Limits

github.com/royceanton

Notes

Syntax:

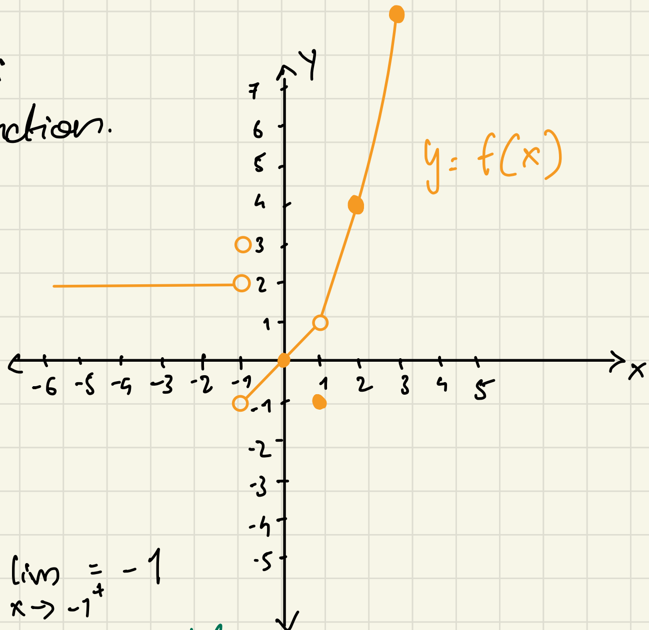
$$\lim_{x \to a} f(x) = L$$

Limits helps define the behavior of a function's independent variable as it approaches a specific value.

We can usually see them applied around vertical & horizontal asymptodes
Helps use approximate an exact value for simulations

Applying limits on:
Piece wise defined function.

$$f(x) = \begin{cases} 2, & x < -1 \\ 3, & x = -1 \\ x, & -1 < x < 1 \\ -1, & x = 1 \\ x^2, & x > 1 \end{cases}$$



Here:

$f(-1) = 3$

function value given/defined in question.

But:

$\lim_{x \to -1^-} = 2$

Approaching from left side

$\lim_{x \to -1^+} = -1$

Function approached from the right side

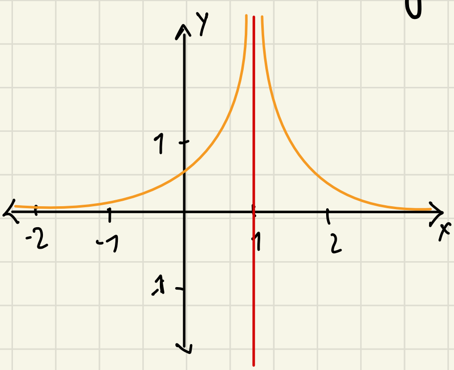
Similarly we have $f(1) = -1$; but

$$\underbrace{\lim_{x \to 1^-} f(x) = 1 \quad \& \quad \lim_{x \to 1^+} f(x) = 1}$$

(This becomes a two sided limit approx)

Therefore $\lim_{x \to 1} f(x) = 1$

Cases where asymptodes occur:



Here we have vertical asymptode at $x = 1$ as function approximates to $+\infty$ when approached from both sides.

$$\lim_{x \to 1} f(x) = +\infty$$

$\frac{1}{0} = \infty$ or undefined

$\frac{0}{1} = 0$

Limit Laws

$$\lim_{x \to a} c = c$$

(limit of constant is the constant itself)

$$\lim_{x \to a} \left[f(x) + g(x) \right] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \left[f(x) - g(x) \right] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)}$$

(limit of n-th root is the n-th root of the limit inside)

$$\lim_{x \to a} [f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$$

$$\lim_{x \to a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$$ given $\lim_{x \to a} g(x) \neq 0$

When dividing & applying limit laws and you find the resut as $\frac{0}{0}$; it is undefined
∴ you must factorize the equation furthear.

$$\lim_{x \to 2} \frac{x^2 - x + 6}{x - 2}$$

checking Num $\lim_{x \to 2} x^2 - x + 6 = 4 - 2 + 6$
$= 8$

checking Denom $\lim_{x - 2} x - 2 = 2 - 2 = 0$

$\lim_{x \to 2^-} \frac{8}{0^-} = -\infty$ $\lim_{x \to 2^+} \frac{8}{0} = +\infty$

Limit does not exist as the limits are different on each sides.

3 cases of applying Limits

$\lim_{x \to a} f(x)$ where $f(x)$ has a numerator $N(x)$ and Denominator $D(x)$, we apply the limit (ie, the specific value it approaches to the numerator and denominator.

3 cases

- $\rightarrow \dfrac{\text{num}}{\text{num}} \rightarrow$ apply limit
- $\rightarrow \dfrac{\text{num}}{0} \rightarrow$ approach from each side
  ie $\lim_{x \to a^-}$ & $\lim_{x \to a^+}$
  - $\downarrow$ agree: $\lim_{x \to a} = \pm\infty$
  - $\downarrow$ disagree: DNE
- $\rightarrow \dfrac{0}{0} \rightarrow$ Factorize or divide $\rightarrow$ Then try applying limit.

if just numerator and you get DNE or infinity ; apply squeeze theorem generally for combo of trigonometric functions eg: $\sin(x) \to -1 \leq \sin(x) \leq 1$

$$\sin\left(\frac{1}{x}\right) \to -1 \leq \sin\left(\frac{1}{x}\right) \leq 1$$

q: $\lim_{x \to 0} x^2 \cdot \sin\left(\frac{1}{x}\right)$

$N(0) = 0 \cdot \sin\left(\frac{1}{0}\right) =$ DNE

$\therefore$ apply squeeze theorem

ie $-1 \leq \sin\left(\frac{1}{x}\right) \leq 1$

x by x^2 on both sides $\Rightarrow -x^2 \leq x^2 \sin\left(\frac{1}{x}\right) \leq x^2$

Apply limits on both ends instead

$\lim_{x \to 0} -x^2 = 0 \iff \lim_{x \to 0} x^2 = 0$

check if they both equal

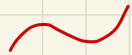
if yes

then $\lim_{x \to 0} x^2 \sin$

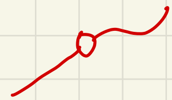
Continuity

Four cases
can apply to
function value
when limit is
applied at that
specific value

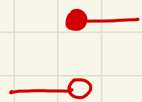
continous



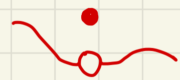
gap
discontinuity



jump
discontinuity
(heaviside functn)



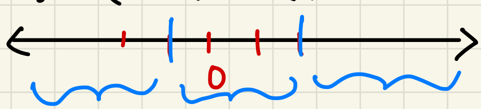
hiccup
(functn $\neq$ limit)



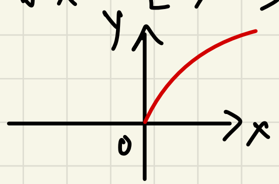
Standard functions Continuity:

Polynomials: continuous on $(-\infty, \infty)$

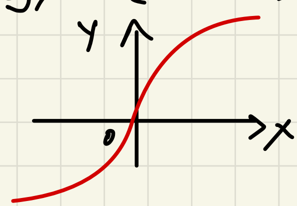
Rational nos: wherever they are defined (ie when defined denominator does not become zer) eg: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$



Root function: $\sqrt{x}$ $[0, \infty)$



$\sqrt[3]{x}$ $(-\infty, \infty)$

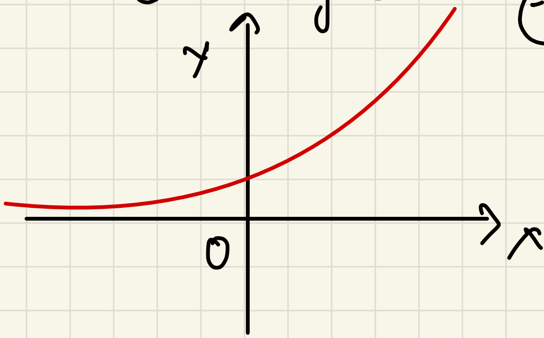


Trig function : Sin, Cos : $(-\infty, \infty)$

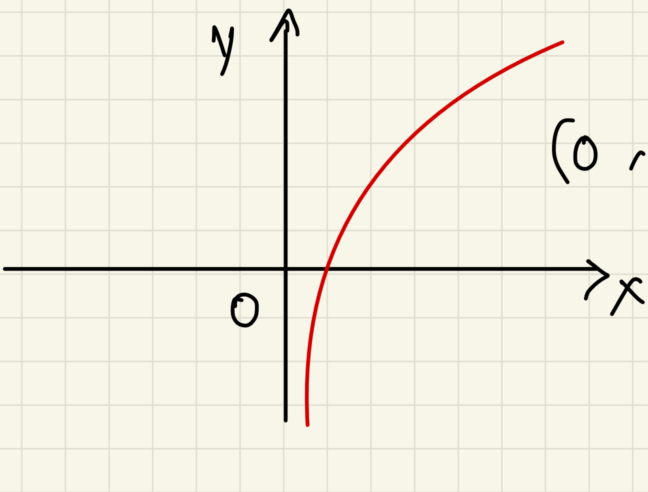
tan : $\ldots \underbrace{\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[}_{\pi}, \underbrace{\left]\frac{\pi}{2}, \frac{3\pi}{2}\right[}_{\pi} \ldots$

(diff.)

Exponents functions : $e^0 = 1$ $y = e^x$ $(-\infty, \infty)$



Log functions : $y = \log x$



$(0, \infty)$

][Included

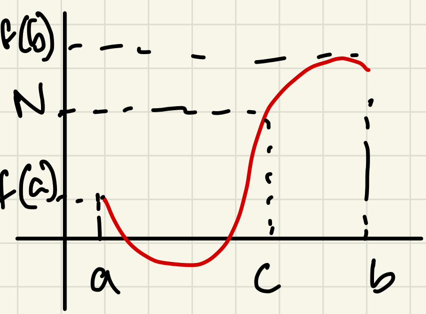
( ) not included

Intermediate Value Theorem

For continuity crossing.

If we have a function $f(x)$ defined b/w $[a, b]$ and there exists a number N b/w $f(a)$ & $f(b)$; then there exists a number $c \in [a, b]$ for which $f(c) = N$

With this proof we can find zero crossings using bisection methods ie we plug in the domain values and check if the function changes sign from -ve to +ve or vice versa; thus we can prove a zero crosing & bisect the interval to estimate the exact point



Eg: Find zero crossing

$$f(x) = 4x^3 - 6x^2 + 3x - 2 = 0$$

$$4x^3 = 6x^2 - 3x + 2$$

$$\frac{4}{6}x^3 = x^2 - \frac{3}{6}x + \frac{2}{6}$$

$$\frac{2}{3}x^3 = x^2 - \frac{1}{2}x + \frac{1}{3}$$

complete the squares.

plug values from -2, 2 for $f(x)$ and find $f(-2), f(-1), f(0), f(1)$ and $f(2)$ and see if sign changes anywhere.

Examples

1) $\lim_{h \to 0} \dfrac{(3+h)^2 - 9}{h}$

$N(a) = (3+0)^2 - 9 = 3^2 - 9 = 0$

$D(a) = 0$

Case 3 $\Rightarrow \dfrac{0}{0}$ $\therefore$ factorise

$\lim_{h \to 0} \dfrac{3^2 + h^2 + 6h - 9}{h}$

$\lim_{h \to 0} \dfrac{\cancel{h}\left(\cancel{\frac{9}{h}} + h + 6 - \cancel{\frac{9}{h}}\right)}{\cancel{h}}$

$\lim_{h \to 0} h + 6 = 6$

2) $\lim_{x \to -4} \left[\dfrac{\frac{1}{4} + \frac{1}{x}}{4+x} \right]$

here $a = -4$

$N(a) = \frac{1}{4} + \frac{1}{-4} = 0$

$D(a) = 4 - 4 = 0$

$\frac{0}{0}$ form ; case 3 ; factorize

$\lim_{x \to -4} \left[\frac{x+4}{4x} \cdot \frac{1}{x+4} \right]$

$\lim_{x \to -4} \left[\frac{1}{4x} \right] = -\frac{1}{16}$

3) $\lim_{x \to -2} \left[\frac{x+2}{x^3+8} \right]$

here $a = -2$

$$\left.\begin{aligned} N(a) &= -2+2 = 0 \\ D(a) &= -8+8 = 0 \end{aligned}\right\} = \frac{0}{0} \quad \text{Case 3}$$

$(a+b)^3$

1

1 1

1 2 1 $= a^3 b^0 \, 3C_0 + a^2 b^1 \, 3C_1 + a^1 b^2 \, 3C_2 + a^0 b^3 \, 3C_3$

1 3 3 1 $= a^3 + a^2 b \frac{3!}{1!(2)!} + ab^2 \frac{3!}{2!\,1!} + b^3 \frac{3!}{3!\,0!}$

$$(a+b)^3 = a^3 + 3a^2 b + 3ab^2 + b^3$$

$$\therefore a^3 + b^3 = (a+b)^3 - 3a^2 b - 3ab^2$$

$$x^3 + 8 = x^3 + 2^3 = (x+2)^2 - 3x^2 \cdot 2 - 3 \cdot x \cdot 2^2$$

$$= (x+2)^2 - 6x^2 - 12x$$

$$= (x+2)^2 - 6x^2 - 12x$$

$$= (x+2)^2 - 6x(x+2)$$

$$\lim_{x \to -2} \left[\frac{x+2}{x^3+8} \right] = \lim_{x \to -2} \frac{(x+2)}{(x+2)\left[x+2 - 6x \right]}$$

$$= \lim_{x \to -2} \frac{1}{x+2-6x}$$

$$= \frac{1}{-2+2+12} = \frac{1}{12}$$

h) $$\lim_{x \to 2} \frac{x^2 - x + 6}{x-2}$$

$N(2) = 4 - 2 + 6 = 8$ $\Rightarrow$ $\underbrace{\frac{\text{num}}{0}}$ cased

$D(2) = 2 - 2 = 0$

$\Downarrow$

check limit on approach from both sides

$$\lim_{x \to 2^-} \frac{8}{2^- - 2} = \frac{8}{0^-} = -\infty$$

$$\lim_{x \to 2^+} \frac{8}{0^+} = +\infty \Leftarrow$$

Left side limit $\neq$ Right approach limit

$\therefore$ DNE

$\lim_{x \to a^-} == \lim_{x \to a^+}$

agree: $\lim_{x \to a} = \pm\infty$

disagree: DNE

5) $\lim_{t \to 0} \dfrac{\sqrt{t^2+9} - 3}{t^2}$

$N(0) = \sqrt{9} - 3 = 3 - 3 = 0$
$D(0) = 0^2 = 0$

$\Big\} \dfrac{0}{0}$ case 3 factorize

$(a+b)^2 = a^2 + b^2 + 2ab$

$(a+b)^2 - 2ab = a^2 + b^2$

$t^2 + 3^2 = (t+3)^2 - 6t$

$$\lim_{t \to 0} \frac{\sqrt{(t+3)^2 - 6t} - 3}{t^2}$$

$$\lim_{t \to 0} \frac{(t+3) - \sqrt{6t} - 3}{t^2} \Rightarrow$$ Still $\dfrac{0}{0}$ form

$\therefore$ Try another approach; perhaps conjugate

$$\lim_{t \to 0} \frac{\sqrt{t^2+9} - 3}{t^2} \cdot \frac{\sqrt{t^2+9} + 3}{\sqrt{t^2+9} + 3}$$

$$= \lim_{t \to 0} \frac{t^2 + 9 - 9}{t^2(\sqrt{t^2+9} + 3)} = \lim_{t \to 0} \frac{t^2}{t^2\sqrt{t^2+9} + 3t^2}$$

$$= \lim_{t \to 0} \frac{1}{\sqrt{t^2+9}+3} = \frac{1}{3+3} = \frac{1}{6}$$

6) $\lim_{x \to 0} |x|$

$\lim_{x \to 0^-} |x| = 0$ Left approaching limit

$\lim_{x \to 0^+} |x| = 0$ Right approaching limit

They both equal $\therefore$ Limit exists.

7) $f(x) = \begin{cases} \sqrt{x-2}, & x > 2 \\ 4-2x, & x < 2 \end{cases}$

$$\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} 4-2x = 4-4 = 0$$

$$\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} \sqrt{x-2} = \sqrt{2-2} = 0$$

one sided limits are same

$\therefore$ Two sided limits exist

8) $f(x) = \dfrac{x^2 - x - 2}{x - 2}$

Check for continuity

At $x = 2$ the functn is not defined
∴ discontinuous at $x = 2$
Now we check the limit when it approaches there

$$\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$$

$$\left.\begin{array}{l} N(2) = 4 - 2 - 2 = 0 \\ D(2) = 2 - 2 = 0 \end{array}\right\} \frac{0}{0} \text{ Case 3 (factorize)}$$

$2a = -1$

$a = -\frac{1}{2}$

$a^2 = \frac{1}{4}$

$$x^2 - x - 2 + \frac{1}{4} - \frac{1}{4}$$

$$x^2 - x + \frac{1}{4} - \frac{1}{4} - 2$$

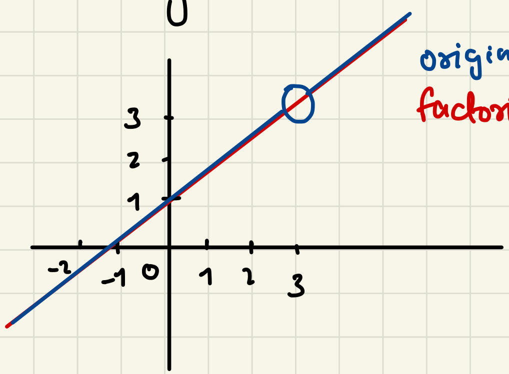
$$= \left(x - \frac{1}{2}\right)^2 - \frac{9}{4}$$

wrong approach by completing squares; better to divide.

$$\begin{array}{r l}
 & x + 1 \\
x-2 & \overline{\big) \, x^2 - x - 2} \\
 & \underline{x^2 - 2x} \\
 & \quad\quad x - 2 \\
 & \quad\quad x - 2
\end{array}$$

$\Rightarrow \lim_{x \to 2} (x+1) = 3$

$\therefore$ By applying limit the function value approaches 3 although the original function is discontinuous



9) $g(x) = \begin{cases} \frac{1}{x^2} & x \neq 0 \\ 1 & x = 0 \end{cases}$

At $x = 0$; function value is 1

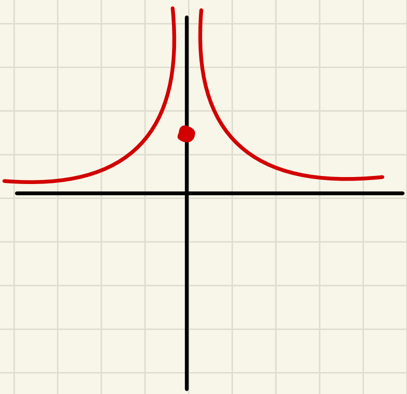
Now we apply limit to see how function behaves as it approaches 0

$$\lim_{x \to 0^-} \frac{1}{x^2} = \frac{1}{(0^-)^2} = +\infty$$

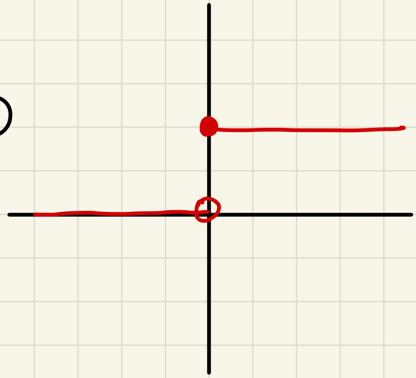
$$\lim_{x \to 0} \frac{1}{x^2} \qquad \lim_{x \to 0^+} \frac{1}{x^2} = \frac{1}{(0^+)^2} = +\infty$$

one sided limits are the same

$\therefore$ two sided limits exists.



10) $h(x) = \begin{cases} 0 & x < 0 \\ 1 & x \geq 0 \end{cases}$



$$\lim_{x \to 0^-} h(x) = 0$$

$$\lim_{x \to 0^+} h(x) = 1$$

Homework

James Stewart 7^{th} Edition

2.2

problems 2-12, 15-18, 29-39

2.3

problems 1-32, 37-52

2.2

2. Explain what it means to say that

$$\lim_{x \to 1^-} f(x) = 3 \qquad \text{and} \qquad \lim_{x \to 1^+} f(x) = 7$$

In this situation is it possible that $\lim_{x\to 1} f(x)$ exists? Explain.

3. Explain the meaning of each of the following.

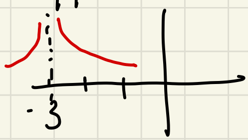
(a) $\lim_{x \to -3} f(x) = \infty$ (b) $\lim_{x \to 4^+} f(x) = -\infty$

2) The limit of function as it approaches 1 from the left side ; the function approximates to a value 3

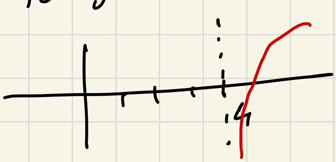
Similarly when the function limit approaches 1 from the +ve side, the function approximates to value 7.

Here the one sided limits are not equal ∴ two sided limits does not exist.

3) a) As the function approaches -3 on the x axis the value of the function tends to approach +ve infinity.



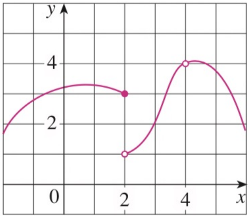
b) As the function approaches 4 from the righ side it tends to go towards -ve infinity



4. Use the given graph of f to state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 2^-} f(x)$ (b) $\lim_{x\to 2^+} f(x)$ (c) $\lim_{x\to 2} f(x)$

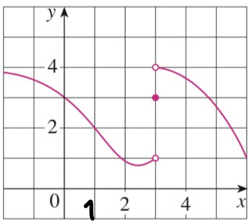
(d) $f(2)$ (e) $\lim_{x\to 4} f(x)$ (f) $f(4)$



5. For the function f whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to 1} f(x)$ (b) $\lim_{x\to 3^-} f(x)$ (c) $\lim_{x\to 3^+} f(x)$

(d) $\lim_{x\to 3} f(x)$ (e) $f(3)$



6. For the function h whose graph is given, state the value of each

4) a) Approach 2 from left; ∴ 3

b) Approach 2 from right; ∴ 1

c) $\lim_{x\to 2} f(x) = \text{DNE}$

d) $f(2) = 3$

e) $\lim_{x\to 4} f(x) = 4$

f) $f(4) = \cancel{4}$ not defined as hole at 4

DNE as two sided limits are not the same

5) a) $\lim_{x\to 1} f(x) = 2$

b) $\lim_{x\to 3^-} f(x) = 1$

c) $\lim_{x\to 3^+} f(x) = 4$

d) $\lim_{x\to 3} f(x) = \text{DNE}$ as one sided limits not same

e) $f(3) = 3$

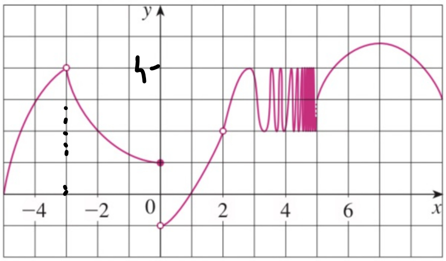
6. For the function h whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x \to -3^-} h(x)$ (b) $\lim_{x \to -3^+} h(x)$ (c) $\lim_{x \to -3} h(x)$

(d) $h(-3)$ (e) $\lim_{x \to 0^-} h(x)$ (f) $\lim_{x \to 0^+} h(x)$

(g) $\lim_{x \to 0} h(x)$ (h) $h(0)$ (i) $\lim_{x \to 2} h(x)$

(j) $h(2)$ (k) $\lim_{x \to 5^+} h(x)$ (l) $\lim_{x \to 5^-} h(x)$



a) $\lim_{x \to -3^-} h(x) = 4$

b) $\lim_{x \to -3^+} h(x) = 4$

c) $\lim_{x \to -3} h(x) = 4$

d) $h(-3) =$ not defined

e) $\lim_{x \to 0^-} h(x) = 1$

f) $\lim_{x \to 0^+} h(x) = -1$

g) $\lim_{x \to 0} h(x) =$ DNE

h) $h(0) = 1$

i) $\lim_{x \to 2} h(x) = 2$

j) $h(2) =$ not defined

k) $\lim_{x \to 5^+} h(x) = 3$

l) $\lim_{x \to 5^-} h(x) = \cancel{3}$

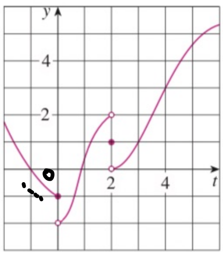
As the function approaches 5 from the left side it keeps oscillating b/w 2 & 4 values and not approaching an approximate value.

7. For the function g whose graph is given, state the value of each quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{t \to 0^-} g(t)$ (b) $\lim_{t \to 0^+} g(t)$ (c) $\lim_{t \to 0} g(t)$

(d) $\lim_{t \to 2^-} g(t)$ (e) $\lim_{t \to 2^+} g(t)$ (f) $\lim_{t \to 2} g(t)$

(g) $g(2)$ (h) $\lim_{t \to 4} g(t)$



a) $\lim_{t \to 0^-} g(t) = -1$

b) $\lim_{t \to 0^+} g(t) = -2$

c) $\lim_{t \to 0} g(t) = \text{DNE}$

d) $\lim_{t \to 2^-} g(t) = 2$

e) $\lim_{t \to 2^+} g(t) = 0$

f) $\lim_{t \to 2} g(t) = \text{DNE}$

g) $g(2) = 1$

h) $\lim_{t \to 4} g(t) = 3$

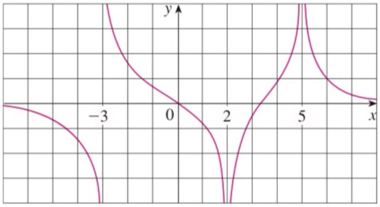
8)

8. For the function R whose graph is shown, state the following.

(a) $\lim_{x\to 2} R(x)$ (b) $\lim_{x\to 5} R(x)$

(c) $\lim_{x\to -3^-} R(x)$ (d) $\lim_{x\to -3^+} R(x)$

(e) The equations of the vertical asymptotes.



a) $\lim_{x\to 2} R(x) = -\infty$

b) $\lim_{x\to 5} R(x) = +\infty$

c) $\lim_{x\to -3^-} R(x) = -\infty$

d) $\lim_{x\to -3^+} R(x) = +\infty$

e) $x=-3$, $x=2$ & $x=5$

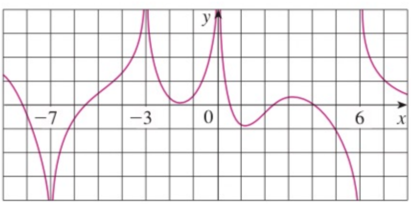
9)

9. For the function f whose graph is shown, state the following.

(a) $\lim_{x\to -7} f(x)$ (b) $\lim_{x\to -3} f(x)$ (c) $\lim_{x\to 0} f(x)$

(d) $\lim_{x\to 6^-} f(x)$ (e) $\lim_{x\to 6^+} f(x)$

(f) The equations of the vertical asymptotes.



a) $\lim_{x\to -7} f(x) = -\infty$

b) $\lim_{x\to -3} f(x) = +\infty$

c) $\lim_{x\to 0} f(x) = +\infty$

d) $\lim_{x\to 6^-} f(x) = -\infty$

e) $\lim_{x\to 6^+} f(x) = +\infty$

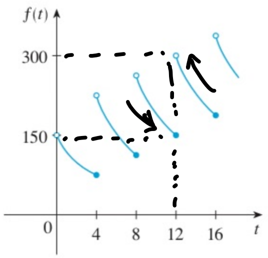
f) $x=-7$; $x=-3$; $x=0$ and $x=6$

10.

10. A patient receives a 150-mg injection of a drug every 4 hours. The graph shows the amount $f(t)$ of the drug in the bloodstream after t hours. Find

$$\lim_{t \to 12^-} f(t) \qquad \text{and} \qquad \lim_{t \to 12^+} f(t)$$

and explain the significance of these one-sided limits.



$\lim_{t \to 12^-} f(t) = 150$

$\lim_{t \to 12^+} f(t) = 300$

one sided limits not equal $\therefore$ two sided limit DNE.

The administation of drug in the patient every 4 hrs leads to a spike in the amount of drug in the body at each time interval that an exact approximat at the time cannot be pin pointed.

11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x \to a} f(x)$ exists.

11. $f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x < 1 \\ 2 - x & \text{if } x \geqslant 1 \end{cases}$

12. $f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leqslant x \leqslant \pi \\ \sin x & \text{if } x > \pi \end{cases}$

11)

11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

11. $f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x < 1 \\ 2 - x & \text{if } x \geqslant 1 \end{cases}$

for $x < -1$

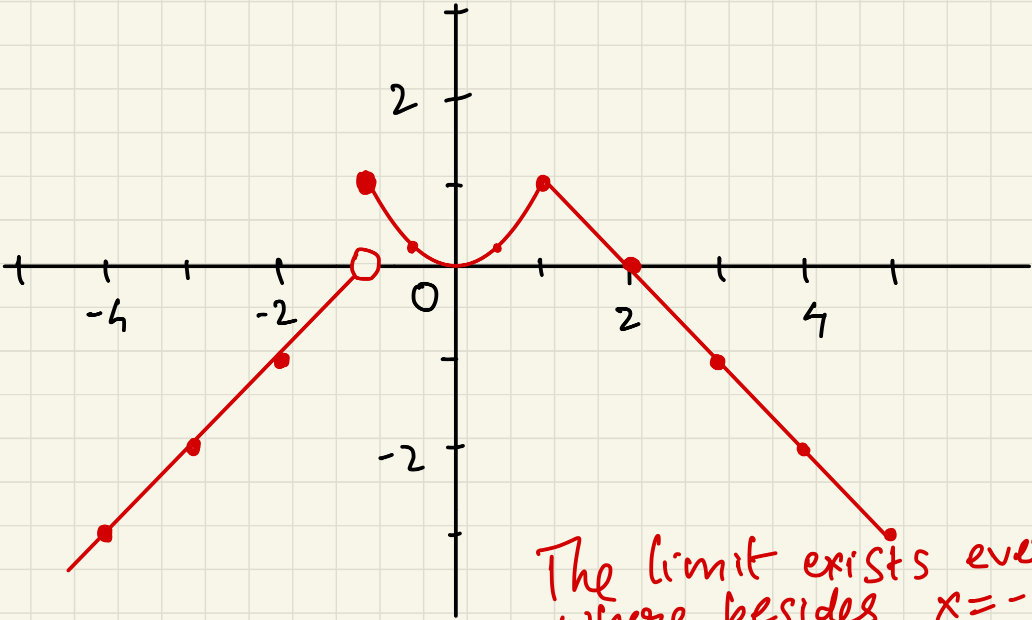
x	-4	-3	-2	-1
y	-3	-2	-1	0

for $-1 \leq x < 1$

x	-1	-0.5	0	0.5	1
y	1	0.25	0	0.25	1

for $x \geq 1$

x	1	2	3	4	5
y	1	0	-1	-2	-3



The limit exists everywhere besides $x = -1$

$\lim_{x\to -1} f(x)$ DNE as $\lim_{x\to -1^-} f(x) = 0$ & $\lim_{x\to -1^+} f(x) = 1$

$\lim_{x\to 1} f(x) = 1$ exists as $\lim_{x\to 1^-} f(x) = 1$

$\lim_{x\to 1}$

12)

11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x\to a} f(x)$ exists.

11. $f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leqslant x < 1 \\ 2 - x & \text{if } x \geqslant 1 \end{cases}$

12. $f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leqslant x \leqslant \pi \\ \sin x & \text{if } x > \pi \end{cases}$

$x < 0$

$1 + \sin x$

$\sin(-\theta) = -\sin\theta$

x	$-\pi$	$-\frac{\pi}{2}$	0
y	$0+1$ 1	$-1+1$ 0	$0+1$ 1

$0 \leq x \leq \pi$

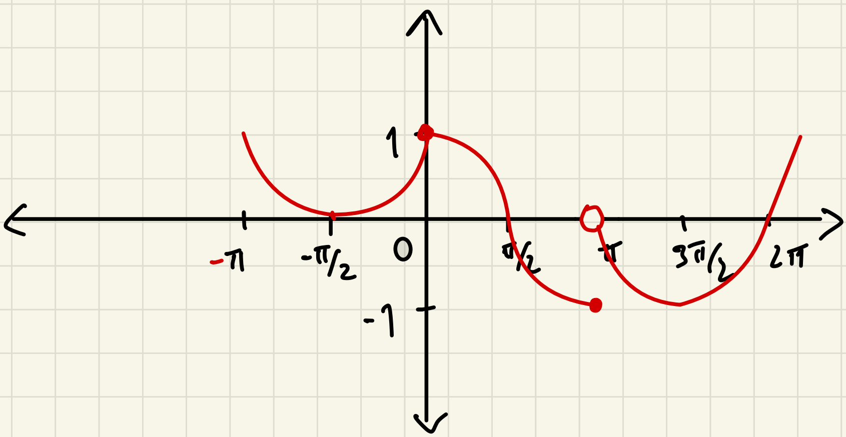
$\cos x$

x	0	$\pi/2$	π
y	1	0	-1

$x > \pi$

$\sin x$

x	π	$3\pi/2$	2π
y	0	-1	0



limit exists every where besides at $x = \pi$

15–18 Sketch the graph of an example of a function f that satisfies all of the given conditions.

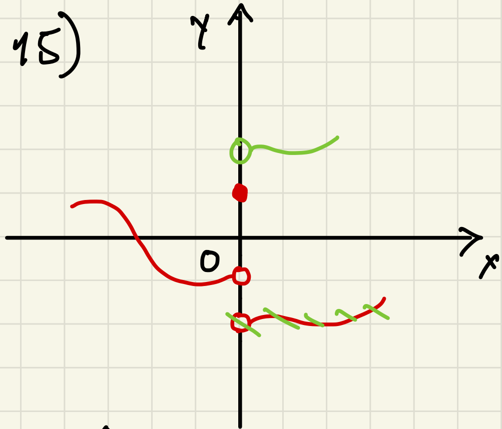
15. $\lim_{x\to 0^-} f(x) = -1, \quad \lim_{x\to 0^+} f(x) = 2, \quad f(0) = 1$

16. $\lim_{x\to 0} f(x) = 1, \quad \lim_{x\to 3^-} f(x) = -2, \quad \lim_{x\to 3^+} f(x) = 2,$
$f(0) = -1, \quad f(3) = 1$

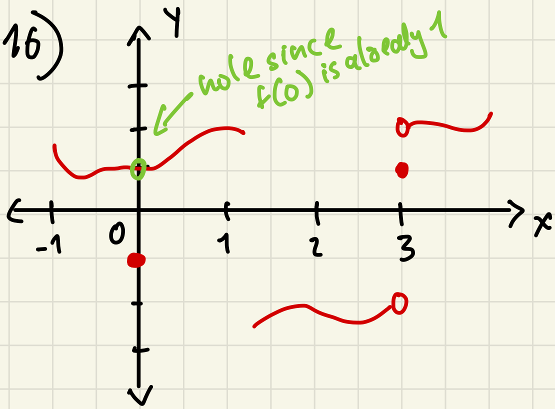
17. $\lim_{x\to 3^+} f(x) = 4, \quad \lim_{x\to 3^-} f(x) = 2, \quad \lim_{x\to -2} f(x) = 2,$
$f(3) = 3, \quad f(-2) = 1$

18. $\lim_{x\to 0^-} f(x) = 2, \quad \lim_{x\to 0^+} f(x) = 0, \quad \lim_{x\to 4^-} f(x) = 3,$
$\lim_{x\to 4^+} f(x) = 0, \quad f(0) = 2, \quad f(4) = 1$

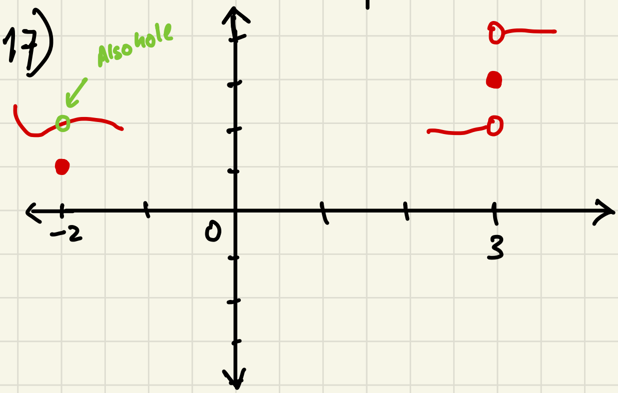
15)



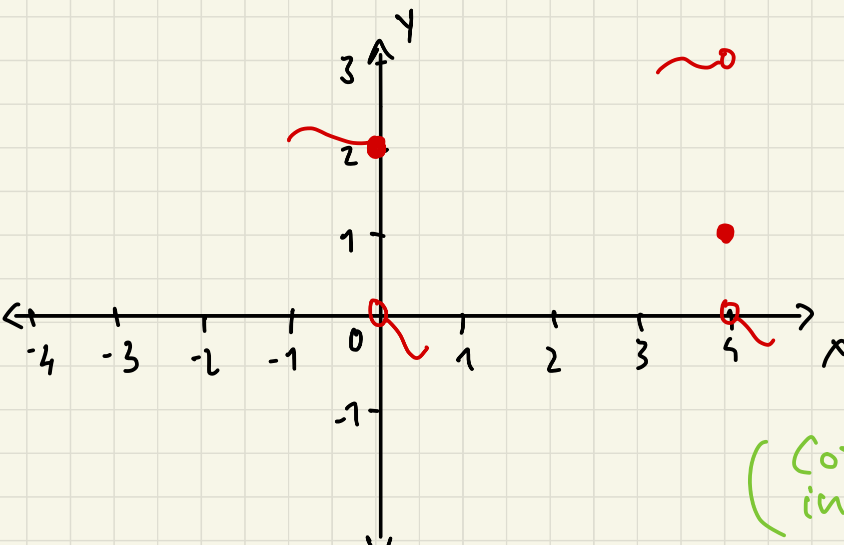
16)



17)



18)



(Corrections in Green)

29–37 Determine the infinite limit.

29. $\lim_{x \to -3^+} \dfrac{x+2}{x+3}$

30. $\lim_{x \to -3^-} \dfrac{x+2}{x+3}$

31. $\lim_{x \to 1} \dfrac{2-x}{(x-1)^2}$

32. $\lim_{x \to 5^-} \dfrac{e^x}{(x-5)^3}$

33. $\lim_{x \to 3^+} \ln(x^2-9)$

34. $\lim_{x \to \pi^-} \cot x$

35. $\lim_{x \to 2\pi^-} x \csc x$

36. $\lim_{x \to 2^-} \dfrac{x^2-2x}{x^2-4x+4}$

37. $\lim_{x \to 2^+} \dfrac{x^2-2x-8}{x^2-5x+6}$

29. $\lim_{x \to -3^+} \dfrac{x+2}{x+3}$

$a = -3$

$N(-3) = -3+2 = -1 = \text{Num}$

$D(-3) = -3+3 = 0$

Case 2: $\dfrac{\text{Num}}{0}$ form

checking one sided limits

$\lim_{x \to -3^+} \dfrac{x+2}{x+3} = \dfrac{-1}{0^+} = -\infty$

30. $\lim_{x \to -3^-} \dfrac{x+2}{x+3}$

$= \dfrac{-3+2}{-3^-+3} = \dfrac{-1}{0^-}$

$= +\infty$

31. $\lim_{x \to 1} \dfrac{2-x}{(x-1)^2}$

Numerator $N(1) = 2-1 = 1$

Denom $D(1) = (1-1)^2 = 0$

Case 2 $\dfrac{\text{Num}}{0}$ form

Checking one sided limits

$\lim_{x \to 1^-} \dfrac{2-x}{(x-1)^2} = \dfrac{1}{(0^-)^2} = +\infty$

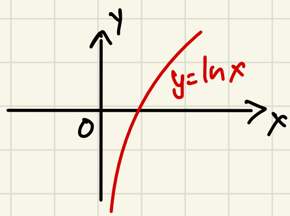
$\lim_{x \to 1^+} \dfrac{2-x}{(x-1)^2} = \dfrac{1}{(0^+)^2} = +\infty$

vertical asymptote at $x = 1$ & limit approaches to $+\infty$

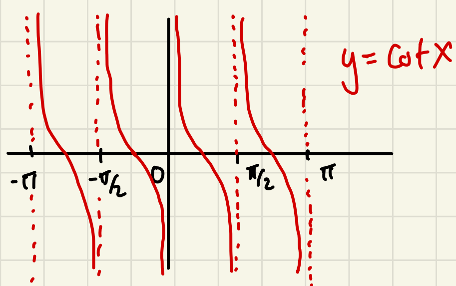
32. $\lim_{x \to 5^-} \dfrac{e^x}{(x-5)^3} = \dfrac{e^5}{(5^- - 5)^3} = \dfrac{e^5}{0^-} = -\infty$

33. $\lim_{x \to 3^+} \ln(x^2-9) = \ln((3^+)^2-9)$

$= \ln 0^+ = -\infty$



34. $\lim_{x \to \pi^-} \cot x = -\infty$



35. $\lim_{x \to 2\pi^-} x \operatorname{cosec} x$

$$\lim_{x \to 2\pi^-} \frac{x}{\sin x} = \frac{2\pi}{0^-} = -\infty$$

	π 180	270	2π 360
sin	0	-1	0
cos	-1	0	1
tan	0	∞	0
cot	∞	0	∞

36. $\lim_{x \to 2^-} \frac{x^2 - 2x}{x^2 - 4x + 4}$

$N(2) = 4 - 4 = 0$

$D(2) = 4 - 8 + 4 = 0$

$\frac{0}{0}$ form ; case 3

$\therefore$ factorize

$$\lim_{x \to 2^-} \frac{x^2\left(1 - \frac{2}{x}\right)}{x^2\left(1 - \frac{4}{x} + \frac{4}{x^2}\right)} = \lim_{x \to 2^-} \frac{1 - \frac{2}{x}}{1 - \frac{4}{x} + \frac{4}{x^2}}$$

also $\frac{0}{0}$ $\therefore$ another approach

$$\lim_{x \to 2^-} \frac{x^2 - 2x}{(x-2)^2} = \lim_{x \to 2^-} \frac{x(x-2)}{(x-2)^2}$$

$$= \lim_{x \to 2^-} \frac{x}{x-2}$$

$$= \frac{2}{2^- - 2} = \frac{2}{0^-} = -\infty$$

38) $\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6}$

$N(2) = 4 - 4 - 8 = -8$

$D(2) = 4 - 10 + 6 = 0$

case 2 $\frac{\text{num}}{0}$ form

$\therefore$ we check the Onesided limits

$\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 5x + 6} = \cancel{\frac{-8}{0^+}} = \cancel{-\infty}$

Correct ans is $+\infty$
why

$-2 \times -3 = 6$

$-2 + -3 = -5$

$\lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 2x - 3x + 6} = \lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x(x-2) - 3(x-2)}$

$= \lim_{x \to 2^+} \frac{x^2 - 2x - 8}{(x-2)(x-3)}$

$= \lim_{x \to 2^+} (x-2) = 2^+ - 2 = 0^+$

$\lim_{x \to 2^+} (x-3) = 2 - 3 = -1$

$\therefore \lim_{x \to 2^+} \frac{x^2 - 2x - 8}{x^2 - 2x - 3x + 6} = \frac{-8}{-0^+} = +\infty$

38)

38. (a) Find the vertical asymptotes of the function

$$y = \frac{x^2+1}{3x-2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

$$y = \frac{x^2+1}{3x-2x^2}$$

denom = 0 for vertical asymptode

$$3x - 2x^2 = 0$$

$$x(3-2x) = 0$$

$$x = 0 \qquad \frac{3}{2} = x$$

asymptodes at $x = 0$ $x = 1.5$

At $x = 0$

$$\lim_{x\to 0} \frac{x^2+1}{3x-2x^2} = \frac{1}{0}$$ case 2 $\frac{\text{num}}{0}$ form

∴ check both side limits

$$\lim_{x\to 0^+} \frac{x^2+1}{3x-2x^2} = \frac{1}{0^+(3-0)} = \frac{1}{0^+} = +\infty$$

$$\lim_{x\to 0^-} \frac{x^2+1}{3x-2x^2} = \frac{1}{0^-(3-0)} = \frac{1}{0^+} = -\infty$$

At $x = 1.5$ or $3/2$

$$\lim_{x\to 1.5} \frac{x^2+1}{3x-2x^2} = \frac{(3/2)^2+1}{\frac{3}{2}\left(3-2\cdot\frac{3}{2}\right)} = \frac{13/4}{\frac{3}{2}\cdot(0)} = \frac{13/4}{0}$$

Case 2 $\frac{\text{num}}{0}$ form

$$\lim_{x\to \frac{3}{2}^+} \frac{x^2+1}{x(3-2x)} = \frac{\left(\frac{3}{2}\right)^2+1}{\frac{3^+}{2}\left(3-2\cdot\frac{3^+}{2}\right)} = \frac{\left(\frac{3}{2}\right)^2+1}{\frac{3^+}{2}(3-3^+)} = \frac{\left(\frac{3}{2}\right)^2+1}{\frac{3^+}{2}\cdot 0^-} = \frac{\left(\frac{3}{2}\right)^2+1}{0^-} = -\infty$$

num larger than 3

$$\lim_{x \to \frac{3}{2}^+} \frac{x^2+1}{3x-2x^2} = -\infty$$

$$\lim_{x \to \frac{3}{2}^-} \frac{x^2-1}{3x-2x^2} = \frac{13/4}{\frac{3}{2}(3-3)} = \frac{13/4}{0^+} = \cancel{-\infty}\ +\infty$$

One sided Limit not the same
Two sided Limit DNE.

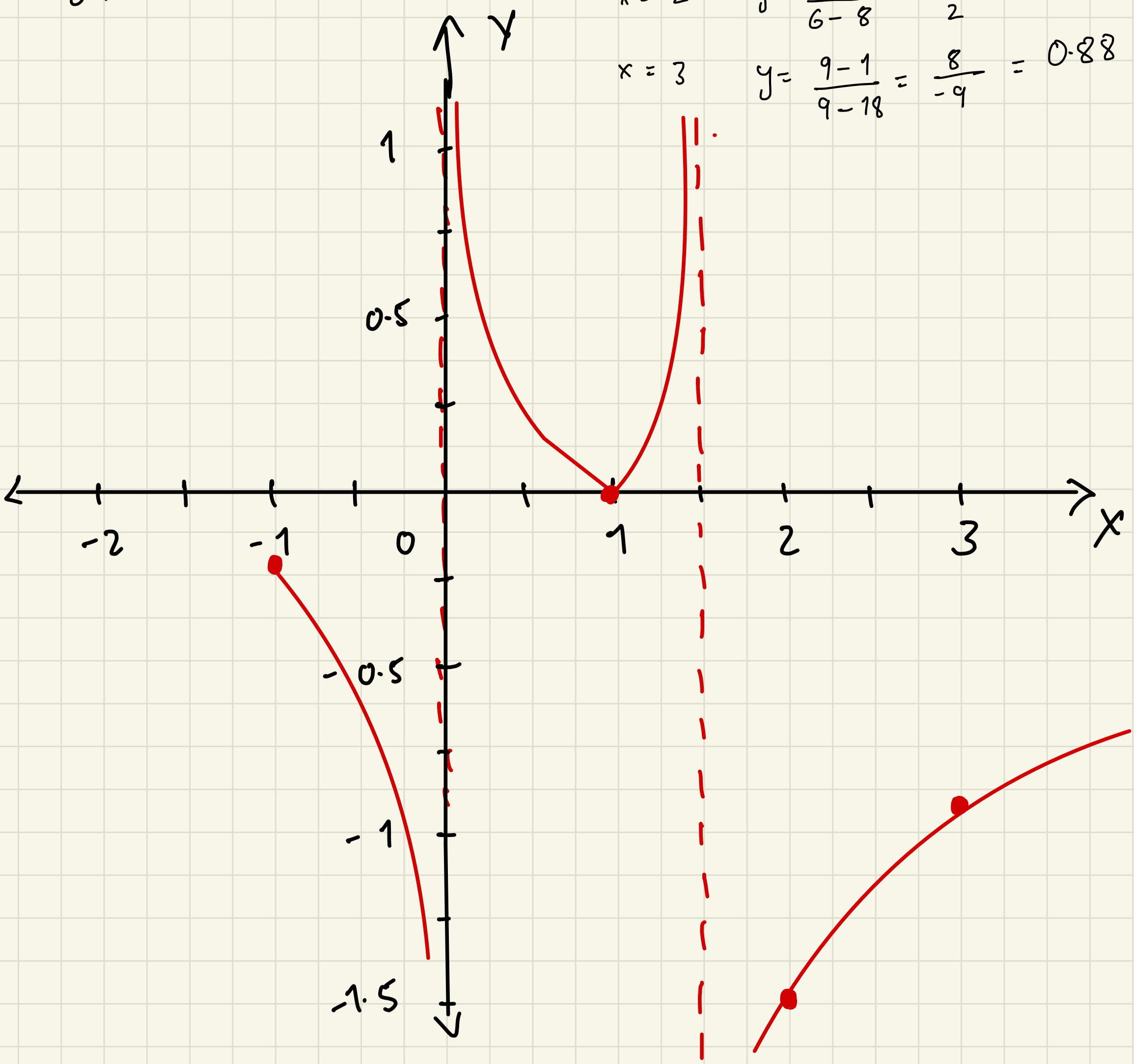
When $x=-1$ $\quad y = \dfrac{1+1}{-3-2} = -\dfrac{1}{5} = -0.2$

$x=1$ $\quad y = \dfrac{1-1}{3-2} = 0 = 0$

$x=2$ $\quad y = \dfrac{4-1}{6-8} = -\dfrac{3}{2} = -1.5$

$x=3$ $\quad y = \dfrac{9-1}{9-18} = \dfrac{8}{-9} = 0.88$

x	-1	1	2	3
y	-0.2	0	-1.5	-0.88



39)

39. Determine $\lim_{x \to 1^-} \frac{1}{x^3 - 1}$ and $\lim_{x \to 1^+} \frac{1}{x^3 - 1}$

(a) by evaluating $f(x) = 1/(x^3 - 1)$ for values of x that approach 1 from the left and from the right,

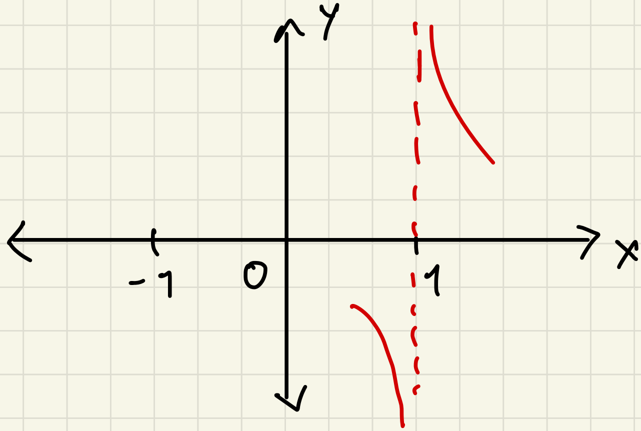
(b) by reasoning as in Example 9, and

(c) from a graph of f.

$$\lim_{x \to 1^-} \frac{1}{x^3 - 1} = \frac{1}{(1^-)^3 - 1} = \frac{1}{0^-} = -\infty$$

$\sim 0.99 - 1 = -ve$

$$\lim_{x \to 1^+} \frac{1}{x^3 - 1} = \frac{1}{(1^+)^3 - 1} = \frac{1}{0^+} = +\infty$$



Exercise 2.3

1. Given that

$$\lim_{x\to 2} f(x) = 4 \qquad \lim_{x\to 2} g(x) = -2 \qquad \lim_{x\to 2} h(x) = 0$$

find the limits that exist. If the limit does not exist, explain why.

(a) $\lim_{x\to 2} [f(x) + 5g(x)]$ (b) $\lim_{x\to 2} [g(x)]^3$

(c) $\lim_{x\to 2} \sqrt{f(x)}$ (d) $\lim_{x\to 2} \dfrac{3f(x)}{g(x)}$

(e) $\lim_{x\to 2} \dfrac{g(x)}{h(x)}$ (f) $\lim_{x\to 2} \dfrac{g(x)h(x)}{f(x)}$

$\lim_{x\to 2} f(x) = 4$

$\lim_{x\to 2} g(x) = -2$

$\lim_{x\to 2} h(x) = 0$

a) $\lim_{x\to 2} f(x) + 5\lim_{x\to 2} g(x)$

$= 4 + 5(-2) = 4 - 10 = -6$

b) $\left[\lim_{x\to 2} g(x)\right]^3 = -2^3 = -8$

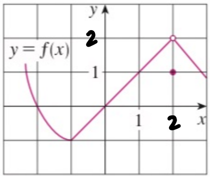
c) $\lim_{x\to 2} \sqrt{f(x)} = \sqrt{\lim_{x\to 2} f(x)} = \sqrt{4} = 2$

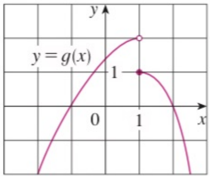
d) $\lim_{x\to 2} \dfrac{3f(x)}{g(x)} = \dfrac{3\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)} = \dfrac{3\cdot 4}{-2} = -6$

e) $\lim_{x\to 2} \dfrac{g(x)}{h(x)} = \dfrac{\lim_{x\to 2} g(x)}{\lim_{x\to 2} h(x)} = \dfrac{4}{0}$ DNE

f) $\dfrac{\lim_{x\to 2} g(x) \cdot \lim_{x\to 2} h(x)}{\lim_{x\to 2} f(x)} = \dfrac{-2\cdot 0}{4} = \dfrac{0}{4} = 0$

2. The graphs of f and g are given. Use them to evaluate each limit, if it exists. If the limit does not exist, explain why.





(a) $\lim_{x\to 2}[f(x)+g(x)]$ (b) $\lim_{x\to 1}[f(x)+g(x)]$

(c) $\lim_{x\to 0}[f(x)g(x)]$ (d) $\lim_{x\to -1}\dfrac{f(x)}{g(x)}$

(e) $\lim_{x\to 2}[x^3 f(x)]$ (f) $\lim_{x\to 1}\sqrt{3+f(x)}$

a) $\lim_{x\to 2} f(x) + \lim_{x\to 2} g(x)$

$= 2 + 0$

$= 2$

b) $\lim_{x\to 1} f(x) + \lim_{x\to 1} g(x)$

$= 1 + \text{DNE}$

= not defined

c) $\lim_{x\to 0} f(x) \cdot \lim_{x\to 0} g(x)$

$= 0 \cdot 1.5$

$= 0$

d) $\dfrac{\lim_{x\to -1} f(x)}{\lim_{x\to -1} g(x)} = \dfrac{-1}{0} =$ not defined

e) $\lim_{x\to 2} x^3 \cdot \lim_{x\to 2} f(x)$

$= \cancel{(1)^3} \cdot 2 = 2$

= As $x \to 2$; $x^3 = 2^3 = 8$

→ That 1 (red point on graph is $f(2)$ & not $\lim_{x\to 2} x$

$= (2)^3 \cdot 2 = 8 \cdot 2 = 16$

f) $\sqrt{\lim_{x\to 1} 3 + \lim_{x\to 1} f(x)}$

$= \sqrt{3 + 1}$

$= \sqrt{4} = 2$

3–9 Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

3. $\lim_{x \to 3} (5x^3 - 3x^2 + x - 6)$

4. $\lim_{x \to -1} (x^4 - 3x)(x^2 + 5x + 3)$

5. $\lim_{t \to -2} \dfrac{t^4 - 2}{2t^2 - 3t + 2}$

6. $\lim_{u \to -2} \sqrt{u^4 + 3u + 6}$

7. $\lim_{x \to 8} (1 + \sqrt[3]{x})(2 - 6x^2 + x^3)$

8. $\lim_{t \to 2} \left(\dfrac{t^2 - 2}{t^3 - 3t + 5} \right)^2$

9. $\lim_{x \to 2} \sqrt{\dfrac{2x^2 + 1}{3x - 2}}$

3) $5 \lim_{x \to 3} x^3 - 3 \lim_{x \to 3} x^2 + \lim_{x \to 3} x - 6$

$= 5(3)^3 - 3(3)^2 + 3 - 6$

$= 135 - 27 + 3 - 6$

$= 135 - 30 = 105$

4) $\left(\lim_{x \to -1} x^4 - 3 \lim_{x \to -1} x\right) \cdot \left(\lim_{x \to -1} x^2 + 5 \lim_{x \to -1} x + \lim_{x \to -1} 3\right)$

$= \left((-1)^4 - 3(-1)\right) \cdot \left((-1)^2 + 5(-1) + 3\right)$

$= (1 + 3)(1 - 5 + 3)$

$= 4 \cdot -1 = \cancel{36}\ -4$

5) $\dfrac{\lim_{t \to -2} t^4 - \lim_{t \to -2} 2}{2 \lim_{t \to -2} t^2 - 3 \lim_{t \to -2} t + \lim_{t \to -2} 2} = \dfrac{(-2)^4 - 2}{2(-2)^2 - 3(-2) + 2}$

$= \dfrac{16 - 2}{8 + 6 + 2} = \dfrac{14}{16} = \dfrac{7}{8}$

6) $\sqrt{\lim_{u\to -2} u^4 + 3\lim_{u\to -2} u + \lim_{u\to -2} 6}$

$= \sqrt{(-2)^4 + 3(-2) + 6}$

$= \sqrt{16 - 6 + 6} = \sqrt{16} = 4$

7. $\left(\lim_{x\to 8} 1 + \sqrt[3]{\lim_{x\to 8} x}\right)\left(\lim_{x\to 8} 2 - 6\lim_{x\to 8} x^2 + \lim_{x\to 8} x^3\right)$

$= \left(1 + \sqrt[3]{8}\right)\left(2 - 6(8)^2 + (8)^3\right)$

$= (1+2)(2 - 384 + 512)$

$= (3)(2+128) = 130 \cdot 3 = 390$

8. $\left(\dfrac{\lim_{t\to 2} t^2 - \lim_{t\to 2} 2}{\lim_{t\to 2} t^3 - 3\lim_{t\to 2} t + \lim_{t\to 2} 5}\right)^2$

$= \left(\dfrac{2^2 - 2}{2^3 - 3(2) + 5}\right)^2$

$$= \left(\frac{4 - 2}{8 - 6 + 5} \right)^2$$

$$= \left(\frac{2}{7} \right)^2 = \frac{4}{49}$$

9. $$\sqrt{\frac{2 \lim\limits_{x \to 2} x^2 + 1}{3 \lim\limits_{x \to 2} x - 2}}$$

$$= \sqrt{\frac{2(2)^2 + 1}{3(2) - 2}} = \sqrt{\frac{8+1}{6-2}} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

10. (a) What is wrong with the following equation?

$$\frac{x^2 + x - 6}{x - 2} = x + 3$$

(b) In view of part (a), explain why the equation

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2} = \lim_{x \to 2} (x + 3)$$

is correct.

10) a) If we apply $x = 2$ to LHS & RHS the LHS is not defined while RHS value $= 5$

LHS $\neq$ RHS

b) However if we apply limit:

$$\lim_{x \to 2} \frac{x^2 + x - 6}{x - 2}$$

Num: $N(2) = 2^2 + 2 - 6 = 0$

Denom: $D(2) = 2 - 2 = 0$

$\frac{0}{0}$ form case 3

$\therefore$ factorize

$$\lim_{x \to 2} \frac{x^2 - 2x + 3x - 6}{x - 2}$$

$$\lim_{x \to 2} \frac{x(x-2) + 3(x-2)}{(x-2)}$$

$$\lim_{x \to 2} (x+3) = \text{RHS}$$

& $\lim_{x \to 2} (x+3)$

$= \lim_{x \to 2} x + 3$

$= 2 + 3 = 5$

11) $\lim_{x \to 5} \frac{x^2 - 6x + 5}{x - 5}$

$$= \frac{\lim_{x \to 5} x^2 - 6 \lim_{x \to 5} x + 5}{\lim_{x \to 5} x - 5}$$

$$= \frac{(5)^2 - 6(5) + 5}{5 - 5} = \frac{25 - 30 + 5}{0} = \frac{0}{0}$$

Case 3 factorize further

$\underline{-1} \times \underline{-5} = 5$

$- + - = -6$

$$\lim_{x \to 5} \frac{x^2 - x - 5x + 5}{x - 5}$$

$$\lim_{x \to 5} \frac{x(x-1) - 5(x-1)}{(x-5)}$$

$$\lim_{x \to 5} (x-1) = 5 - 1 = 4$$

12. $\lim\limits_{x \to 4} \dfrac{x^2 - 4x}{x^2 - 3x - 4} \rightarrow$ $D(4) = (4)^2 - 4(4)$

$= 16 - 16 = 0$

$N(4) = (4)^2 - 3(4) - 4$

$= 16 - 12 - 4 = 0$

factorize $\leftarrow$ form $\frac{0}{0}$ case 3 .

$\downarrow$

$\underline{1} \; x \; \underline{-4} = -4$

$\_ + \_ = -3$

$$\lim_{x \to 4} \frac{x(x-4)}{x^2 + x - 4x - 4}$$

$$\lim_{x \to 4} \frac{x(x-4)}{x(x+1) - 4(x+1)}$$

$$\lim_{x \to 4} \frac{x}{x+1} = \frac{\lim\limits_{x \to 4} x}{\lim\limits_{x \to 4} x + 1} = \frac{4}{4+1} = \frac{4}{5}$$

13. $\lim_{x \to 5} \dfrac{x^2 - 5x + 6}{x - 5}$ → $N(5) = 25 - 25 + 6$
$= 6$

$D(5) = 5 - 5 = 0$

$\dfrac{\text{num}}{0}$ form case 2

as $\lim_{x \to 5} f(x)$ DNE

we check one sided limits.

Left side approach

$$\lim_{x \to 5^-} \frac{x^2 - 5x + 6}{x - 5}$$

$$\frac{\lim_{x \to 5^-} x^2 - 5 \lim_{x \to 5^-} x + 6}{\lim_{x \to 5^-} x - 5}$$

$$= \frac{25 - 25 + 6}{\underbrace{5^-}_{\text{slightly smaller than } 5} - 5} = \frac{6}{0^-} = -\infty$$

Right side approach

$$\lim_{x \to 5^+} \frac{x^2 - 5x + 6}{x - 5} = \frac{6}{5^+ - 5} = \frac{6}{0^+} = +\infty$$

One sided limits not the same

∴ Two sided limits DNE

14. $\lim_{x \to -1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$ $N(-1) = (-1)^2 - 4(-1)$

$= 1 + 4 = 5$

$\dfrac{N(-1)}{D(-1)} = \dfrac{5}{0} \Rightarrow \dfrac{\text{number}}{0}$ form $D(-1) = (-1)^2 - 3(-1) - 4$

$= 1 + 3 - 4$

case 2 apply limits from both sides $= 0$

$1 \quad x^{-4} = -4$

$1 \quad +^{-4} = -3$

$\lim_{x \to -1^-} \dfrac{x(x-4)}{x^2 + x - 4x - 4}$

$= \lim_{x \to -1^-} \dfrac{x(x-4)}{x(x+1) - 4(x+1)}$

$= \lim_{x \to -1^-} \dfrac{x(x-4)}{(x+1)(x-4)}$

$= \lim_{x \to -1^-} \dfrac{x}{x+1} = \dfrac{-1}{-1^- + 1} = \dfrac{-1}{0^-}$

slightly smaller

num slightly smaller than 0

$= -1 \cdot (-\infty) = +\infty$

Also, $\lim_{x \to -1^+} \dfrac{x}{x+1} = \dfrac{-1}{-1^+ + 1} = \dfrac{-1}{0^+} = -\infty$

Two sided limits DNE as one sided limits not same

15. $\lim_{t \to -3} \dfrac{t^2 - 9}{2t^2 + 7t + 3}$

$N(-3) = (-3)^2 - 9 = 9 - 9 = 0$

$D(-3) = 2(-3)^2 + 7(-3) + 3$

$= 2(9) - 21 + 3$

$= 18 - 21 + 3 = 21 - 21 = 0$

Case 3 factorize

$\underline{1} \times \underline{6} = 6$

$\_ + \_ = 7$

$\lim_{t \to -3} \dfrac{(t+3)(t-3)}{2t^2 + t + 6t + 3}$

$\lim_{t \to -3} \dfrac{(t+3)(t-3)}{t(2t+1) + 3(2t+1)}$

$\lim_{t \to -3} \dfrac{t-3}{2t+1} \Rightarrow N(-3) = -3 - 3 = -6$

$D(-3) = 2(-3) + 1 = -5$

$\lim_{t \to -3} \dfrac{t-3}{2t+1} = \dfrac{-6}{-5} = \dfrac{6}{5}$

96. $\lim_{x \to -1} \dfrac{2x^2 + 3x + 1}{x^2 - 2x - 3}$

$N(-1) = 2(-1)^2 + 3(-1) + 1$
$= 2 - 3 + 1 = 3 - 3 = 0$

$D(-1) = (-1)^2 - 2(-1) - 3$
$= 1 + 2 - 3 = 0$

case 3 $\frac{0}{0}$ form $\therefore$ factorize

$2x^2 + 3x + 1$

$\underline{1} \times \underline{2} = 2$
$\underline{1} + \underline{2} = 3$

$x^2 - 2x - 3$

$\underline{1} \times \underline{-3} = -3$
$\underline{1} + \underline{-3} = -2$

$\lim_{x \to -1} \dfrac{2x^2 + x + 2x + 1}{x^2 + x - 3x - 3}$

$\lim_{x \to -1} \dfrac{x(2x+1) + 1(2x+1)}{x(x+1) - 3(x+1)}$

$\lim_{x \to -1} \dfrac{(2x+1)(x+1)}{(x+1)(x-3)}$

$\lim_{x \to -1} \dfrac{2x+1}{x-3} = \dfrac{2(-1)+1}{-1-3}$

$= \dfrac{-2+1}{-4} = \dfrac{-1}{-4}$

$= \dfrac{1}{4}$

17.

17. $\lim_{h\to 0} \frac{(-5+h)^2 - 25}{h}$

$$N(0) = (-5+0)^2 - 25$$
$$= 25 - 25 = 0$$

$$D(0) = 0$$

case 3 $\frac{0}{0}$ form $\Rightarrow$ factorize

$$\lim_{h\to 0} \frac{(-5)^2 + 2(-5)(h) + h^2 - 25}{h}$$

$$\lim_{h\to 0} \frac{25 - 10h + h^2 - 25}{h}$$

$$\lim_{h\to 0} \frac{h^2 - 10h}{h}$$

$$\lim_{h\to 0} \frac{h(h-10)}{h}$$

$$\lim_{h\to 0} h - 10 = 0 - 10 = -10$$

18. $\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$

$^nC_r = \frac{n!}{r!(n-r!)}$

0	1
1	1 1
2	1 2 1
3	1 3 3 1
	$^3C_0$ $^3C_1$ $^3C_2$ $^3C_3$

$^nC_r\, a b$

$a^3 b^0\, {}^3C_0 + a^2 b^1\, {}^3C_1 + a^1 b^2\, {}^3C_2 + a^0 b^3\, {}^3C_3$

$a^3 + a^2 b \frac{3!}{1!\,2!} + ab^2 \frac{3!}{2!\,1!} + b^3$

$= a^3 + 3a^2 b + 3ab^2 + b^3$

$N(0) = (2+0)^3 - 8 = 8 - 8 = 0$

$D(0) = 0$

$\frac{0}{0}$ form $\Rightarrow$ case 3

$\therefore$ we factorize

$$\lim_{h\to 0} \frac{(2+h)^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{2^3 + 3\cdot(2)^2 h + 3(2)(h)^2 + h^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{8 + 12h + 6h^2 + h^3 - 8}{h}$$

$$= \lim_{h\to 0} \frac{12h + 6h^2 + h^3}{h}$$

$$= \lim_{h\to 0} 12 + 6h + h^2 = 12$$

19. $\lim_{x \to -2} \dfrac{x+2}{x^3+8}$?

$N(2) = -2+2 = 0$
$D(2) = -8+8 = 0$
} case 3 $\frac{0}{0}$ form
Factorize!

$\lim_{x \to -2} \dfrac{x\left(1+\frac{2}{x}\right)}{x\left(x^2+\frac{8}{x}\right)}$

also not a easy solution approach

Approach carefully & slowly with all 4 ways.

$\lim_{x \to -2} \dfrac{1}{\dfrac{x^3+0x^2+0x+8}{x+2}}$

$$\begin{array}{r|l} & x^2+2x-4 \\ \hline x+2 & x^3+0x^2+0x+8 \\ & x^3+2x^2 \\ \hline & -2x^2+0x \\ & 2x^2+4x \\ \hline & -4x+8 \\ & -4x-8 \\ \hline & 16 \end{array}$$

$\Rightarrow x^2+2x-4 \quad \dfrac{16}{x+2}$

$\Rightarrow \dfrac{(x^2+2x-4)(x+2)+16}{x+2}$

$\lim_{x \to -2} \dfrac{x+2}{(x^2+2x-4)(x+2)+16} = \dfrac{0}{0+16} = \dfrac{0}{16} = 0$

Another approach multiplying w/ conjugate

$$\therefore \lim_{x \to -2} \frac{x+2}{x^3+8} \frac{(x^3-8)}{(x^3-8)}$$

$$= \lim_{x \to -2} \frac{x^4-8x+2x^3-16}{(x^3)^2-8^2}$$

$$= \frac{(-2)^4+8(-2)+2(-2)^3-16}{((-2)^3)^2-64}$$

$$= 16-16+-16-16$$

When $\frac{0}{0}$ form

- Try to take x common
- Try to factorize
- Try division
- Try $(a+b)^3$ $(a+b)^2$ relations.

19. $\lim_{x \to -2} \frac{x+2}{x^3+8}$ $N(-2) = -2+2 = 0$

$D(-2) = (-2)^3 + 8$

$= -8 + 8 = 0$

$\frac{0}{0}$ form $\therefore$ factorize

$$\lim_{x \to -2} \frac{x+2}{x^3+2^3}$$

$$(a+b)^3 = 3C_0 a^3 b^0 + 3C_1 a^2 b^1 + 3C_2 a b^2 + 3C_3 a^0 b^3$$

0 1 $= 1a^3 \cdot 1 + 3a^2 b + 3ab^2 + 1 \cdot 1b^3$

1 1 1

2 1 2 1 $= a^3 + 3a^2 b + 3ab^2 + b^3$

3 1 3 3 1 $= a^3 + b^3 + 3a^2 b + 3ab^2$

$$\therefore a^3 + b^3 = (a+b)^3 - 3a^2 b - 3ab^2$$

$$x^3 + 2^3 = (x+2)^3 - 3x^2 \cdot 2 - 3 \cdot x \cdot 2^2$$

$$x^3 + 2^3 = (x+2)^3 - 6x^2 - 12x$$

$$= (x+2)^3 - 6x(x+2)$$

Now $\lim_{x \to -2} \frac{x+2}{x^3+8^3} = \lim_{x \to -2} \frac{x+2}{x^3+2^3}$

$$\lim_{x\to -2} \frac{(x+2)}{(x+2)\left[(x+2)^2 - 6x\right]}$$

$$= \lim_{x\to -2} \frac{1}{(x+2)^2 - 6x} = \frac{1}{0-6(-2)} = \frac{1}{12}$$

20) $\lim_{t\to 1} \frac{t^4 - 1}{t^3 - 1}$ = $N(1) = 1-1=0$

$D(1) = 1-1=0$

$\frac{0}{0}$ form case 3 $\therefore$ factorize

$$\lim_{t\to 1} \frac{(t^2)^2 - 1^2}{t^3 - 1^3} = \lim_{t\to 1} \frac{(t^2+1)(t^2-1)}{t^3-1^3}$$

$$(a+b)^3 = {}^3C_0 a^3 b^0 + {}^3C_1 a^2 b^1 + {}^3C_2 a^1 b^2 + {}^3C_3 a^0 b^3$$

$$= 1 \cdot a^3 + 3a^2 b + 3ab^2 + 1 \cdot b^3$$

1

1 1

1 2 1

1 3 3 1

$\therefore (a+(-b))^3 = a^3 - 3a^2 b + 3ab^2 - b^3$

$(a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$

$$a^3 - b^3 = (a-b)^3 + 3a^2b - 3ab^2$$

$$t^3 - 1^3 = (t-1)^3 + 3t^2 - 3t$$

$$= (t-1)^3 + 3t(t-1)$$

$$\lim_{t \to 1} \frac{t^4 - 1}{t^3 - 1} =$$

$$\lim_{t \to 1} \frac{(t^2+1)(t^2-1)}{(t-1)\left[(t-1)^2 + 3t\right]}$$

$$\lim_{t \to 1} \frac{(t^2+1)(t+1)}{(t-1)^2 + 3t} = \frac{(1+1)(1+1)}{0+3}$$

$$= \frac{4}{3}$$

21) $\lim_{h \to 0} \dfrac{\sqrt{9+h} - 3}{h}$

$\dfrac{N(a)}{D(a)} = \dfrac{3-3}{0} = \dfrac{0}{0} =$ Cas 3 factorize

$\lim_{h \to 0} \dfrac{(\sqrt{9+h} - 3)}{h} \dfrac{(\sqrt{9+h} + 3)}{(\sqrt{9+h} + 3)}$

$\lim_{h \to 0} \dfrac{9+h - 3}{h(\sqrt{9+h} + 3)}$

$\lim_{h \to 0} \dfrac{6+h}{h(\sqrt{9+h} + 3)} = \dfrac{6}{0} =$ Case 2 num form $\frac{\text{num}}{0}$

∴ check one sided limits.

DNE

$\lim_{h \to 0^+} \dfrac{6+h}{h(\sqrt{9+h} + 3)} = \dfrac{6}{0^+} = +\infty$

$\lim_{h \to 0^-} \dfrac{6+h}{h(\sqrt{9+h} + 3)} = \dfrac{6}{0^-} = -\infty$

One sided limits not equal

Two sided limits DNE

22. $\lim_{u \to 2} \dfrac{\sqrt{4u+1}-3}{u-2}$

$N(2) = \sqrt{9} - 3 = 0$ $\quad \dfrac{0}{0}$ factorize

$D(2) = 2-2 = 0$

$$\lim_{u \to 2} \frac{(\sqrt{4u+1}-3)}{u-2} \frac{(\sqrt{4u+1}+3)}{(\sqrt{4u+1}+3)}$$

$$\lim_{u \to 2} \frac{4u+1-9}{(u-2)(\sqrt{4u+1}+3)}$$

$$\lim_{u \to 2} \frac{4u-8}{(u-2)(\sqrt{4u+1}+3)}$$

$$\lim_{u \to 2} \frac{4\cancel{(u-2)}}{\cancel{(u-2)}(\sqrt{4u+1}+3)} = \frac{4}{\sqrt{8+1}+3} = \frac{4}{6} = \frac{2}{3}$$

23) $\lim_{x \to -4} \dfrac{\frac{1}{4} + \frac{1}{x}}{4+x}$

$N(-4) = \frac{1}{4} - \frac{1}{4} = 0$ $\quad \frac{0}{0}$ form case 3

$D(-4) = 4 - 4 = 0$ $\quad$ factorize

$\lim_{x \to -4} \dfrac{x+4}{4x} \cdot \dfrac{1}{4+x} = \lim_{x \to -4} \dfrac{1}{4x} = -\dfrac{1}{16}$

24) $\lim_{x \to -1} \dfrac{x^2 + 2x + 1}{x^4 - 1}$

$N(-1) = 1 - 2 + 1 = 0$ $\quad \frac{1}{} \times \frac{1}{} = 1$

$D(-1) = 1 - 1 = 0$ $\quad \frac{1}{} + \frac{1}{} = 2$

$\lim_{x \to -1} \dfrac{x^2 + x + x + 1}{(x^2)^2 - 1^2}$

$\lim_{x \to -1} \dfrac{x(x+1) + 1(x+1)}{(x^2+1)(x^2-1)}$

$\lim_{x \to -1} \dfrac{(x+1)^2}{(x^2+1)(x+1)(x-1)} = \lim_{x \to -1} \dfrac{(x+1)}{(x^2+1)(x-1)}$

$$= \lim_{x\to -1} \frac{(x+1)}{(x^2+1)(x-1)} = \frac{0}{(1+1)(-1-1)} = \frac{0}{-4} = 0$$

25) $$\lim_{t\to 0} \frac{\sqrt{1+t} - \sqrt{1-t}}{t}$$

$$\left.\begin{aligned} N(0) &= \sqrt{1} - \sqrt{1} = 0 \\ D(0) &= 0 \end{aligned}\right\} \frac{0}{0} \text{ form}$$

factorize

$$\lim_{t\to 0} \frac{\left(\sqrt{1+t} - \sqrt{1-t}\right)\left(\sqrt{1+t} + \sqrt{1-t}\right)}{t\left(\sqrt{1+t} + \sqrt{1-t}\right)}$$

$$\lim_{t\to 0} \frac{1+t - (1-t)}{t\sqrt{1+t} + t\sqrt{1-t}}$$

$$\lim_{t\to 0} \frac{2\cancel{t}}{\cancel{t}\left(\sqrt{1+t} + \sqrt{1-t}\right)} = \frac{2}{\sqrt{1} \cancel{0} + \sqrt{1}} = \cancel{\frac{num}{0}} \; \cancel{\text{case } 2}$$

$$2/2 = 1$$

26) $\lim_{t \to 0} \left(\frac{1}{t} - \frac{1}{t^2+t} \right)$

$\lim_{t \to 0} \frac{1}{t} - \lim_{t \to 0} \frac{1}{t^2+t}$

$\frac{N(0)}{D(0)} = \frac{1}{0}$ not defined

$\frac{N(0)}{D(0)} = \frac{1}{0}$ not defined

case 2 $\frac{\text{num}}{0}$ form

case 2 $\frac{\text{num}}{0}$ form

$\lim_{t \to 0^+} \frac{1}{t} = \frac{1}{0^+} = +\infty$

$\lim_{t \to 0^+} \frac{1}{t^2+t} = \frac{1}{0^+} = +\infty$

$\lim_{t \to 0^-} \frac{1}{t} = \frac{1}{0^-} = -\infty$

$\lim_{t \to 0^-} \frac{1}{t^2+t} = \frac{1}{0^-} = -\infty$

individual two sided limits DNE

However original question:

$\lim_{t \to 0} \frac{t^2+t - t}{t(t^2+t)}$

$\lim_{t \to 0} \frac{t^2}{t(t^2+t)}$

$\lim_{t \to 0} \frac{t^2}{t^2(t+1)}$

$\lim_{t \to 0} \frac{1}{t+1} = \frac{1}{0+1} = \frac{1}{1} = 1$

27) $\lim_{x \to 16} \dfrac{4-\sqrt{x}}{16x - x^2}$

$N(16) = 4-4=0$

$D(16) = 16^2-16^2=0$

$\frac{0}{0}$ form

case 3

← factorize

$= \lim_{x \to 16} \dfrac{4-\sqrt{x}}{x(16-x)}$

$= \lim_{x \to 16} \dfrac{4-\sqrt{x}}{x(4^2-(\sqrt{x})^2)}$

$= \lim_{x \to 16} \dfrac{4-\sqrt{x}}{x(4+\sqrt{x})(4-\sqrt{x})}$

$= \lim_{x \to 16} \dfrac{1}{x(4+\sqrt{x})} = \dfrac{1}{16(4+4)} = \dfrac{1}{16 \cdot 8} = \dfrac{1}{128}$

28) $\lim_{h \to 0} \dfrac{(3+h)^{-1} - 3^{-1}}{h}$

$$\left.\begin{array}{l} N(0) = 3^{-1} - 3^{-1} = 0 \\ D(0) = 0 \end{array}\right\} \frac{0}{0} \text{ form}$$

case 3

factorize

$\lim_{h \to 0} \left[\dfrac{1}{(3+h)} - \dfrac{1}{3} \div h\right]$

$\lim_{h \to 0} \left[\dfrac{3-(3+h)}{3(3+h)} \cdot \dfrac{1}{h}\right]$

$\lim_{h \to 0} \left[\dfrac{3-3-h}{3h(3+h)}\right]$

$= \lim_{h \to 0} \left[\dfrac{-1}{3(3+h)}\right] = \dfrac{-1}{9} = -\dfrac{1}{9}$

29) $\lim_{t \to 0} \left(\frac{1}{t\sqrt{1+t}} - \frac{1}{t} \right)$

$= \lim_{t \to 0} \frac{1}{t\sqrt{1+t}} - \lim_{t \to 0} \frac{1}{t}$

$= N(0) = \frac{1}{0} \quad - \quad \lim_{t \to 0} \frac{1}{t} = \frac{1}{0}$

we have to approach limits from both sides

Case 2 $\frac{\text{num}}{0}$ form

individually.

$= \lim_{t \to 0} \left(\frac{t - t\sqrt{1+t}}{t^2\sqrt{1+t}} \right)$ But simplified.

$= N(0) = 0 \quad \frac{0}{0}$ form case 3

$D(0) = 0$ factorize

$\lim_{t \to 0} \left(\frac{t(1 - 1\sqrt{1+t})}{t^2\sqrt{1+t}} \right)$

$\lim_{t \to 0} \frac{1 - \sqrt{1+t}}{t\sqrt{1+t}} \frac{(1+\sqrt{1+t})}{(1+\sqrt{1+t})}$

$\lim_{t \to 0} \frac{1 - (1+t)}{t\sqrt{1+t}(1+\sqrt{1+t})}$

$\lim_{t \to 0} \frac{1 - 1 - t}{t\sqrt{1+t}(1+\sqrt{1+t})}$

$\lim_{t \to 0} \frac{-t}{t\sqrt{1+t}(1+\sqrt{1+t})}$

$\lim_{t \to 0} \frac{-1}{\sqrt{1+t}(1+\sqrt{1+t})} = \frac{-1}{\sqrt{1}(1+\sqrt{1})} = \frac{-1}{1(2)} = -\frac{1}{2}$

30) $$\lim_{x \to -4} \frac{\sqrt{x^2+9} - 5}{x+4}$$

$$\left.\begin{aligned} N(-4) &= \sqrt{16+9} - 5 = 0 \\ D(-4) &= -4+4 = 0 \end{aligned}\right\}$$ case 3 $\frac{0}{0}$ factorize

$$\lim_{x \to -4} \frac{\left(\sqrt{x^2+9} - 5\right)}{x+4} \frac{\left(\sqrt{x^2+9} + 5\right)}{\left(\sqrt{x^2+9} + 5\right)}$$

$$\lim_{x \to -4} \frac{x^2+9-25}{(x+4)\left(\sqrt{x^2+9}+5\right)}$$

$$\lim_{x \to -4} \frac{x^2-16}{(x+4)\left(\sqrt{x^2+9}+5\right)} = \lim_{x \to -4} \frac{(x+4)(x-4)}{(x+4)\left(\sqrt{x^2+9}+5\right)}$$

$$= \lim_{x \to -4} \frac{x-4}{\sqrt{x^2+9}+5} = \frac{-4-4}{\sqrt{25}+5} = \frac{-8}{10} = \frac{-4}{5}$$

31) $$\lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$$

$$= \begin{aligned} N(0) &= x^3 - x^3 = 0 \\ D(0) &= 0 \end{aligned}$$ $\frac{0}{0}$ Case 3 factorize

$$\lim_{h \to 0} \frac{x^3 + h^3 + 3xh(x+h) - x^3}{h}$$

$$\lim_{h \to 0} \frac{h^3 + 3x^2h + 3xh^2}{h} = \lim_{h \to 0} \frac{h(h^2 + 3x^2 + 3xh)}{h}$$

$$\lim_{h \to 0} h^2 + 3x^2 + 3xh = 3x^2$$

32)

$$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$\lim_{h\to 0} \frac{x^2 - (x+h)^2}{x^2 (x+h)^2} \times \frac{1}{h}$$

$N(0) = x^2 - x^2 = 0$

$D(0) = 0$

$\frac{0}{0}$ case 3

factorize

$$\lim_{h\to 0} \frac{x^2 - (x+h)^2}{x^2 h (x+h)^2}$$

$$\lim_{h\to 0} \frac{(x^2 - (x+h)^2)(x^2 + (x+h)^2)}{x^2 h (x+h)^2 (x^2 + (x+h)^2)}$$

Do Not Do this approach simplyfy !!

$$\lim_{h\to 0} \frac{(x^2)^2 - ((x+h)^2)^2}{x^2 h (x+h)^2 (x^2 + (x+h)^2)}$$

$$\lim_{h\to 0} \frac{x^4 - (x+h)^4}{x^2 h (x+h)^2 (x^2 + (x+h)^2)}$$

0	1				
1	1	1			
2	1	2	1		
3	1	3	3	1	
4	1	4	6	4	1
	$4C_0$	$4C_1$	$4C_2$	$4C_3$	$4C_4$

$$4C_0 a^4 b^0 + 4C_1 a^3 b^1 + 4C_2 a^2 b^2 + 4C_3 a^1 b^3 + 4C_3 a^0 b^4$$

$$= 1a^4 + 4a^3 b + 6a^2 b^2 + 4ab^3 + 1b^4$$

$(x+h)^4$

$$= x^4 + 4x^3 h + 6x^2 h^2 + 4xh^3 + h^4$$

$$\lim_{h\to 0} \frac{\cancel{x^4} - \cancel{x^4} - 4x^3h - 6x^2h^2 - 4xh^3 + h^{\cancel{3}4}}{x^2 h (x+h)^2 (x^2 + (x+h)^2)}$$

$$\lim_{h\to 0} \frac{-4x^3 - 6x^2h - 4xh^2 + h^{\cancel{2}3}}{x^2 (x+h)^2 (x + (x+h)^2)}$$

$$= \frac{-4x^3 - 0 - 0 + 0}{x^2 \, x^2 (x + x^2)}$$

$$= \frac{-4x^3}{x^4(x+x^2)} = \frac{-4}{x(x+x^2)} = \frac{-4}{x^3+x^2}$$

32. $$\lim_{h\to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h}$$

$$\lim_{h\to 0} \frac{x^2 - (x+h)^2}{x^2(x+h)^2} \cdot \frac{1}{h}$$

$$\lim_{h\to 0} \frac{x^2 - [x^2 + 2hx + h^2]}{x^2 h (x+h)^2}$$

$$\lim_{h \to 0} \frac{x^2 - x^2 - 2hx - h^2}{x^2 h (x+h)^2}$$

$$\lim_{h \to 0} \frac{-h(2x-h)}{x^2 h (x+h)^2}$$

$$\lim_{h \to 0} \frac{-2x+h}{x^2 (x+h)^2} = \frac{-2x}{x^2 (x)^2} = \frac{-2x}{x^4} = \frac{-2}{x^3}$$

37 - 52 HW

37. If $4x - 9 \leqslant f(x) \leqslant x^2 - 4x + 7$ for $x \geqslant 0$, find $\lim_{x \to 4} f(x)$.

38. If $2x \leqslant g(x) \leqslant x^4 - x^2 + 2$ for all x, evaluate $\lim_{x \to 1} g(x)$.

37

$$f(x) \geq 4x - 9$$

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} 4x - 9$$

$$= 16 - 9 = 7$$

$$\lim_{x \to 4} f(x) = \lim_{x \to 4} x^2 - 4x + 7$$

$$= 16 - 16 + 7 = 7$$

$$\therefore \lim_{x \to 4} f(x) = 7$$

38

$$\lim_{x \to 1} 2x = 2$$

$$\lim_{x \to 1} x^4 - x^2 + 2$$

$$= 1 - 1 + 2 = 2$$

$$\therefore \lim_{x \to 1} g(x) = 1$$

39. Prove that $\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0$.

40. Prove that $\lim_{x \to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0$.

39.

$$\lim_{x \to 0} x^4 \cos \frac{2}{x} = 0 \cdot \cos \frac{2}{0} = \text{DNE}$$

Apply squeeze theorem

Cosine $\Rightarrow$ $-1 \leq \cos\left(\frac{2}{x}\right) \leq 1$

$\times$ by x^4 $\Rightarrow$ $-x^4 \leq x^4 \cos\left(\frac{2}{x}\right) \leq x^4$

checking limit on both ends:

$$\lim_{x \to 0} -x^4 = 0 \qquad \lim_{x \to 0} x^4 = 0$$

Both the end limits are same hence $\lim_{x \to 0} x^4 \cos\left(\frac{2}{x}\right) = 0$

40. $\lim_{x \to 0^+} \sqrt{x}\, e^{\sin\left(\frac{\pi}{x}\right)} = 0$ } Prove this

We have $\quad -1 \leq \sin\left(\frac{\pi}{x}\right) \leq 1$

raising by e $\Rightarrow \quad e^{-1} \leq e^{\sin\left(\frac{\pi}{x}\right)} \leq e^{1}$

x by $\sqrt{x}$ $\Rightarrow \quad \sqrt{x}\, e^{-1} \leq \sqrt{x}\, e^{\sin\left(\frac{\pi}{x}\right)} \leq \sqrt{x} \cdot e$

$$\frac{1}{e}\sqrt{x} \leq \sqrt{x}\, e^{\sin\left(\frac{\pi}{x}\right)} \leq e\sqrt{x}$$

Checking limits on both ends

$$\lim_{x \to 0^+} \frac{1}{e}\sqrt{x} = \frac{\sqrt{0^+}}{e} = 0$$

$$\lim_{x \to 0^+} e\sqrt{x} = e\sqrt{0^+} = 0$$

$$\therefore \lim_{x \to 0^+} \sqrt{x}\, e^{\sin(\pi/x)} = 0$$

41) $\lim_{x \to 3} \left(2x + |x-3| \right)$

$= \lim_{x \to 3} 2x + \lim_{x \to 3} |x-3|$

$|x-3| = \begin{cases} -(x-3) & < 3 \\ +(x-3) & \geq 3 \end{cases}$

$= \lim_{x \to 3} 2x = 2(3)$
$= 6$

$\lim_{x \to 3^-} -x+3$
$= \lim_{x \to 3^-} 3-x = 0^+$

$\lim_{x \to 3^+} +(x+3)$
$= \cancel{\lim_{x \to 3^+}} \ x \cancel{+} 3 = \cancel{6}$

※ Not plus!

$\lim_{x \to 3^+} x-3 = 3^+ - 3 = 0^+$

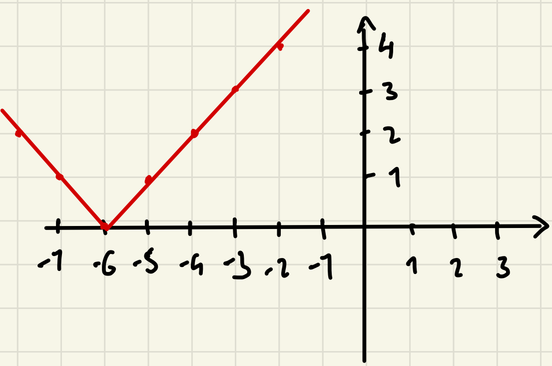
$\Rightarrow \quad 6 + 0^+ = 6 \ ?$

42. $\lim_{x \to -6} \dfrac{2x+12}{|x+6|}$

Applying limit directly $\quad N(-6) = -12 + 12 = 0 \qquad \dfrac{0}{0}$ factorize

$D(-6) = |-6+6| = 0$

Also $\quad |x+6| = \begin{cases} -(x+6) & \cancel{\leq}\ < -6 \\ +(x+6) & \geq 6 \end{cases}$



$\lim_{x \to -6} \dfrac{2(x+6)}{|x+6|}$

$\lim_{x \to -6^+} \dfrac{2(x+6)}{(x+6)} \qquad \lim_{x \to -6^-} \dfrac{2(x+6)}{-(x+6)}$

$= 2 \qquad\qquad = -2$

One sided limits not same

$\therefore$ Two sided limits DNE for this function.

43. $$\lim_{x \to 0.5^-} \frac{2x - 1}{|2x^3 - x^2|}$$

Applying limit directly $N(0.5) = 2 \cdot \frac{1}{2} - 1 = 1 - 1 = 0$ $\qquad \frac{0}{0}$ form

$$D(0.5) = 2 \cdot \left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 = 0$$

Also $|2x^3 - x^2| = |x^2(2x-1)|$

$$|x^2(2x-1)| = \begin{cases} -x^2(2x-1) & x < 0.5 \\ x^2(2x-1) & x \geq 0.5 \end{cases}$$

Therefore

$$\lim_{x \to 0.5^-} \frac{2x - 1}{-x^2(2x-1)} = \lim_{x \to 0.5^-} \frac{1}{-x^2}$$

$$= \lim_{x \to \left(\frac{1}{2}\right)^-} \frac{1}{-x^2}$$

$$= \frac{1}{-\left(\frac{1}{2}\right)^2} = \frac{-1}{\frac{1}{4}} = -4$$

$$\lim_{x \to 0.5^-} \frac{2x-1}{|2x^3 - x^2|} = -4$$

44. $$\lim_{x \to -2} \frac{2-|x|}{2+x}$$

$N(-2) = 2 - |-2| = 2 - 2 = 0$ $\quad \frac{0}{0}$ factorize case 3

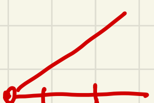
$D(-2) = 2 + -2 = 2 - 2 = 0$

we have $|x| = \begin{cases} -x, & x < -2 \\ +x, & x \geq -2 \end{cases}$

$$\lim_{x \to -2^-} \frac{2-(-x)}{2+x} = \lim_{x \to -2^-} \frac{2+x}{2+x} = \lim_{x \to -2^-} 1 = 1$$

They are -ve as it gets closer $-x$

$$\lim_{x \to -2^+} \frac{2-(\cancel{+x})}{2+x} = \cancel{\lim_{x \to} \frac{2-x}{2+x}} \Rightarrow \text{~~stuck.~~}$$



-ve(x) two $+x \to -x$

But

the actual way is $$\lim_{x \to -2^+} \frac{2-(-x)}{2+x}$$

$$= \lim_{x \to -2^+} \frac{2+x}{2+x} = 1$$

why? Because

Approaching -2 from the right minutely

the values are same as that of $|x| = x$ but just multiplied by -1 $\therefore$ $-1(x) = -x$

eg: take $\lim_{x \to -2^+}$, a value close as -1.5 or -1.99999 which is just $-1 \cdot (x)$

45. $$\lim_{x \to 0^-} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$

$$|x| = \begin{cases} -x & x < 0 \\ +x & x \geq 0 \end{cases}$$

$$\lim_{x \to 0^-} \left(\frac{|x| - x}{x|x|} \right)$$

$-1 \quad -0.1 \quad -0.01 \quad 0$

all are -ve $\therefore -x$

$$\lim_{x \to 0^-} \left(\frac{-x - x}{x(-x)} \right)$$

Some really close number which is same as $|x| = x$ but opposite sign as it is on the -ve number line $\therefore -1 \cdot x = -x$

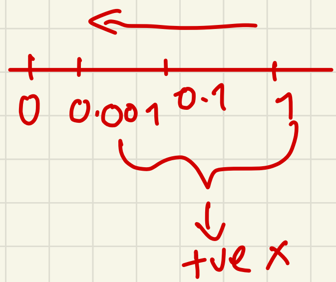
$$= \lim_{x \to 0^-} \frac{-2x}{-x^2}$$

$$= \lim_{x \to 0^-} \frac{2}{x} = \frac{2}{0^-} = 2 \cdot (-\infty)$$

$$= \cancel{+}\infty$$

$$= -\infty$$

46. $$\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{|x|} \right)$$



$$= \lim_{x \to 0^+} \left(\frac{|x| - x}{x|x|} \right)$$

we know $|x| = \begin{cases} -x, & x < 0 \\ x, & x \geq 0 \end{cases}$

$$= \lim_{x \to 0^+} \frac{x - x}{x^2} = \lim_{x \to 0^+} \frac{0}{x^2} = 0$$

47.

47. The *signum* (or sign) *function*, denoted by sgn, is defined by

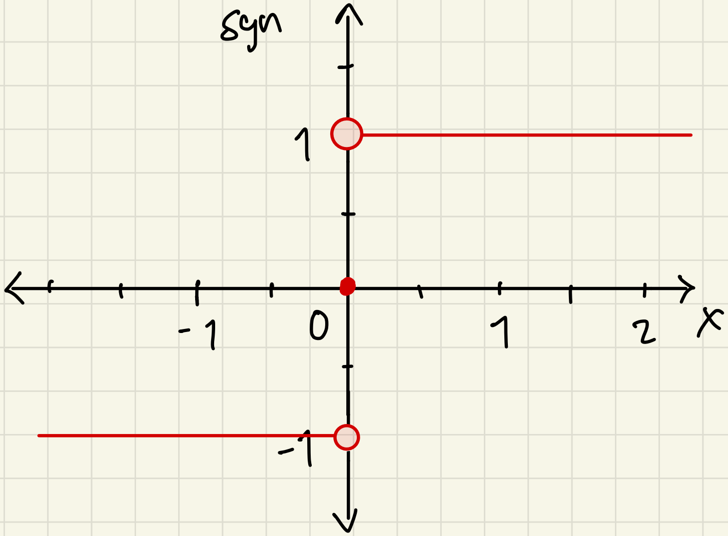
$$\operatorname{sgn} x = \begin{cases} -1 & \text{if } x < 0 \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(a) Sketch the graph of this function.

(b) Find each of the following limits or explain why it does not exist.

(i) $\lim_{x \to 0^+} \operatorname{sgn} x$ (ii) $\lim_{x \to 0^-} \operatorname{sgn} x$

(iii) $\lim_{x \to 0} \operatorname{sgn} x$ (iv) $\lim_{x \to 0} |\operatorname{sgn} x|$



1) $\lim_{x \to 0^+} \operatorname{sgn} x = 1$

2) $\lim_{x \to 0^-} \operatorname{sgn} x = -1$

3) $\lim_{x \to 0} \operatorname{sgn} = \text{DNE}$ one sided limits are not the same

4) $\lim_{x \to 0} |\operatorname{sgn} x| = \cancel{\text{DNE}}$?

~~because one sided limits different~~

or

or 0 because $\lim_{x \to 0} |0| = 0$

for $|\operatorname{sgn} x| = \begin{cases} -1 & x < 0 \\ +1 & x \geq 0 \end{cases}$ given in question

$\therefore \lim_{x \to 0^-} |\operatorname{sgn} x| = |-1| = 1$

$$\lim_{x \to 0^+} |\operatorname{sgn} x| = |1| = 1$$

$$\therefore \lim_{x \to 0} |\operatorname{sgn} x| = 1$$

(48)

48. Let

$$f(x) = \begin{cases} x^2 + 1 & \text{if } x < 1 \\ (x-2)^2 & \text{if } x \geqslant 1 \end{cases}$$

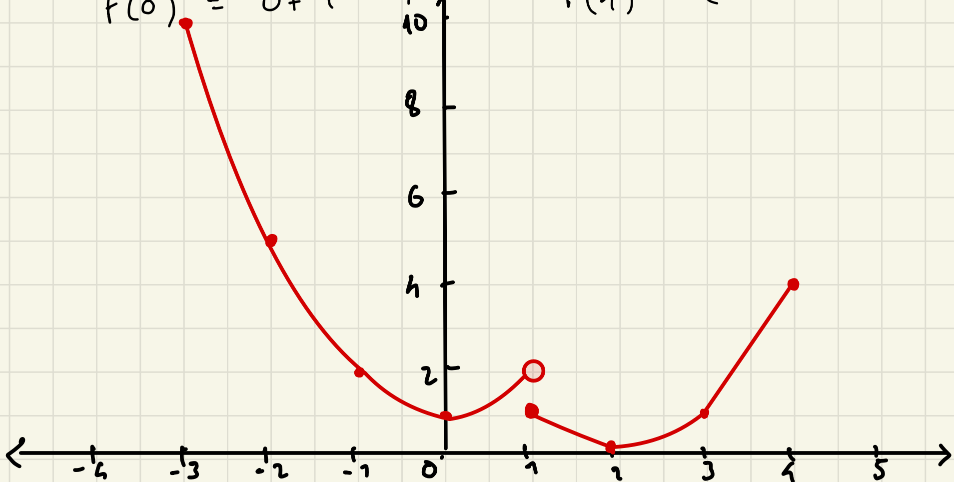
(a) Find $\lim_{x \to 1^-} f(x)$ and $\lim_{x \to 1^+} f(x)$.
(b) Does $\lim_{x \to 1} f(x)$ exist?
(c) Sketch the graph of f.

a) $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^-} x^2 + 1 = 1 + 1 = 2$

$\lim_{x \to 1^+} f(x) = \lim_{x \to 1^+} (x\cancel{^2} - 2)^2 = (1-2)^2 = 1$

b) $\lim_{x \to 1} f(x)$ DNE as one sided limits does not equal

c) $f(-3) = 9 + 1 = 10$ $\quad$ $f(1) = (1-2)^2 = 1$
$f(-2) = 4 + 1 = 5$ $\quad$ $f(2) = (0)^2 = 0$
$f(-1) = 1 + 1 = 2$ $\quad$ $f(3) = (1)^2 = 1$
$f(0) = 0 + 1 = 1$ $\quad$ $f(4) = (4-2)^2 = 4$



49.

49. Let $g(x) = \dfrac{x^2 + x - 6}{|x - 2|}$.

(a) Find

(i) $\lim_{x \to 2^+} g(x)$ (ii) $\lim_{x \to 2^-} g(x)$

(b) Does $\lim_{x \to 2} g(x)$ exist?

(c) Sketch the graph of g.

given $g(x) = \dfrac{x^2 + x - 6}{|x-2|}$

$$|x-2| = \begin{cases} -(x-2), & x<2 \\ +(x-2), & x \geq 2 \end{cases}$$

$$|x-2| = \begin{cases} 2-x, & x<2 \\ x-2, & x \geq 2 \end{cases}$$

a)

$$\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} \frac{x^2 + x - 6}{x-2}$$

$$\lim_{x \to 2^+} \frac{x^2 + 3x - 2x - 6}{x-2}$$

$\underline{3} \times \underline{-2} = -6$

$\underline{3} + \underline{-2} = 1$

$$\lim_{x \to 2^+} \frac{x(x+3) - 2(x+3)}{(x-2)}$$

$$\lim_{x \to 2^+} \frac{(x+3)(x-2)}{(x-2)}$$

$$\lim_{x \to 2^+} x+3 = 3+2 = 5$$

$$\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} \frac{x^2 + x - 6}{2 - x}$$

1 1.9 1.999 2

+ve

so just x

$$= \lim_{x \to 2^-} \frac{(x+3)(x-2)}{2-x}$$

$$= \lim_{x \to 2^-} \frac{(x+3)}{-1}$$

$$= \lim_{x \to 2^-} -3 - x = -3 - 2^- = -5^-$$

b) One sided limits are not same; hence two sided limits DNE

c) $x = -2, -1, 0, 1, 2$ $\quad$ $x = 2, 3, 4, 5$

$f(x) = -3 - x$ $\quad$ $f(x) = x + 3$

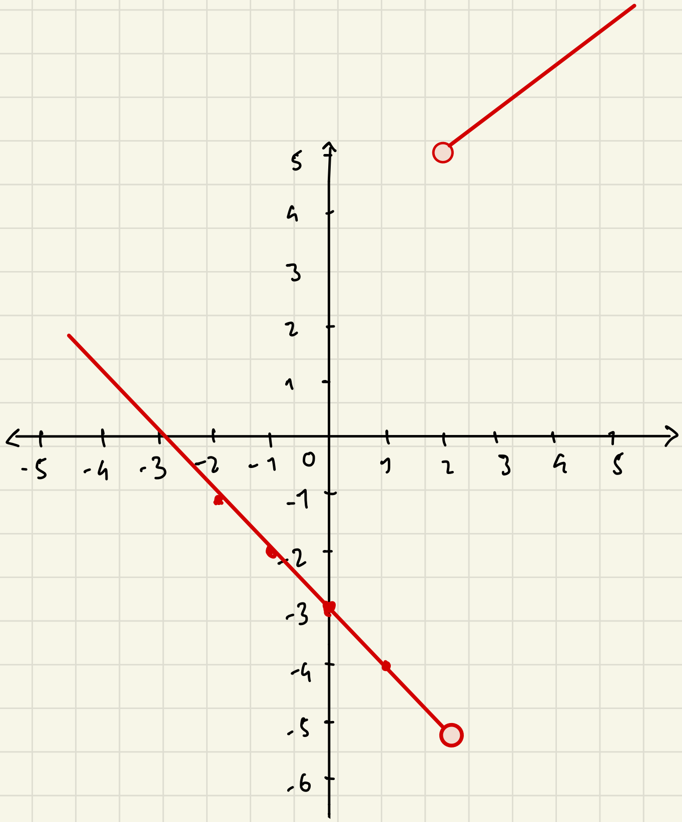
$f(-2) = -3 - (-2) = -1$ $\quad$ $f(2) = 2 + 3 = 5$

$f(-1) = -3 - (-1) = -2$ $\quad$ $f(3) = 3 + 3 = 6$

$f(0) = -3 - (0) = -3$ $\quad$ $f(4) = 4 + 3 = 7$

$f(1) = -3 - 1 = -4$ $\quad$ $f(5) = 5 + 3 = 8$

$f(2) = -3 - 2 = -5$



5
4
3
2
1
0
-1
-2
-3
-4
-5
-6
-5
-4
-3
-2
-1
1
2
3
4
5

5. Limits [20 pts]

a. [2 pts] Indicate which of the following statements are correct (+) or incorrect (-)

+	-	
☐	☒	The equation $\lim_{x \to a} f(x) = +\infty$ indicates that the function has a limit at $x = a$
☒	☐	A polynomial is a special case of a rational function
☒	☐	The limit of a quotient of two functions is the quotient of the limits of the functions
☐	☒	The squeeze theorem is only relevant for finding the limit of an oscillating function

b. $\lim_{x \to -2} \sqrt[4]{x^2 - 5x + 2}$

$= \sqrt[4]{\lim_{x \to -2} (x^2 - 5x + 2)}$

$= \sqrt[4]{\lim_{x \to -2} x^2 - \lim_{x \to -2} 5x + \lim_{x \to -2} 2}$

$= \sqrt[4]{(-2)^2 - 5(-2) + 2}$

$= \sqrt[4]{4 + 10 + 4}$

$= \sqrt[4]{18}$

c) $\lim_{t \to 4} \dfrac{t-4}{t^2-t-12}$

$N(4) = 4-4 = 0$ $\quad \frac{0}{0}$ case 3

$D(4) = 16-4-12 = 0$

factorize further

Denom: $-\underline{4} \times \underline{3} = -12$

$-\underline{4} + \underline{3} = -1$

$\lim_{t \to 4} \dfrac{t-4}{t^2-4t+3t-12}$

$\lim_{t \to 4} \dfrac{t-4}{t(t-4)+3(t-4)}$

$\lim_{t \to 4} \dfrac{t-4}{(t-4)(t+3)}$

$\lim_{t \to 4} \dfrac{1}{t+3} = \dfrac{1}{7}$

d) $\lim_{x \to 0} \left[x^2 \sin\left(\frac{3}{x}\right) + 2 \right]$

Applying limit directly we have
0 in denominator which leads
to DNE
case

Applying squeeze theorem

we know
for Sin
function

$$-1 \leq \sin\left(\frac{3}{x}\right) \leq 1$$

$$-x^2 \leq x^2 \sin\left(\frac{3}{x}\right) \leq x^2$$

$$-x^2 + 2 \leq x^2 \sin\left(\frac{3}{x}\right) + 2 \leq x^2 + 2$$

Checking the limit on both ends

$$\left.\begin{aligned} \lim_{x \to 0} -x^2 + 2 &= 2 \\ \lim_{x \to 0} x^2 + 2 &= 2 \end{aligned}\right\} \text{Both limits are equal}$$

$$\therefore \lim_{x \to 0} x^2 \sin\left(\frac{3}{x}\right) + 2 = 2$$

A 1

G X1

txb Today at 5:27 PM

please help me find the x tyy