Formula Sheet: Intro Math

github.com/soyceanton

Numbers

$\mathbb{N}$ - Natural nos : 0, 1, 2 ...

New Zealand

$\mathbb{Z}$ → Integers : ... -3, -2, 0, 1, 2 ...

Queensland

$\mathbb{Q}$ → Rational nos : $\frac{3}{10}$, 2.111

Roy

$\mathbb{R}$ → Irrational : e , π , $\sqrt{2}$

Cynda

$\mathbb{C}$ → Complex : $4+3i$

$i = \sqrt{-1}$ $\quad i^2 = -1$

$z = a + ib$ ← cartesian form

$z = r(\cos\theta + i\sin\theta)$ ← polar form

$z = re^{i\theta}$ ← exponential form

where $r = \sqrt{a^2+b^2}$

$\tan\theta = \frac{b}{a}$

real imaginary together

$(a+ib) + (c+id) = (a+c) + (b+d)i$

$(a+ib) - (c+id) = (a-c) + (b-d)i$

$(a+ib)(c+id)$

$= ac + (a)(id) + bc + (b)(id)$

$$\frac{a+ib}{c+id} = \frac{a+ib}{c+id}\,\frac{(c-id)}{(c-id)}$$

n^{th} power $\quad z^n = r^n e^{(i\theta)n}$

n^{th} root $\quad z^{\frac{1}{n}} = r^{\frac{1}{n}} e^{\frac{i\theta}{n}}$

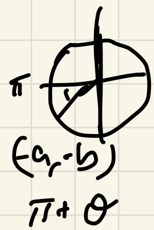
n n^{th} root

$$w_n = r^{\frac{1}{n}} \cdot e^{i\left(\frac{\theta}{n} + k\frac{2\pi}{n}\right)}$$

where Argument $= \frac{\theta}{n}$

periodicity $= \frac{2\pi}{n}$

also make sure to locate the initial complex numbers quadrand to find the appropriate $\tan\theta$ inverse ; θ value



plot the argument on the graph and add periodicity to annotate $w_0, w_1, w_2 \dots$

Trigonometry

$\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

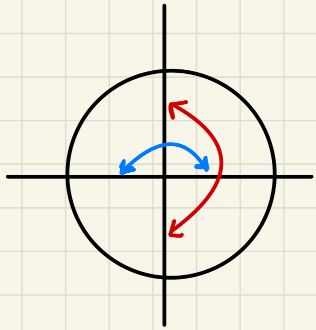
$\tan(-\theta) = -\tan\theta$

Inverse trig restrictions:

$\theta = \sin^{-1}\left(\frac{m}{m}\right) \quad \frac{\pi}{2}, -\frac{\pi}{2}$

$\theta = \cos^{-1}\left(\frac{m}{m}\right) \quad 0, \pi$

$\theta = \tan^{-1}\left(\frac{m}{m}\right) = \frac{\pi}{2}, \frac{\pi}{2}$



SLEs

Reduced → Yes → No solution
pivot on rhs

Reduced → Yes → Solution possible
based on pivot on lhs

Solution possible → One solution
each column on lhs has exactly one pivot
→ Apply Backsubstitution
normal

Solution possible → Infinitely many solution
One of the columns on the lhs does not have a pivot (pivot free column)
→ Apply Backsubstitution
with params $t, s \in R$

Reduced → No → Continue

each column has one pivot → unique soln

one of the colum does not have a pivot → infinitely many solns
introduce params

pivot appears on rhs → no solution.

Trick Multiplication:

51 × 63 = 30 $\overset{2+}{}$ 1 3 = 3213

15 + 6 = 21

For Fast Mutiplication.

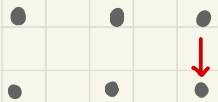
$22 \times 32 = 6\ \overset{1}{\underline{0}}\ 4 = 704$

$51 \times 63 = 30\ \overset{2}{\underline{1}}\ 3 = 3213$

$15 + 6 = 21$ ← carry to left

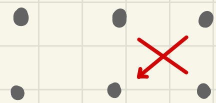
$$\begin{array}{r} 213 \\ \times \quad 323 \\ \hline 6\overset{1}{8},799 \end{array}$$

For unit



$3 \times 3 = 9$

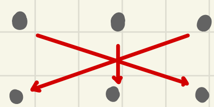
For 10^{th}



$(3 \times 2) + (1 \cdot 3)$
$6 + 3 = 9$

add them

For 100^{th}

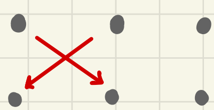


$9 + 6 + 2 = 17$

add them.

carry this

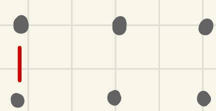
For 1000^{th}



add the carry before !

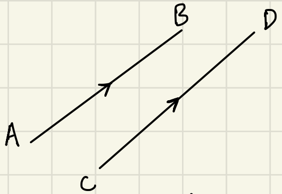
$4 + 3 + 1 = 8$

For last part

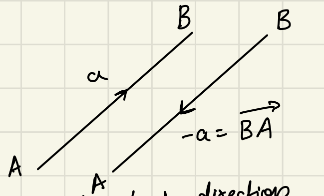


$2 \cdot 3 = 6$

Vectors, Lines & Planes



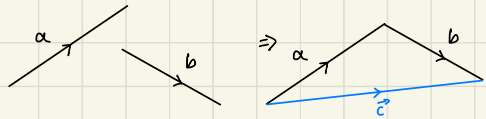
Equal Vectors



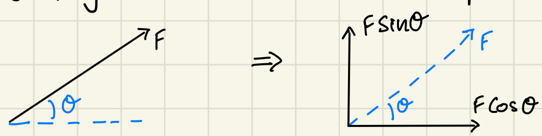
defined only by direction and magnitude and not the space

$$\vec{a} + \vec{b} = \vec{b} + \vec{a}$$

$$\vec{a} + (\vec{b} + \vec{c}) = (\vec{a} + \vec{b}) + \vec{c}$$

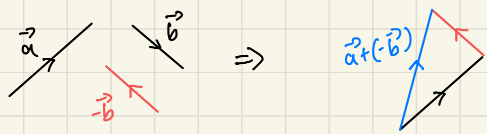


Resolving Forces into two components:



Subtraction of vectors

$$\vec{a} - \vec{b} \Rightarrow \vec{a} + (-\vec{b})$$



Multiplying Vectors:

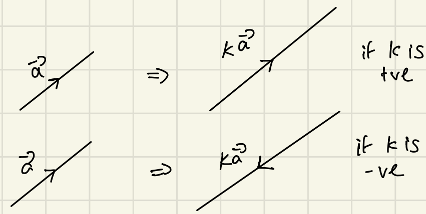
usually multiplied with a +ve scalar

$\therefore k\vec{a}$

$$k(\vec{a} + \vec{b}) = k\vec{a} + k\vec{b}$$

$$(k + l)\vec{a} = k\vec{a} + l\vec{a}$$

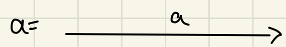
$$k(l)\vec{a} = (kl)\vec{a}$$



Unit Vectors:

described as the vector that has magnitude 1.

if $\vec{a}$ has magnitude 3; unit vector in the direction of $\vec{a}$ is $\frac{1}{3}\vec{a}$

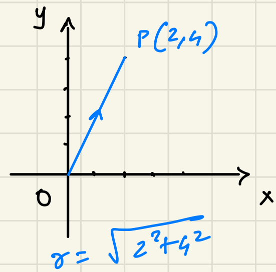


$$\hat{a} = \frac{\vec{a}}{|\vec{a}|}$$

$$\hat{a} = \frac{1}{3}\vec{a}$$

Position Vectors

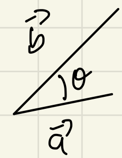
eg: $P(2,4) \Rightarrow 2\hat{i} + 4\hat{j}$ or $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$



Since they start at origin:
their length can be defined as:

$r = \sqrt{a^2+b^2}$

Scalar Product (Dot Product)



$a \cdot b = |\vec{a}|\,|\vec{b}| \cos\theta$

$i \cdot i = 1 \qquad i \cdot j = 0$

$j \cdot j = 1 \qquad j \cdot k = 0$

$k \cdot k = 1 \qquad k \cdot i = 0$

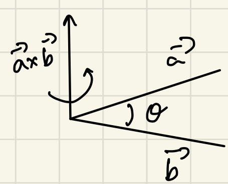
scalar product of perpendicular vectors are 0

For any vector $\vec{r}$, $|\vec{r}|^2 = r \cdot r$

Scalar Proj of b on a: $\text{comp}_a b = \dfrac{a \cdot b}{|a|}$

Vector Proj of b on a: $\left(\dfrac{a \cdot b}{|a|}\right) \cdot \dfrac{a}{|a|}$

Vector Product



$\vec{a} \times \vec{b} = |\vec{a}|\,|\vec{b}| \sin\theta \, \hat{e}$

$|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}| \sin\theta$

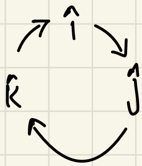
$\vec{a} \times \vec{b} = -\vec{b} \times \vec{a}$

(goes in the opposite direction)

$\hat{i} \times \hat{i} = 0$

$\hat{j} \times \hat{j} = 0$

$\hat{k} \times \hat{k} = 0$



$\hat{i} \times \hat{j} = \hat{k}$

$\hat{j} \times \hat{k} = \hat{i}$

$\hat{k} \times \hat{i} = \hat{j}$

$\hat{j} \times \hat{i} = -\hat{k}$

in anticlockwise signs would be -ve

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \end{vmatrix}$$

$$\hat{i} \begin{vmatrix} b_1 & c_1 \\ b_2 & c_2 \end{vmatrix} - \hat{j} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix} + \hat{k} \begin{vmatrix} a_1 & c_1 \\ a_2 & c_2 \end{vmatrix}$$

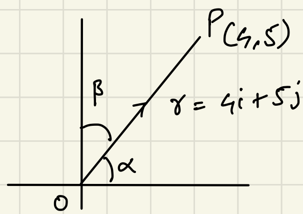
The direction ratio and direction cosines

direction ratio:
$4:5$

direction cosine:

l; $\cos\alpha = \frac{4}{\sqrt{41}}$

m; $\cos\beta = \frac{5}{\sqrt{41}}$



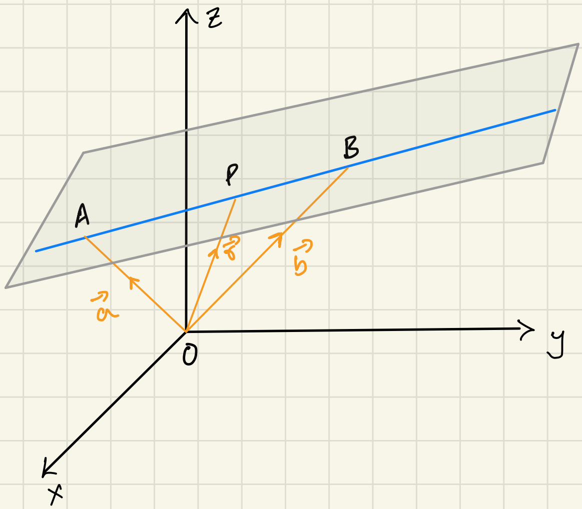
For any vector $r = a\hat{i} + b\hat{j} + c\hat{k}$; its direction ratio is $a : b : c$

direction cosines are:

$$l = \frac{a}{\sqrt{a^2+b^2+c^2}} \qquad m = \frac{b}{\sqrt{a^2+b^2+c^2}} \qquad n = \frac{c}{\sqrt{a^2+b^2+c^2}}$$

and $l^2 + m^2 + n^2 = 1$

Vector Equation of a line



$\vec{OA} = \vec{a}$
$\vec{OP} = \vec{r}$
$\vec{OB} = \vec{b}$

$\vec{AP} = t\,\vec{AB}$

∴ expression for $\vec{AB}$ in terms of $\vec{a}$ & $\vec{b}$

$$\vec{AB} = (-\vec{a}) + \vec{b}$$
$$= \vec{b} - \vec{a}$$

expression for r in terms of $\vec{a}, \vec{b}$ & t

$$\vec{r} = \vec{OP}$$
$$\vec{r} = \vec{OA} + \vec{AP}$$
$$= \vec{a} + t\,\vec{AB}$$
$$\boxed{\vec{r} = \vec{a} + t(\vec{b} - \vec{a})}$$

t is the parameter which we can vary to move the point through the line.

ie when $t = 0$; $\vec{r} = \vec{a}$ locates point A

when $t = 1$; $\vec{r} = \vec{b}$ locates point B

Cartesian Form

$$\therefore \quad \vec{r} = \vec{a} + t(\vec{b} - \vec{a})$$

$$= \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + t \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \\ b_3 - a_3 \end{pmatrix}$$

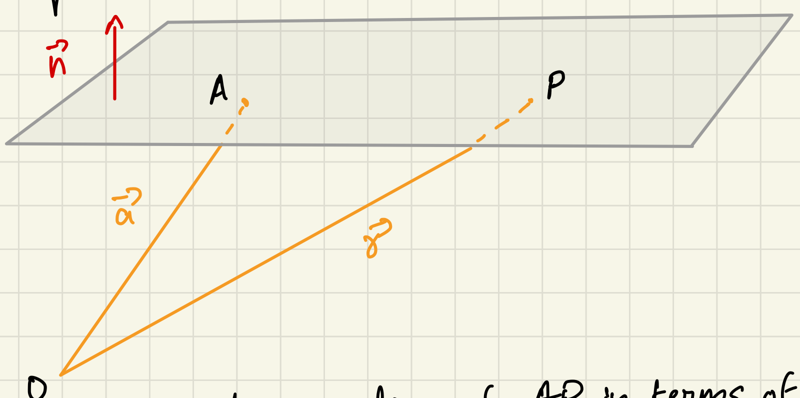
$$\therefore \quad t = \frac{x - a_1}{b_1 - a_1}$$

$$t = \frac{y - a_2}{b_2 - a_2}$$

$$t = \frac{z - a_3}{b_3 - a_3}$$

$$\therefore \quad \frac{x - a_1}{b_1 - a_1} = \frac{y - a_2}{b_2 - a_1} = \frac{z - a_3}{b_3 - a_3}$$

Equation of a Plane



The vector equation of AP in terms of $\vec{a}$ & $\vec{r}$:

$$\vec{AP} = (-\vec{a}) + \vec{r}$$

$$\vec{AP} = \vec{r} - \vec{a}$$

Relation b/w $\vec{AP}$ and normal vector $\vec{n}$:
They both are perpendicular; ∴ their dot product will be equal to 0.

ie $\vec{AP} \cdot \vec{n} = 0$

$(\vec{r} - \vec{a})\vec{n} = 0$

$\boxed{\vec{n}\vec{r} = \vec{n}\vec{a}}$ Equation of the plane

when $\vec{n}$ is a unit vector; $\hat{n}\vec{a}$ denotes the perpendicular distance of plane from the origin; denoted by d

∴ when $\vec{n} \Rightarrow \hat{n}.a = d$

∴ $\boxed{\vec{n}\vec{r} = d}$

where d is the ⊥r distance d from Origin.

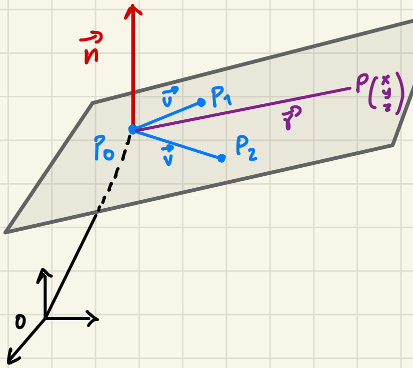
Dealing with planes with more than 2 points.

We know the vector equation of a line:

$r = \vec{a} + \lambda(\vec{b} - \vec{a})$

In the case of plane; $\vec{r}$ would be the planar mapping of the line on the plane which is formed by the two directional vectors $\vec{u}$ & $\vec{v}$. The directional vectors are made by utilizing two of the points from the

3 given point. The other one would be mapped to the planar map to arbitary point $P\begin{pmatrix}x\\y\\z\end{pmatrix}$ and it would also have the normal vector $\vec{n}$ to the plane.



Parametric form:

$$\vec{r} = \vec{P_0} + \vec{u}s + \vec{v}t$$

$$\begin{pmatrix}x\\y\\z\end{pmatrix} = \vec{P_0} + \vec{u}s + \vec{v}t$$

Normal form:

We can see $\vec{u} \times \vec{v} = \vec{n}$

and dot product of $\vec{n}$ & $\overrightarrow{P_0P} = 0$

$$\therefore \quad \vec{n} \cdot \vec{r} = 0$$

$$\vec{n}(P - P_0) = 0$$

$$\vec{n}\left(\begin{pmatrix}x\\y\\z\end{pmatrix} - P_0\right) = 0$$

Conversion

param form → (utilize $\vec{u}$ & $\vec{v}$ to find $\vec{n}$ by $\vec{u} \times \vec{v}$ & rewrite) → normal form

normal form → (open up & expand & then group) → Coordinat form

Coordinat form → (use SLE & pivots) → param form

Coordinate form:

The normal vector in normal form can be represented as $\vec{n} = \begin{pmatrix}a\\b\\c\end{pmatrix}$

$$\therefore \begin{pmatrix}a\\b\\c\end{pmatrix}\left(\begin{pmatrix}x\\y\\z\end{pmatrix} - P_0\right) = 0$$

$$\begin{pmatrix}a\\b\\c\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} - \underbrace{\begin{pmatrix}a\\b\\c\end{pmatrix}\begin{pmatrix}P_0\end{pmatrix}}_{\text{scalar product} = d} = 0$$

$$ax + by + cz = d$$

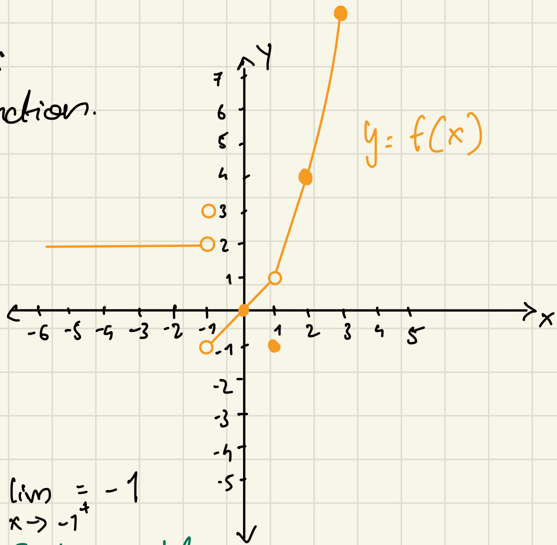
Limits

$$\lim_{x \to a} f(x) = L$$

Limits helps define the behavior of a function's independent variable as it approaches a specific value.

Applying limits on:
piece wise defined function.

$$f(x) = \begin{cases} 2 & , x < -1 \\ 3 & , x = -1 \\ x & , -1 < x < 1 \\ -1 & , x = 1 \\ x^2 & , x > 1 \end{cases}$$



Here:

$f(-1) = 3$

function value given/defined in question.

But:

$\lim_{x \to -1^-} = 2$

Approaching from left side

$\lim_{x \to -1^+} = -1$

Function approached from the right side

Limit Laws

$\frac{1}{0} = \infty$ or undefined

$\frac{0}{1} = 0$

$$\lim_{x \to a} c = c$$

(limit of constant is the constant itself)

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} [f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

(limit of n-th root is the n-th root of the limit inside)

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$ given $\lim_{x \to a} g(x) \neq 0$

3 cases

- $\rightarrow \frac{\text{num}}{\text{num}} \rightarrow$ apply limit
- $\rightarrow \frac{\text{num}}{0} \rightarrow$ approach from each side

  ie $\lim_{x \to a^-}$ & $\lim_{x \to a^+}$

  - agree: $\lim_{x \to a} = \pm\infty$
  - disagree: DNE
- $\rightarrow \frac{0}{0} \rightarrow$ Factorize or divide $\rightarrow$ Then try applying limit.

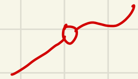
if just numerator and you get DNE or infinity ; apply squeeze theorem generally for combo of trigonometric functions eg: $\sin(x) \rightarrow -1 \leq \sin(x) \leq 1$

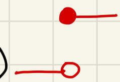
$\sin\left(\frac{1}{x}\right) \rightarrow -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

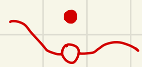
Continuity

Four cases can apply to function value when limit is applied at that specific value

continous 

gap discontinuity 

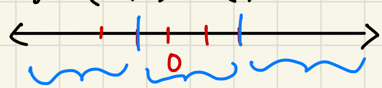
jump discontinuity (heaviside functn) 

hiccup (functn $\neq$ limit) 

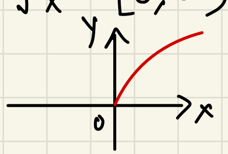
Standard functions Continuity :

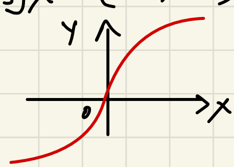
Polynomials : continuous on $(-\infty, \infty)$

Rational nos : whoreever they are defined (ie when defined denominator does not become zer) eg: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$



Root function: $\sqrt{x}$ $[0, \infty)$ $\sqrt[3]{x}$ $(-\infty, \infty)$



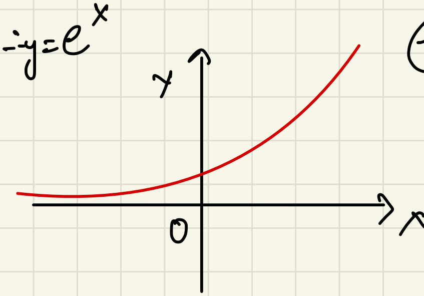


Trig function : Sin, Cos : $(-\infty, \infty)$

tan : $\ldots \left]-\frac{\pi}{2}, \frac{\pi}{2}\right[, \left]\frac{\pi}{2}, \frac{3\pi}{2}\right[\ldots$

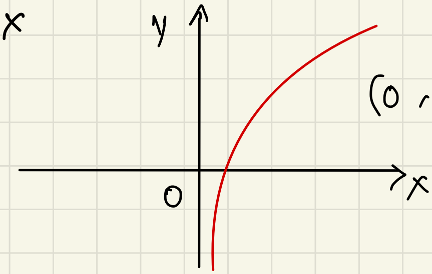
π (diff.) π

Exponents functions : $e^0 = 1$ $\therefore y = e^x$



$(-\infty, \infty)$

Log functions : $y = \log x$



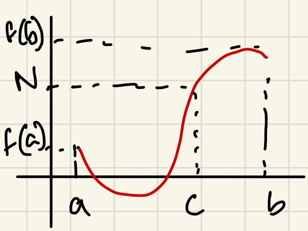
$(0, \infty)$

][Included
( ) not included

Intermediate Value Theorem

For continuity crossing.

With this proof we can find zero crossings using bisection methods ie we plug in the domain values and check if the function changes sign from -ve to +ve or vice versa, thus we can prove a zero crosing & bisect the interval to estimate the exact point



Eg: Find zero crossing

$f(x) = 4x^3 - 6x^2 + 3x - 2 = 0$

$4x^3 = 6x^2 - 3x + 2$

$\frac{4}{6}x^3 = x^2 - \frac{3}{6}x + \frac{2}{6}$

$\frac{2}{3}x^3 = x^2 - \frac{1}{2}x + \frac{1}{3}$

complete the squares.

plug values from -2, 2 for $f(x)$ and find $f(-2), f(-1), f(0), f(1)$ and $f(2)$ and see if sign changes anywhere.

Herons method

find square root

$$x_{i+1} = \frac{1}{2}\left(x_i + \frac{a}{x_i} \right)$$

$a \rightarrow$ number to find root of

$x_i \rightarrow$ initial guess

Newtons method

find functions zero crossing

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

$x_i \rightarrow$ initial guess.

Differentiation

1) Why derivative of constant is zero?

Lets have point $f(x) = c$

From what we know the slope would be defined as:

$$f'(x) = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

Here it would mean

$$f'(x) = \lim_{\Delta x \to 0} \frac{c - c}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{0}{\Delta x}$$

$$= \lim_{\Delta x \to 0} 0 = 0$$

2) $\frac{d}{dx} x^n = n x^{n-1}$

3) $\frac{d}{dx} \sqrt{x} = \frac{1}{2\sqrt{x}}$

4) $\frac{d}{dx} (f(x) + g(x)) = \frac{d}{dx} f(x) + \frac{d}{dx} g(x)$

5) $\frac{d}{dx} (\underset{u(x)}{\underset{\downarrow}{u}} \cdot \underset{v(x)}{\underset{\downarrow}{v}}) = u \cdot \frac{dv}{dx} + v \cdot \frac{du}{dx}$

6) $\frac{d}{dx} \left(\frac{u}{v}\right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v}$

7) $\frac{d}{dx} \sin x = \cos x$

8) $\frac{d}{dx} \cos x = -\sin x$

9) $\frac{d}{dx} e^x = e^x$

10) $\frac{d}{dx} \ln x = \frac{1}{x}$ $\{\ln x = \log_e x\}$

11) $\frac{d}{dx} (\tan x) = \sec^2 x = 1 + \tan^2 x$

12) Chain Rule: evaluate inner most paranthesis first then move out.

ie $h(x) = f(g(x))$

then $h'(x) = g'(x) \cdot f'(g(x))$

$= f'(g(x)) \cdot g'(x)$

13) In implicit differention; take y as a function of x & apply chain rule to get something $\cdot \frac{dy}{dx}$ form. check example

14) $\frac{d}{dx} kx = k$

15) $\frac{d}{dx} kx^n = k \cdot nx^{n-1}$

16) $\frac{d}{dx} e^{kx} = k \cdot e^{kx}$

17) $\frac{d}{dx} \ln kx = k \cdot \frac{1}{kx} = \frac{1}{x}$

18) $\frac{d}{dx} \sin kx = k \cdot \cos kx$

19) $\frac{d}{dx} \sin(kx+c) = k \cdot \cos(kx+c)$

20) $\frac{d}{dx} \cos kx = k \cdot (-\sin(kx))$

21) $\cos(kx+c) = k(-\sin(kx+c))$

22) $\tan kx = k \cdot \sec^2(kx) = k(1+\tan^2(kx))$

23) $\tan(kx+c) = k \cdot \sec^2(kx+c)$

24) $\sec kx = k \sec(kx)\tan(kx) = k(\sec^2(kx))$

25) $\sin^{-1}(kx) = k \cdot \dfrac{1}{\sqrt{1-(kx)^2}}$

26) $\dfrac{1}{k}\tan^{-1}\left(\dfrac{x}{k}\right) = \dfrac{1}{k} \cdot \dfrac{1}{k} \cdot \dfrac{1}{1+(x/k)^2} + \tan^{-1}\left(\dfrac{x}{k}\right) \cdot 0$

$$= \frac{1}{k^2} \frac{1 \cdot k^2}{k^2+x^2} = \frac{1}{k^2+x^2}$$

Curve sketching

Domain & zero crossing $\rightarrow$ factorize $f(x)$ & find points $x_1, x_2, x_3 \dots$

Find what's going on at the edges of domain points by applying limits

(Helps find asymptodes if they exist)

monotonicity and its intervals → first derivative $f'(x)$

↳ equate $f'(x) = 0$

↳ CN's

Use critical Nums (CN's) to split domain.

↳ (,) (,) (,)

Apply limit on the point x is not defined

→ find second derivative $f''(x)$

SC-1

Sign table 1
intervals
$f'(x)$ factors
↓
increasing or decreasing

SC-2

Sign table 2
intervals
$f''(x)$ values
↓
local max
local min

FVC

Function value chart

$f(x)$ $f'(x)$ values with CN's & its domain end points.

Concavity → equate $f''(x) = 0$

↳ find new intervals

↳ Create sign chart for the intervals

↳ Apply smiley faces

functions representations :

$f'(x)$ & $f''(x)$ ⇒ { undefined / defined everywhere

undefined at $x = ...$

value of x at defined xaxis value

Integration

1. $\int \sin x\,dx = -\cos x + C$
2. $\int \cos x\,dx = \sin x + C$
3. $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$
4. $\int 1\,dx = x + C$
5. $\int \sec^2 x\,dx = \tan x + C$
6. $\int \sec x \tan x\,dx = \sec x + C$
7. $\int \operatorname{cosec}^2 x = -\cot x + C$
8. $\int \operatorname{cosec} x \cot x = -\operatorname{cosec} x + C$
9. $\int \frac{1}{\sqrt{1-x^2}}\,dx = \sin^{-1}(x) + C$
10. $\int \frac{1}{\sqrt{1-x^2}}\,dx = -\cos^{-1}(x) + C$
11. $\int \frac{1}{1+x^2}\,dx = \tan^{-1}(x) + C$
12. $\int \frac{1}{1+x^2}\,dx = -\cot^{-1}(x) + C$
13. $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = \sec^{-1}(x) + C$

(14) $\int \frac{1}{|x|\sqrt{x^2-1}}\,dx = -\operatorname{cosec}^{-1}(x)+C$

(15) $\int e^x\,dx = e^x + C$

(16) $\int \frac{1}{x}\,dx = \ln x + C$

$\frac{d}{dx}a^x = a^x \log a$

(17) $\int a^x\,dx = \frac{a^x}{\ln x} + C$

* Special Integrals

$\int \tan x\,dx$ — $\times$ & $\div$ by $\sec x$ then u-sub

$\int \sec x\,dx$ — $\times$ & $\div$ by $\sec x + \tan x$ & u-sub

$\int \frac{1}{x^3+x}\,dx$ — take x^3 common & u-sub the bracket

* $\int uv = uv - \int v\,du$

u choose LIATE

or

D		I
+	$\searrow$	
−	$\searrow$	
+	$\leftrightarrow$	ϵ
−		

stop when:

when integrable

or

when $D \to 0$

or function question part repeats.

* Use trig identities to convert the question

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \operatorname{cosec}^2\theta$$

$$\sin 2\theta = 2\sin\theta\cos\theta$$

$$= \frac{2\tan\theta}{1+\tan^2\theta}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta$$

$$= 2\cos^2\theta - 1$$

$$= 1 - 2\sin^2\theta$$

* $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

* $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$$

* $\int \left(\begin{matrix}\text{exp in} \\ u=\sin x\end{matrix}\right) \cos x\,dx$

* $\int \left(\begin{matrix}\text{expression in} \\ u=\cos x\end{matrix}\right) \sin x\,dx$

* $\int \left(\begin{matrix}\text{exp in} \\ u=\tan x\end{matrix}\right) \sec^2 x\,dx$

* $\int \left(\begin{matrix} \text{exp in} \\ u = \sec x \end{matrix} \right) \sec x \tan x \, dx$

For $\int \sec^3 x \, dx$ don't split; try to apply IBP whenever possible

* Apply Trig - Substitution when you see the form x^2+a^2, x^2-a^2 or a^2-x^2

$x^2+a^2 \rightarrow a(\tan^2\theta + 1) = (\sec^2\theta)a$

$x^2-a^2 \rightarrow a(\sec^2\theta - 1) = a(\tan^2\theta)$

$a^2-x^2 \rightarrow \sin^2\theta + \cos^2\theta = 1$

$1 - \sin^2\theta = \cos^2\theta$

$a(1-\sin^2\theta) = (\cos^2\theta)a$

rephrase eqn in $\theta \, d\theta$ and solve for θ & replace θ with original x to find the integral.

* Partial Fraction.

→ when degree on top >= degree on bottom
do polynomial division.

→ Two Linear:

$$\frac{\sim}{x^2-5x+4} = \frac{\sim}{(x-1)(x-4)} = \frac{A}{(x-1)} + \frac{B}{(x-4)}$$

→ Irreducable quadratic:

$$\frac{\sim}{(x+3)(x^2+4)} = \frac{A}{x+3} + \frac{Bx+C}{x^2+4}$$

→ $\frac{\sim}{x^2+6x+13} = \frac{\sim}{\quad} \leftarrow$ irreducable = comple through squares

$2a = 6x$

and then square a

$a^2 =$ magic number

to add or subtract.

→ Repeating Squares:

$$\frac{\sim}{x^3(x^2+1)^2} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x^3} + \frac{Dx+E}{x^2+1} + \frac{Fx+G}{(x^2+1)^2}$$