Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 9 & 10 & 7 \\ 9 & 11 & 9 & 8 \\ 11 & 6 & 8 & 11 \\ 9 & 7 & 6 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 108$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 3 & 2 \\ 2 & 1 & 2 \\ 4 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/5 & -7/5 & 4/5 \\ 2/5 & 1/5 & -2/5 \\ -2/5 & 9/5 & -3/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & -2 & 2 \\ -3 & -5 & 7 \\ -6 & -4 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 5\,\lambda^2 + 2\,\lambda + 24$.

(b) $\lambda_1 = 2, \lambda_2 = -3, \lambda_3 = -4$.

(c) $v_2 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -3/2 \\ 0 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -2/3 \\ 1/3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 7 & 11 & 9 \\ 8 & 8 & 11 & 8 \\ 9 & 9 & 9 & 10 \\ 9 & 8 & 6 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 117$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 4 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/2 & 1/4 & -7/4 \\ -1/2 & 1/4 & 1/4 \\ -1/2 & -1/4 & 3/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & -5 & -3 \\ 4 & 2 & 3 \\ 7 & 7 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - \lambda^2 + 14\,\lambda + 24$.

(b) $\lambda_1 = 4, \lambda_2 = -2, \lambda_3 = -3$.

(c) $v_4 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 9 & 7 & 8 \\ 8 & 11 & 8 & 11 \\ 6 & 8 & 10 & 10 \\ 7 & 11 & 9 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -128$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 2 & 4 \\ 3 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/4 & -1/2 & 3/2 \\ 1/4 & 3/2 & -5/2 \\ 1/4 & -1/2 & 1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & -2 & -8 \\ -10 & 5 & -8 \\ -8 & 8 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 5\,\lambda^2 + 29\,\lambda - 105.$

(b) $\lambda_1 = 7, \lambda_2 = 3, \lambda_3 = -5.$

(c) $v_7 = \begin{pmatrix} 1 \\ 1/3 \\ -4/3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 6 & 7 & 7 \\ 10 & 11 & 9 & 10 \\ 7 & 8 & 7 & 9 \\ 6 & 10 & 10 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = -118$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & 1/3 \\ -1/2 & 1/2 & 1/2 \\ 1/3 & 1/3 & -2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & -2 & -2 \\ -5 & -5 & 2 \\ -2 & -8 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 37\,\lambda - 84$.

(b) $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = -7$.

(c) $v_4 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -1 \\ -3/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 3/2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 11 & 11 & 8 \\ 9 & 11 & 10 & 11 \\ 8 & 10 & 10 & 11 \\ 9 & 7 & 8 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 138$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 3 & 2 \\ 2 & 3 & 1 \\ 1 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/4 & 5/4 & -3/4 \\ -1/4 & 1/4 & 1/4 \\ 5/4 & -9/4 & 3/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & -9 & -9 \\ 4 & 7 & 3 \\ -2 & -6 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 6\,\lambda^2 + 27\,\lambda - 140$.

(b) $\lambda_1 = 7, \lambda_2 = 4, \lambda_3 = -5$.

(c) $v_7 = \begin{pmatrix} 1 \\ -1/3 \\ -4/3 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ -2/3 \\ -2/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1/3 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 10 & 10 & 8 \\ 6 & 11 & 8 & 10 \\ 11 & 8 & 11 & 7 \\ 10 & 8 & 11 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 2 \\ 4 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/3 & -5/3 & 4/3 \\ 2/3 & -2/3 & 1/3 \\ -5/3 & 11/3 & -7/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 9 & 3 & -5 \\ -2 & 4 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 15\,\lambda^2 - 74\,\lambda + 120$.

(b) $\lambda_1 = 6, \lambda_2 = 5, \lambda_3 = 4$.

(c) $v_6 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 9 & 9 & 9 \\ 11 & 7 & 10 & 11 \\ 11 & 8 & 8 & 6 \\ 10 & 9 & 10 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -142$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 4 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/6 & -5/6 & 3/2 \\ 1/6 & -7/6 & 3/2 \\ 1/6 & 11/6 & -5/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & -4 & 2 \\ -3 & -3 & -4 \\ 3 & -3 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 7\,\lambda^2 + 14\,\lambda + 120.$

(b) $\lambda_1 = 4, \lambda_2 = -5, \lambda_3 = -6.$

(c) $v_4 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 7 & 11 & 11 \\ 7 & 10 & 9 & 6 \\ 6 & 10 & 11 & 8 \\ 11 & 7 & 8 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -105$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 3 & 4 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/2 & 1/4 & 7/4 \\ 3/2 & 1/4 & -9/4 \\ -1/2 & -1/4 & 5/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -4 & -4 & 7 \\ 4 & -2 & 2 \\ 2 & -4 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 24\,\lambda + 36$.

(b) $\lambda_1 = 6, \lambda_2 = -2, \lambda_3 = -3$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 6 & 9 & 9 \\ 10 & 11 & 8 & 11 \\ 11 & 10 & 8 & 7 \\ 11 & 8 & 11 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 102$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/2 & 1/2 & -1/2 \\ 1/3 & -2/3 & 1/3 \\ -2/3 & 1/3 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -5 & -2 & 2 \\ -6 & -8 & 6 \\ -7 & -6 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 9\,\lambda^2 - 26\,\lambda - 24$.

(b) $\lambda_1 = -2, \lambda_2 = -3, \lambda_3 = -4$.

(c) $v_{-2} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 3/2 \\ 2 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 11 & 10 & 9 \\ 8 & 7 & 9 & 11 \\ 9 & 6 & 10 & 10 \\ 10 & 10 & 9 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 115$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 3 & 4 & 4 \\ 2 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/7 & 1/7 & -4/7 \\ -4/7 & 6/7 & -3/7 \\ 1/7 & -5/7 & 6/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & -9 & 9 \\ 4 & -2 & -4 \\ 4 & -9 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - \lambda^2 + 44\,\lambda + 84$.

(b) $\lambda_1 = 7, \lambda_2 = -2, \lambda_3 = -6$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 9 & 11 & 10 \\ 9 & 10 & 9 & 6 \\ 7 & 10 & 10 & 9 \\ 7 & 10 & 9 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 106$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -1/5 & 3/5 & -1/5 \\ -3/5 & -6/5 & 7/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & -9 & -8 \\ -2 & -3 & -8 \\ 2 & -2 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 4\,\lambda^2 + 27\,\lambda - 90$.

(b) $\lambda_1 = 6, \lambda_2 = 3, \lambda_3 = -5$.

(c) $v_6 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
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1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 11 & 6 & 10 \\ 8 & 7 & 9 & 8 \\ 8 & 11 & 7 & 8 \\ 6 & 8 & 9 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 116$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 4 & 2 & 1 \\ 1 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/7 & 3/7 & 1/7 \\ 11/7 & -4/7 & -6/7 \\ -10/7 & 3/7 & 8/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & -7 & 6 \\ -2 & -3 & 6 \\ -4 & 4 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 3\,\lambda^2 + 18\,\lambda + 40$.

(b) $\lambda_1 = 4, \lambda_2 = -2, \lambda_3 = -5$.

(c) $v_4 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1 \\ 1/2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 9 & 10 & 10 \\ 8 & 8 & 8 & 9 \\ 11 & 11 & 7 & 10 \\ 9 & 11 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -107$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 2 & 2 \\ 1 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/2 & 1/4 & 1/2 \\ 3/2 & 1/4 & -5/2 \\ -1/2 & -1/4 & 3/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & -6 & -6 \\ -4 & -9 & -6 \\ 6 & 6 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 13\,\lambda^2 - 52\,\lambda - 60.$

(b) $\lambda_1 = -2, \lambda_2 = -5, \lambda_3 = -6.$

(c) $v_{-2} = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1/2 \\ -1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 2/3 \\ -1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 11 & 7 & 11 \\ 8 & 7 & 8 & 7 \\ 9 & 7 & 9 & 8 \\ 11 & 9 & 7 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -148$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 3 & 2 & 2 \\ 4 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & -1/3 & 2/3 \\ 2/3 & -2/3 & 1/3 \\ 1/3 & 5/3 & -4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & 5 & -4 \\ -6 & -7 & 6 \\ 6 & 5 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 13\,\lambda^2 - 50\,\lambda - 56.$

(b) $\lambda_1 = -2, \lambda_2 = -4, \lambda_3 = -7.$

(c) $v_{-2} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -2/3 \\ 2/3 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 9 & 7 & 8 \\ 6 & 7 & 10 & 6 \\ 8 & 8 & 6 & 8 \\ 7 & 11 & 8 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 140$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 3 & 3 \\ 2 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/7 & 3/7 & -3/7 \\ -3/7 & 5/7 & 2/7 \\ 4/7 & -9/7 & 2/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & 2 & 6 \\ 3 & 5 & -6 \\ -9 & -5 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 34\,\lambda + 56$.

(b) $\lambda_1 = 7, \lambda_2 = -2, \lambda_3 = -4$.

(c) $v_7 = \begin{pmatrix} 0 \\ 1 \\ -1/3 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1 \\ -2/3 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 8 & 7 & 7 \\ 11 & 7 & 7 & 10 \\ 11 & 8 & 8 & 11 \\ 6 & 6 & 9 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 106$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 4 & 2 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/5 & 1/5 & 2/5 \\ 3/5 & 2/5 & -11/5 \\ -2/5 & -3/5 & 14/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -9 & -6 & -8 \\ 7 & 10 & 2 \\ 7 & 3 & 9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 10\,\lambda^2 - 11\,\lambda - 70$.

(b) $\lambda_1 = 7, \lambda_2 = 5, \lambda_3 = -2$.

(c) $v_7 = \begin{pmatrix} 1 \\ -2 \\ -1/2 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 10 & 11 \\ 10 & 9 & 11 & 9 \\ 10 & 11 & 10 & 8 \\ 8 & 8 & 11 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -142$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 3 & 2 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/3 & 1/3 & -2/3 \\ -1/3 & 4/3 & -2/3 \\ -2/3 & -4/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & 8 & 6 \\ -3 & -8 & -3 \\ -6 & -6 & -9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 14\,\lambda^2 - 63\,\lambda - 90$.

(b) $\lambda_1 = -3, \lambda_2 = -5, \lambda_3 = -6$.

(c) $v_{-3} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -3/4 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 11 & 7 & 10 \\ 7 & 11 & 9 & 6 \\ 11 & 9 & 10 & 10 \\ 10 & 6 & 9 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 141$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 1 & 4 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/3 & 2/3 & -4/3 \\ -2/3 & 1/3 & 1/3 \\ 1/3 & -2/3 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & 7 & -10 \\ -2 & 6 & -4 \\ 3 & -3 & 8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 11\,\lambda^2 - 38\,\lambda + 40$.

(b) $\lambda_1 = 5, \lambda_2 = 4, \lambda_3 = 2$.

(c) $v_5 = \begin{pmatrix} 1 \\ 2/3 \\ -1/3 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 10 & 7 & 9 \\ 10 & 11 & 7 & 8 \\ 6 & 8 & 7 & 6 \\ 9 & 9 & 10 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -121$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 3 & 3 & 2 \\ 2 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & 1/3 & 1/3 \\ 1/3 & 1/3 & -2/3 \\ 1/2 & -1/2 & 1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -10 & 6 & -6 \\ -5 & 7 & -2 \\ 3 & 3 & 2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - \lambda^2 + 22\,\lambda + 40.$

(b) $\lambda_1 = 5, \lambda_2 = -2, \lambda_3 = -4.$

(c) $v_5 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1/3 \\ -1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 1/3 \\ -2/3 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 10 & 10 & 6 \\ 8 & 8 & 10 & 10 \\ 11 & 9 & 10 & 7 \\ 11 & 8 & 10 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -120$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 4 & 3 \\ 4 & 3 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/7 & -2/7 & 1/7 \\ -11/7 & 3/7 & 2/7 \\ 13/7 & -1/7 & -3/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & 2 & -7 \\ -5 & 9 & -7 \\ -5 & 5 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 8\,\lambda^2 + 5\,\lambda - 84.$

(b) $\lambda_1 = 7, \lambda_2 = 4, \lambda_3 = -3.$

(c) $v_7 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 11 & 11 & 10 \\ 9 & 11 & 7 & 10 \\ 6 & 7 & 11 & 8 \\ 7 & 6 & 7 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -5/4 & 3/4 & 1/4 \\ -3/2 & 3/2 & -1/2 \\ 7/4 & -5/4 & 1/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 10 & -6 & -4 \\ 8 & -3 & -5 \\ -4 & 2 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 13\,\lambda^2 - 54\,\lambda + 72$.

(b) $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = 3$.

(c) $v_6 = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1/2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 6 & 11 \\ 7 & 7 & 8 & 7 \\ 11 & 9 & 7 & 9 \\ 11 & 9 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 2 \\ 2 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/3 & 1/3 & -2/3 \\ 1/3 & 5/3 & -4/3 \\ -2/3 & -4/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & 9 & 9 \\ -2 & 4 & 9 \\ 2 & -2 & -7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 10\,\lambda^2 - 11\,\lambda + 70$.

(b) $\lambda_1 = 2, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-5} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 7 & 7 & 8 \\ 11 & 6 & 6 & 10 \\ 6 & 7 & 9 & 9 \\ 8 & 10 & 9 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 133$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 3 & 4 \\ 1 & 2 & 2 \\ 4 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/3 & -5/3 & -2/3 \\ 5/3 & -4/3 & -4/3 \\ -7/3 & 8/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -9 & -2 & -4 \\ 9 & -3 & 9 \\ 3 & 2 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 14\,\lambda^2 - 63\,\lambda - 90.$

(b) $\lambda_1 = -3, \lambda_2 = -5, \lambda_3 = -6.$

(c) $v_{-3} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 3/2 \\ -3/2 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 8 & 7 & 10 \\ 10 & 9 & 9 & 6 \\ 11 & 10 & 8 & 9 \\ 8 & 8 & 9 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -119$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 2 & 4 & 4 \\ 3 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & 1/6 & 2/3 \\ 5/3 & -2/3 & -2/3 \\ -4/3 & 5/6 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & -8 & -4 \\ 4 & 9 & 2 \\ -8 & -8 & -5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 41\,\lambda - 105$.

(b) $\lambda_1 = 5, \lambda_2 = 3, \lambda_3 = -7$.

(c) $v_5 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -1/2 \\ 2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 9 & 8 & 6 \\ 7 & 6 & 11 & 11 \\ 6 & 8 & 7 & 6 \\ 8 & 6 & 10 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 130$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 4 & 2 & 1 \\ 4 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/3 & -2/3 & -1/3 \\ -8/3 & 5/3 & 1/3 \\ -4/3 & 1/3 & 2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 9 & 2 & 2 \\ -3 & 3 & -3 \\ -3 & -2 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 16\,\lambda^2 - 81\,\lambda + 126$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 3$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_6 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -2 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 9 & 11 & 6 \\ 10 & 8 & 11 & 7 \\ 7 & 7 & 10 & 7 \\ 7 & 10 & 7 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -112$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 3 \\ 3 & 2 & 1 \\ 4 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/7 & 6/7 & -5/7 \\ -5/7 & -3/7 & 6/7 \\ 1/7 & -5/7 & 3/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & -3 & 6 \\ 4 & 4 & -2 \\ 6 & -3 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 10\,\lambda^2 - 3\,\lambda - 126$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = -3$.

(c) $v_7 = \begin{pmatrix} 1 \\ 2/3 \\ 1 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -1 \\ -3/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 6 & 11 \\ 8 & 7 & 9 & 7 \\ 11 & 6 & 11 & 6 \\ 11 & 9 & 10 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = -138$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 3 & 4 \\ 4 & 4 & 3 \\ 2 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 13/6 & -4/3 & -7/6 \\ -5/3 & 4/3 & 2/3 \\ -2/3 & 1/3 & 2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & 3 & 6 \\ 2 & -3 & 4 \\ -8 & 10 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.
   (a) Calculate $\det(A - \lambda I)$.
   (b) Find the eigenvalues for A.
   (c) Find the eigenvectors for A.
   (d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 22\,\lambda - 40$.

(b) $\lambda_1 = 4, \lambda_2 = 2, \lambda_3 = -5$.

(c) $v_4 = \begin{pmatrix} 1 \\ 2/3 \\ 2/3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2/3 \\ 1/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 9 & 9 \\ 7 & 10 & 8 & 9 \\ 7 & 8 & 11 & 11 \\ 10 & 9 & 8 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 136$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 3 & 2 \\ 1 & 2 & 3 \\ 2 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/6 & -1/6 & 5/6 \\ 5/6 & -1/6 & -7/6 \\ -1/2 & 1/2 & 1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & -2 & 2 \\ 3 & 3 & 4 \\ -3 & 4 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 2\,\lambda^2 + 43\,\lambda + 140$.

(b) $\lambda_1 = 7, \lambda_2 = -4, \lambda_3 = -5$.

(c) $v_7 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -3/4 \\ 3/4 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 6 & 10 & 9 \\ 6 & 9 & 11 & 7 \\ 8 & 7 & 11 & 7 \\ 6 & 10 & 8 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 142$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 4 & 4 & 1 \\ 3 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/5 & 1/5 & 1/5 \\ 1/5 & 2/5 & -3/5 \\ 4/5 & -7/5 & 8/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & -2 & -3 \\ 10 & 5 & 3 \\ 10 & 10 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 4\,\lambda^2 + 11\,\lambda + 30$.

(b) $\lambda_1 = 3, \lambda_2 = -2, \lambda_3 = -5$.

(c) $v_3 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 8 & 10 & 8 \\ 6 & 11 & 6 & 10 \\ 6 & 11 & 7 & 7 \\ 6 & 8 & 8 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 1 & 2 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/2 & -3/2 & 1/2 \\ 5/4 & -9/4 & 1/4 \\ -3/2 & 7/2 & -1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & -3 & 6 \\ -2 & -7 & 6 \\ -3 & -5 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 6\,\lambda^2 + 7\,\lambda + 60$.

(b) $\lambda_1 = 3, \lambda_2 = -4, \lambda_3 = -5$.

(c) $v_3 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -4/3 \\ -1/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 2/3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 10 & 6 & 8 \\ 6 & 8 & 11 & 9 \\ 8 & 8 & 9 & 9 \\ 11 & 6 & 11 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -148$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 3 & 1 \\ 2 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -7/3 & 2/3 & 4/3 \\ 1/3 & 1/3 & -1/3 \\ 4/3 & -2/3 & -1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & 5 & 4 \\ 9 & -3 & -9 \\ -3 & 5 & 10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 10\,\lambda^2 - 3\,\lambda - 126$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = -3$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 8 & 6 & 10 \\ 6 & 6 & 6 & 9 \\ 7 & 7 & 7 & 7 \\ 7 & 6 & 10 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 4 & 3 & 4 \\ 3 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -4/3 & 1/3 & 1/3 \\ 2/3 & 1/3 & -2/3 \\ 5/6 & -1/3 & 1/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & 3 & -3 \\ 3 & 3 & 4 \\ 3 & -3 & 10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 17\,\lambda^2 - 94\,\lambda + 168$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 4$.

(c) $v_7 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ -1/3 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 6 & 6 & 7 \\ 9 & 8 & 9 & 6 \\ 11 & 6 & 10 & 9 \\ 11 & 10 & 7 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -120$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 3 & 3 & 2 \\ 1 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/7 & 4/7 & -1/7 \\ -4/7 & 1/7 & 5/7 \\ 9/7 & -4/7 & -6/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -5 & 8 & -3 \\ -3 & 6 & -3 \\ 9 & -8 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 8\,\lambda^2 - 4\,\lambda - 48$.

(b) $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = -2$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ -1/3 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 10 & 7 & 8 \\ 8 & 6 & 9 & 6 \\ 7 & 6 & 10 & 10 \\ 11 & 8 & 10 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -134$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 2 & 1 & 2 \\ 1 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/7 & 5/7 & -2/7 \\ 4/7 & 1/7 & -6/7 \\ -1/7 & -2/7 & 5/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 5 & -2 & -8 \\ 9 & -2 & -6 \\ 3 & -7 & -9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 6\,\lambda^2 + 37\,\lambda + 210$.

(b) $\lambda_1 = 6, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_6 = \begin{pmatrix} 1 \\ 3/2 \\ -1/2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1/3 \\ 4/3 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ 3/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 8 & 7 & 9 \\ 11 & 10 & 8 & 10 \\ 11 & 10 & 7 & 9 \\ 10 & 11 & 11 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -138$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/5 & 1/5 & -2/5 \\ -3/5 & -6/5 & 7/5 \\ -1/5 & 3/5 & -1/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & 4 & 2 \\ -10 & 6 & 2 \\ 3 & -4 & -7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 9\,\lambda^2 - 8\,\lambda + 60$.

(b) $\lambda_1 = 2, \lambda_2 = -5, \lambda_3 = -6$.

(c) $v_2 = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ -1/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 6 & 6 & 11 \\ 6 & 7 & 9 & 8 \\ 9 & 10 & 9 & 9 \\ 10 & 10 & 9 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 141$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 3 & 2 \\ 4 & 3 & 4 \\ 2 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 13/7 & -5/7 & -6/7 \\ -4/7 & 1/7 & 4/7 \\ -10/7 & 6/7 & 3/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & -6 & -10 \\ -3 & 3 & -3 \\ 2 & 2 & 8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 13\,\lambda^2 - 54\,\lambda + 72$.

(b) $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = 3$.

(c) $v_6 = \begin{pmatrix} 1 \\ -3/2 \\ 1/2 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 3/2 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 6 & 9 & 6 \\ 6 & 8 & 10 & 6 \\ 6 & 8 & 9 & 11 \\ 7 & 6 & 7 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 4 & 2 \\ 3 & 3 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/7 & -1/7 & 4/7 \\ 5/7 & -1/7 & -3/7 \\ -9/7 & 6/7 & 4/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & -2 & 2 \\ -4 & -5 & 2 \\ -2 & -2 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 15\,\lambda^2 - 71\,\lambda - 105$.

(b) $\lambda_1 = -3, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_{-3} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 6 & 9 \\ 11 & 9 & 7 & 6 \\ 11 & 11 & 7 & 11 \\ 9 & 8 & 7 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -125$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 3 & 3 \\ 3 & 3 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -6/5 & 1/5 & 3/5 \\ 7/5 & -2/5 & -1/5 \\ -3/5 & 3/5 & -1/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & -2 & 2 \\ 3 & -3 & -3 \\ -7 & -4 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 13\,\lambda^2 - 54\,\lambda - 72.$

(b) $\lambda_1 = -3, \lambda_2 = -4, \lambda_3 = -6.$

(c) $v_{-3} = \begin{pmatrix} 1 \\ -3/2 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -3/2 \\ 1/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 10 & 7 & 6 \\ 7 & 6 & 7 & 10 \\ 10 & 10 & 7 & 9 \\ 8 & 10 & 8 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 1 & 3 \\ 1 & 4 & 1 \\ 3 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -4/7 & -10/7 & 11/7 \\ -1/7 & 1/7 & 1/7 \\ 8/7 & 13/7 & -15/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 6 & 3 & 2 \\ -6 & -3 & -4 \\ 6 & -3 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 30\,\lambda - 72.$

(b) $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = -6.$

(c) $v_4 = \begin{pmatrix} 1 \\ -2 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 8 & 8 & 11 \\ 8 & 9 & 6 & 6 \\ 11 & 11 & 11 & 9 \\ 6 & 8 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 1 & 3 & 1 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/2 & 3/2 & -5/2 \\ -1/4 & 1/4 & 1/4 \\ -3/4 & -5/4 & 7/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -4 & 9 & 2 \\ 8 & -3 & 8 \\ 9 & -9 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.
(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 4\,\lambda^2 + 27\,\lambda + 90$.

(b) $\lambda_1 = 5, \lambda_2 = -3, \lambda_3 = -6$.

(c) $v_5 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1/3 \\ -1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 10 & 10 & 8 \\ 6 & 7 & 10 & 10 \\ 9 & 11 & 8 & 10 \\ 7 & 9 & 7 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = -148$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 4 & 3 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/3 & 1/3 & -4/3 \\ -4/3 & 1/3 & 2/3 \\ -4/3 & -2/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & -3 & 6 \\ 6 & -10 & 10 \\ 6 & -5 & 5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 2\,\lambda^2 + 33\,\lambda + 90$.

(b) $\lambda_1 = 6, \lambda_2 = -3, \lambda_3 = -5$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 0 \\ 1 \\ 1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 6 & 7 \\ 11 & 10 & 10 & 7 \\ 9 & 6 & 10 & 6 \\ 10 & 10 & 11 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 129$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 1 & 1 & 3 \\ 3 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & 1/3 \\ 5/3 & 2/3 & -4/3 \\ -2/3 & 1/3 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & 10 & 3 \\ -5 & 7 & 3 \\ 2 & -4 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 6\,\lambda^2 + 7\,\lambda - 60$.

(b) $\lambda_1 = 5, \lambda_2 = 4, \lambda_3 = -3$.

(c) $v_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 2/3 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1/2 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 7 & 7 & 6 \\ 10 & 11 & 10 & 6 \\ 10 & 10 & 6 & 6 \\ 8 & 10 & 6 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 4 & 1 \\ 4 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -11/3 & 10/3 & 4/3 \\ 2/3 & -1/3 & -1/3 \\ 14/3 & -13/3 & -4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & -9 & -6 \\ 4 & -9 & -3 \\ 6 & -6 & -9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 14\,\lambda^2 - 63\,\lambda - 90$.

(b) $\lambda_1 = -3, \lambda_2 = -5, \lambda_3 = -6$.

(c) $v_{-3} = \begin{pmatrix} 1 \\ 1/3 \\ 2/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 2/3 \\ 2/3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 7 & 6 \\ 7 & 9 & 9 & 9 \\ 9 & 9 & 7 & 7 \\ 6 & 6 & 11 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -114$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 1 & 3 \\ 2 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/3 & 2/3 & -7/6 \\ -1/3 & -2/3 & 2/3 \\ -1/3 & 1/3 & 1/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & -2 & 2 \\ 7 & 2 & -7 \\ 9 & -2 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 3\,\lambda^2 + 28\,\lambda - 60$.

(b) $\lambda_1 = 6, \lambda_2 = 2, \lambda_3 = -5$.

(c) $v_6 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 6 & 7 & 6 \\ 7 & 10 & 9 & 8 \\ 8 & 6 & 7 & 8 \\ 11 & 8 & 11 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -116$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 4 & 3 & 4 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/5 & 3/5 & -2/5 \\ -4/5 & -1/5 & 4/5 \\ 6/5 & -1/5 & -1/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 8 & -9 & -6 \\ 5 & -6 & -6 \\ 5 & -2 & -10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.
(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 8\,\lambda^2 + 5\,\lambda + 84$.

(b) $\lambda_1 = 3, \lambda_2 = -4, \lambda_3 = -7$.

(c) $v_3 = \begin{pmatrix} 1 \\ 1/3 \\ 1/3 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 1 \\ 1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 11 & 11 & 9 \\ 6 & 9 & 8 & 10 \\ 10 & 9 & 9 & 7 \\ 7 & 6 & 9 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = -111$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 1 \\ 4 & 2 & 1 \\ 1 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/6 & 1/6 & -1/6 \\ -7/6 & 5/6 & 1/6 \\ 5/3 & -4/3 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 5 & 2 & -3 \\ 6 & 9 & -6 \\ 6 & 6 & -4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 10\,\lambda^2 - 31\,\lambda + 30.$

(b) $\lambda_1 = 5, \lambda_2 = 3, \lambda_3 = 2.$

(c) $v_5 = \begin{pmatrix} 0 \\ 1 \\ 2/3 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 8 & 11 & 7 \\ 10 & 10 & 8 & 6 \\ 11 & 8 & 11 & 6 \\ 7 & 9 & 9 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 140$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 3 & 2 \\ 1 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/3 & -2/3 & 2/3 \\ 1/3 & 1/3 & -4/3 \\ -2/3 & 1/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 6 & -6 & 2 \\ 10 & -10 & 2 \\ 3 & -6 & 5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 14\,\lambda - 24$.

(b) $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = -4$.

(c) $v_3 = \begin{pmatrix} 1 \\ 1 \\ 3/2 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 7 & 6 & 9 \\ 6 & 9 & 6 & 7 \\ 9 & 7 & 6 & 9 \\ 11 & 7 & 7 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 4 & 1 \\ 4 & 3 & 3 \\ 3 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/7 & -2/7 & 9/7 \\ 5/7 & 1/7 & -8/7 \\ -1/7 & 4/7 & -4/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & -6 & 7 \\ -2 & -3 & -3 \\ -2 & -10 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 2\,\lambda^2 + 43\,\lambda + 140$.

(b) $\lambda_1 = 7, \lambda_2 = -4, \lambda_3 = -5$.

(c) $v_7 = \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 8 & 8 & 8 \\ 9 & 7 & 8 & 11 \\ 7 & 7 & 11 & 10 \\ 6 & 9 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -112$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 4 & 1 \\ 3 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -5/2 & 1/6 & 11/6 \\ 3/2 & 1/6 & -7/6 \\ 3/2 & -1/6 & -5/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & -3 & 3 \\ -10 & -3 & -3 \\ -4 & -4 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - \lambda^2 + 44\,\lambda + 84.$

(b) $\lambda_1 = 7, \lambda_2 = -2, \lambda_3 = -6.$

(c) $v_7 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1 \\ -3 \end{pmatrix}, v_{-6} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 7 & 10 \\ 10 & 9 & 9 & 10 \\ 6 & 10 & 11 & 10 \\ 10 & 9 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -101$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/7 & 4/7 & -5/7 \\ -6/7 & 1/7 & 4/7 \\ 5/7 & -2/7 & -1/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -5 & -5 & -2 \\ -2 & -2 & 2 \\ -9 & -5 & 2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 5\,\lambda^2 + 22\,\lambda + 56$.

(b) $\lambda_1 = 4, \lambda_2 = -2, \lambda_3 = -7$.

(c) $v_4 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 7 & 7 & 9 \\ 9 & 9 & 9 & 10 \\ 10 & 7 & 10 & 10 \\ 6 & 8 & 10 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 132$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 1 & 1 \\ 4 & 2 & 3 \\ 3 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/5 & 1/5 & -1/5 \\ 7/5 & -13/5 & 8/5 \\ -6/5 & 9/5 & -4/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & 3 & -2 \\ 6 & -4 & -4 \\ 9 & 3 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 9\,\lambda^2 - 8\,\lambda + 60.$

(b) $\lambda_1 = 2, \lambda_2 = -5, \lambda_3 = -6.$

(c) $v_2 = \begin{pmatrix} 1 \\ 1/3 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 8 & 9 & 6 \\ 7 & 9 & 10 & 11 \\ 10 & 10 & 11 & 10 \\ 7 & 10 & 9 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 144$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 2 & 4 \\ 2 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/7 & -5/7 & 8/7 \\ 5/7 & 2/7 & -6/7 \\ -2/7 & 2/7 & 1/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & 9 & -7 \\ -3 & 5 & -7 \\ -3 & 3 & -5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 7\,\lambda^2 - 2\,\lambda + 40.$

(b) $\lambda_1 = 2, \lambda_2 = -4, \lambda_3 = -5.$

(c) $v_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 3/2 \\ 3/2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 8 & 7 \\ 7 & 7 & 8 & 9 \\ 8 & 9 & 10 & 7 \\ 9 & 11 & 8 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -142$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 2 \\ 3 & 3 & 1 \\ 4 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -5/7 & 2/7 & 4/7 \\ 2/7 & 2/7 & -3/7 \\ 9/7 & -5/7 & -3/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 7 & -9 & -9 \\ 2 & -4 & -9 \\ -2 & 2 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 10\,\lambda^2 - 11\,\lambda - 70.$

(b) $\lambda_1 = 7, \lambda_2 = 5, \lambda_3 = -2.$

(c) $v_7 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_5 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 7 & 10 & 6 \\ 8 & 6 & 9 & 8 \\ 10 & 11 & 11 & 6 \\ 10 & 7 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 144$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 3 & 4 \\ 1 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 10/7 & 2/7 & -9/7 \\ 2/7 & -1/7 & 1/7 \\ -9/7 & 1/7 & 6/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 10 & 2 & -3 \\ -2 & 5 & 2 \\ 4 & 2 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 18\,\lambda^2 - 107\,\lambda + 210$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 5$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 6 & 7 \\ 8 & 8 & 10 & 11 \\ 6 & 8 & 6 & 8 \\ 10 & 10 & 7 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 136$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 4 & 3 \\ 1 & 1 & 1 \\ 4 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/5 & 2/5 & 1/5 \\ 3/5 & -11/5 & 2/5 \\ -2/5 & 14/5 & -3/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & 5 & -5 \\ -2 & -9 & 5 \\ -2 & -2 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 13\,\lambda^2 - 50\,\lambda - 56$.

(b) $\lambda_1 = -2, \lambda_2 = -4, \lambda_3 = -7$.

(c) $v_{-2} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 7 & 10 & 8 \\ 10 & 8 & 10 & 8 \\ 8 & 6 & 7 & 10 \\ 7 & 8 & 10 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -130$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 1 \\ 3 & 2 & 1 \\ 4 & 2 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/3 & 2/3 & -1/6 \\ 1/3 & 1/3 & -1/3 \\ 1/3 & -5/3 & 7/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & 9 & -7 \\ -2 & 8 & -8 \\ -3 & 6 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.
(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 6\,\lambda^2 + 19\,\lambda + 84$.

(b) $\lambda_1 = 4, \lambda_2 = -3, \lambda_3 = -7$.

(c) $v_4 = \begin{pmatrix} 1 \\ 3/2 \\ 1/2 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 2/3 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 11 & 9 \\ 7 & 8 & 6 & 8 \\ 8 & 8 & 8 & 8 \\ 9 & 6 & 9 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -144$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 4 \\ 2 & 1 & 1 \\ 3 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/5 & -1/5 & 3/5 \\ 3/5 & 9/5 & -7/5 \\ 1/5 & -2/5 & 1/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -10 & -6 & 6 \\ 8 & 8 & -4 \\ -6 & -3 & 5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 3\,\lambda^2 + 18\,\lambda - 40.$

(b) $\lambda_1 = 5, \lambda_2 = 2, \lambda_3 = -4.$

(c) $v_5 = \begin{pmatrix} 1 \\ -2 \\ 1/2 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1/2 \\ 1/2 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 10 & 6 & 8 \\ 8 & 7 & 10 & 9 \\ 10 & 8 & 6 & 7 \\ 11 & 9 & 10 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -118$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 3 & 4 \\ 2 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/3 & -1/3 & 4/3 \\ -2/3 & 1/3 & 2/3 \\ 2/3 & 1/6 & -7/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 6 & -2 & -2 \\ 7 & -3 & 2 \\ -8 & 8 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 6\,\lambda^2 + 27\,\lambda - 140.$

(b) $\lambda_1 = 7, \lambda_2 = 4, \lambda_3 = -5.$

(c) $v_7 = \begin{pmatrix} 1 \\ 1/2 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-5} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 6 & 11 & 6 \\ 8 & 8 & 6 & 8 \\ 10 & 6 & 7 & 10 \\ 11 & 11 & 7 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 120$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 4 \\ 2 & 1 & 3 \\ 1 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/3 & -4/3 & -4/3 \\ 1/6 & -1/3 & 1/6 \\ -7/6 & 4/3 & 5/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 8 & 6 & -6 \\ -6 & -2 & -2 \\ -4 & -2 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 4\,\lambda^2 + 20\,\lambda - 48$.

(b) $\lambda_1 = 6, \lambda_2 = 2, \lambda_3 = -4$.

(c) $v_6 = \begin{pmatrix} 1 \\ -2/3 \\ -1/3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -4/3 \\ -1/3 \end{pmatrix}, v_{-4} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 11 & 6 & 11 \\ 7 & 6 & 11 & 8 \\ 10 & 9 & 9 & 9 \\ 10 & 10 & 7 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 128$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 2 \\ 1 & 4 & 4 \\ 4 & 4 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -4/7 & -1/7 & 4/7 \\ 13/7 & -2/7 & -6/7 \\ -12/7 & 4/7 & 5/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & -8 & 4 \\ 4 & -9 & 2 \\ -3 & 3 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 14\,\lambda^2 - 59\,\lambda - 70$.

(b) $\lambda_1 = -2, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_{-2} = \begin{pmatrix} 1 \\ 1/2 \\ -1/4 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1/2 \\ -3/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 6 & 8 & 6 \\ 11 & 9 & 7 & 6 \\ 10 & 6 & 10 & 7 \\ 8 & 6 & 6 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -120$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 1 & 3 & 1 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 6/5 & -9/5 & 1/5 \\ -1/5 & 4/5 & -1/5 \\ -3/5 & 2/5 & 2/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 7 & -2 & 2 \\ -2 & 8 & -2 \\ -2 & 3 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 18\,\lambda^2 - 107\,\lambda + 210$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 5$.

(c) $v_7 = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, v_6 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 8 & 9 & 9 \\ 11 & 7 & 10 & 6 \\ 9 & 8 & 10 & 10 \\ 6 & 10 & 8 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -110$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 3 & 1 \\ 1 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/2 & 1/6 & -7/6 \\ -5/2 & 1/6 & 11/6 \\ 3/2 & -1/6 & -5/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -9 & 6 & 9 \\ 5 & 4 & -3 \\ -4 & 6 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - \lambda^2 + 32\,\lambda + 60$.

(b) $\lambda_1 = 6, \lambda_2 = -2, \lambda_3 = -5$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -1/3 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1/3 \\ 2/3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 7 & 10 & 10 \\ 6 & 7 & 6 & 6 \\ 6 & 9 & 10 & 10 \\ 7 & 6 & 9 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -112$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 2 & 4 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/3 & 8/3 & -10/3 \\ -1/3 & 2/3 & -1/3 \\ 2/3 & -7/3 & 8/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -4 & -2 & -2 \\ -9 & 3 & -2 \\ 8 & -8 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 4\,\lambda^2 + 27\,\lambda + 90$.

(b) $\lambda_1 = 5, \lambda_2 = -3, \lambda_3 = -6$.

(c) $v_5 = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1 \\ -3/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 8 & 10 & 6 \\ 10 & 8 & 7 & 11 \\ 10 & 7 & 11 & 8 \\ 11 & 6 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 3 \\ 2 & 1 & 3 \\ 3 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 8/7 & -4/7 & -3/7 \\ -1/7 & -3/7 & 3/7 \\ -5/7 & 6/7 & 1/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & -9 & -2 \\ 2 & -7 & -2 \\ 10 & -9 & -8 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 11\,\lambda^2 - 16\,\lambda + 84.$

(b) $\lambda_1 = 2, \lambda_2 = -6, \lambda_3 = -7.$

(c) $v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 1 \\ 1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 8 & 10 \\ 7 & 11 & 8 & 11 \\ 6 & 7 & 6 & 9 \\ 7 & 6 & 9 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = -119$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 4 \\ 2 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/6 & 2/3 & -2/3 \\ 5/6 & -2/3 & -1/3 \\ -2/3 & 1/3 & 2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 6 & 3 & -2 \\ -5 & 7 & 5 \\ 9 & 3 & -5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 8\,\lambda^2 + 5\,\lambda - 84$.

(b) $\lambda_1 = 7, \lambda_2 = 4, \lambda_3 = -3$.

(c) $v_7 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 10 & 9 & 8 \\ 11 & 10 & 9 & 9 \\ 8 & 11 & 10 & 7 \\ 11 & 7 & 9 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 114$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 4 \\ 4 & 4 & 3 \\ 2 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 13/3 & -4/3 & -10/3 \\ -10/3 & 4/3 & 7/3 \\ -4/3 & 1/3 & 4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & 4 & 4 \\ 4 & -2 & -4 \\ -2 & -2 & -4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 8\,\lambda^2 - 4\,\lambda + 48$.

(b) $\lambda_1 = 2, \lambda_2 = -4, \lambda_3 = -6$.

(c) $v_2 = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ 1/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 10 & 11 & 11 \\ 9 & 10 & 11 & 11 \\ 8 & 6 & 11 & 6 \\ 7 & 6 & 10 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 3 & 3 \\ 2 & 4 & 1 \\ 4 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/2 & -3/2 & -3/2 \\ -2/3 & 2/3 & 1/3 \\ -7/3 & 4/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 10 & 8 & 4 \\ -3 & -9 & -6 \\ -6 & 6 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 7\,\lambda^2 - 36$.

(b) $\lambda_1 = 6, \lambda_2 = 3, \lambda_3 = -2$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -3 \\ 3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 9 & 10 & 9 \\ 9 & 10 & 11 & 10 \\ 6 & 8 & 8 & 9 \\ 8 & 6 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -104$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 4 & 4 & 3 \\ 2 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/3 & -7/6 & 5/6 \\ -1/3 & 1/6 & 1/6 \\ -4/3 & 5/3 & -4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & 3 & 3 \\ -10 & 4 & 10 \\ 10 & 3 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 5\,\lambda^2 + 38\,\lambda - 168$.

(b) $\lambda_1 = 7, \lambda_2 = 4, \lambda_3 = -6$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 9 & 7 & 8 \\ 7 & 6 & 10 & 9 \\ 7 & 7 & 6 & 8 \\ 9 & 11 & 8 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -134$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 2 & 3 & 1 \\ 1 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/3 & -4/3 & -1/3 \\ -7/6 & 5/3 & 1/6 \\ 5/6 & -4/3 & 1/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 5 & -9 & -9 \\ 8 & -8 & -6 \\ 4 & -8 & -10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 13\,\lambda^2 - 50\,\lambda - 56.$

(b) $\lambda_1 = -2, \lambda_2 = -4, \lambda_3 = -7.$

(c) $v_{-2} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ 4/3 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 10 & 8 & 9 \\ 6 & 6 & 8 & 9 \\ 7 & 11 & 8 & 8 \\ 9 & 8 & 8 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 128$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 2 & 2 & 3 \\ 3 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 4/3 & 8/3 & -7/3 \\ -1/3 & -5/3 & 4/3 \\ -2/3 & -1/3 & 2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & 4 & -3 \\ -9 & 10 & -3 \\ -10 & 4 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 11\,\lambda^2 - 16\,\lambda - 84.$

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = -2.$

(c) $v_7 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ 3 \\ 1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 6 & 11 & 7 \\ 10 & 7 & 11 & 9 \\ 6 & 10 & 11 & 8 \\ 9 & 7 & 8 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -105$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 3 & 2 \\ 1 & 3 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/4 & 7/4 & -5/4 \\ -1/4 & 1/4 & 1/4 \\ 3/2 & -5/2 & 3/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & -5 & -5 \\ 6 & -9 & -5 \\ -6 & 6 & 2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 5\,\lambda^2 + 2\,\lambda + 24.$

(b) $\lambda_1 = 2, \lambda_2 = -3, \lambda_3 = -4.$

(c) $v_2 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-4} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 8 & 9 & 8 \\ 10 & 6 & 9 & 11 \\ 8 & 10 & 11 & 9 \\ 7 & 11 & 9 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 106$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 2 & 1 \\ 2 & 1 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/5 & -1/5 & 3/5 \\ 4/5 & -1/5 & -2/5 \\ -1/5 & 4/5 & -2/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -10 & 6 & -6 \\ 5 & -4 & 10 \\ 8 & -3 & 9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 5\,\lambda^2 + 38\,\lambda + 168$.

(b) $\lambda_1 = 6, \lambda_2 = -4, \lambda_3 = -7$.

(c) $v_6 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 9 & 10 & 11 \\ 6 & 11 & 6 & 11 \\ 8 & 8 & 9 & 9 \\ 11 & 7 & 6 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 126$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 3 \\ 2 & 2 & 1 \\ 3 & 4 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & 2/3 & 1/3 \\ 5/6 & 1/6 & -2/3 \\ -1/3 & -2/3 & 2/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & 5 & 8 \\ -6 & 3 & 8 \\ -7 & 7 & 5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 19\,\lambda + 30$.

(b) $\lambda_1 = 5, \lambda_2 = -2, \lambda_3 = -3$.

(c) $v_5 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 2/3 \\ 1/3 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 10 & 11 & 6 \\ 7 & 7 & 11 & 10 \\ 6 & 10 & 6 & 9 \\ 6 & 11 & 6 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 127$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 3 \\ 4 & 1 & 3 \\ 3 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 5/4 & 3/4 & -3/2 \\ 7/4 & 1/4 & -3/2 \\ -9/4 & -3/4 & 5/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & -6 & -4 \\ -2 & -7 & -2 \\ -4 & 6 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.
   (a) Calculate $\det(A - \lambda I)$.
   (b) Find the eigenvalues for A.
   (c) Find the eigenvectors for A.
   (d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 11\,\lambda^2 - 16\,\lambda + 84$.

(b) $\lambda_1 = 2, \lambda_2 = -6, \lambda_3 = -7$.

(c) $v_2 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 3/2 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 9 & 11 & 10 \\ 8 & 8 & 11 & 7 \\ 9 & 6 & 9 & 9 \\ 6 & 10 & 8 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -135$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 4 \\ 1 & 3 & 2 \\ 2 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/7 & -4/7 & 4/7 \\ -3/7 & 5/7 & 2/7 \\ 4/7 & -2/7 & -5/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 7 & -2 & -2 \\ 2 & 3 & -2 \\ 3 & -2 & 2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 12\,\lambda^2 - 47\,\lambda + 60$.

(b) $\lambda_1 = 5, \lambda_2 = 4, \lambda_3 = 3$.

(c) $v_5 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 1/2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 7 & 9 & 6 \\ 11 & 6 & 8 & 6 \\ 8 & 8 & 9 & 6 \\ 9 & 9 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 117$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 2 & 1 \\ 3 & 1 & 2 \\ 2 & 1 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/6 & 5/6 & -1/2 \\ 5/6 & -7/6 & 1/2 \\ -1/6 & -1/6 & 1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & -9 & 5 \\ -9 & -3 & 5 \\ -5 & -5 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 3\,\lambda^2 + 40\,\lambda + 84$.

(b) $\lambda_1 = 6, \lambda_2 = -2, \lambda_3 = -7$.

(c) $v_6 = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 9 & 7 & 7 \\ 7 & 11 & 11 & 6 \\ 10 & 9 & 9 & 7 \\ 7 & 10 & 10 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 116$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 2 & 4 \\ 1 & 4 & 1 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -9/4 & -3/2 & 7/2 \\ 1/4 & 1/2 & -1/2 \\ 5/4 & 1/2 & -3/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 6 & 2 & 8 \\ 6 & -7 & 2 \\ -6 & 3 & -6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 7\,\lambda^2 + 6\,\lambda + 72.$

(b) $\lambda_1 = 3, \lambda_2 = -4, \lambda_3 = -6.$

(c) $v_3 = \begin{pmatrix} 1 \\ 1/2 \\ -1/2 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 1 \\ -3/2 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 7 & 7 & 10 \\ 6 & 6 & 7 & 9 \\ 10 & 11 & 8 & 10 \\ 6 & 10 & 7 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -120$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 1 & 4 & 3 \\ 4 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -3/2 & 1/2 & 1/2 \\ -9/4 & 5/4 & 1/4 \\ 7/2 & -3/2 & -1/2 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -7 & -4 & -2 \\ 10 & 10 & 6 \\ -5 & -8 & -6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 3\,\lambda^2 + 10\,\lambda + 24$.

(b) $\lambda_1 = 3, \lambda_2 = -2, \lambda_3 = -4$.

(c) $v_3 = \begin{pmatrix} 1 \\ -4 \\ 3 \end{pmatrix}, v_{-2} = \begin{pmatrix} 0 \\ 1 \\ -2 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 7 & 6 & 7 \\ 6 & 7 & 11 & 10 \\ 10 & 7 & 7 & 6 \\ 9 & 9 & 11 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -104$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 3 & 4 \\ 4 & 4 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -10/7 & 1/7 & 13/7 \\ 11/7 & 1/7 & -15/7 \\ -4/7 & -1/7 & 8/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & 5 & -7 \\ 4 & -7 & 7 \\ 4 & -10 & 10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 3\,\lambda^2 + 10\,\lambda + 24$.

(b) $\lambda_1 = 3, \lambda_2 = -2, \lambda_3 = -4$.

(c) $v_3 = \begin{pmatrix} 1 \\ -1 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 8 & 9 & 10 \\ 8 & 9 & 7 & 9 \\ 11 & 9 & 11 & 8 \\ 9 & 9 & 9 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = 146$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 2 & 1 & 3 \\ 3 & 1 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/3 & -1/3 & 2/3 \\ 5/3 & -1/3 & -1/3 \\ -1/3 & 2/3 & -1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 9 & 4 & 5 \\ -5 & -2 & -5 \\ -6 & -4 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 5\,\lambda^2 + 2\,\lambda - 24.$

(b) $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = -2.$

(c) $v_4 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ -3/2 \\ -1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 10 & 7 & 8 \\ 11 & 6 & 8 & 7 \\ 7 & 11 & 6 & 10 \\ 9 & 10 & 7 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -118$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/7 & 5/7 & -2/7 \\ -1/7 & -2/7 & 5/7 \\ 3/7 & -1/7 & -1/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & 2 & -8 \\ 7 & -9 & -10 \\ 2 & 2 & -6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 11\,\lambda^2 - 16\,\lambda + 84$.

(b) $\lambda_1 = 2, \lambda_2 = -6, \lambda_3 = -7$.

(c) $v_2 = \begin{pmatrix} 1 \\ 1/3 \\ 1/3 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -3/2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 11 & 9 & 7 \\ 10 & 9 & 8 & 9 \\ 10 & 11 & 9 & 10 \\ 7 & 10 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 141$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 4 & 2 \\ 1 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & 5/3 & -4/3 \\ -1/6 & 1/6 & 1/6 \\ 4/3 & -7/3 & 5/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & 6 & 3 \\ -4 & 8 & 3 \\ 7 & -7 & -5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 22\,\lambda - 40$.

(b) $\lambda_1 = 4, \lambda_2 = 2, \lambda_3 = -5$.

(c) $v_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1/3 \\ 2/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 8 & 6 & 8 \\ 7 & 10 & 8 & 7 \\ 11 & 11 & 6 & 11 \\ 7 & 10 & 8 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = -138$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 1 \\ 3 & 1 & 3 \\ 3 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 11/7 & -3/7 & -2/7 \\ -6/7 & 1/7 & 3/7 \\ -9/7 & 5/7 & 1/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & -10 & -10 \\ -2 & 3 & 9 \\ 2 & 4 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 3\,\lambda^2 + 40\,\lambda - 84$.

(b) $\lambda_1 = 7, \lambda_2 = 2, \lambda_3 = -6$.

(c) $v_7 = \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ -1/4 \\ 1/4 \end{pmatrix}, v_{-6} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 8 & 7 & 11 \\ 11 & 8 & 11 & 6 \\ 11 & 9 & 10 & 6 \\ 11 & 9 & 8 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 122$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 2 \\ 2 & 2 & 3 \\ 1 & 2 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/3 & -2/3 & 1/3 \\ 1/3 & -4/3 & 5/3 \\ -2/3 & 5/3 & -4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & -8 & 10 \\ -6 & -3 & 6 \\ -5 & -4 & 9 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 37\,\lambda - 84$.

(b) $\lambda_1 = 4, \lambda_2 = 3, \lambda_3 = -7$.

(c) $v_4 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ -1/2 \\ 1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 3/4 \\ 1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 8 & 10 & 7 \\ 6 & 10 & 9 & 8 \\ 6 & 6 & 11 & 7 \\ 10 & 7 & 8 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 127$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 4 \\ 4 & 3 & 4 \\ 2 & 4 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -7/3 & 4/3 & 4/3 \\ -4/3 & 1/3 & 4/3 \\ 10/3 & -4/3 & -7/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -4 & -3 & 2 \\ -6 & 3 & -6 \\ -9 & -3 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 6\,\lambda^2 + 37\,\lambda - 210$.

(b) $\lambda_1 = 7, \lambda_2 = 5, \lambda_3 = -6$.

(c) $v_7 = \begin{pmatrix} 1 \\ -3 \\ 1 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ -3 \\ 0 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ 4/3 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 8 & 11 & 8 \\ 10 & 6 & 6 & 11 \\ 7 & 8 & 10 & 6 \\ 11 & 10 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -112$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & 3 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/7 & 3/7 & -4/7 \\ -1/7 & 6/7 & -1/7 \\ -1/7 & -8/7 & 6/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & -4 & 10 \\ 3 & 3 & -3 \\ 3 & -4 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 4\,\lambda^2 + 39\,\lambda - 126$.

(b) $\lambda_1 = 7, \lambda_2 = 3, \lambda_3 = -6$.

(c) $v_7 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-6} = \begin{pmatrix} 1 \\ -1/2 \\ -1/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 9 & 8 & 7 \\ 10 & 11 & 11 & 10 \\ 8 & 7 & 10 & 9 \\ 10 & 6 & 6 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 115$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 3 & 1 \\ 3 & 4 & 2 \\ 3 & 1 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/2 & -1/2 & 1/2 \\ 3/4 & -1/4 & -1/4 \\ -9/4 & 7/4 & -1/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & 6 & -10 \\ -2 & -9 & 9 \\ -2 & -4 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 7\,\lambda^2 - 2\,\lambda + 40$.

(b) $\lambda_1 = 2, \lambda_2 = -4, \lambda_3 = -5$.

(c) $v_2 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_{-4} = \begin{pmatrix} 1 \\ 1/2 \\ 1/2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ -1/2 \\ 0 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 9 & 11 & 11 \\ 10 & 10 & 8 & 10 \\ 8 & 7 & 6 & 10 \\ 6 & 6 & 6 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = -138$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 2 \\ 2 & 3 & 1 \\ 3 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 11/3 & -2/3 & -5/3 \\ -5/3 & 2/3 & 2/3 \\ -7/3 & 1/3 & 4/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 10 & 5 & -8 \\ 4 & 7 & -4 \\ 4 & 5 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 15\,\lambda^2 - 68\,\lambda + 84$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 2$.

(c) $v_7 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ 4/3 \\ 4/3 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 10 & 9 & 11 \\ 11 & 7 & 7 & 6 \\ 8 & 11 & 8 & 8 \\ 9 & 8 & 8 & 10 \end{pmatrix}$$

Solution:

$$\det(A) = -111$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 3 & 1 & 4 \\ 1 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -14/3 & 10/3 & 1/3 \\ -2/3 & 1/3 & 1/3 \\ 11/3 & -7/3 & -1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -3 & 4 & -2 \\ 4 & -3 & -2 \\ -7 & 8 & 4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 2\,\lambda^2 + 29\,\lambda - 42$.

(b) $\lambda_1 = 3, \lambda_2 = 2, \lambda_3 = -7$.

(c) $v_3 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \\ -1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 7 & 9 & 9 \\ 8 & 6 & 9 & 6 \\ 8 & 9 & 9 & 6 \\ 7 & 11 & 6 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 135$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 1 & 4 \\ 2 & 3 & 1 \\ 3 & 4 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/6 & -5/2 & 11/6 \\ -1/6 & 3/2 & -5/6 \\ 1/6 & 3/2 & -7/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 7 & 6 & 2 \\ -3 & -6 & -3 \\ 2 & 6 & 7 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 8\,\lambda^2 + 3\,\lambda - 90$.

(b) $\lambda_1 = 6, \lambda_2 = 5, \lambda_3 = -3$.

(c) $v_6 = \begin{pmatrix} 1 \\ -1/2 \\ 1 \end{pmatrix}, v_5 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 9 & 7 & 7 \\ 8 & 7 & 9 & 7 \\ 10 & 10 & 9 & 10 \\ 10 & 9 & 10 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -102$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 2 \\ 2 & 3 & 2 \\ 1 & 1 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -5/2 & 3/2 & 1/2 \\ 3/2 & -1/2 & -1/2 \\ 1/4 & -1/4 & 1/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -2 & -4 & -3 \\ -5 & -7 & -5 \\ -10 & 10 & 3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 6\,\lambda^2 + 13\,\lambda + 42$.

(b) $\lambda_1 = 3, \lambda_2 = -2, \lambda_3 = -7$.

(c) $v_3 = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix}, v_{-2} = \begin{pmatrix} 1 \\ 3 \\ -4 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 10 & 7 & 10 \\ 7 & 7 & 8 & 8 \\ 9 & 10 & 10 & 8 \\ 8 & 6 & 8 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 144$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 1 & 3 \\ 2 & 2 & 1 \\ 3 & 4 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/7 & 9/7 & -5/7 \\ -3/7 & -3/7 & 4/7 \\ 2/7 & -5/7 & 2/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 10 & -9 & 3 \\ 5 & -4 & 3 \\ 3 & -5 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 12\,\lambda^2 - 47\,\lambda + 60.$

(b) $\lambda_1 = 5, \lambda_2 = 4, \lambda_3 = 3.$

(c) $v_5 = \begin{pmatrix} 1 \\ 2/3 \\ 1/3 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 2/3 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 7 & 9 & 11 & 8 \\ 6 & 8 & 6 & 11 \\ 6 & 11 & 11 & 11 \\ 10 & 7 & 8 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = 143$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 2 & 3 & 2 \\ 1 & 4 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -2/3 & 8/3 & -5/3 \\ -2/3 & 5/3 & -2/3 \\ 5/3 & -14/3 & 8/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 9 & -5 & 6 \\ 6 & -2 & 10 \\ 3 & -3 & 10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 17\,\lambda^2 - 94\,\lambda + 168$.

(b) $\lambda_1 = 7, \lambda_2 = 6, \lambda_3 = 4$.

(c) $v_7 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix}, v_6 = \begin{pmatrix} 1 \\ -3 \\ -3 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 6 & 6 & 11 \\ 6 & 11 & 11 & 11 \\ 6 & 6 & 9 & 8 \\ 10 & 6 & 10 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -146$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 2 & 4 & 1 \\ 3 & 1 & 3 \\ 3 & 2 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -4/7 & -6/7 & 11/7 \\ 3/7 & 1/7 & -3/7 \\ 3/7 & 8/7 & -10/7 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -8 & 10 & 6 \\ -3 & 5 & 6 \\ 10 & -4 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 6\,\lambda^2 + 37\,\lambda + 210$.

(b) $\lambda_1 = 6, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_6 = \begin{pmatrix} 1 \\ 1 \\ 2/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 3/2 \\ -2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 1 \\ -3/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 8 & 10 & 9 & 10 \\ 6 & 10 & 8 & 6 \\ 11 & 11 & 8 & 8 \\ 11 & 9 & 7 & 11 \end{pmatrix}$$

Solution:

$$\det(A) = -108$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 2 \\ 3 & 2 & 4 \\ 4 & 3 & 4 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 2/3 & 1/3 & -2/3 \\ -2/3 & -4/3 & 5/3 \\ -1/6 & 2/3 & -1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 2 & -9 & -9 \\ 3 & -8 & -3 \\ 3 & -5 & -10 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 16\,\lambda^2 - 83\,\lambda - 140$.

(b) $\lambda_1 = -4, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_{-4} = \begin{pmatrix} 1 \\ 1 \\ -1/3 \end{pmatrix}, v_{-5} = \begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 10 & 7 & 8 & 9 \\ 8 & 11 & 6 & 6 \\ 7 & 11 & 7 & 6 \\ 9 & 11 & 9 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = -128$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 3 \\ 3 & 1 & 1 \\ 2 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/5 & 2/5 & 1/5 \\ -1/5 & -3/5 & 6/5 \\ 4/5 & 2/5 & -9/5 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -6 & -3 & -9 \\ 10 & 7 & 9 \\ 3 & 3 & 6 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 7\,\lambda^2 + 6\,\lambda - 72.$

(b) $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = -3.$

(c) $v_6 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ -1/3 \\ -1 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 10 & 7 & 9 \\ 10 & 8 & 10 & 9 \\ 9 & 9 & 6 & 11 \\ 6 & 6 & 6 & 7 \end{pmatrix}$$

Solution:

$$\det(A) = 114$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 4 & 1 & 3 \\ 1 & 3 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 7/6 & 1/6 & -5/6 \\ 5/6 & -1/6 & -1/6 \\ -11/6 & 1/6 & 7/6 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} -9 & 4 & -2 \\ -2 & -3 & -2 \\ 5 & -5 & -2 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 14\,\lambda^2 - 59\,\lambda - 70$.

(b) $\lambda_1 = -2, \lambda_2 = -5, \lambda_3 = -7$.

(c) $v_{-2} = \begin{pmatrix} 1 \\ 1 \\ -3/2 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 6 & 7 & 6 & 9 \\ 7 & 11 & 7 & 9 \\ 11 & 7 & 9 & 9 \\ 8 & 7 & 8 & 9 \end{pmatrix}$$

Solution:

$$\det(A) = 144$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 4 & 2 & 3 \\ 3 & 2 & 1 \\ 2 & 2 & 2 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 1/3 & 1/3 & -2/3 \\ -2/3 & 1/3 & 5/6 \\ 1/3 & -2/3 & 1/3 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 4 & 9 & -9 \\ -2 & 4 & 2 \\ -2 & 9 & -3 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + 5\,\lambda^2 + 26\,\lambda - 120.$

(b) $\lambda_1 = 6, \lambda_2 = 4, \lambda_3 = -5.$

(c) $v_6 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_4 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-5} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}.$

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 11 & 8 & 8 & 9 \\ 7 & 9 & 8 & 11 \\ 11 & 7 & 9 & 10 \\ 9 & 6 & 6 & 8 \end{pmatrix}$$

Solution:

$$\det(A) = 117$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 3 & 4 & 1 \\ 4 & 3 & 1 \\ 1 & 2 & 1 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} -1/4 & 1/2 & -1/4 \\ 3/4 & -1/2 & -1/4 \\ -5/4 & 1/2 & 7/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & 2 & 6 \\ -3 & -4 & -3 \\ 7 & 5 & -4 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.

(b) Find the eigenvalues for A.

(c) Find the eigenvectors for A.

(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 - 5\,\lambda^2 + 29\,\lambda + 105$.

(b) $\lambda_1 = 5, \lambda_2 = -3, \lambda_3 = -7$.

(c) $v_5 = \begin{pmatrix} 1 \\ -1/2 \\ 1/2 \end{pmatrix}, v_{-3} = \begin{pmatrix} 1 \\ -3/2 \\ -1/2 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ -1/2 \\ -3/2 \end{pmatrix}$.

(d) Yes.

Linear Algebra - Assignment 1
Prof. Roberto Panai

1. Calculate the determinant of A, where

$$A = \begin{pmatrix} 9 & 7 & 10 & 9 \\ 7 & 6 & 8 & 8 \\ 6 & 10 & 6 & 8 \\ 8 & 7 & 11 & 6 \end{pmatrix}$$

Solution:

$$\det(A) = 116$$

2. Find the inverse of A, if it exists, using the Gauss-Jordan elimination. Where

$$A = \begin{pmatrix} 1 & 1 & 2 \\ 3 & 4 & 3 \\ 1 & 2 & 3 \end{pmatrix}$$

Solution:

$$A^{-1} = \begin{pmatrix} 3/2 & 1/4 & -5/4 \\ -3/2 & 1/4 & 3/4 \\ 1/2 & -1/4 & 1/4 \end{pmatrix}$$

3. Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ a linear map and

$$A = \begin{pmatrix} 3 & 10 & -10 \\ -2 & 3 & 2 \\ -2 & 10 & -5 \end{pmatrix}$$

the matrix of T with respect to the canonical basis.

(a) Calculate $\det(A - \lambda I)$.
(b) Find the eigenvalues for A.
(c) Find the eigenvectors for A.
(d) Is A diagonalizable?

Solution:

(a) $\det(A - \lambda I) = -\lambda^3 + \lambda^2 + 41\,\lambda - 105$.

(b) $\lambda_1 = 5, \lambda_2 = 3, \lambda_3 = -7$.

(c) $v_5 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, v_{-7} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$.

(d) Yes.