

$2 \sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0 \qquad \cos x = \frac{1}{2}$

$x = \sin^{-1}(0) \qquad x = \cos^{-1}\left(\frac{1}{2}\right)$

$= 0 \qquad = \pm \frac{\pi}{3}$

$\therefore \left(2\pi \pm \frac{\pi}{3}\right)$

Here we are asked below 2π $\therefore$ only $-\frac{\pi}{3}$

ie $2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

$0, \frac{\pi}{3}, \frac{5\pi}{3}, 2\pi$

9. $y = 1 + \sin 2x$

given $1+$ $\therefore$ we shift everything up the y axis by 1 unit.

we know $y = a \sin bx + c$

where $a \rightarrow$ amplitude

$b \rightarrow$ period cycle

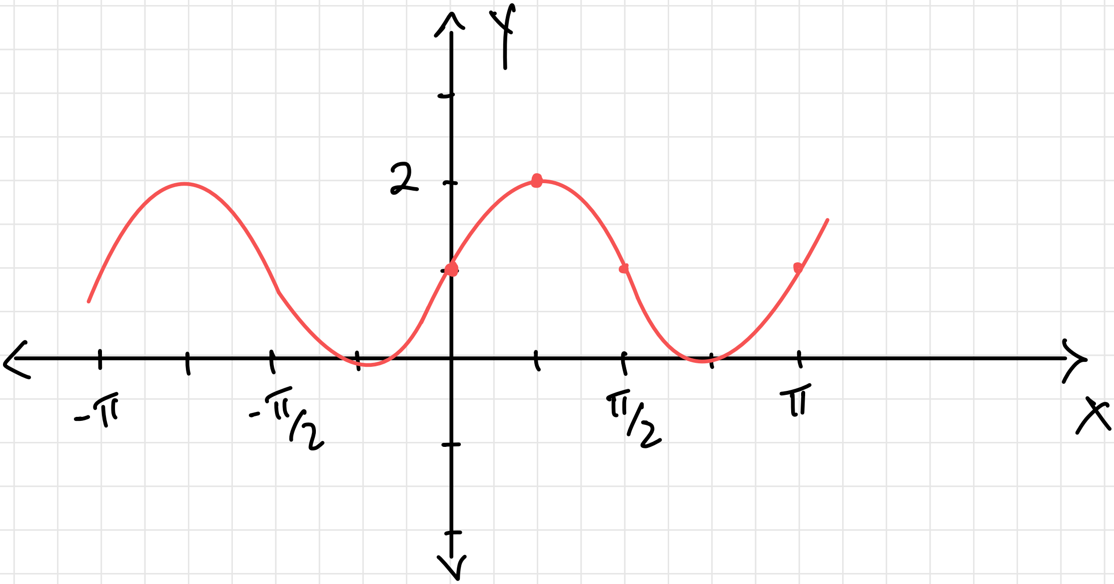
$c \rightarrow$ base line

here $a = 1$, $b = 2$ $\therefore$ $T = \frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

repeats every π

and base line is at $c = 1$

x	y	x	y
0	$1 + \sin 0 = 1$	$\pi/4$	$1 + \sin \frac{\pi}{2} = 1 + 1 = 2$
$\pi/2$	$1 + \sin \pi = 1$	$3\pi/4$	$1 + \sin \frac{6\pi}{4}$
π	$1 + \sin 2\pi = 1$		



	0	30°	45°	60°	90°	180°	270°	360°
Sin	0	$1/2$	$\sqrt{2}/2$	$\sqrt{3}/2$	1	0	-1	0
Cos	1	$\sqrt{3}/2$	$\sqrt{2}/2$	$1/2$	0	-1	0	1
tan	0	$1/\sqrt{3}$	1	$\sqrt{3}$	∞	0	∞	0
Cot	∞	$\sqrt{3}$	1	$1/\sqrt{2}$	0	∞	0	∞