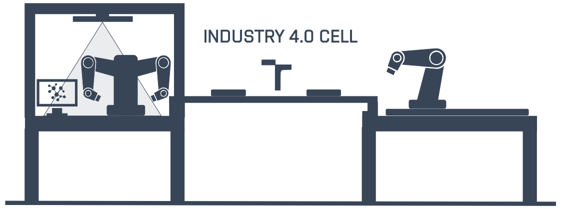
INSTITUTE OF AUTOMATION AND COMPUTER SCIENCE

Programming for robots and manipulators
Lecture 4

Roman Parak

NETME Centre



FACULTY OF MECHANICAL ENGINEERING institute of automation and computer science

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Content

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Introduction

Brief Methodology of Robotics Control

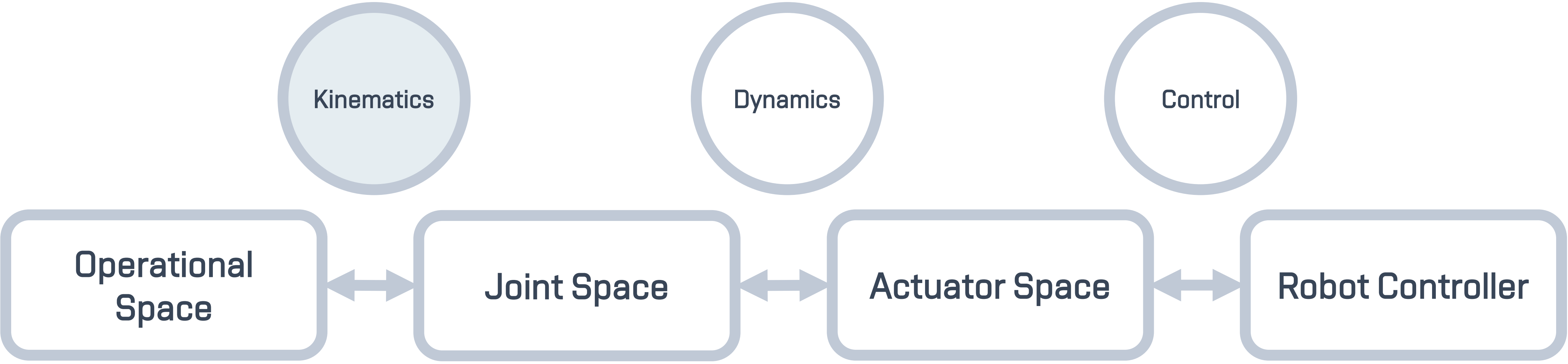
Kinematics analyzes the geometry of a motion analytically, e.g. of a robot:

- With respect to a fixed reference co-ordinate system
- Without regard to the forces or moments that cause the motion.
- Essential concepts are position and orientation (Euler Angles, Quaternions).

Kinematics describes the analytical relationship between the joint positions and the end-effector position and orientation.

Differential kinematics describes the analytical relationship between the joint motion and the end-effector motion in terms of velocities.

Brief Methodology of Robotics Control



Forward (Direct) Kinematics
Joint relations (rotations, translations) for the robot arm.

Task: What is the orientation and position of the end effector?

$$\theta_1, \theta_2, \dots, \theta_n \xrightarrow{\boldsymbol{H} = f(\boldsymbol{q})} TCP \begin{cases} x, y, z,\ R_x, R_y, R_z \\ x, y, z,\ q_x, q_y, q_z, q_w \end{cases}$$

Inverse kinematics
The desired end effector position and orientation.

Task: What are the joint rotations and orientations to achieve this?

$$\theta_1, \theta_2, \dots, \theta_n \xleftarrow{\boldsymbol{q} = f^{-1}(\boldsymbol{H})} TCP \begin{cases} x, y, z,\ R_x, R_y, R_z \\ x, y, z,\ q_x, q_y, q_z, q_w \end{cases}$$

Forward Kinematics

Forward Kinematics

The forward kinematics problem is concerned with the relationship between the individual joints of the robot manipulator and the position and orientation of the tool or end-effector.

Stated more formally, the forward kinematics problem is to determine the position and orientation of the end-effector, given the values for the joint variables of the robot.

The joint variables are the angles between the links in the case of revolute or rotational joints, and the link extension in the case of prismatic or sliding joints.

$$\theta_1, \theta_2, \dots, \theta_n \xrightarrow{\boldsymbol{H} = f(\boldsymbol{q})} TCP \begin{cases} x, y, z,\ R_x, R_y, R_z \\ x, y, z,\ q_x, q_y, q_z, q_w \end{cases}$$

The Denavit–Hartenberg parameters are the four parameters associated with a particular convention for attaching reference frames to the links of a spatial kinematic chain, or robot manipulator.

Link Length a

The length of the mutually perpendicular line, denoted by the scalar a_{i-1}, is called the link length of link $i-1$. Despite its name, this link length does not necessarily correspond to the actual length of the physical link.

Link Twist α

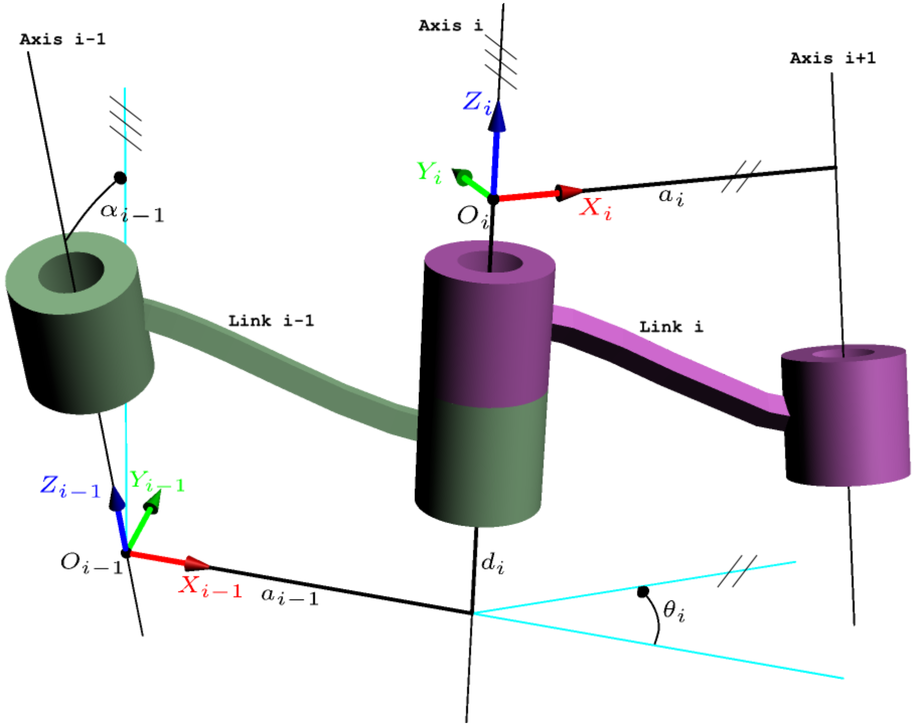
The link twist α_{i-1} is the angle from $\hat{z}_{i-1}$ to $\hat{z}_i$, measured about $\hat{x}_{i-1}$.

Link Offset d

The link offset d_i is the distance from the intersection of $\hat{x}_{i-1}$ and $\hat{z}_i$ to the origin of the link – i frame (the positive direction is defined to be along the $\hat{z}_i$ – axis).

Joint Angle θ

The joint angle θ_i is the angle from $\hat{x}_{i-1}$ to $\hat{x}_i$, measured about the $\hat{z}_i$ – axis.



DH Representation

In this convention, each homogeneous transformation $\boldsymbol{H_i}$ is represented as a product of four basic transformations.

$$H_i = R_{z,\theta_i} Trans_{z,d_i} Trans_{x,a_i} R_{x,\alpha_i}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i & 0 & 0 \\ s\theta_i & c\theta_i & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c\alpha_i & -s\alpha_i & 0 \\ 0 & s\alpha_i & c\alpha_i & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} c\theta_i & -s\theta_i c\alpha_i & s\theta_i s\alpha_i & a_i c\theta_i \\ s\theta_i & c\theta_i c\alpha_i & -c\theta_i s\alpha_i & a_i s\theta_i \\ 0 & s\alpha_i & c\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Translation Part

Rotation Part

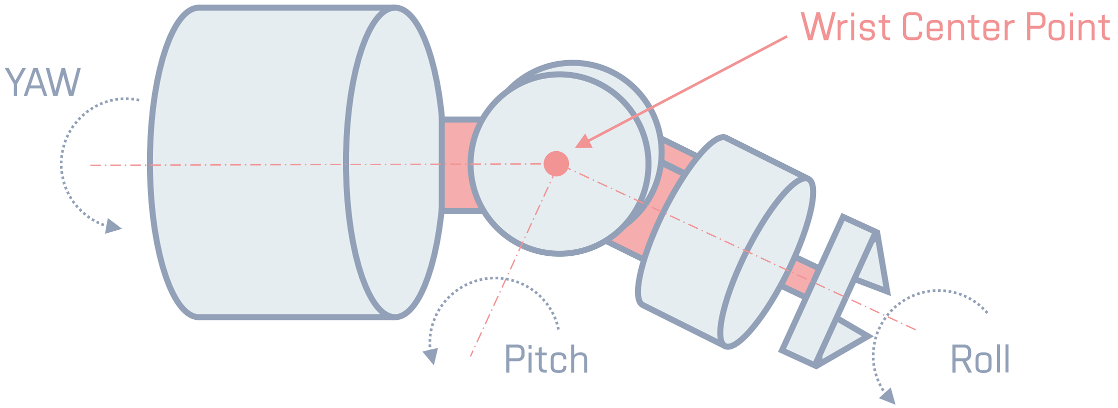
Inverse Kinematics

Inverse kinematics deals with the problem of finding the required joint angles to produce a certain desired position and orientation of the end-effector. Finding the inverse kinematics solution for a general manipulator can be a very tricky task (Generally, they are non-linear equations).

Close-form solutions may not be possible and multiple, infinity, or impossible solutions can arise. Nevertheless, special cases have a closed-form solution and can be solved.

$$\boxed{\theta_1, \theta_2, \ldots, \theta_n} \xleftarrow{\boldsymbol{q} = f^{-1}(\boldsymbol{H})} \boxed{TCP \begin{cases} x, y, z, \; R_x, R_y, R_z \\ x, y, z, \; q_x, q_y, q_z, q_w \end{cases}}$$

The sufficient condition for solving a six-axis manipulator is that it must have three consecutive revolute axes that intersect at a common point: Pieper condition.



Methods for Solving the Inverse Kinematics Problem

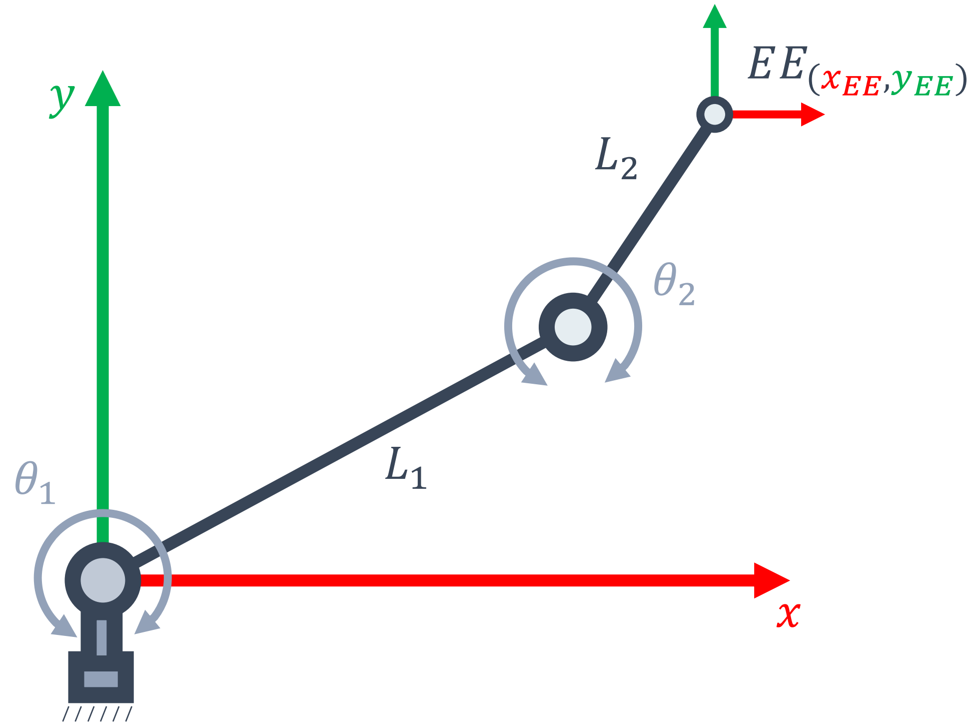
Analytical methods (Closed-form solutions)

- Algebraic methods.
- Geometric methods.

Numerical methods

- Symbolic elimination methods: involve analytical manipulations to eliminate variables from a system of nonlinear equations to reduce it to a smaller
- Continuation methods: involve tracking a solution path from a start system with known solutions to a target system.
- Iterative methods: are in general based on Newton-Raphson method for finding roots using 1st order approximation of the original algebraic equation. They converge in a single solution (from several possible) based on the initial guess.

Example



Forward Kinematics

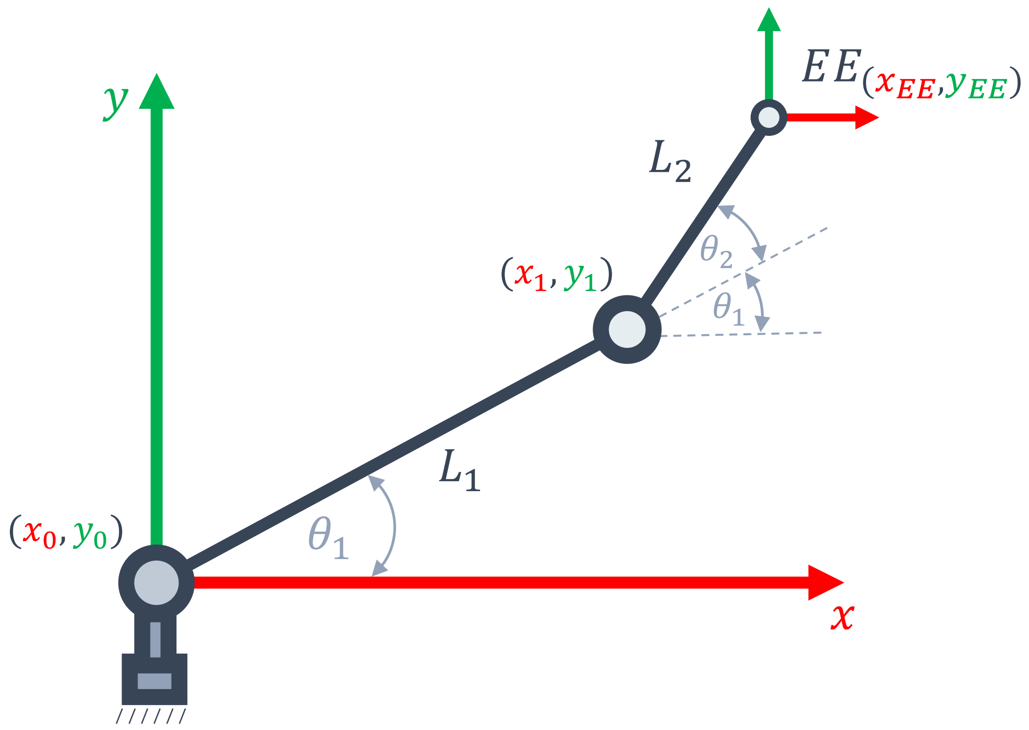
$(\theta_1, \theta_2) \rightarrow (x, y)$

Inverse Kinematics

$(\theta_1, \theta_2) \leftarrow (x, y)$

Forward Kinematics

$(\theta_1, \theta_2) \rightarrow (x_{EE}, y_{EE})$

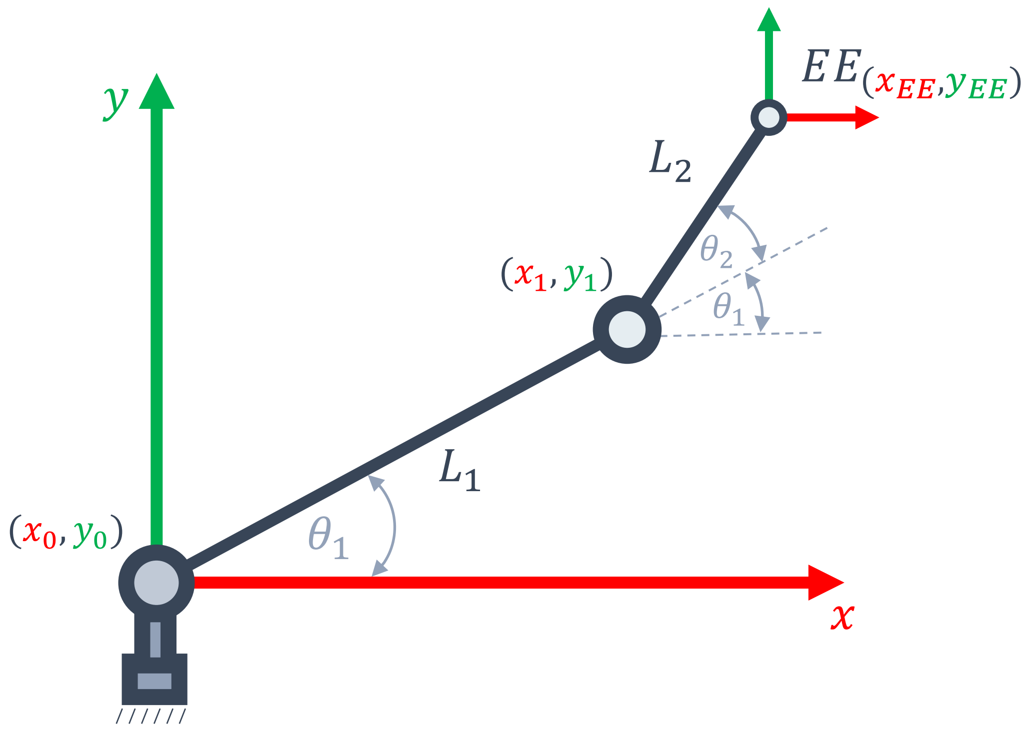


a) DH Parameters – Table?

b) Position of the x_1, y_1 and x, y?

Forward Kinematics

$(\theta_1, \theta_2) \rightarrow (x_{EE}, y_{EE})$

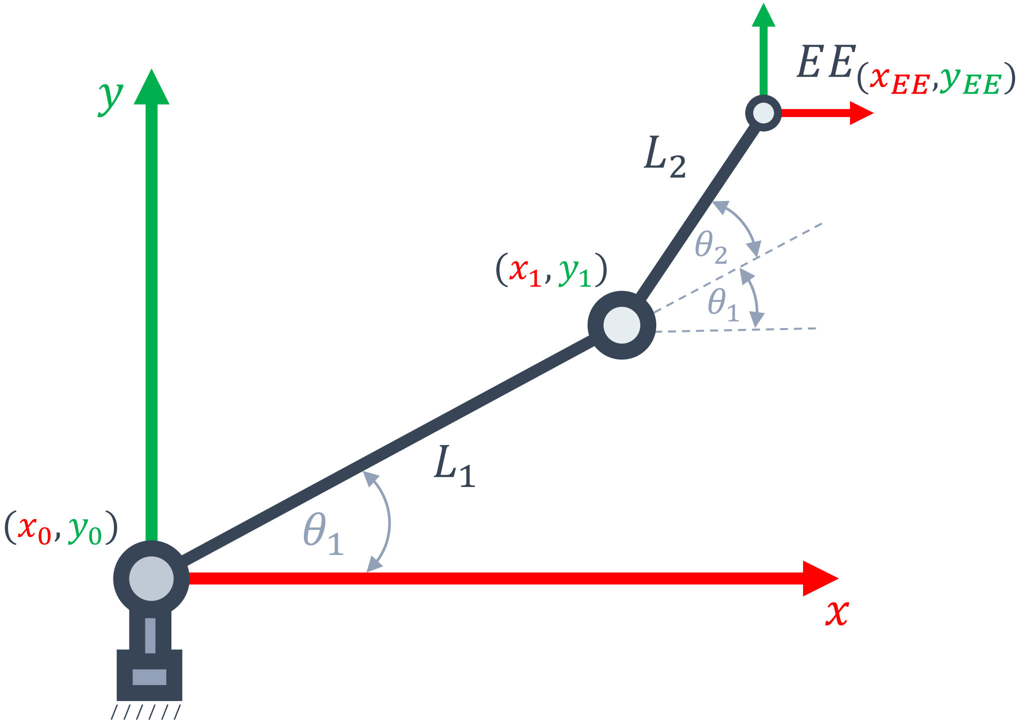


a) DH Parameters – Table?

Link	a_i	α_i	d_i	θ_i
1	$a_1 = L_1$	0	0	θ_1
2	$a_2 = L_2$	0	0	θ_2

Forward Kinematics

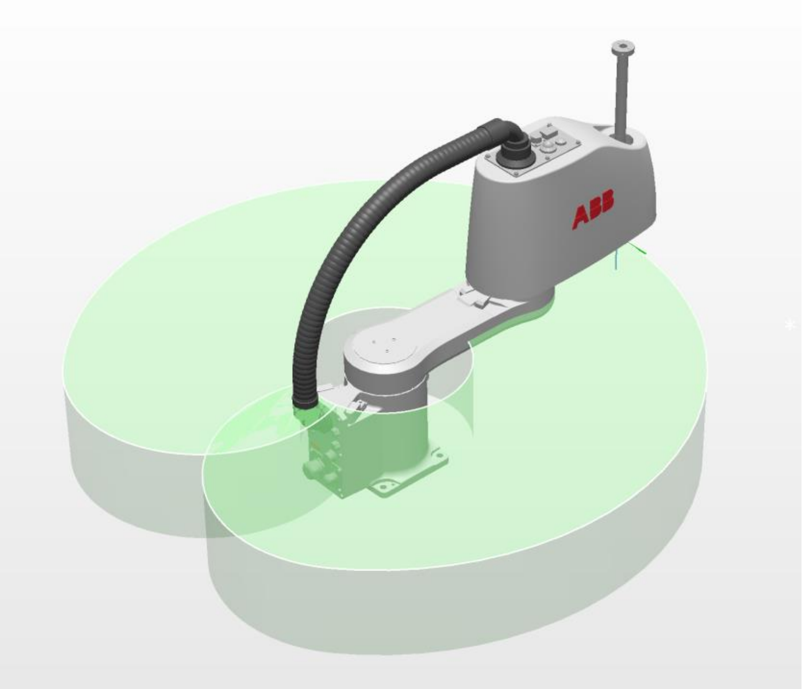
$(\theta_1, \theta_2) \rightarrow (x_{EE}, y_{EE})$



b) Position of the x_1, y_1 and x, y?

$$x_1 = L_1 cos\theta_1$$
$$y_1 = L_1 sin\theta_1$$

$$x = L_1 cos\theta_1 + L_2 cos(\theta_1 + \theta_2)$$
$$y = L_1 sin\theta_1 + L_2 sin(\theta_1 + \theta_2)$$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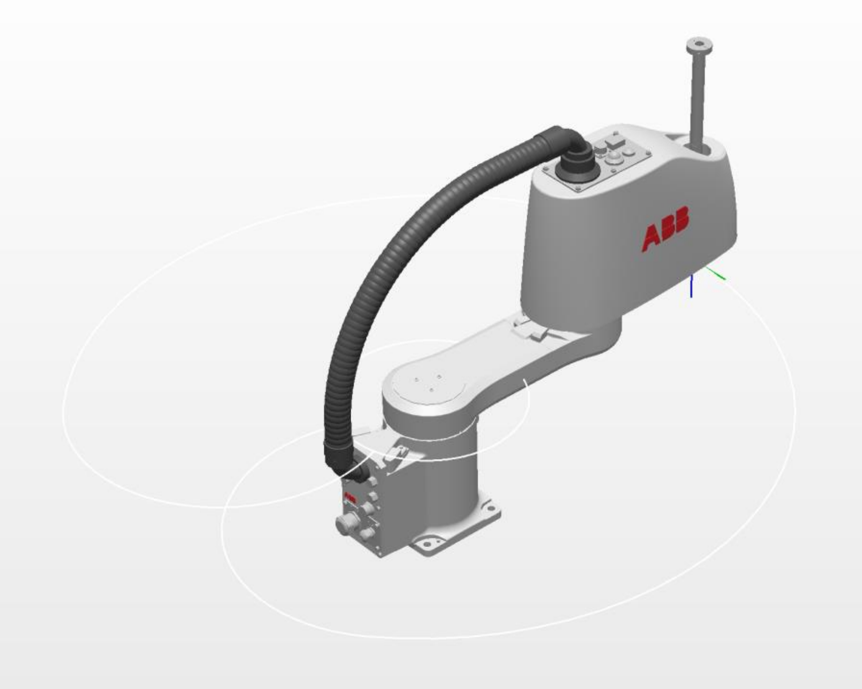
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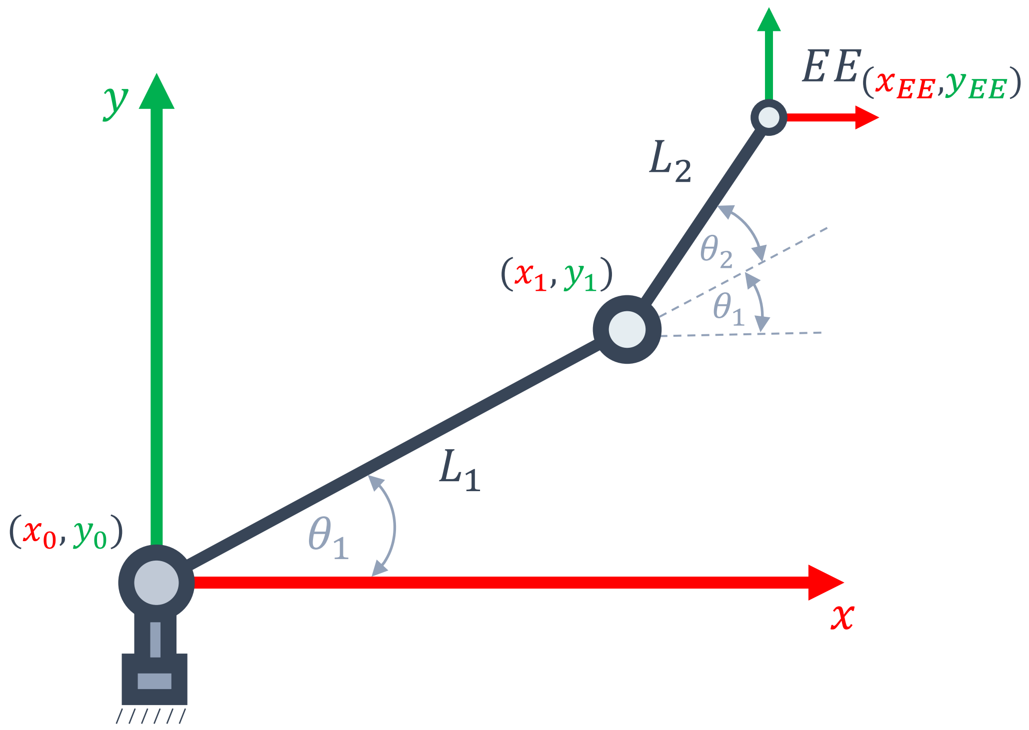
Inverse Kinematics

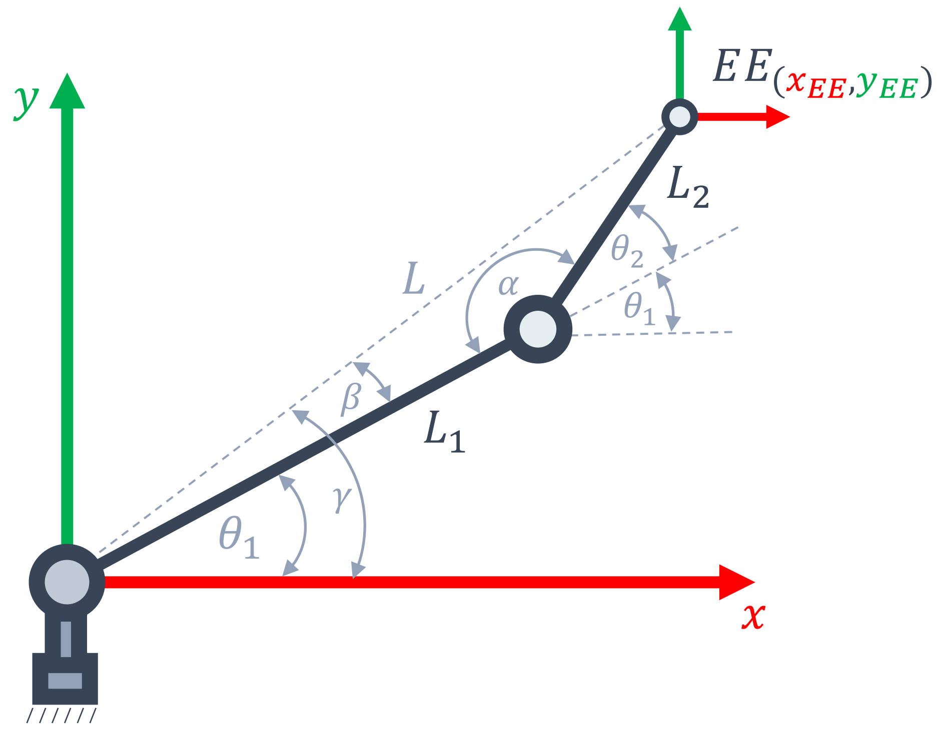
$(\theta_1, \theta_2) \leftarrow (x_{EE}, y_{EE})$



a) Rotation of the θ_1, θ_2?

Inverse Kinematics

$(\theta_1, \theta_2) \leftarrow (x_{EE}, y_{EE})$



$$R_{2DOF} \begin{cases} \theta_1 = \gamma - \beta \\ \theta_2 = \pi - \alpha \end{cases}$$

Law of cosines (Cosine Theorem)

$$L = \sqrt{x^2 + y^2}$$

$$L^2 = L_1^2 + L_2^2 - 2L_1L_2cos\alpha$$

$$L_2^2 = L_1^2 + L^2 - 2L_1Lcos\beta$$

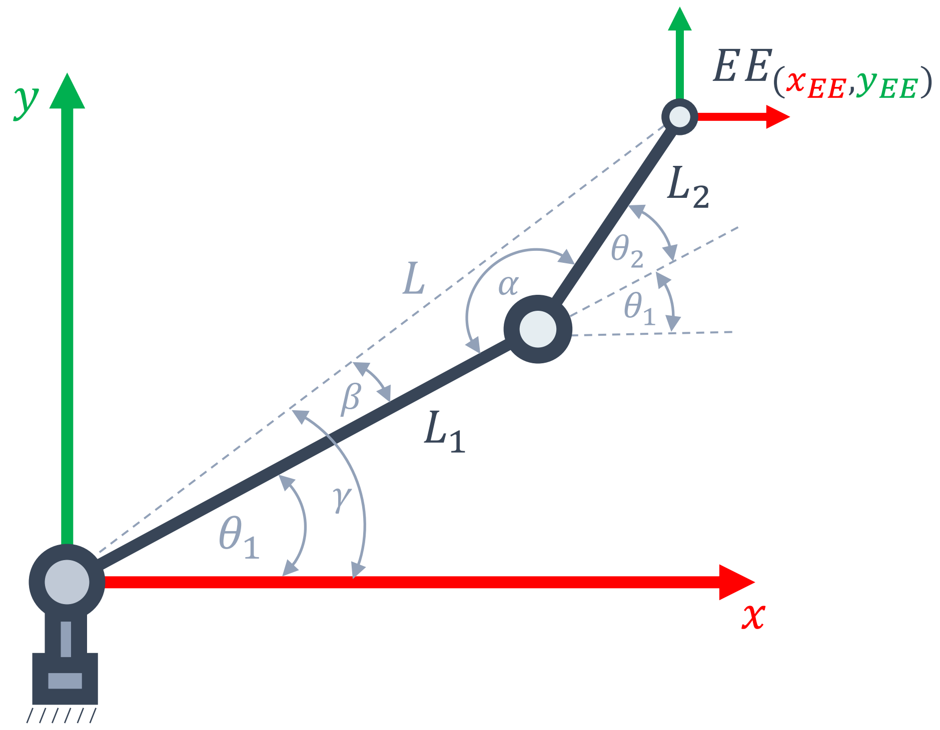
$$cos\alpha = \frac{L_1^2 + L_2^2 - L^2}{2L_1L_2}$$

$$cos\beta = \frac{L_1^2 + L^2 - L_2^2}{2L_1L}$$

$$tan\gamma = \frac{y}{x}$$

Inverse Kinematics

$(\theta_1, \theta_2) \leftarrow (x_{EE}, y_{EE})$



$$R_{2DOF}\begin{cases}\theta_1 = \gamma - \beta \\ \theta_2 = \pi - \alpha\end{cases}$$

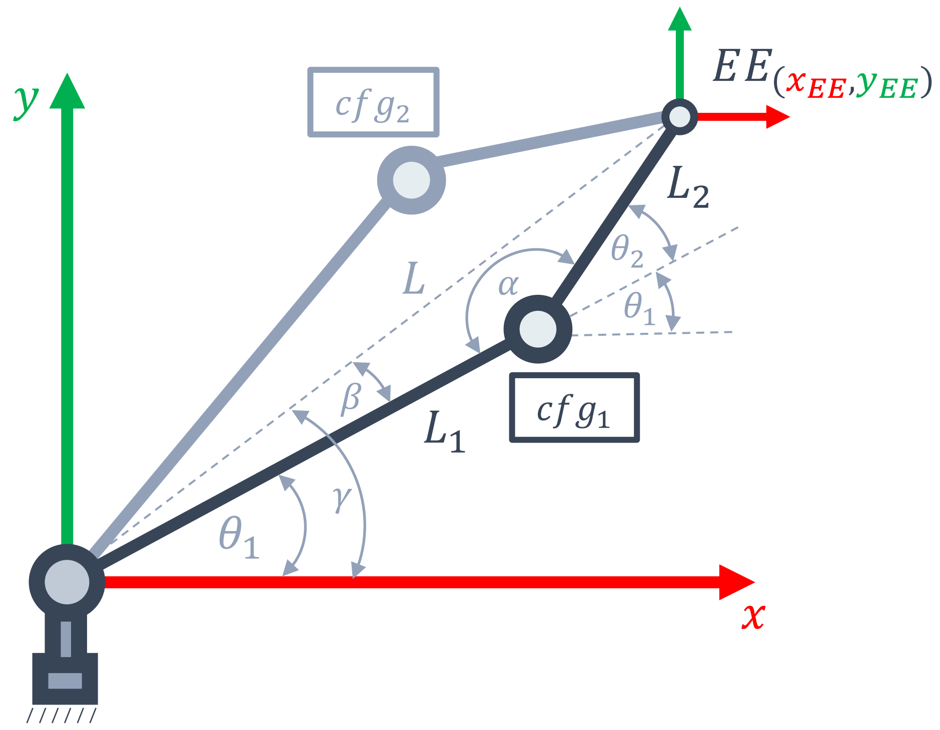
Inverse Trigonometric Functions

$$\theta_1 = arctan\frac{y}{x} - \arccos\frac{L_1^2 + L^2 - L_2^2}{2L_1L}$$

$$\theta_2 = \pi - \arccos\frac{L_1^2 + L_2^2 - L^2}{2L_1L_2}$$

Inverse Kinematics

$(\theta_1, \theta_2) \leftarrow (x_{EE}, y_{EE})$



$$cfg_1 \begin{cases} \theta_1 = \gamma - \beta \\ \theta_2 = \pi - \alpha \end{cases}$$

$$cfg_2 \begin{cases} \theta_1 = \gamma + \beta \\ \theta_2 = \alpha - \pi \end{cases}$$

Thank You!

Questions?

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