FACULTY OF MECHANICAL ENGINEERING

institute of automation and computer science

Cybernetics and Robotics Division

NETME Centre

FACULTY OF MECHANICAL ENGINEERING institute of automation and computer science

Programming for robots and manipulators
Lecture 5

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IACS

INSTITUTE OF AUTOMATION AND COMPUTER SCIENCE

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2. Differential Kinematics
3. Dynamics
4. Robotic Control
5. Why do we need knowledge of Kinematics and Dynamics?

Introduction

Robotic Control

Kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation.

Differential Kinematics describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities.

Robot dynamics is concerned with the relationship between the forces acting on a robot mechanism and the accelerations they produce.

Typically, the robot mechanism is modelled as a rigid-body system, in which case robot dynamics is the application of rigid-body dynamics to robots.

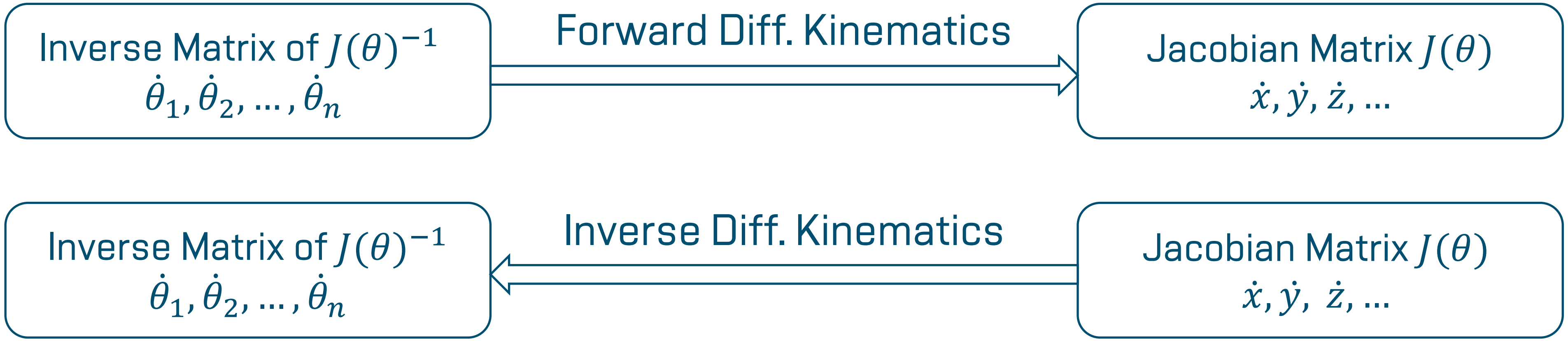
The two main problems in robot dynamics are:
A) Forward dynamics – Given the forces, work out the accelerations.
B) Inverse dynamics – Given the accelerations, work out the forces.

Differential Kinematics

Differential Kinematics

Differential kinematics relates the velocities of the manipulator components. These velocities may be the velocities at the joints of the manipulator or the velocities of one or more links in the manipulator kinematic chain.

The standard approach to differential kinematics is to relate joint and end-effector velocities through the **Jacobian matrix**, which allows the calculation of the end-effector velocities given the joint velocities (Forward differential kinematics) or, to determine the joint velocities in order to move the end-effector with a prescribed speed (Inverse differential kinematics).



Inverse Matrix of $J(\theta)^{-1}$
$\dot{\theta}_1, \dot{\theta}_2, \dots, \dot{\theta}_n$

Forward Diff. Kinematics

Jacobian Matrix $J(\theta)$
$\dot{x}, \dot{y}, \dot{z}, \dots$

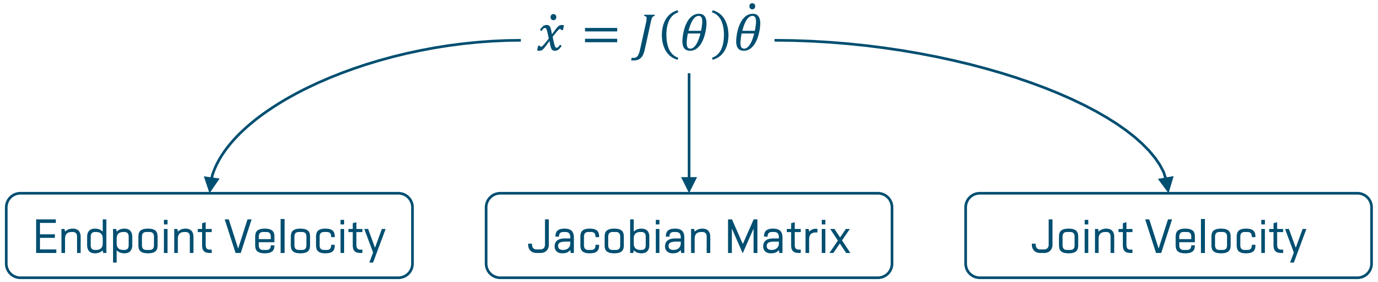
Inverse Matrix of $J(\theta)^{-1}$
$\dot{\theta}_1, \dot{\theta}_2, \dots, \dot{\theta}_n$

Inverse Diff. Kinematics

Jacobian Matrix $J(\theta)$
$\dot{x}, \dot{y}, \ \dot{z}, \dots$

Differential Kinematics

Jacobian is a multidimensional form of the derivative, in robotics, Jacobian is a time-varying linear transformations, which determines the relationship between joint velocity (input) and end-effector velocity (output).



$$\dot{x} = J(\theta)\dot{\theta}$$

Endpoint Velocity

Jacobian Matrix

Joint Velocity

Most manipulator have values of θ where the Jacobian becomes singular. If the Jacobian matrix is non singular, we can calculate "Inverse Jacobian matrix".

$$\dot{\theta} = J(\theta)^{-1}\dot{x}$$

The Jacobian is singular when its determinant is equal to 0 → $\det(J) = 0$.

Differential Kinematics

The Jacobian, J, is defined as:

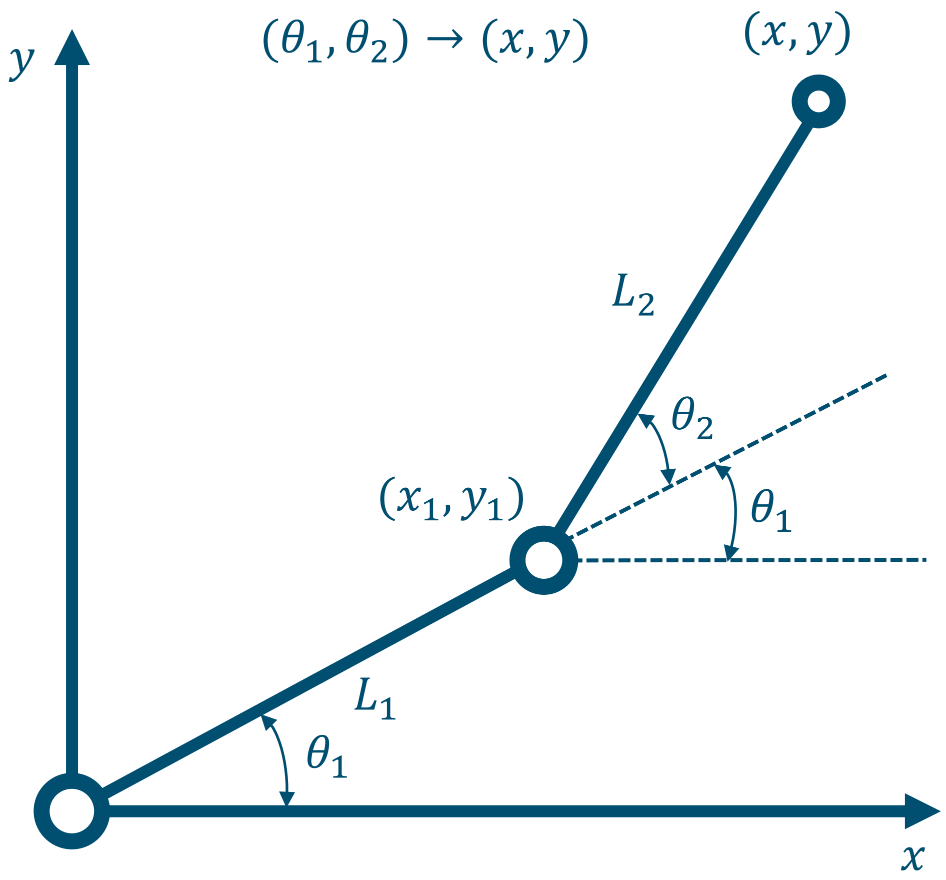
$$J(\theta) = \begin{bmatrix} \frac{\partial x_1}{\partial \theta_1} & \cdots & \frac{\partial x_1}{\partial \theta_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_m}{\partial \theta_1} & \cdots & \frac{\partial x_m}{\partial \theta_n} \end{bmatrix}$$

where is the m × n analytical Jacobian matrix.

Jacobians are useful for planning and executing smooth trajectories, determining singular configurations, executing coordinated anthropomorphic motion, deriving dynamic equations of motion, and transforming forces and torques from the end-effector to the manipulator joints.

Example

Example: Differential Kinematics



Forward Kinematics

$$x = L_1 cos\theta_1 + L_2 cos(\theta_1 + \theta_2)$$
$$y = L_1 sin\theta_1 + L_2 sin(\theta_1 + \theta_2)$$

Jacobian Matrix

$$\begin{bmatrix} p\dot{x} \\ p\dot{y} \end{bmatrix} = J(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Example: Differential Kinematics

Jacobian Matrix

$$\begin{bmatrix} p\dot{x} \\ p\dot{y} \end{bmatrix} = J(\theta_1, \theta_2) \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} =$$

$$= \begin{bmatrix} -L_1 sin\theta_1 - L_2 sin(\theta_1 + \theta_2) & -L_2 sin(\theta_1 + \theta_2) \\ L_1 cos\theta_1 + L_2 cos(\theta_1 + \theta_2) & L_2 cos(\theta_1 + \theta_2) \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Singular Jacobian

$$\det\left(\begin{bmatrix} -L_1 sin\theta_1 - L_2 \sin(\theta_1 + \theta_2) & -L_2 \sin(\theta_1 + \theta_2) \\ L_1 cos\theta_1 + L_2 \cos(\theta_1 + \theta_2) & L_2 \cos(\theta_1 + \theta_2) \end{bmatrix}\right) = 0$$

Dynamics

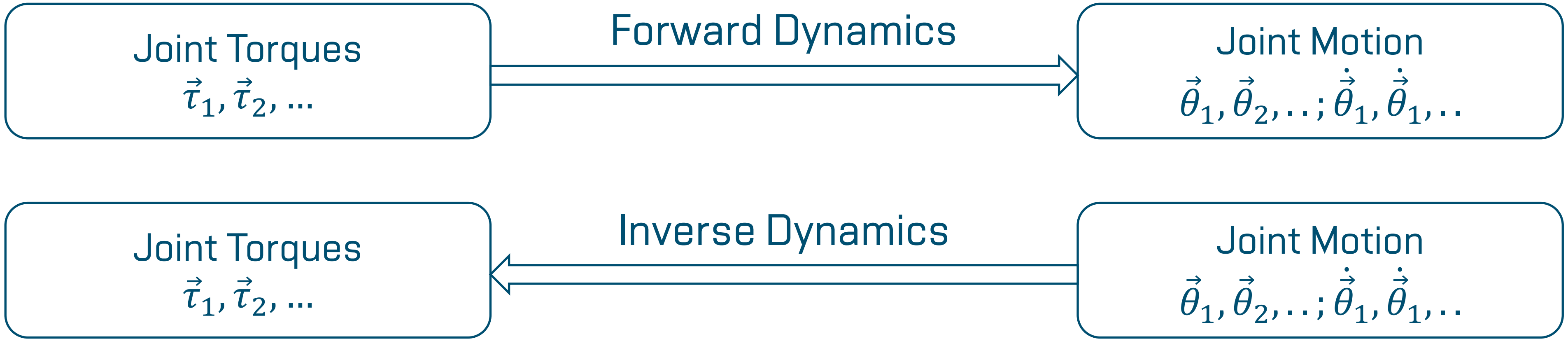
Robotic Dynamics

The dynamics of a system describes how the controls θ_t influence the change-of-state of the system:

$$x_{t+1} = f(x_t, \theta_t)$$

The notation x_t refers to the dynamic state of the system: e.g., joint positions and velocities $x_t = (\theta_t, \dot{\theta}_t)$. Function f is an arbitrary function, often smooth.

Describes the relationship between Forces ($\vec{f}_n$)/Torques ($\vec{\tau}_n$):



Joint Torques
$\vec{\tau}_1, \vec{\tau}_2, \ldots$

Forward Dynamics

Joint Motion
$\vec{\theta}_1, \vec{\theta}_2, ..; \dot{\vec{\theta}}_1, \dot{\vec{\theta}}_1, ..$

Joint Torques
$\vec{\tau}_1, \vec{\tau}_2, \ldots$

Inverse Dynamics

Joint Motion
$\vec{\theta}_1, \vec{\theta}_2, ..; \dot{\vec{\theta}}_1, \dot{\vec{\theta}}_1, ..$

Robotic Dynamics

For many applications with fixed-based robots we need to find a multi-body dynamics formulated as:

$$M(\theta)\ddot{\theta} + b\left(\theta, \dot{\theta}\right) + g(\theta) = \tau + J_c(\theta)^T F_C$$

$M(\theta)$ → Generalized mass matrix (orthogonal).

$\theta, \dot{\theta}, \ddot{\theta}$ → Generalized position, velocity and acceleration vectors.

$b(\theta, \dot{\theta})$ → Coriolis and centrifugal terms.

$g(\theta)$ → Gravitational terms.

τ → External generalized forces.

$J_c(\theta)$ → Geometric Jacobian corresponding to the external forces.

F_C → External Cartesian forces (e.g. from contacts).

Robotic Dynamics

There are typically two ways to derive the equation of motion for an open-chain robot:

Lagrangian Method
Energy-based method.
Dynamic equations in closed form.
Often used for study of dynamic properties and analysis of control methods.

Newton-Euler Method
Balance of forces/torques.
Dynamic equations in numeric/recursive form.
Often used for numerical solution of forward/inverse dynamics.

Many other formal methods based on basic principles in mechanics are available for the derivation of the robot dynamic model:
Principle of d'Alembert, of Hamilton, of virtual works, etc.

Lagrangian Method

Lagrange Method is one of the analytical methods used to describe the motion of physical systems (closely tied to both the d′Alembert and Hamilton principles).

The method is centered around three fundamental concepts:

1. The definition of generalized coordinates θ and generalized velocities $\dot{\theta}$, which may or may not encode the information regarding the constraints applicable to the system.

2. A scalar function called the Lagrangian function $\mathcal{L}$. For mechanical systems, itis exactly the difference between the total Kinetic energy T and the total Potential energy U, of the system at each instant:

$$\mathcal{L} = T - U$$

Lagrangian Method

3. The so-called Euler-Lagrange equation, also known as the Euler-Lagrange of the second kind, which applies to the Lagrangian function $\mathcal{L}$ and to the total external generalized forces τ:

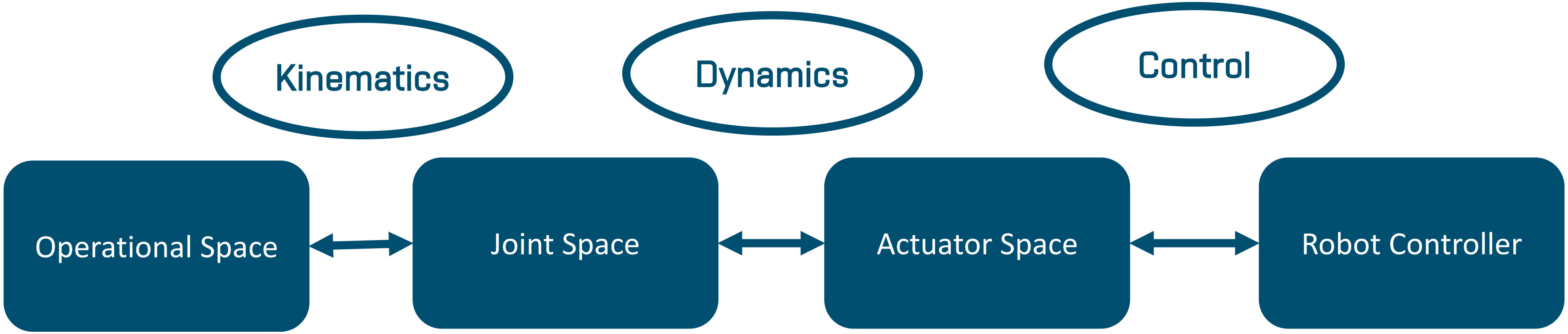
$$\frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{\theta}}\right)-\left(\frac{\partial\mathcal{L}}{\partial\theta}\right)=\tau$$

In the most general case, the Lagrangian is a function of the generalized coordinates and velocities θ and $\dot{\theta}$, and it may also have an explicit dependence on time t, hence we redefine the aforementioned scalar energy functions as:

$$T=T(t,\theta,\dot{\theta})$$
$$U=U(t,\theta)$$
$$\mathcal{L}=\mathcal{L}(t,\theta,\dot{\theta})$$

Robotic Control

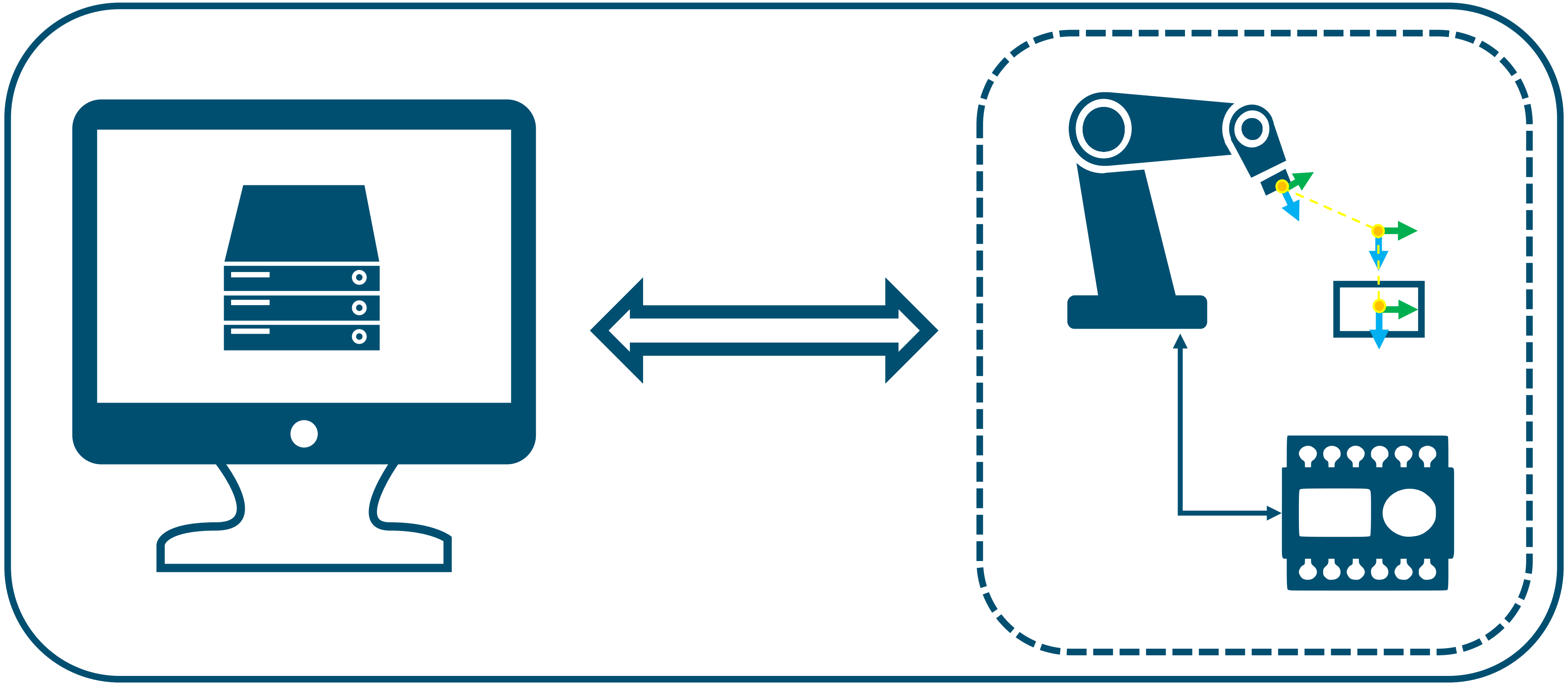
Robotic Control



Why do we need knowledge of Kinematics and Dynamics?

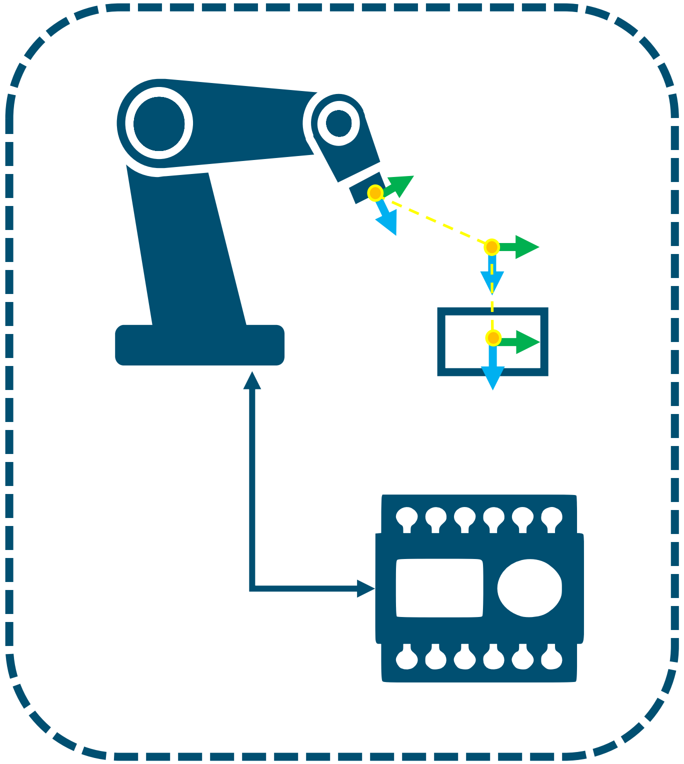
Design and Implementation of a Robotic Application

Budget: $$$$$
Saved: ?



Design and Implementation of a Robotic Application

Budget: $$$$$
Saved: ?



Robotic Construction

Controller

Additional Hardware

?

Design and Implementation of a Robotic Application

Budget: $$$$$
Saved: ?

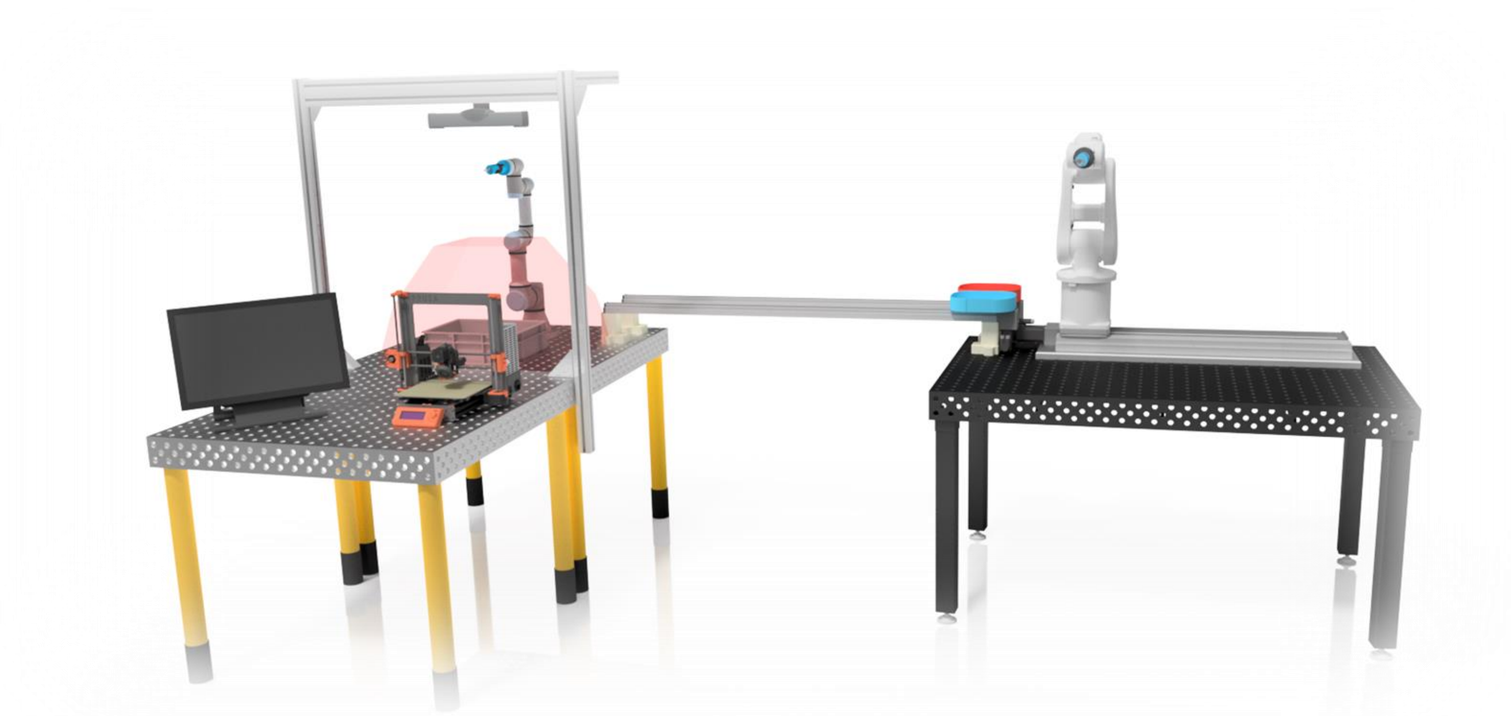
Simulation Software

Control Software

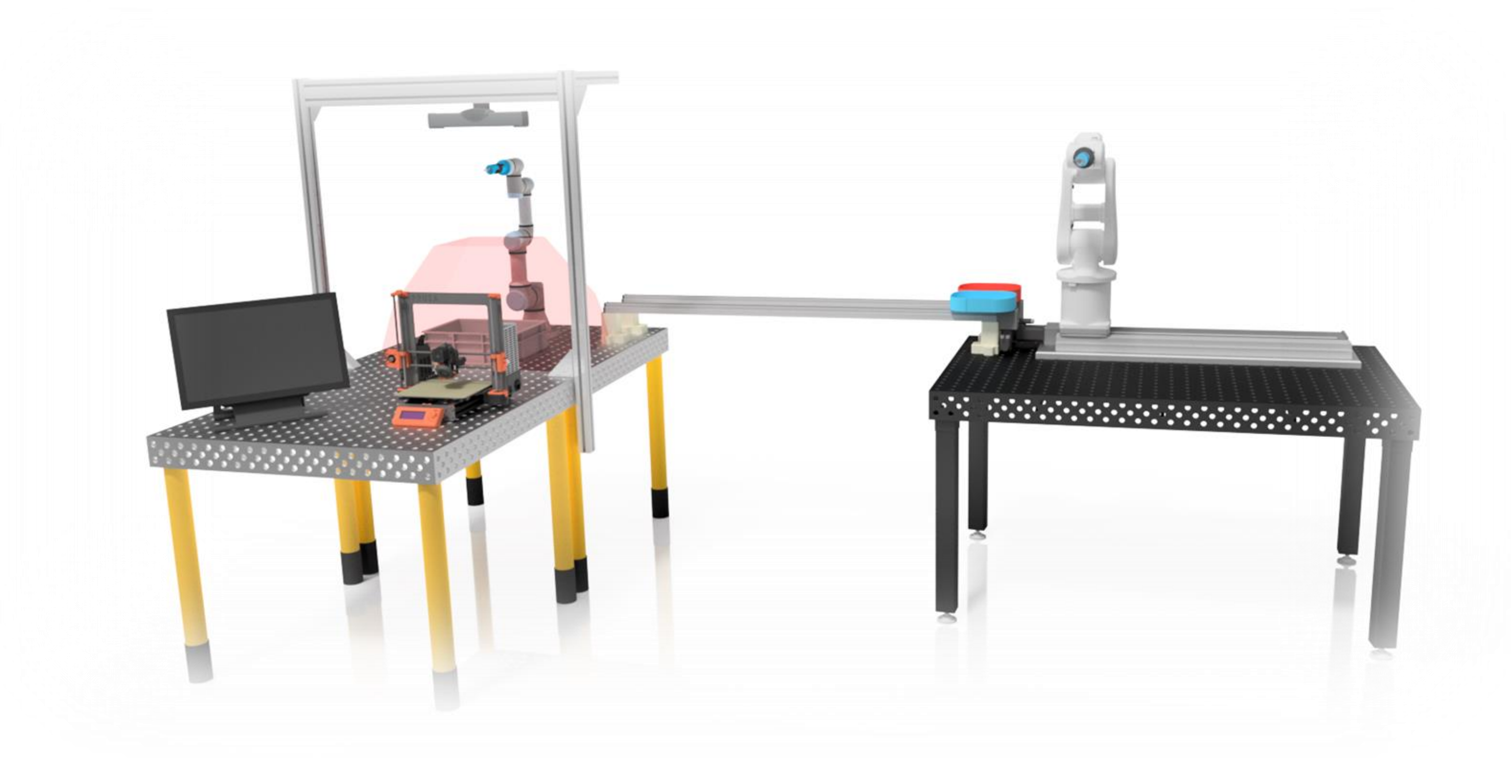
Additional Libraries

?

Thank You!



Questions?



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