and Twitter cats

# Applied mathematics with a quantum computer

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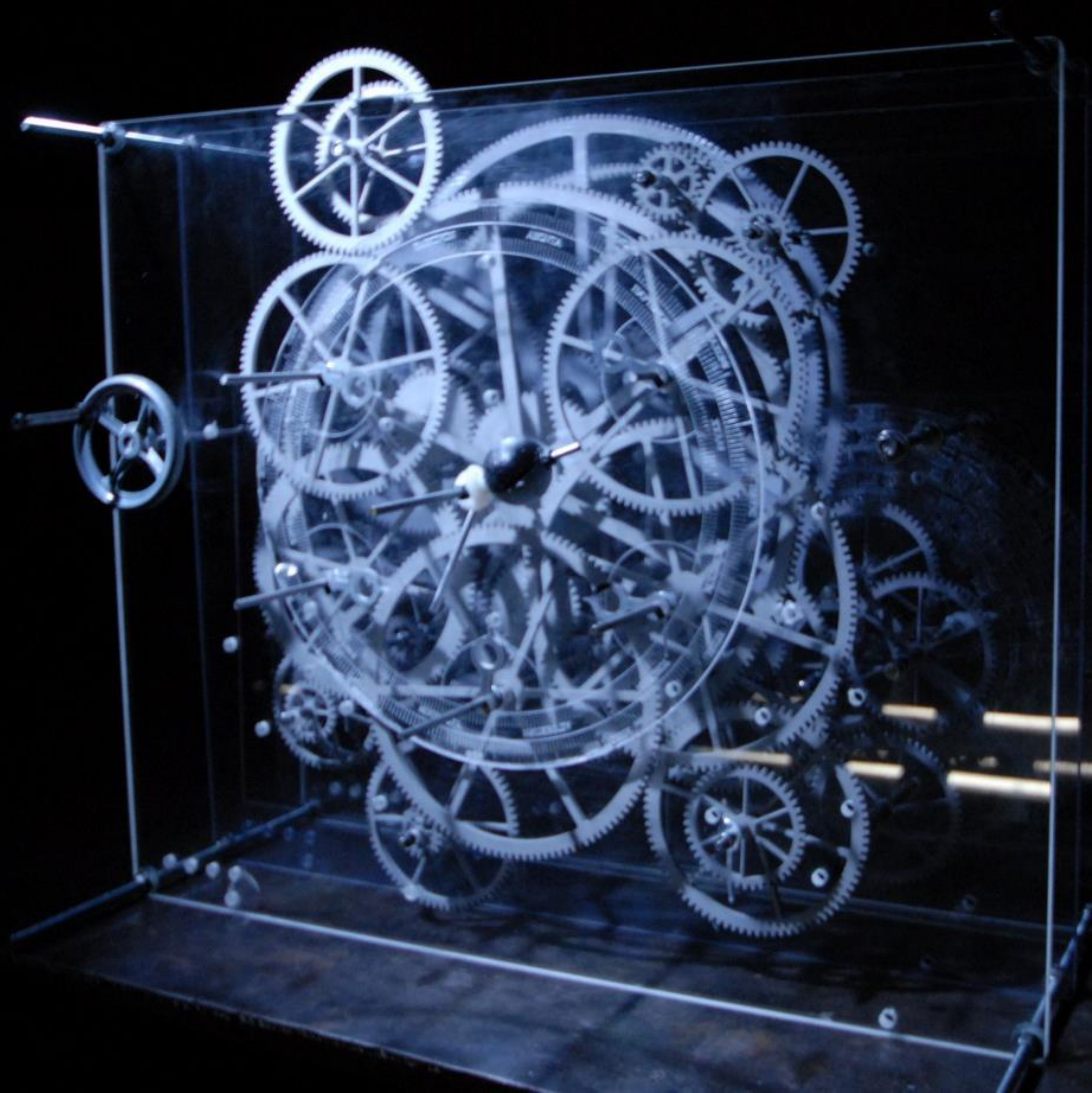
**Antikythera Mechanism**
**200-100 BC**



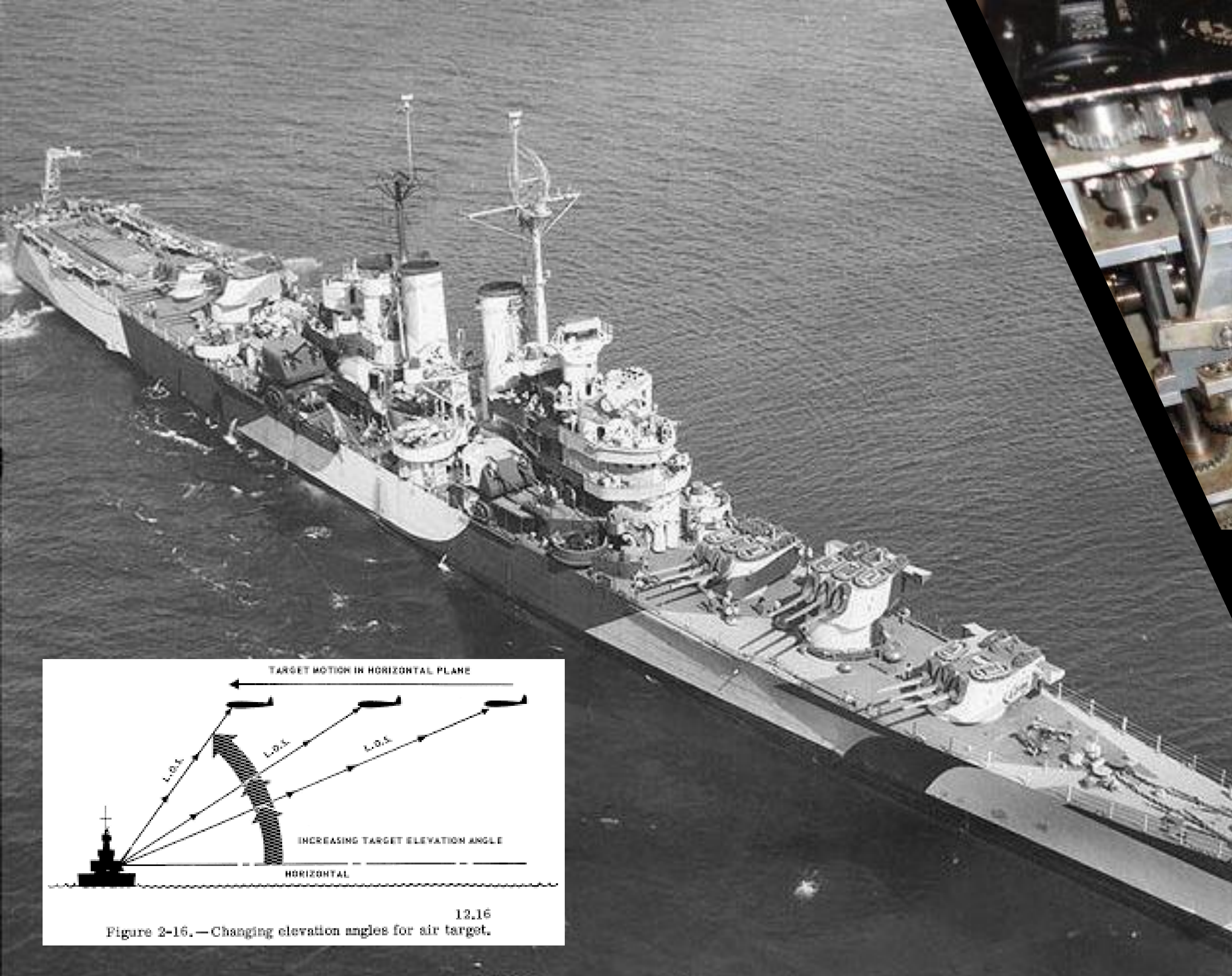
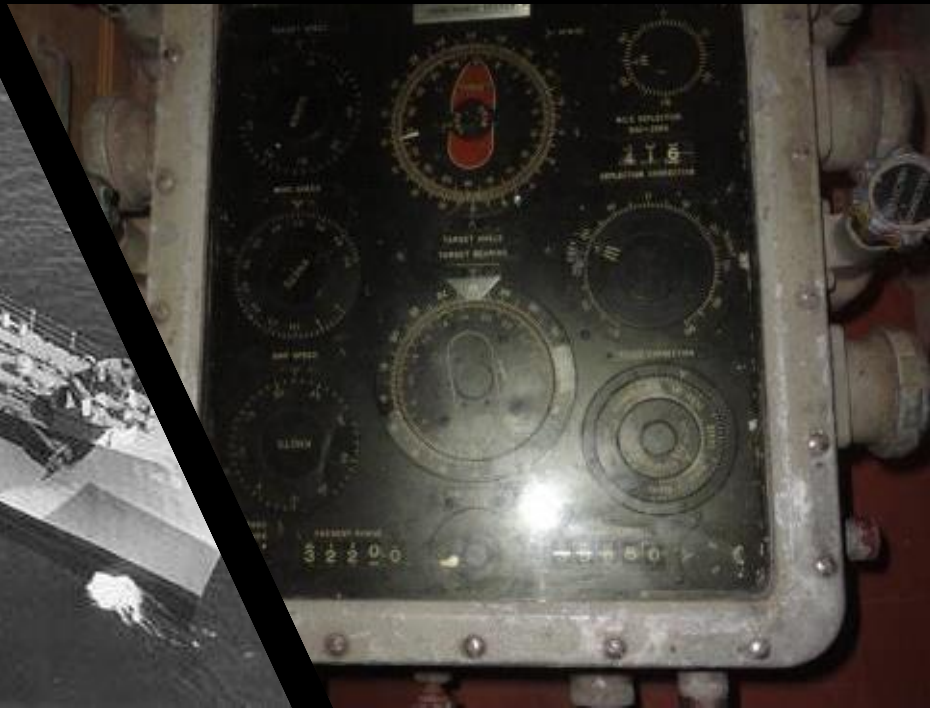
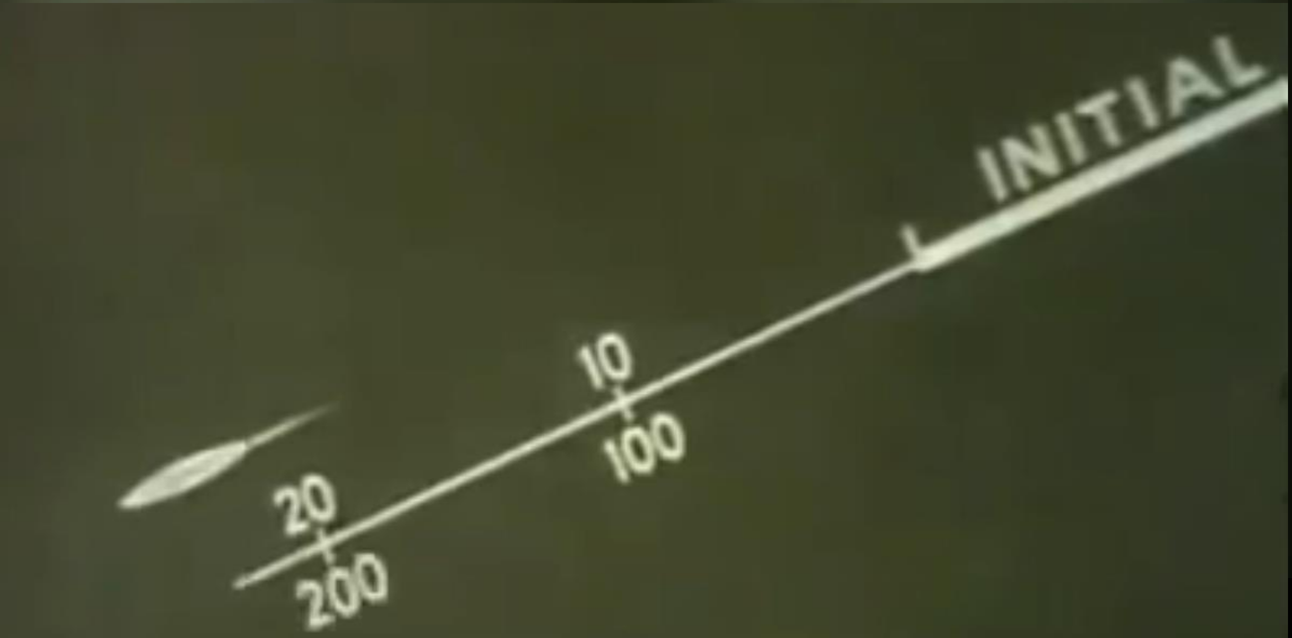


Figure 2-16.—Changing elevation angles for air target.



Source: https://www.earlycomputers.com/cgi-bin/item-report-main.cgi?20140910 Date ~1936



https://youtu.be/s1i-dnAH9Y4


UPLINK ACTY
TEMP
NO ATT
GIMBAL LOCK
STBY
PROG
KEY REL
RESTART
OPR ERR
TRACKER
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00
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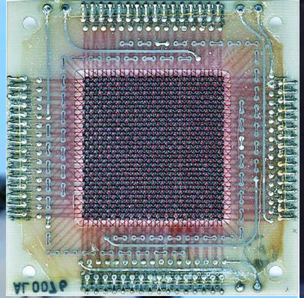

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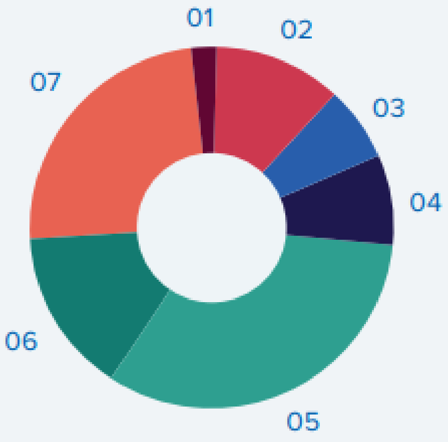
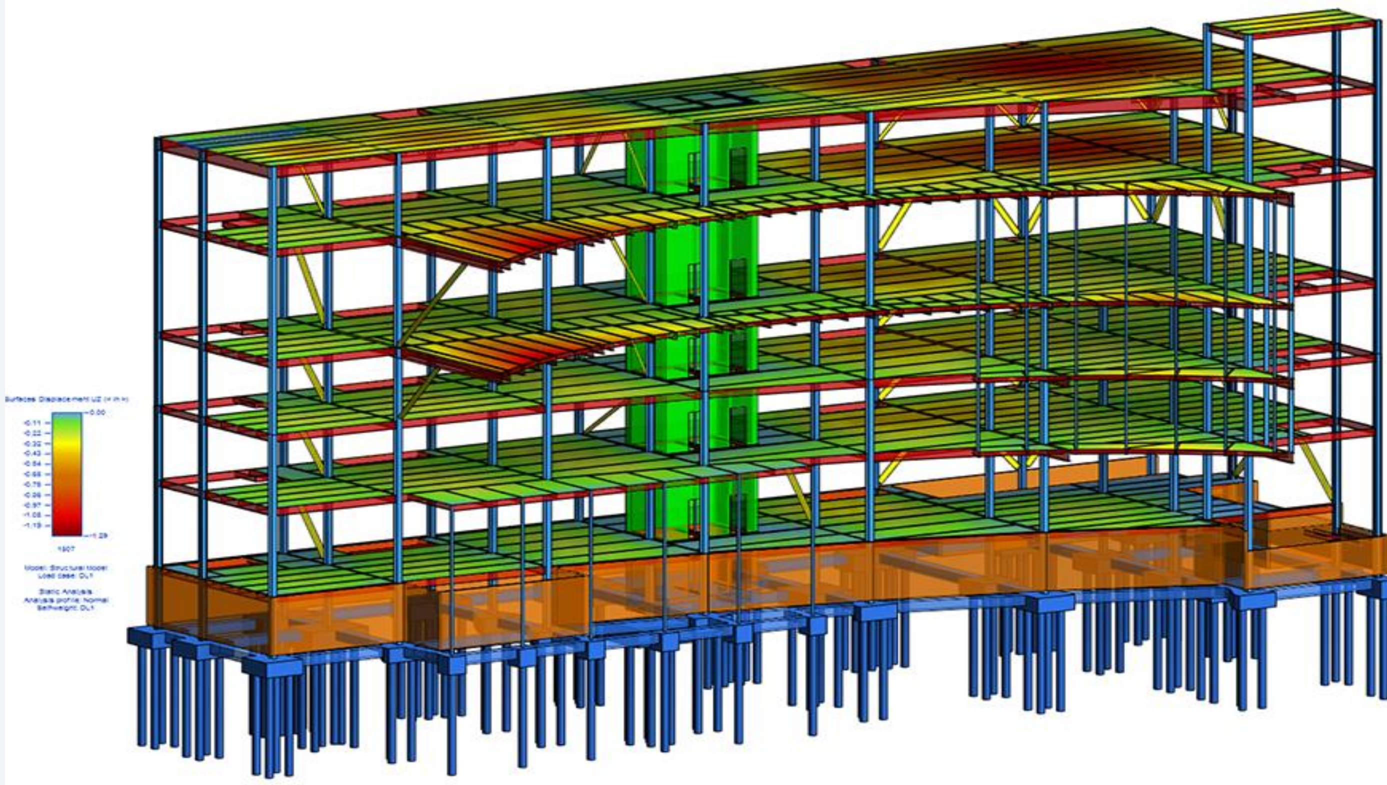
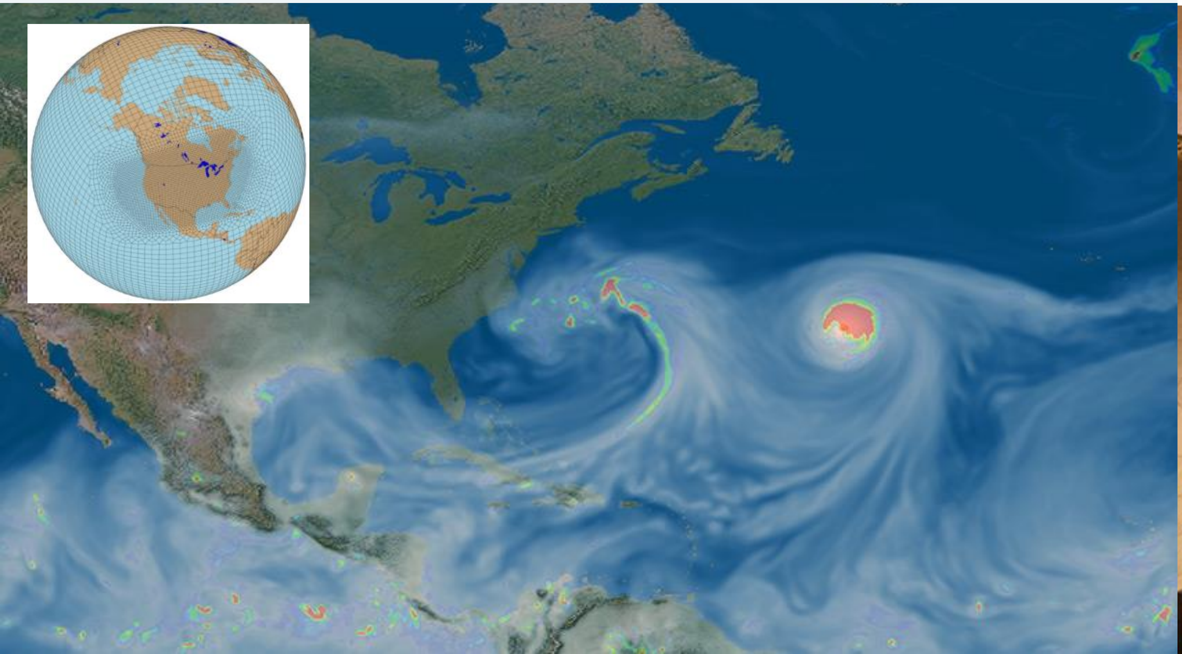
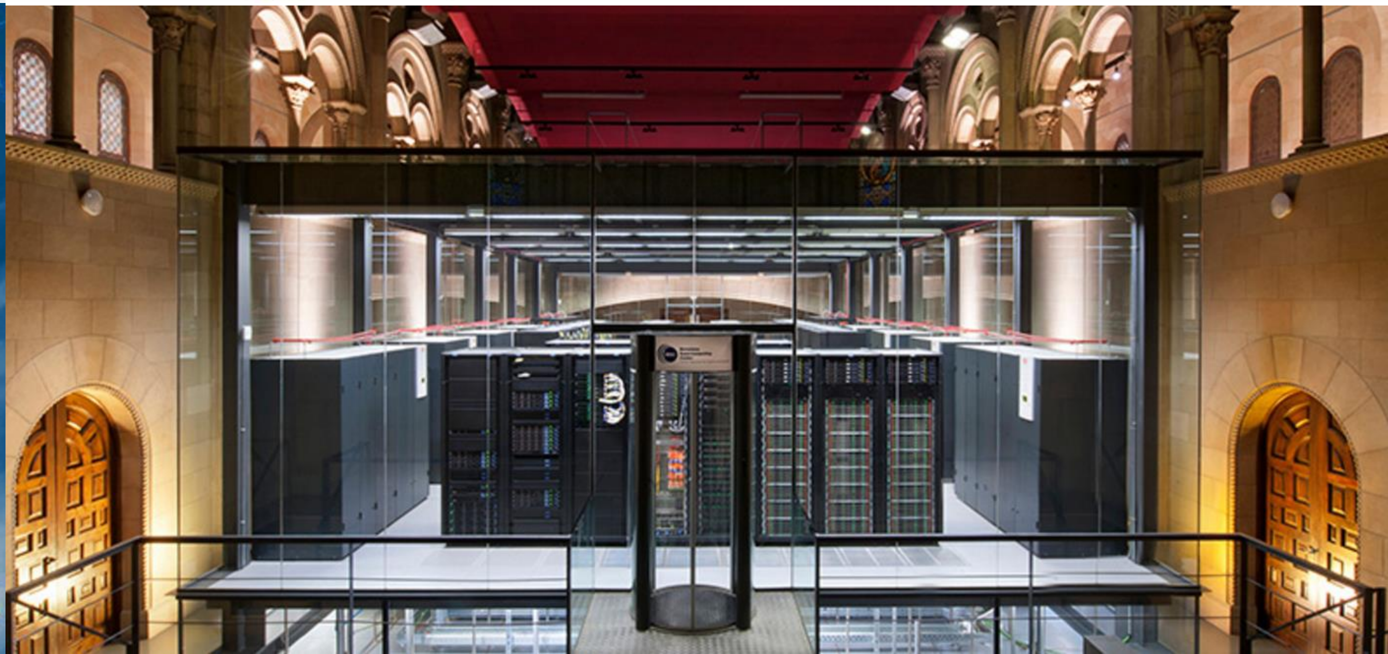
# 220M

| | | |
|---|---|---|
| 01 | Biological Sciences | 2% |
| 02 | Chemistry | 11% |
| 03 | Earth Science | 7% |
| 04 | Energy Technologies | 8% |
| 05 | Engineering | 33% |
| 06 | Materials Science | 15% |
| 07 | Physics | 24% |








IBM Q™


Google AI
Quantum

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**Many interesting problems to address…**

**Quantum advantage?**

| | | |
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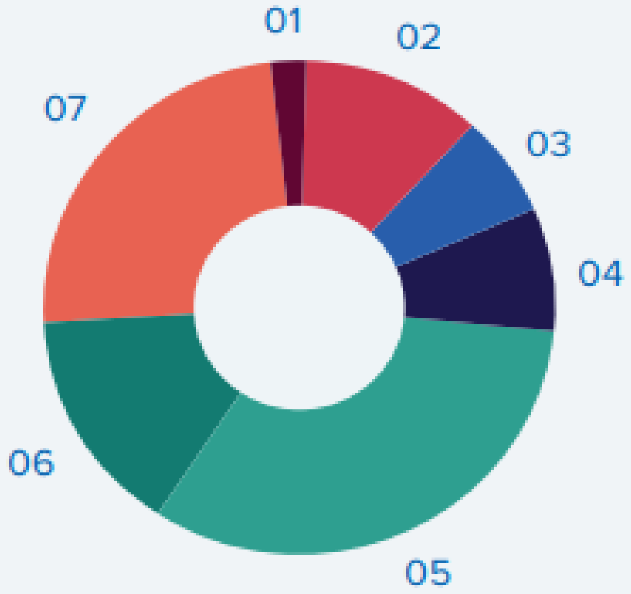


# Problem domain: numerical analysis

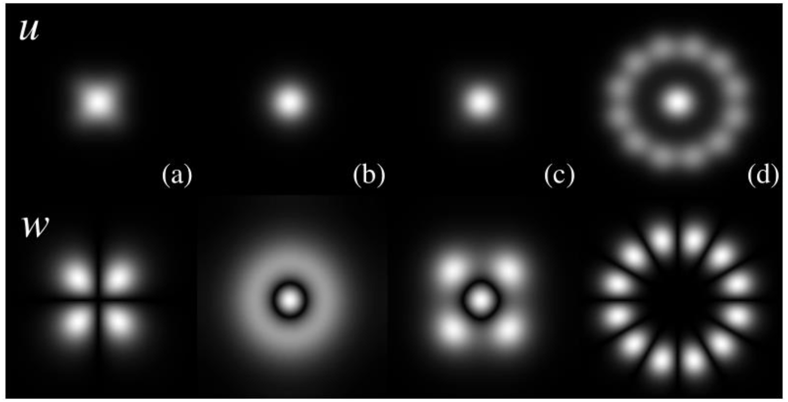
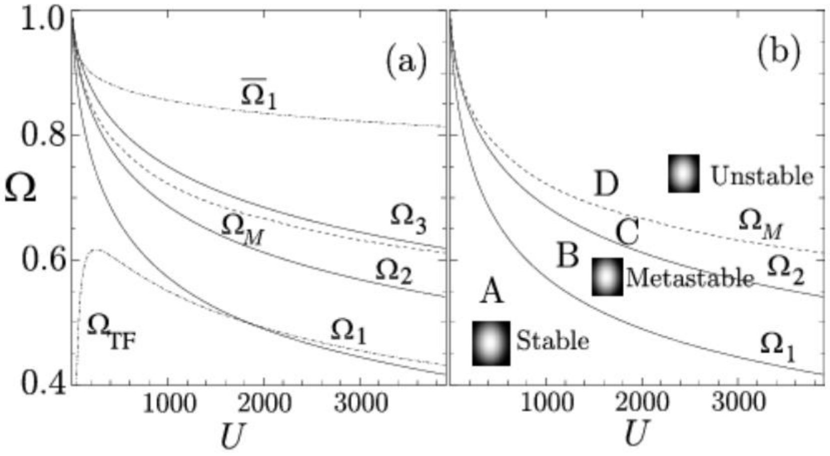


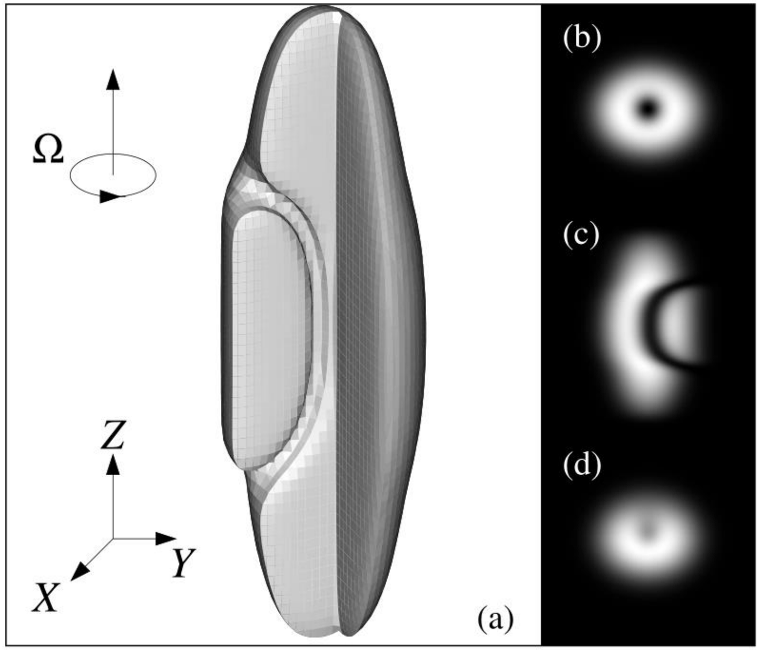

FIG. 5.3. *Different unstable multi–solitonic configurations arising from Eq. (5.14) with a change of the initial conditions for fixed* $\mu_u = -1, \mu_w = -0.3, \kappa = 0.5$.

*JJGR & Pérez-García, V.M., SIAM J. Sci. Comp. (2001)*

$$\frac{\partial \psi}{\partial t} = \left[-\frac{1}{2}\Delta + V(x) + U|\psi|^2 - i\hbar\partial_\theta\right]\psi(x,t)$$

Nonlinear Schrödinger equations, applications, mathematical properties, new numerical techniques, etc.

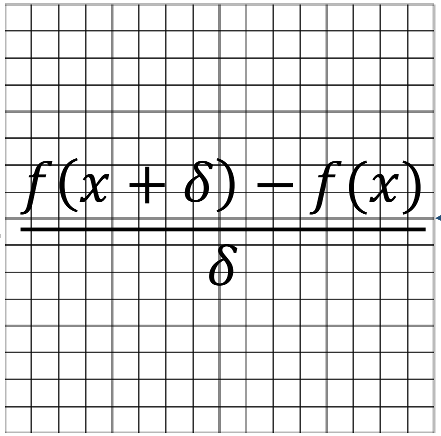


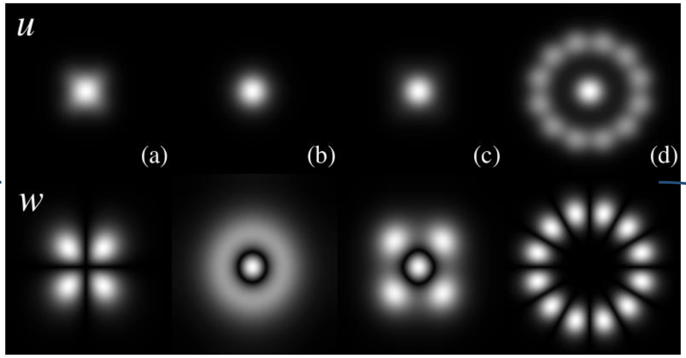

*JJGR & Pérez-García, V.M., PRA (1999)*

# Many classical encodings

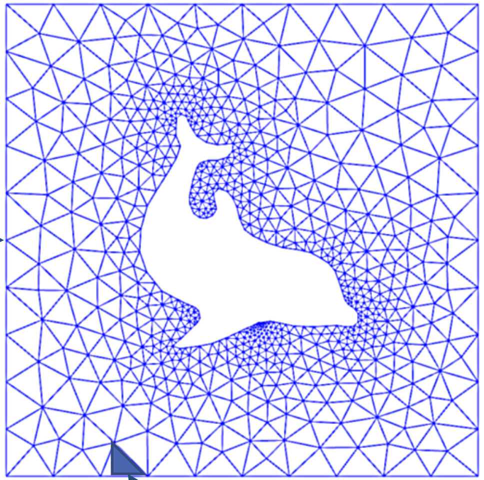
## Finite differences



$$\frac{df}{dx} \simeq \frac{f(x+\delta) - f(x)}{\delta}$$



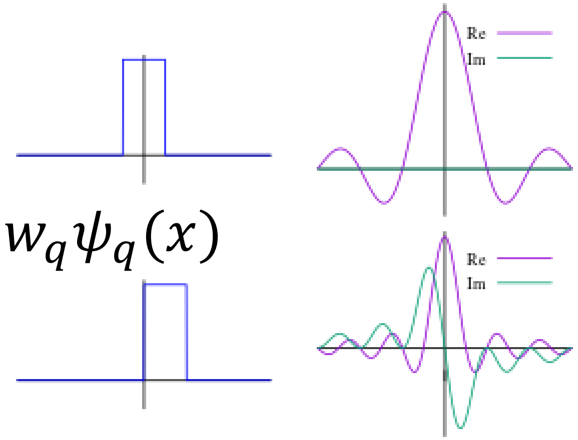

## Finite element



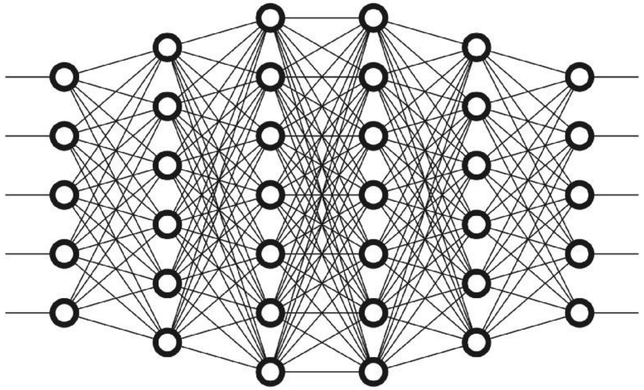
$$f(x)_m \simeq \sum_n c_n^m x^n$$

## Spectral method

$$f(x) = \sum w_q \psi_q(x)$$

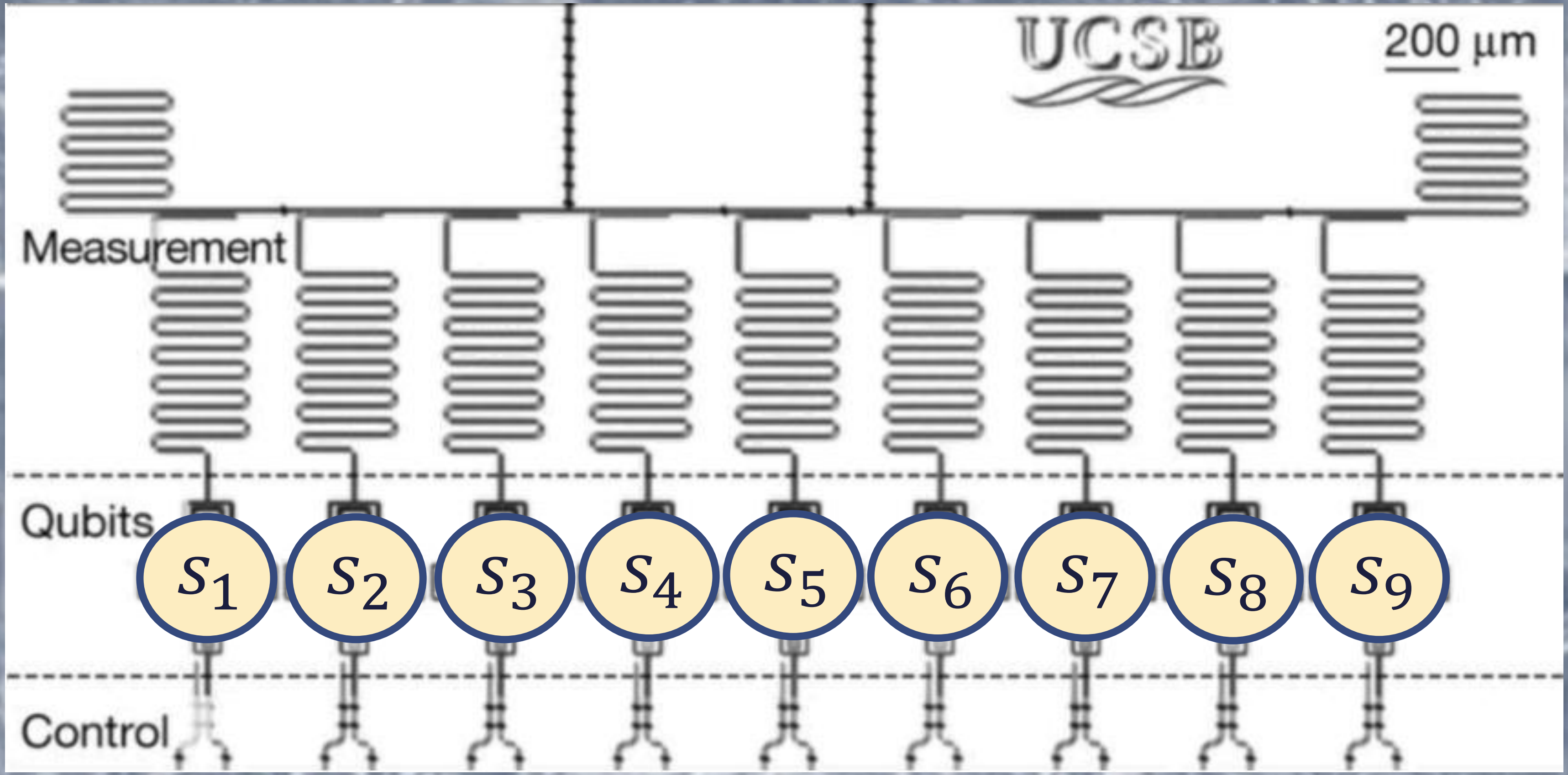



## Variational methods



$$f(x_1, x_2, \dots)$$

# Quantum register



# Quantum register

Quantum superposition

$$|\psi\rangle = \sum_{s} \psi(s_1, s_2, \dots)|s_1, s_2, \dots\rangle$$

**Exponential capacity**

$$\overbrace{\underbrace{\{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}}^{2^4 = 16} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\} \times \{0,1\}}^{2^9 = 512}$$

$$s_1 \quad s_2 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \quad s_7 \quad s_8 \quad s_9$$

# Quantum algorithms

$$|\psi\rangle = \sum_{s} \psi(s_1, s_2, \dots)|s_1, s_2, \dots\rangle$$

$$|\psi\rangle = \psi(s_1^{opt}, s_2^{opt}, \dots)|s_1^{opt}, s_2^{opt}, \dots\rangle$$

Construction of complex wavefunctions that solve or encode problems.

$$\langle O_1\rangle, \langle O_1 O_2\rangle, \dots$$

Interrogating those wavefunctions for properties or features.

# Quantum image processing

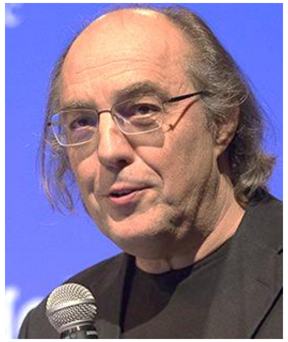


*Image compression and entanglement,*
J. I. Latorre
quant-ph/0510031

# Quantum image processing

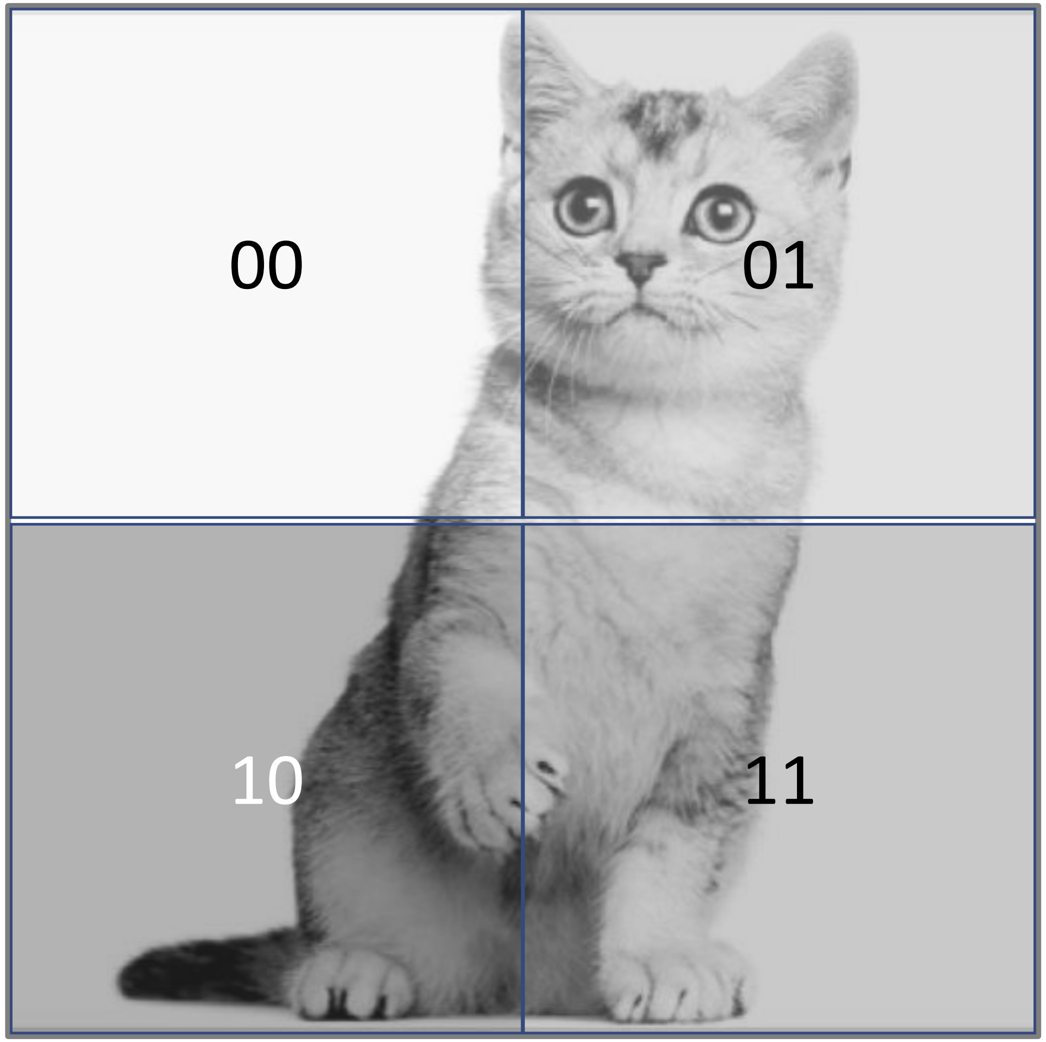
B&W image encoded in pure intensity, with an integer between 0 and 255

```
I( 0)= 213
I( 1)= 213
I( 2)= 213
…
I( 100)= 213
…
I(1300)= 214
…
I(2600)= 254
…
I(2700)= 254
…
I(4530)= 253
…
```

**512 x 512 pixels**
**= 262144 values**
**= 9 + 9 qubits**

# Image encoding

$s_2 \sim$ x coordinate

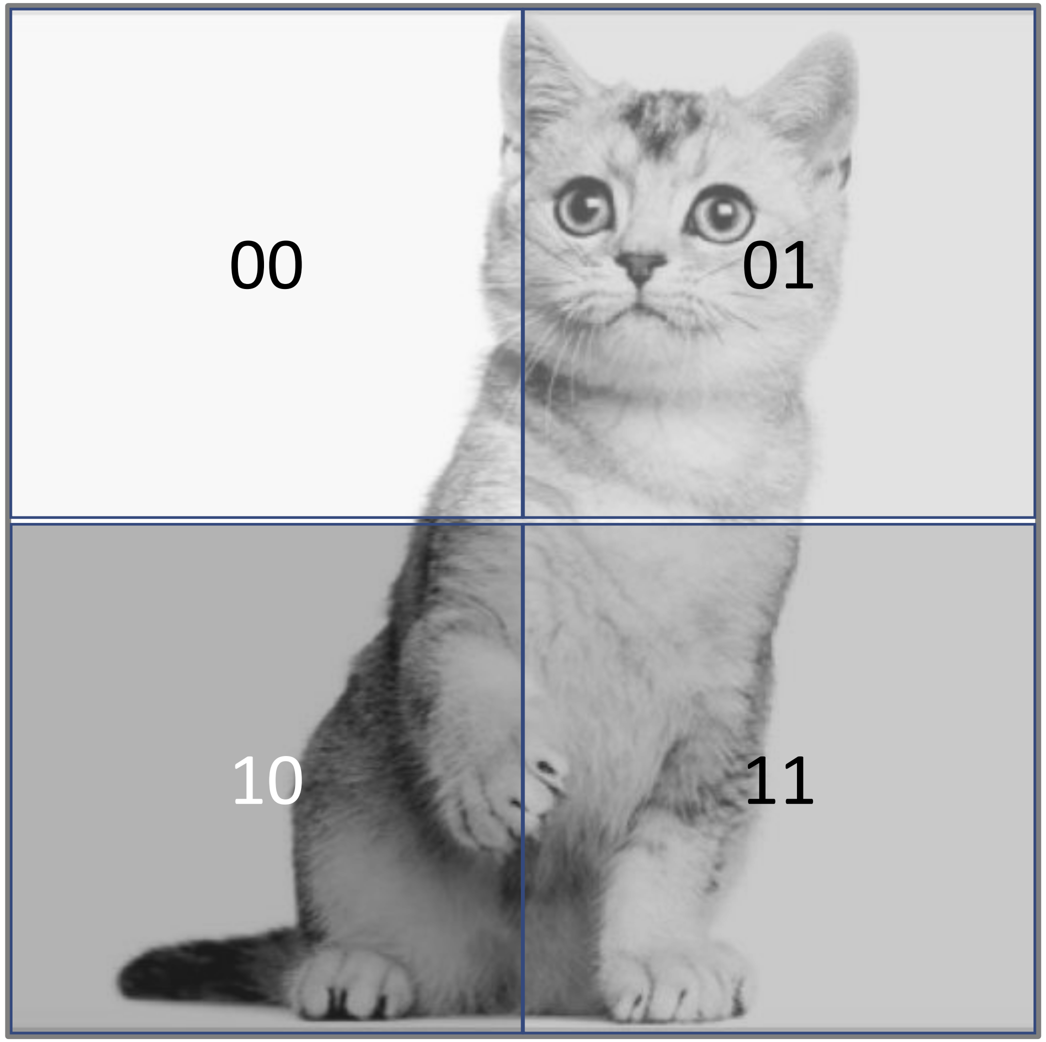



$s_1 \sim$ y coordinate

For instance, using 2x2 pixels, 4 values

$$I(s_1, s_2) \sim I(00), I(01), I(10), I(11)$$

# Image encoding




For instance, using 2x2 pixels, 4 values

$$I(s_1, s_2) \sim I(00), I(01), I(10), I(11)$$

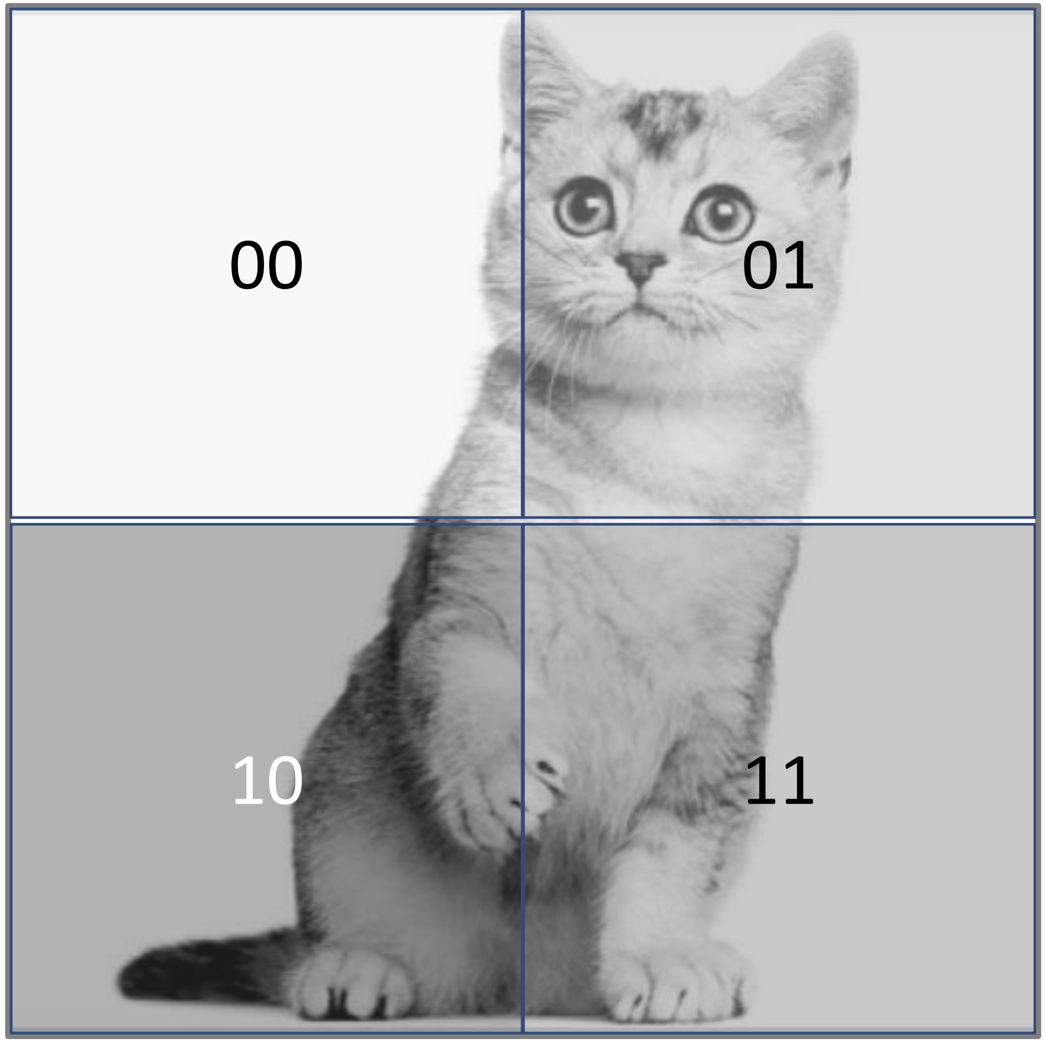
We identify "I" with the probability of finding the quantum register in a given state

$$|\psi\rangle \sim \sum_{\vec{s}_2} \sqrt{I(s_1, s_2)}|s_1, s_2\rangle$$

Explicitely

$$|\psi\rangle \sim \sqrt{I(0,0)}|0,0\rangle + \sqrt{I(0,1)}|0,1\rangle + \sqrt{I(1,0)}|1,0\rangle + \sqrt{I(1,1)}|1,1\rangle$$

# Normalization



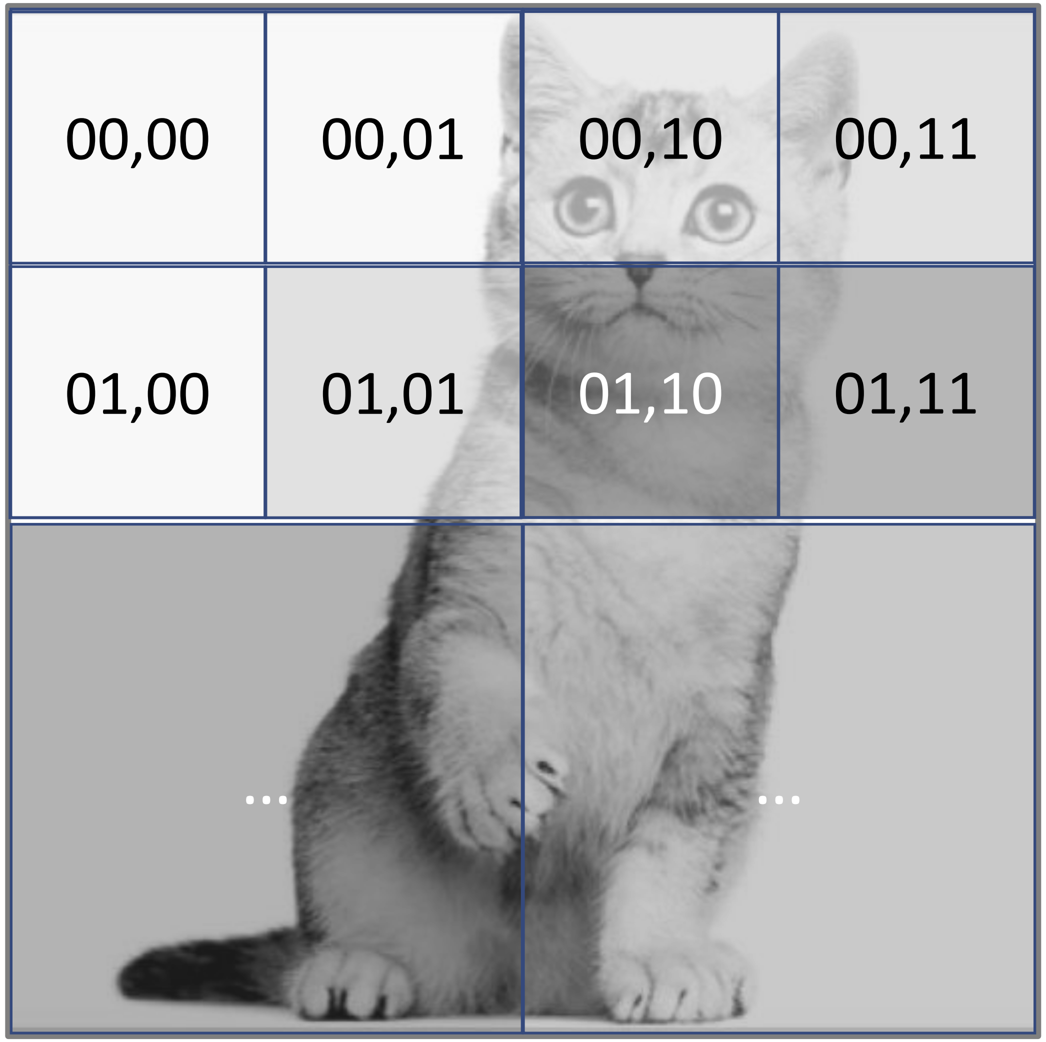

Original and encoding do not perfectly match:

$$|\psi\rangle = \sum_{\vec{s}_2} \sqrt{\frac{I(s_1, s_2)}{I_T}} |s_1, s_2\rangle$$

Probability is normalized with respect to total intensity

$$I_T = \sum_{\vec{s}_2} I(s_1, s_2)$$

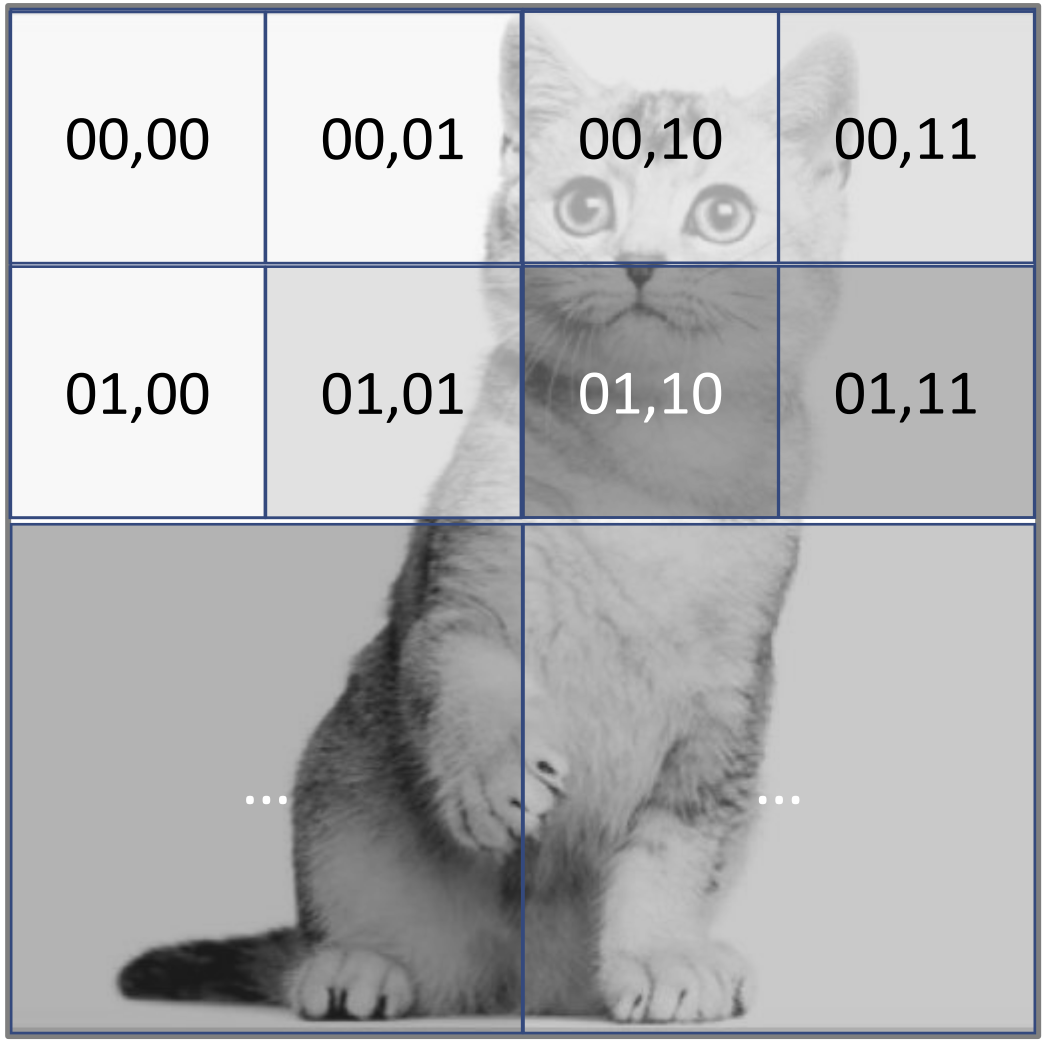
# +1 qubit = double resolution




**1 qubits x 1 qubits** **4 pixels**

**2 qubits x 2 qubits** **16 pixels**

# Quantum register ordering




Different orders: coordinate first

| $s_1^1$ | $s_1^2$ | $s_1^3$ | … | $s_2^1$ | $s_2^2$ | $s_2^3$ | … | (A) |
|---|---|---|---|---|---|---|---|---|

**Precision first**

| $\boldsymbol{s_1^1}$ | $\boldsymbol{s_2^1}$ | $s_1^2$ | $s_2^2$ | $s_1^2$ | $s_2^2$ | … | … | (B) |
|---|---|---|---|---|---|---|---|---|

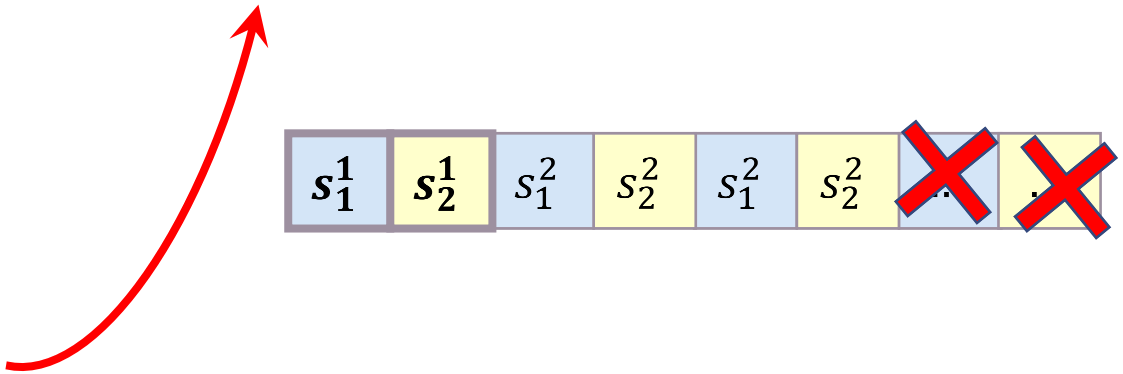
Implicit renormalization.
Most important length scales appear first, correlated details at the end.

# “Forgetting” information

**What happens when qubits disappear?**

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\boldsymbol{s_1^1}$ | $\boldsymbol{s_2^1}$ | $s_1^2$ | $s_2^2$ | $s_1^2$ | $s_2^2$ | ~~…~~ | ~~…~~ |

**8+8**




**9 qubits + 9 qubits**

$$|\psi\rangle = \sum \psi(\vec{s})|s_1 \dots s_{18}\rangle$$

**9 qubits + 9 qubits**

$$|\psi\rangle = \sum \psi(\vec{s})|s_1 \dots s_{18}\rangle$$

**8+8**

Original state:

$$|\psi\rangle = \sum \psi(\vec{s})|s_1 \dots s_{18}\rangle$$

Separate bits that we ignore:

$$|\psi\rangle = \sum \psi(\vec{s}_{16}, \vec{r}_2)|\vec{s}_{16}\rangle|\vec{r}\rangle$$

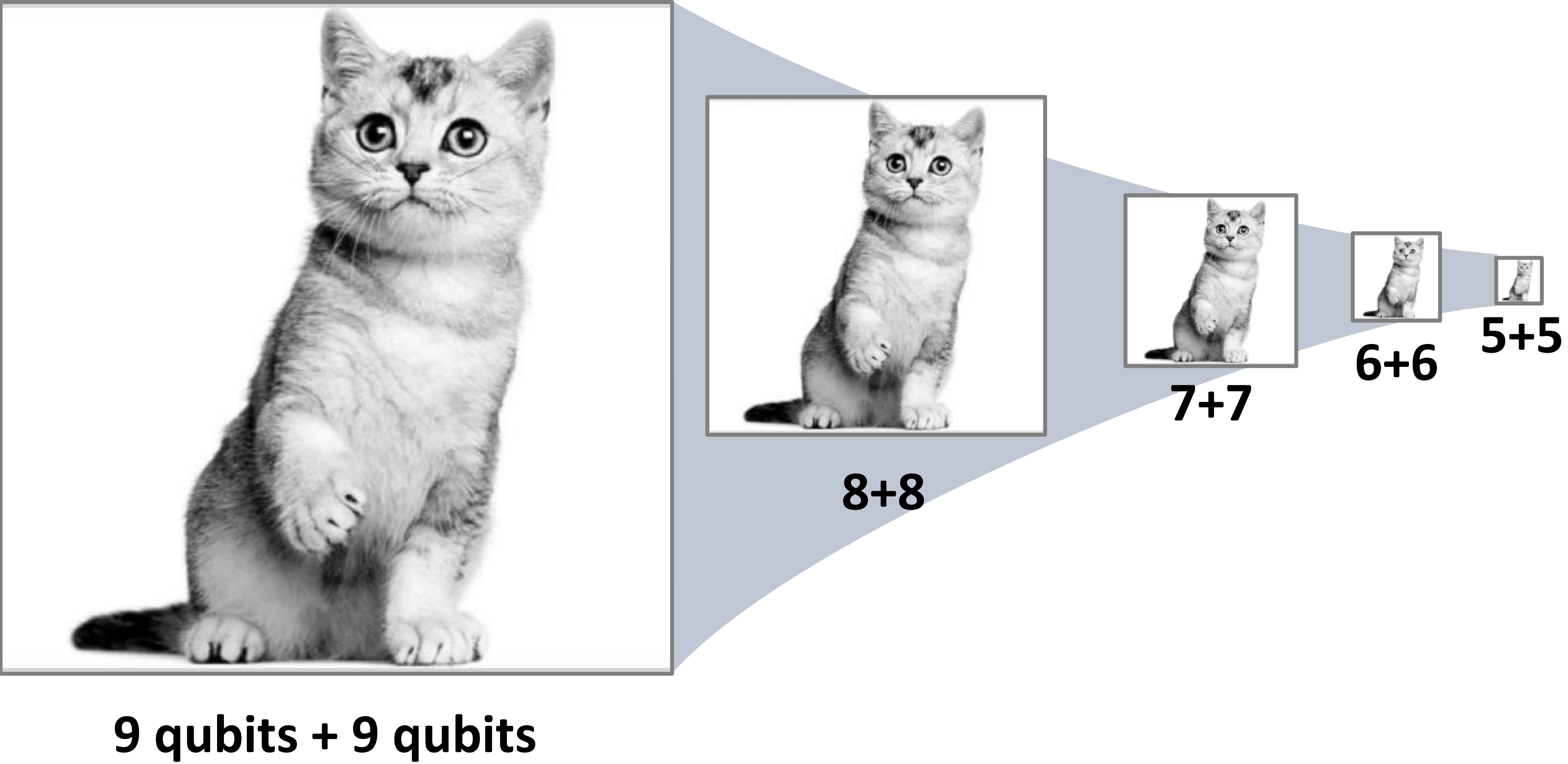
Image is the probability of a state:

$$\rho(\vec{s}_{16}, \vec{s}'_{16}) = \sum_{\vec{r}_2} \psi(\vec{s}_{16}, \vec{r}_2)\psi^*(\vec{s}_{16}, \vec{r}_2)$$

$$I(\vec{s}_{16}) = \rho(\vec{s}_{16}, \vec{s}_{16})$$

This is related to the density matrix of remaining qubits:

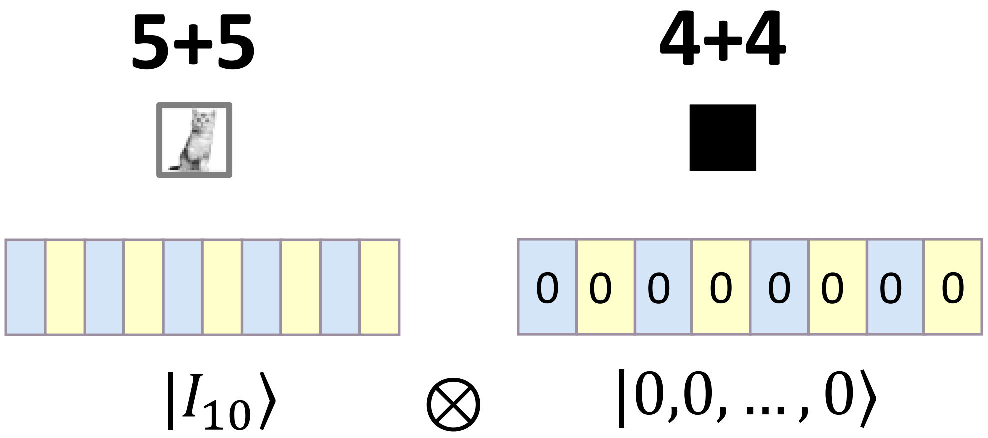
$$\rho = \sum \rho(\vec{s}_{16}, \vec{s}'_{16})|\vec{s}_{16}\rangle\langle\vec{s}'_{16}|$$




# Resampling

**9 qubits + 9 qubits**

We bring in fresh qubits with no information about details (all zero)




# Upscaling

9 qubits + 9 qubits

5+5

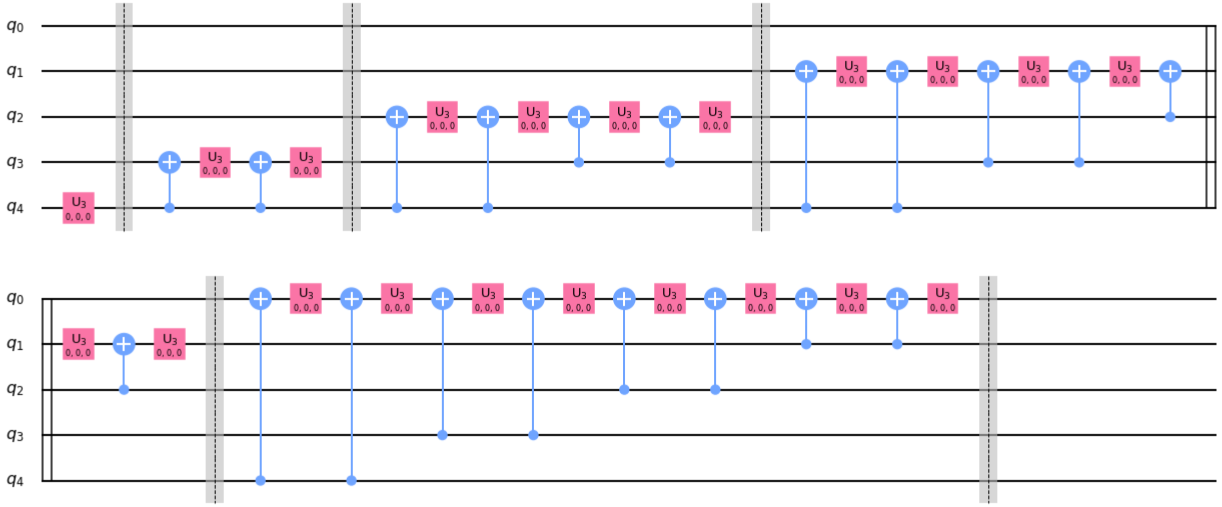
# Resampling

# A word of caution!

A lot of information!

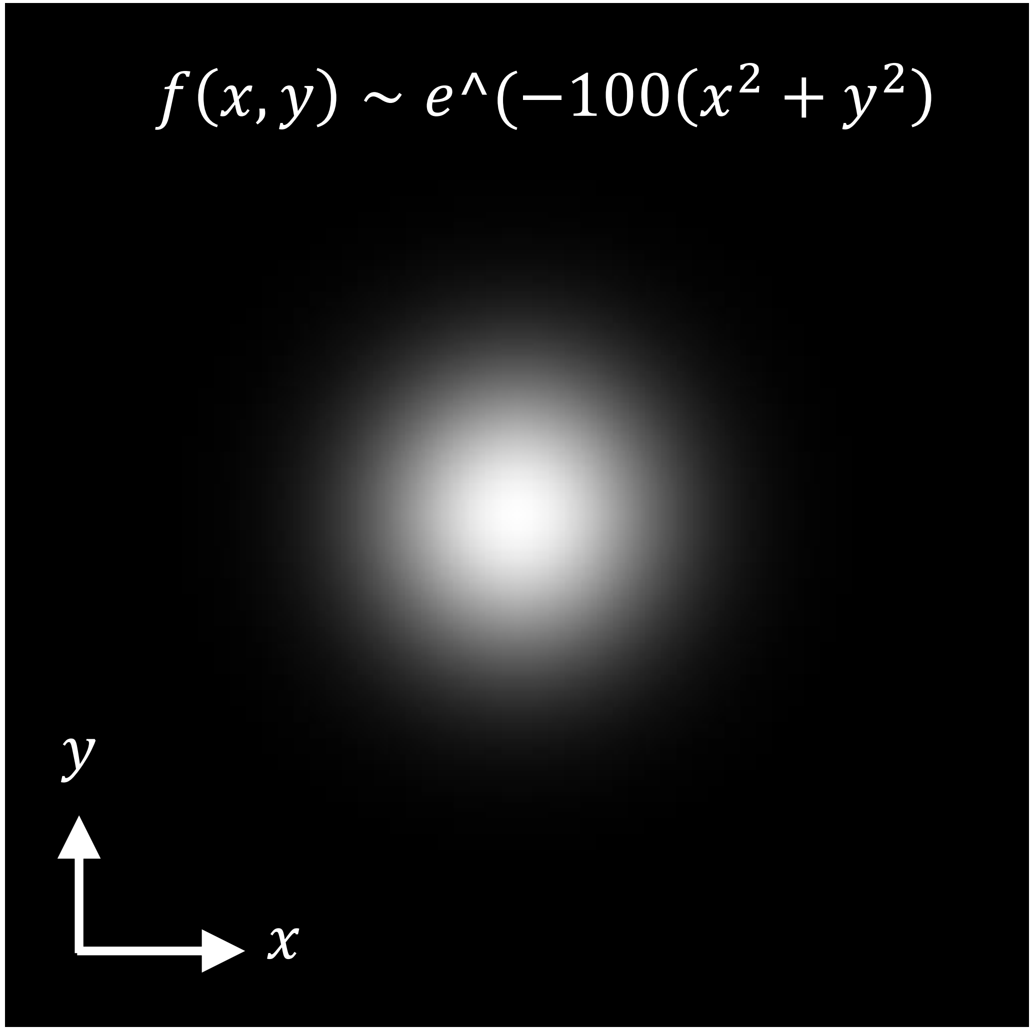
Cost of reading+writing?

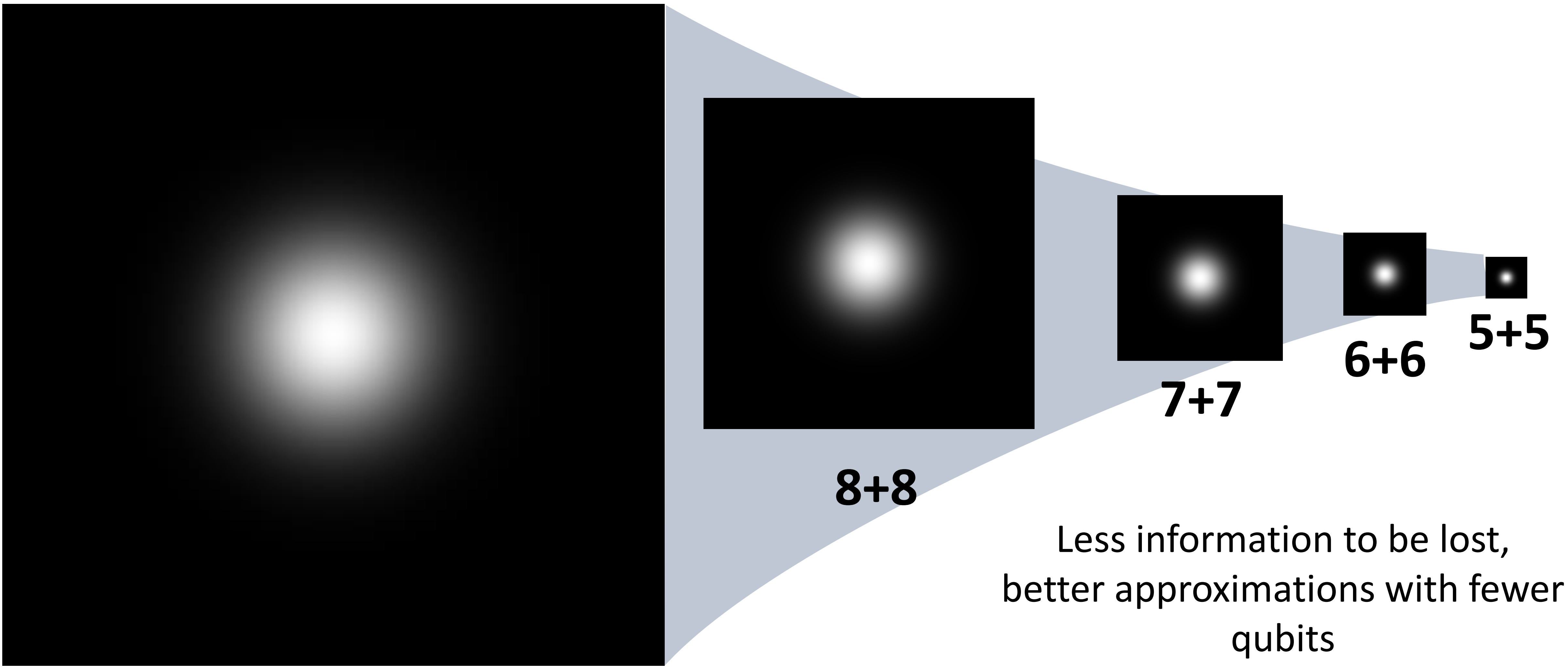
- Can not write wavefunction
- Need to find a quantum circuit




Unclear advantage!!!

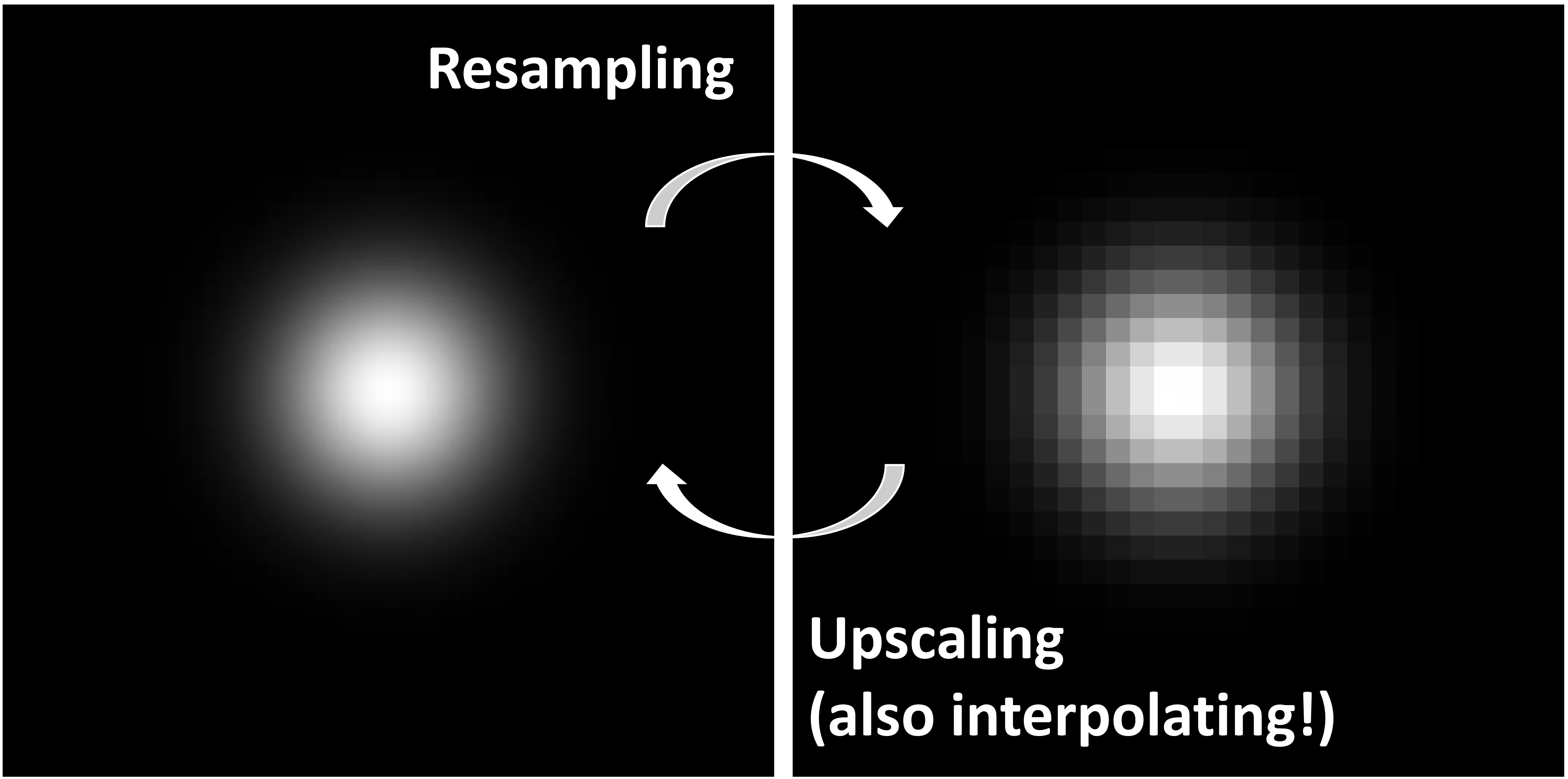
# Generic functions






**9 qubits + 9 qubits**

# Downsampling

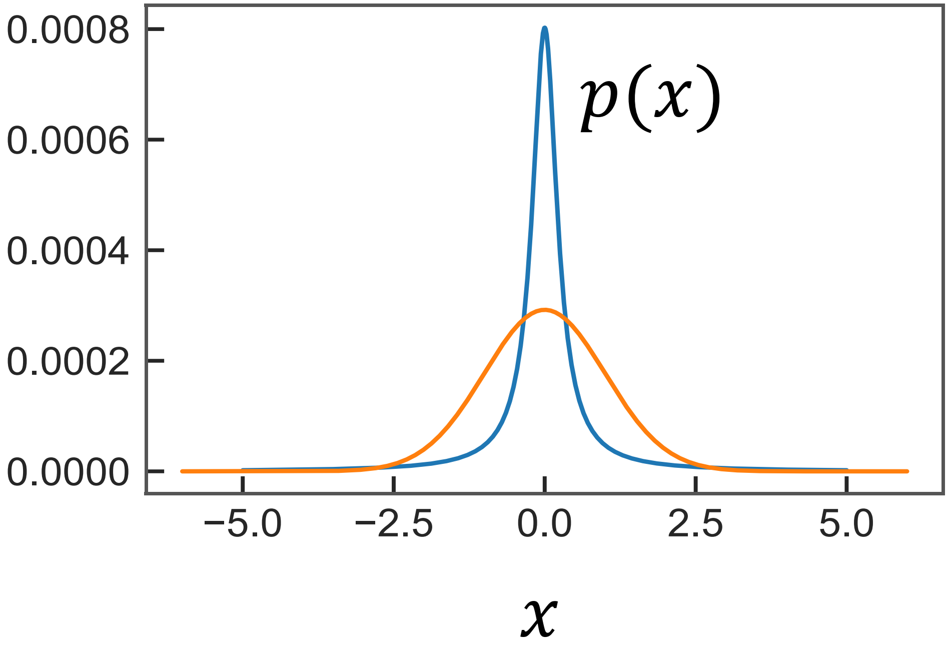



9 qubits + 9 qubits

5+5

# Downsampling

# Quantum register encoding




Discretize space

$$x = a + (b - a)s$$

with integer labels

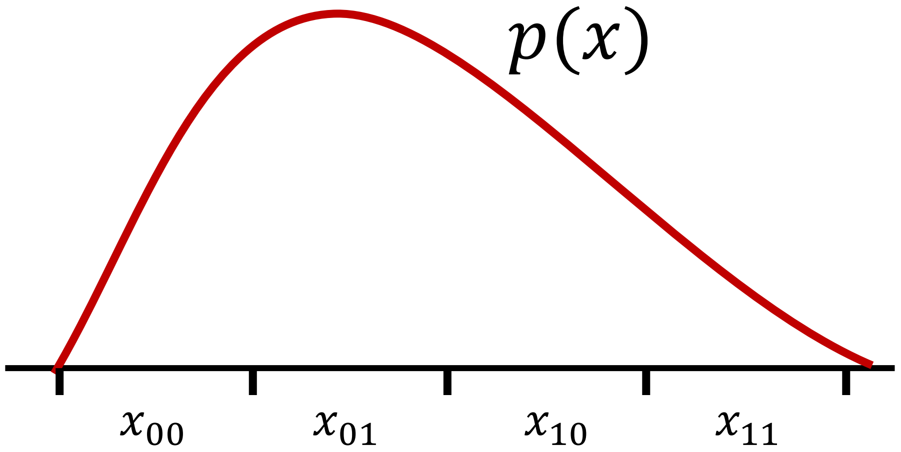
$$s = \sum_{k=1} \frac{s_1^k}{2^k}$$

Encode probability as register amplitude

$$|\psi\rangle = \sum_{\{s_1,s_2,\dots\}} \sqrt{p(x_s)}\,|s_1, s_2, \dots\rangle$$

*L. Grover, T. Rudolph arXiv:quant-ph/0208112*

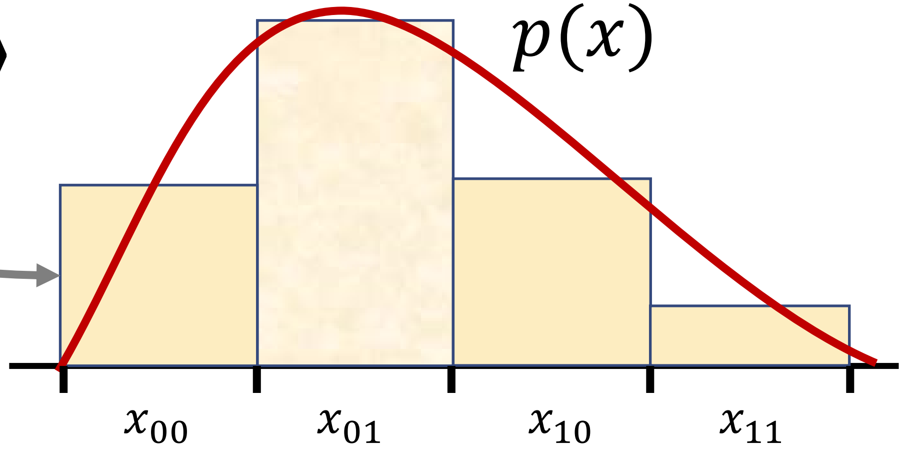
# Discretized functions




**2 qubits**

# Wavefunction encoding

$$|\psi\rangle = \psi_{00}|00\rangle + \psi_{01}|01\rangle + \psi_{10}|10\rangle + \psi_{11}|11\rangle$$

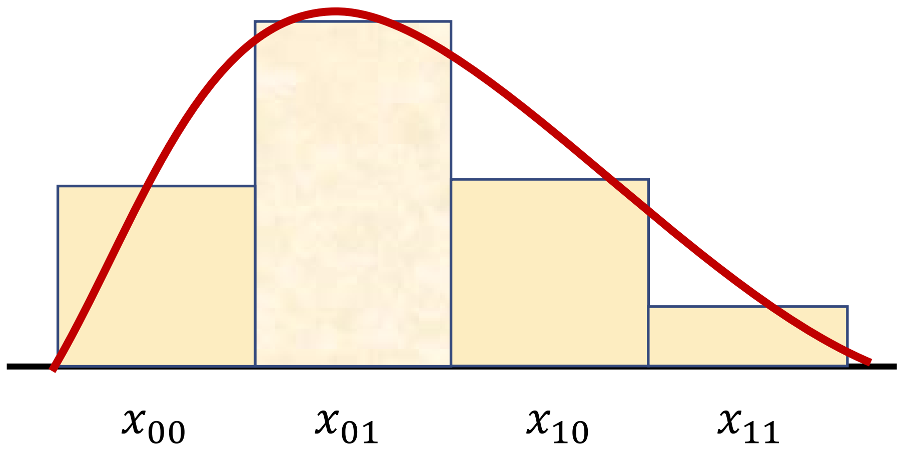



**2 qubits**

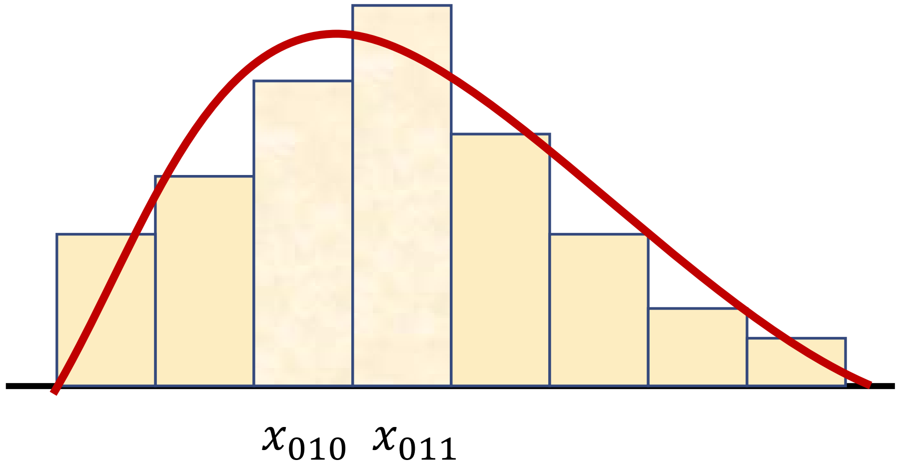
*L. Grover, T. Rudolph arXiv:quant-ph/0208112*

# Probability state creation

**2 qubits**



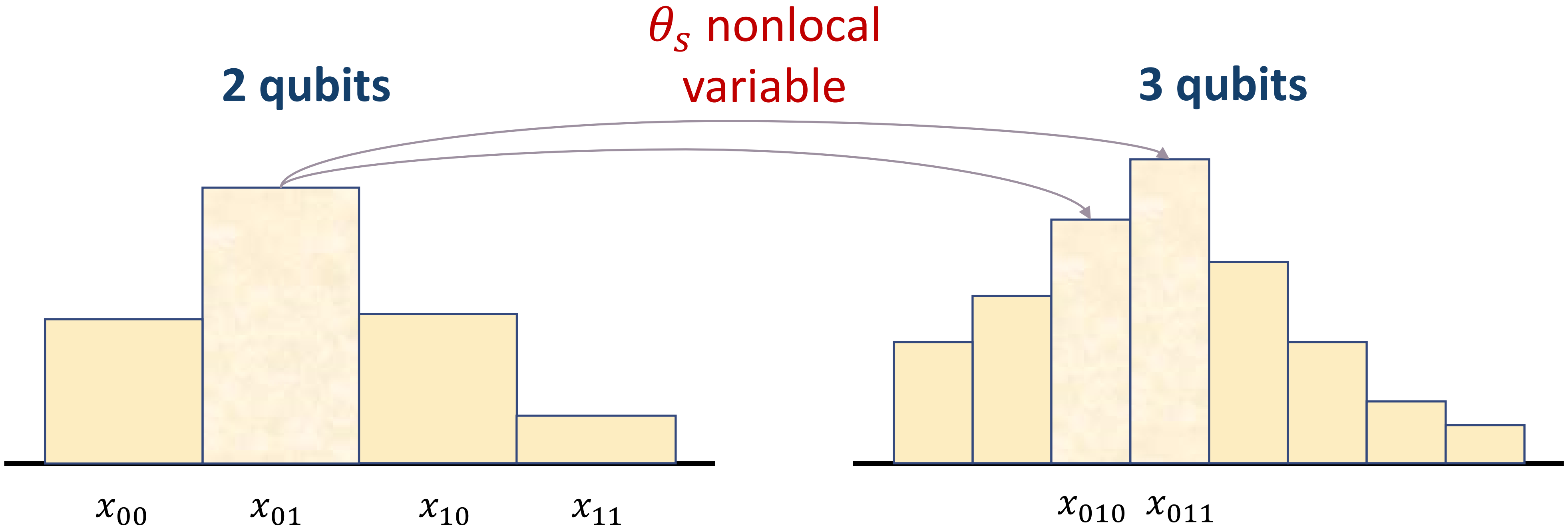

**3 qubits**




$$\left.\begin{aligned} p(x_{010}) &= \cos(\theta_{01})^2\, p(x_{01}) \\ p(x_{011}) &= \sin(\theta_{01})^2\, p(x_{01}) \end{aligned}\right\} \longrightarrow \left|\psi^{(3)}\right\rangle = U\left|\psi^{(2)}\right\rangle$$

*L. Grover, T. Rudolph arXiv:quant-ph/0208112*

# Probability state creation

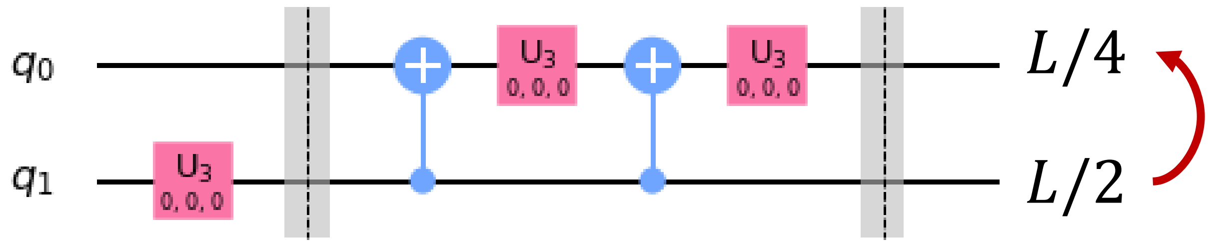



$$\left.\begin{aligned} p(x_{010}) &= \cos(\theta_{01})^2\, p(x_{01}) \\ p(x_{011}) &= \sin(\theta_{01})^2\, p(x_{01}) \end{aligned}\right\} \longrightarrow \left|\psi^{(3)}\right\rangle = U_{2\to 3}\left|\psi^{(2)}\right\rangle|0\rangle$$

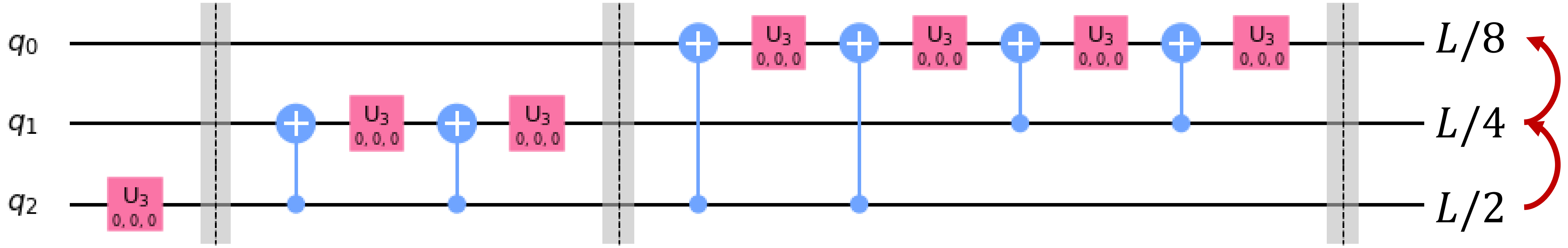
Unitary transformation Conserves probabiliy

*L. Grover, T. Rudolph arXiv:quant-ph/0208112*

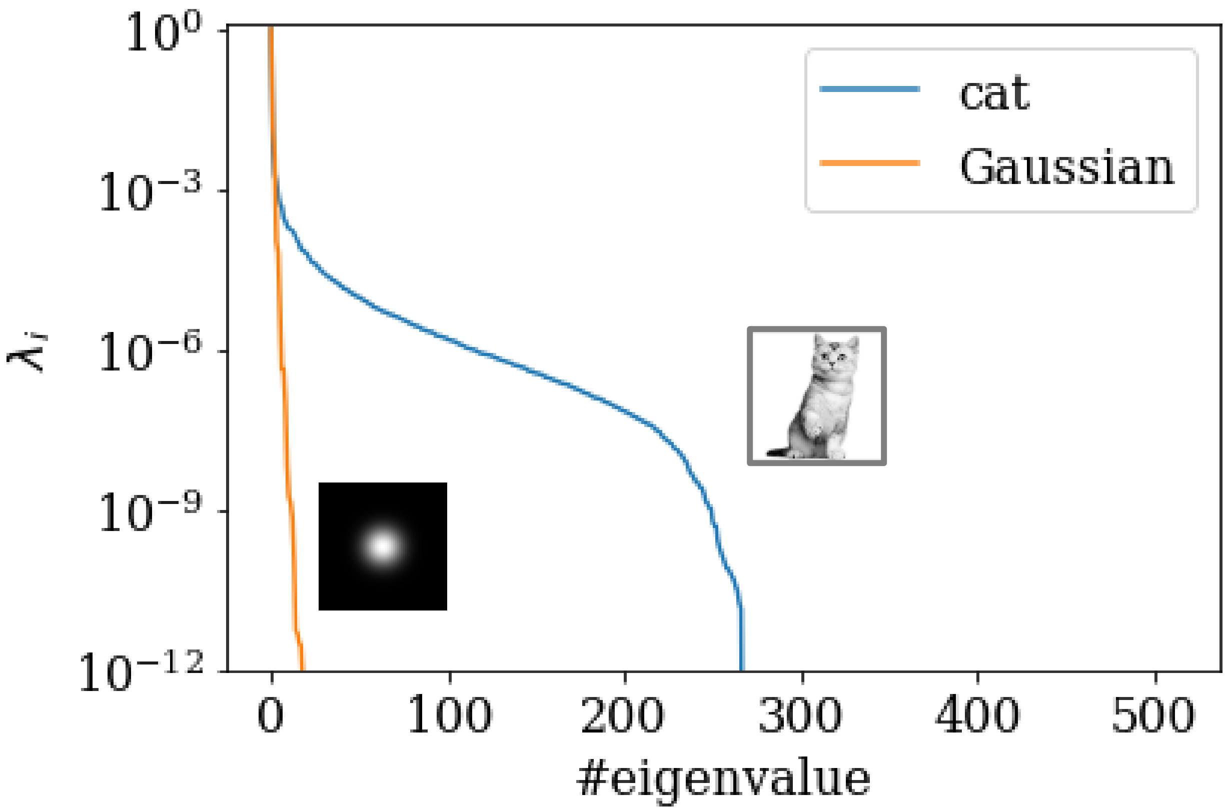
# GR-like circuit




Gates propagate "information" or correlations between circuits. The farther / stronger correlations, the more gates we need



# Amount of information & entanglement




We analyse how half of the qubits are entangled with the other half, computing the Schmidt decomposition

$$|\psi\rangle = \sum_i \lambda_i \left|\phi_i^{N/2}\right\rangle \otimes |\xi_i^{N/2}\rangle$$

The more complex the encoded function, the more states we need.

# Quantifying complexity

Schmidt decomposition: an optimal decomposition of bipartite systems

$$|\psi\rangle = \sum_{\alpha} \lambda_{\alpha}^{1/2} |\phi_{\alpha}\rangle |\psi_{\alpha}\rangle \qquad \langle \phi_{\alpha} | \phi_{\beta} \rangle = \langle \xi_{\alpha} | \xi_{\beta} \rangle = \delta_{\alpha\beta}$$

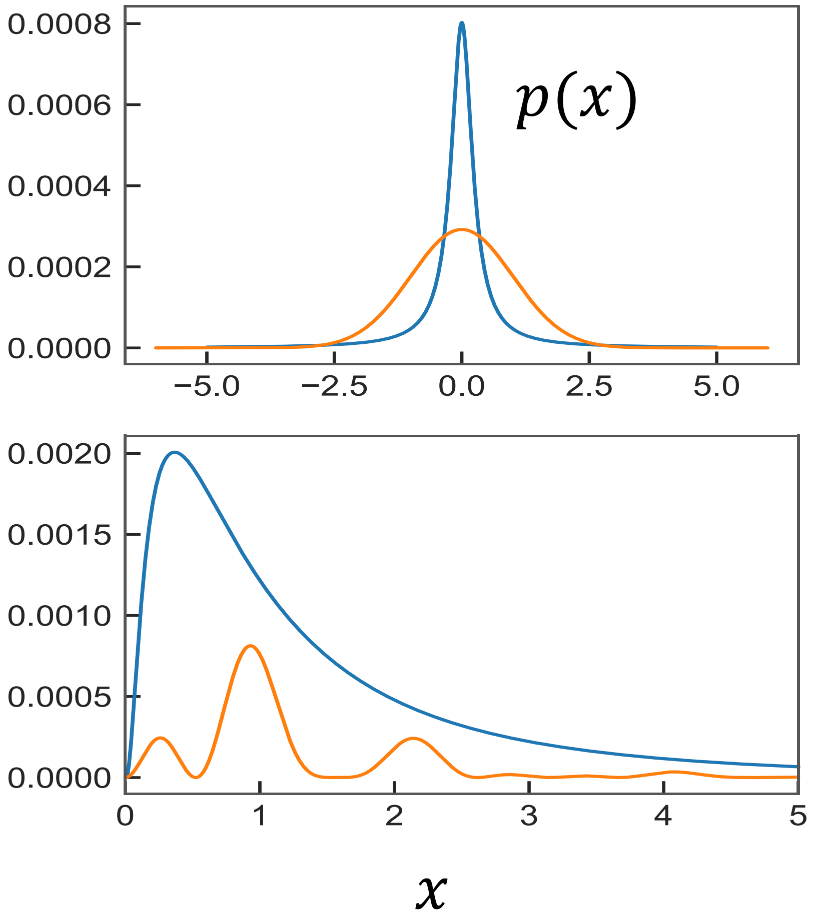
Directly related to entanglement entropy

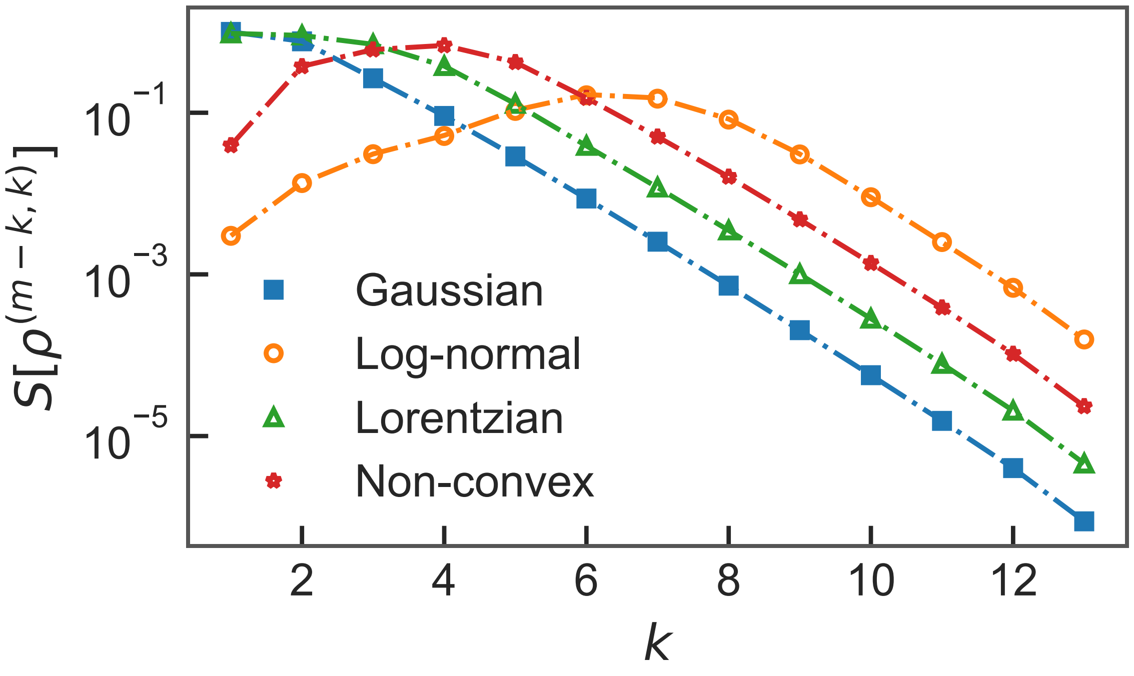
$$\rho_1 = \mathrm{tr}_2(|\psi\rangle\langle\psi|) = \sum_{s_2} \psi(s_1, s_2)\psi(s_1', s_2)^* |s_1\rangle\langle s_1'|$$

$$S = \mathrm{tr}(\rho \log_2(\rho)) = -\sum_{\alpha} \lambda_{\alpha} \log_2(\lambda_{\alpha})$$

More entanglement ~ more complex unitary operations to build the state

# Smooth functions as low-entanglement states



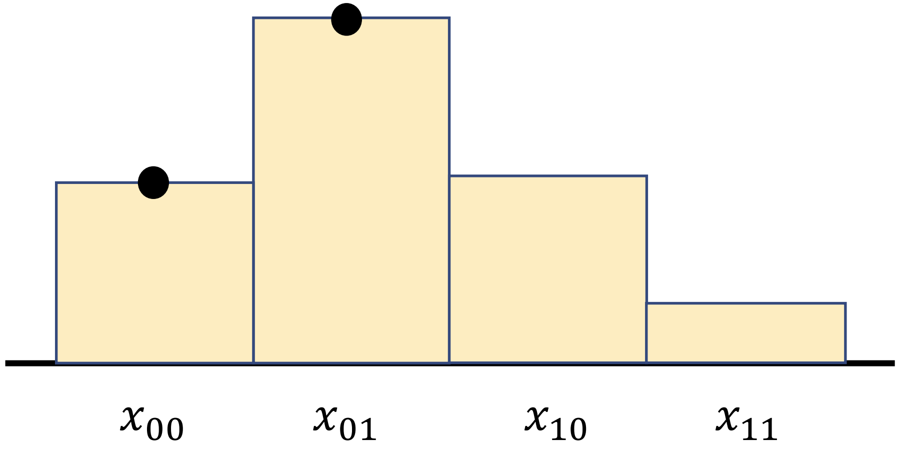




Smooth, differentiable functions require an exponentially small entanglement entropy for each added qubit. **Efficient construction** in a quantum computer.

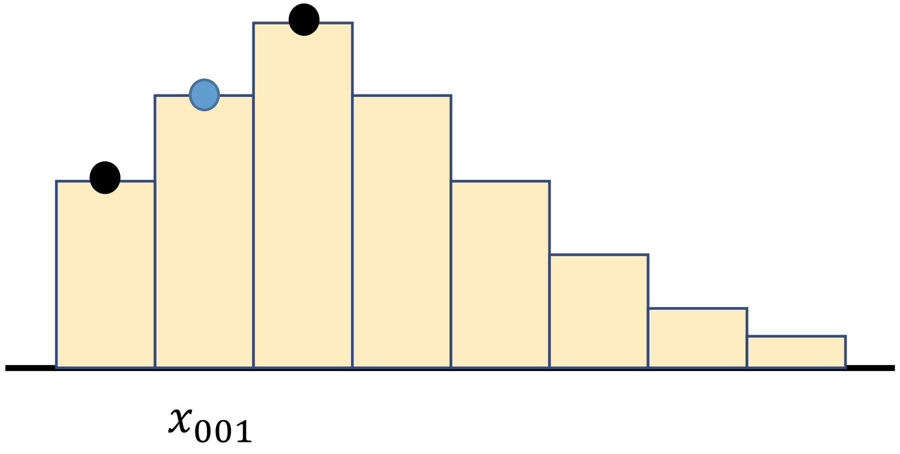
*JJGR, arXiv:1909.06619*

# Intuition

**2 qubits**




**3 qubits**

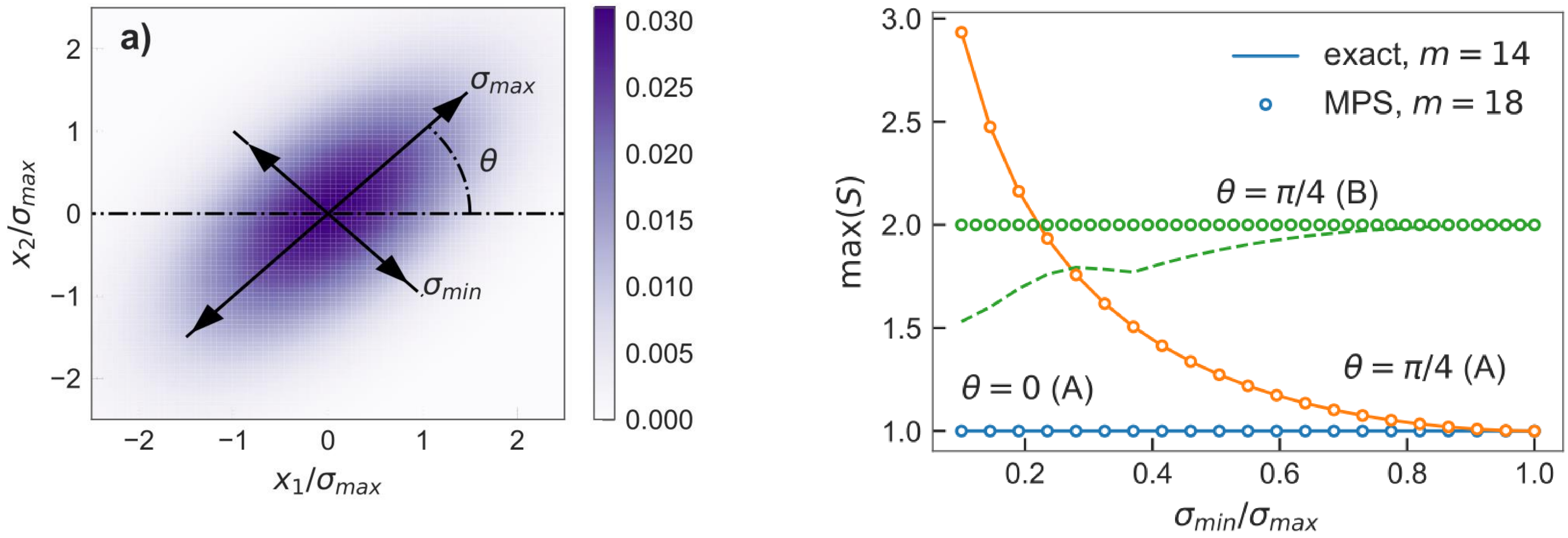



As we refine the discretization, the angles are exponentially small perturbations of a constant.

$$x_{001} \simeq \frac{1}{2}(x_{00} + x_{01})$$

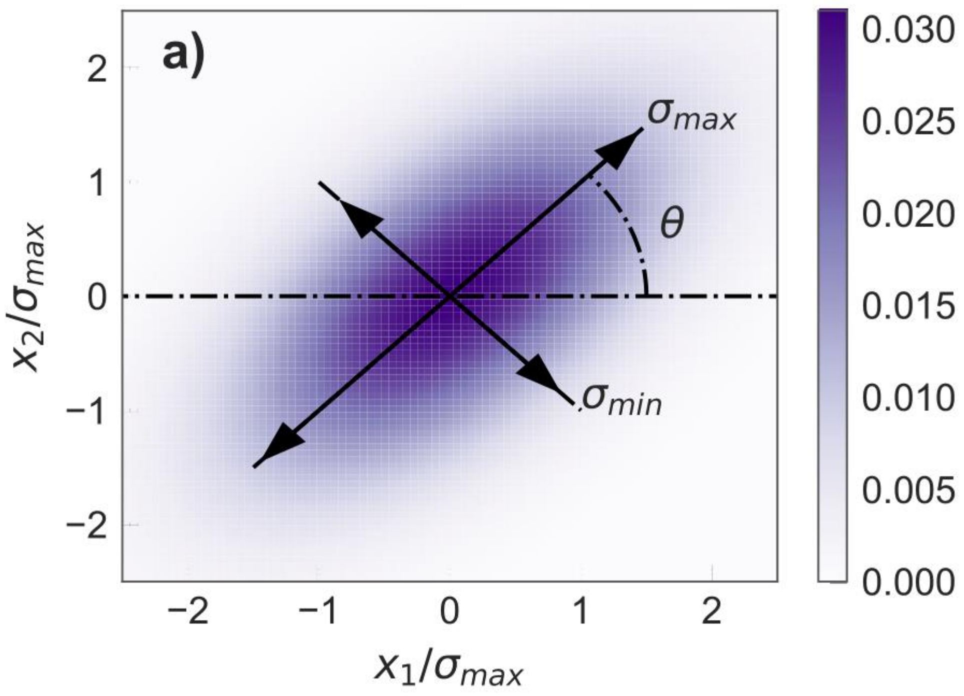
$$\theta_s \simeq ct. + \delta x \simeq ct. + O(2^{-n})$$

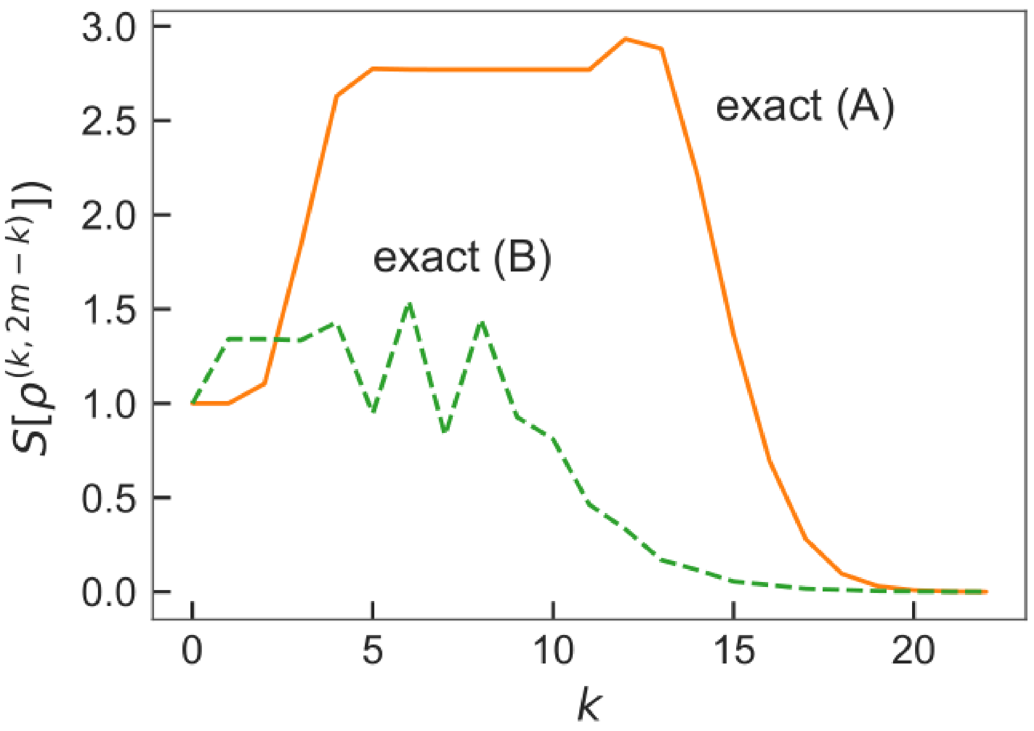
# Higher dimensions




The amount of entanglement is still small, although diverges with correlations

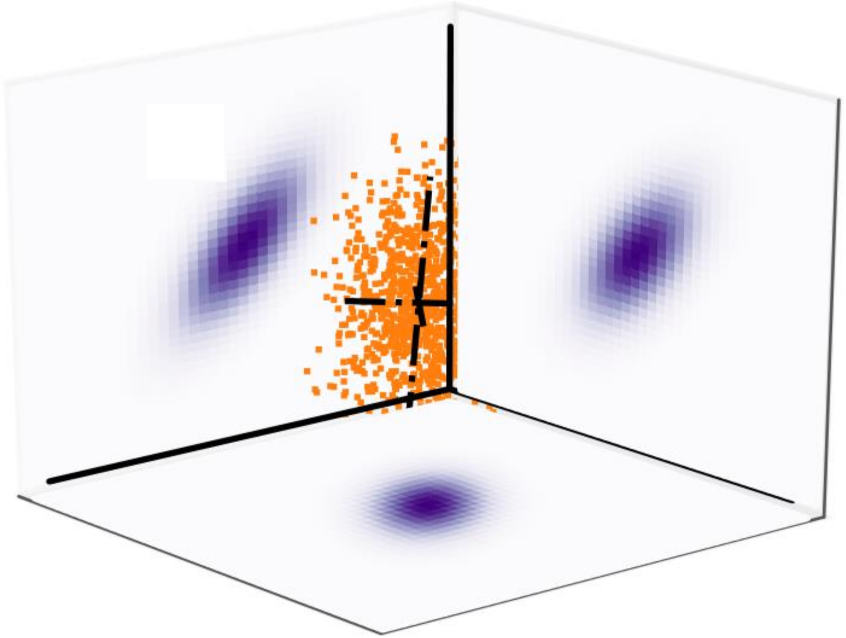
# Higher dimensions

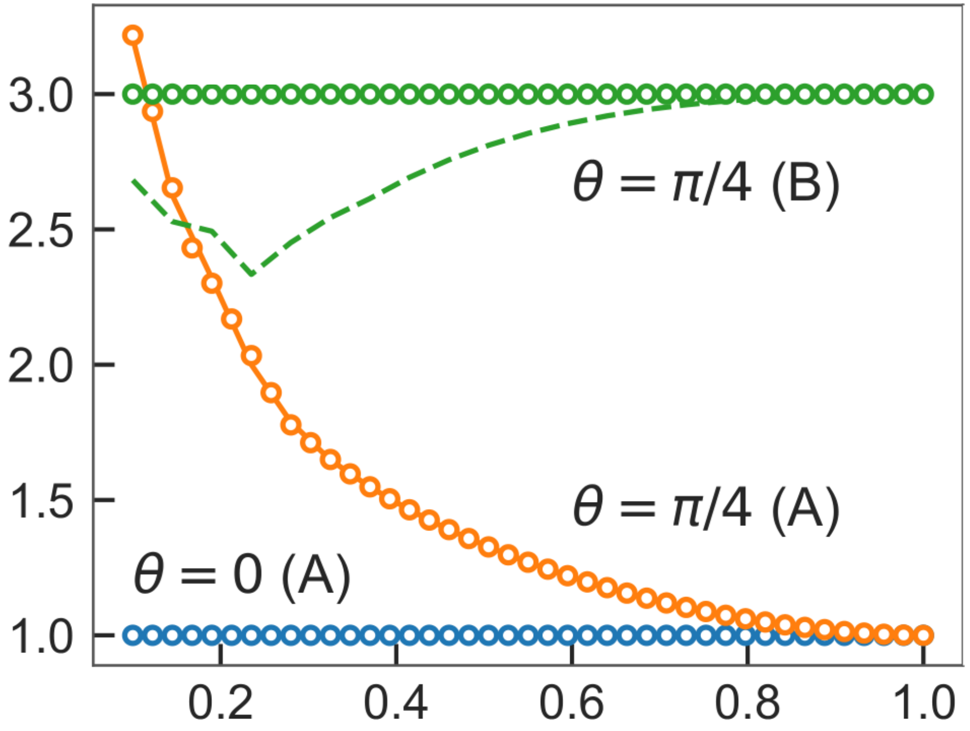



The amount of entanglement is still small, although diverges with correlations.
It improves significantly when we shift to the renormalization order.

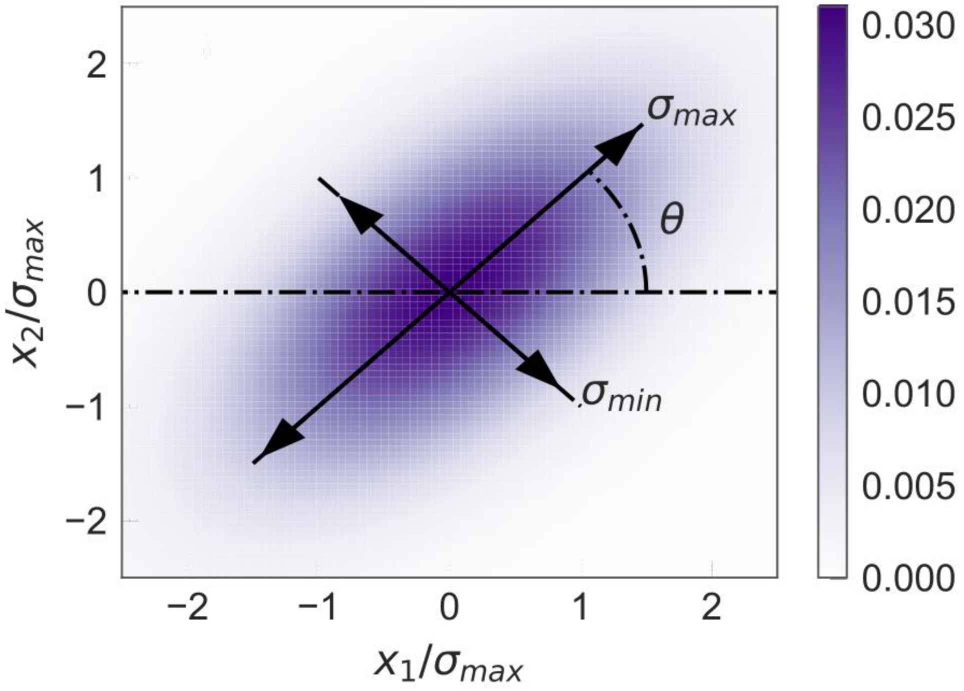
# Higher dimensions






Scales up to higher dimensions. Maximum entanglement grows as 2^d, d=dimension

# Quantum Numerical Analysis




*Q. Algorithm*

$$|\psi\rangle = \sum_{\{s_1,s_2\}} f(x_{s_1}x_{s_2})|s_1,s_2\rangle, s_i \in 2^m - 1$$

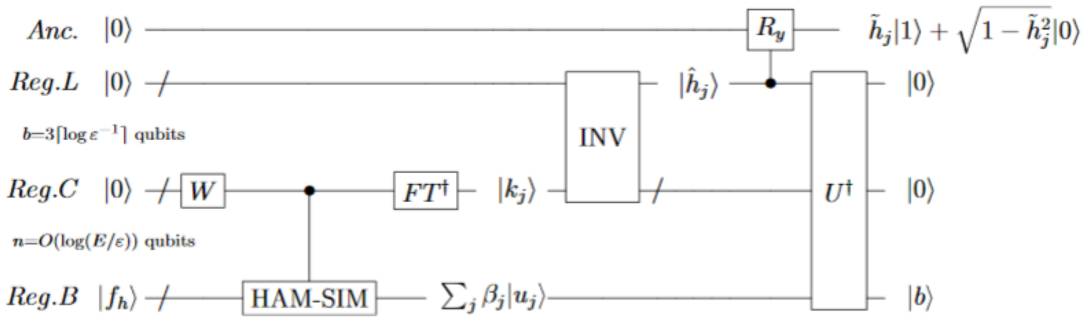
*Q. Algorithm*

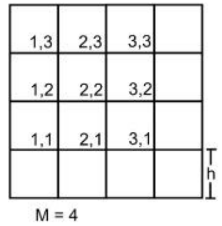
Expected values, risk analysis, PDE's

Encode, manipulate and interrogate multivariate functions in quantum registers.

# Algorithms for Poisson equation...

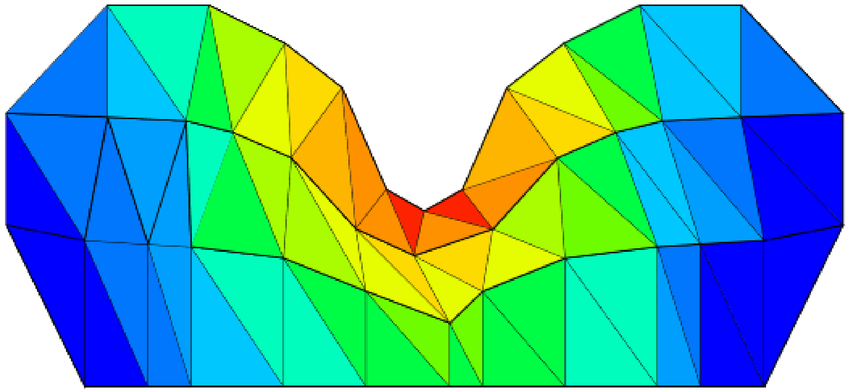
*Quantum algorithm and circuit design solving the Poisson equation*
Y. Cao et al, NJP 15 (2013)




Implement Poisson operator in finite differences using Hamiltonian simulation; invert eigenvalues to obtain solution.

*Quantum algorithms and the finite element method*
A. Montanaro, S. Pallister, PRA 93 (2016)



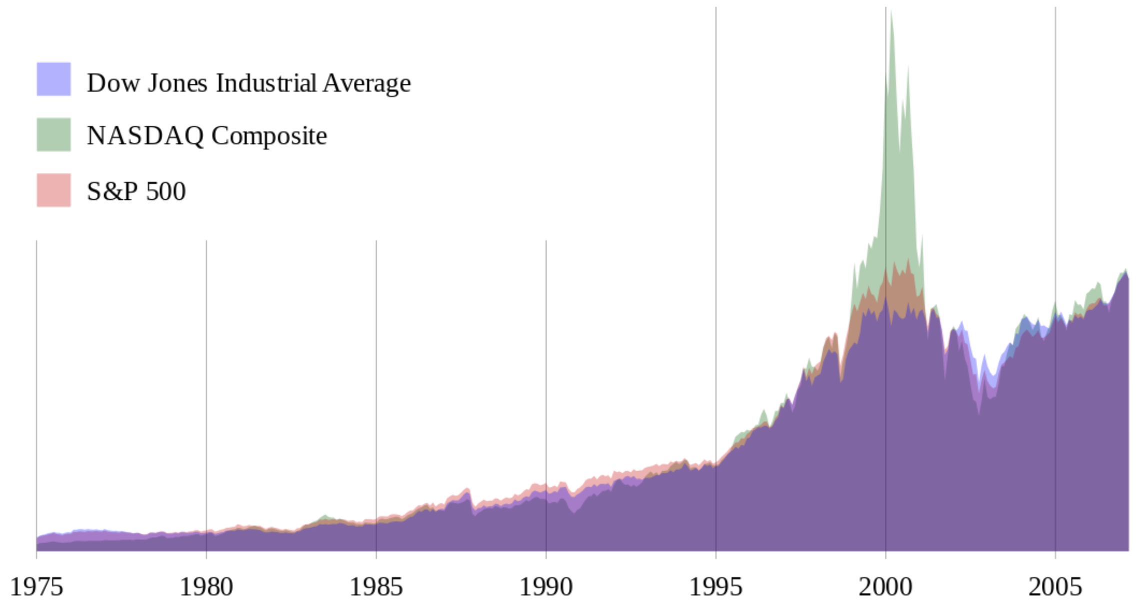
$$u(x_s) = (H^{-1})_{ss'}\, f(x_{s'})$$

Use finite elements to construct H, f; use HHL to invert problem

$$-\Delta u(x) = f(x)$$

# ...and in quantum finance

## Stock market




## Probability distribution

$$p(s_1, s_2, \dots, s_n)$$

## Quantum register

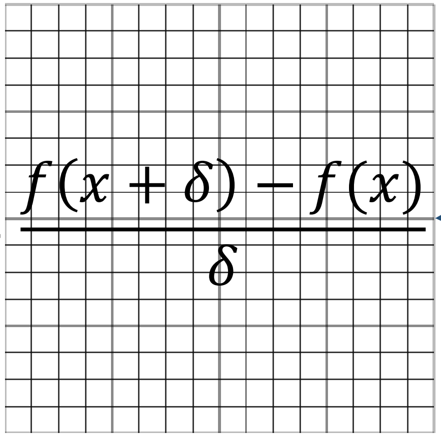
$$|\psi\rangle = \sum_s \sqrt{p(s_1, \dots, s_n)}|s_1, \dots s_n\rangle$$

## Amplitude estimation

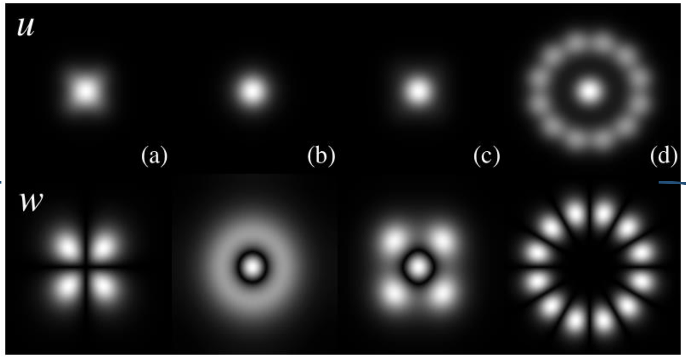
$$\langle s_i \rangle, \langle s_i s_j \rangle, \dots$$

# Many classical encodings

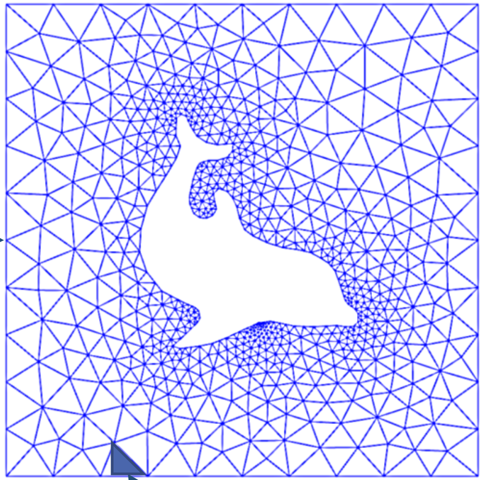
## Finite differences



$$\frac{df}{dx} \simeq \frac{f(x+\delta) - f(x)}{\delta}$$

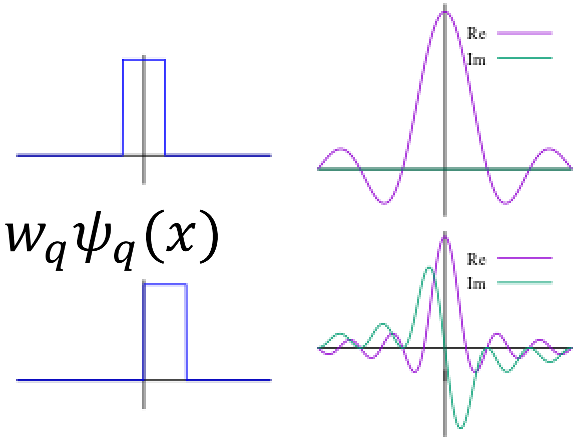



## Finite element



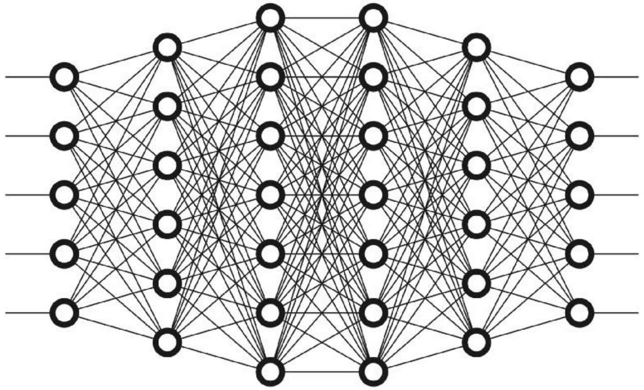
$$f(x)_m \simeq \sum_n c_n^m x^n$$

## Spectral method




$$f(x) = \sum w_q \psi_q(x)$$

## Variational methods



$$f(x_1, x_2, \dots)$$

# Algorithms

| Problem | Algorithm | Type | Cost |
|---|---|---|---|
| Expected value | Monte Carlo | C | $\mathcal{O}(1/\varepsilon^2)$ |
| " | Amplitude estimation | Q | $\mathcal{O}(1/\varepsilon)$ |
| " | MPS | QI | $\mathcal{O}(-N\chi^3\log(\varepsilon))$ |
| Fourier transform | QFT | Q | $\mathcal{O}(N^2m^2)$ |
| " | FFT | C | $\mathcal{O}(Nm2^{Nm})$ |
| " | MPS QFT | QI | $\mathcal{O}(Nm \times \mathrm{Simp}_{Nm})$ |
| Interpolation | Linear $(k=1)$ | C | $\mathcal{O}(2^{Nm})$ |
| " | MPS Linear $(k=1)$ | QI | $\sim \mathrm{Simp}_{Nm}$ |
| " | FFT | C | $\mathcal{O}(N(m+k)2^{N(m+k)})$ |
| " | MPS QFT | QI | $\sim 3 \times \mathrm{QFT}_{N(m+k)}$ |
| PDE Evolution | Finite differences | C | $\mathcal{O}(T_{\mathrm{cgs}}2^{2Nm})$ |
| " | MPS differences | QI | $\mathcal{O}(T_{\mathrm{cgs}} \times \mathrm{Simp}_{Nm})$ |
| " | FFT method | C | $\mathcal{O}((Nm+1)2^{Nm})$ |
| " | MPS QFT | QI | $\sim 2 \times \mathrm{QFT}_{N(m+k)}$ |
| State construct | GR-like (Sect. 3.3) | Q | $\mathcal{O}(Nm\chi^2)$ |
| " | Explicit wavefunction | C | $\mathcal{O}(2^{Nm})$ |
| " | MPS | QI | $\mathcal{O}(T_{\mathrm{steps}} \times \mathrm{Simp}_{Nm})$ |

*JJGR, arXiv:1909.06619*

# Fourier analysis

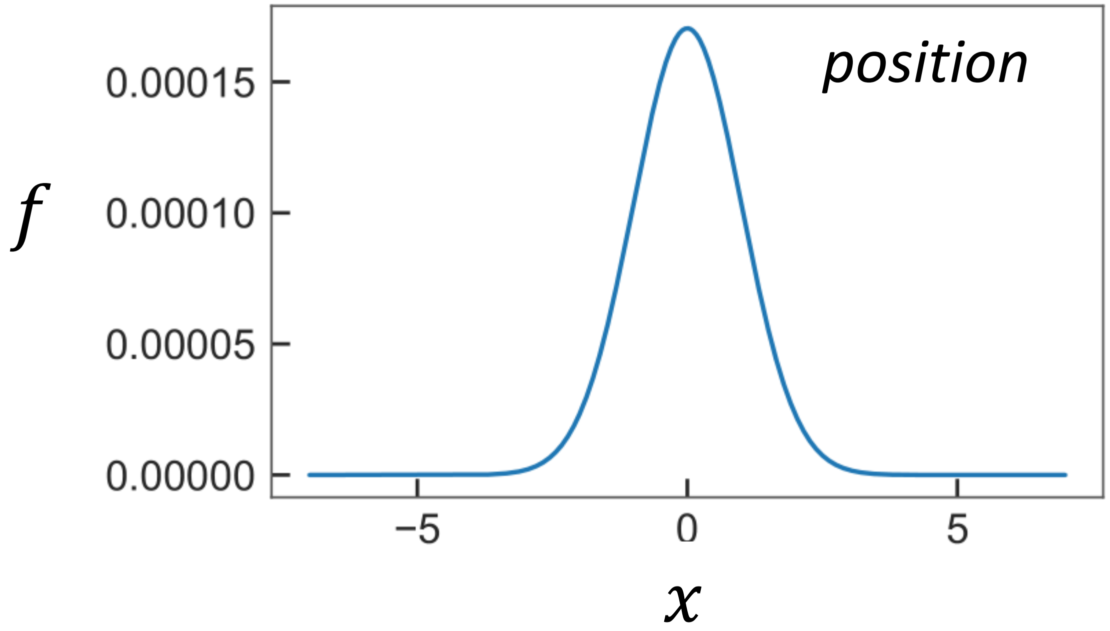
An expansion of functions in a harmonic basis

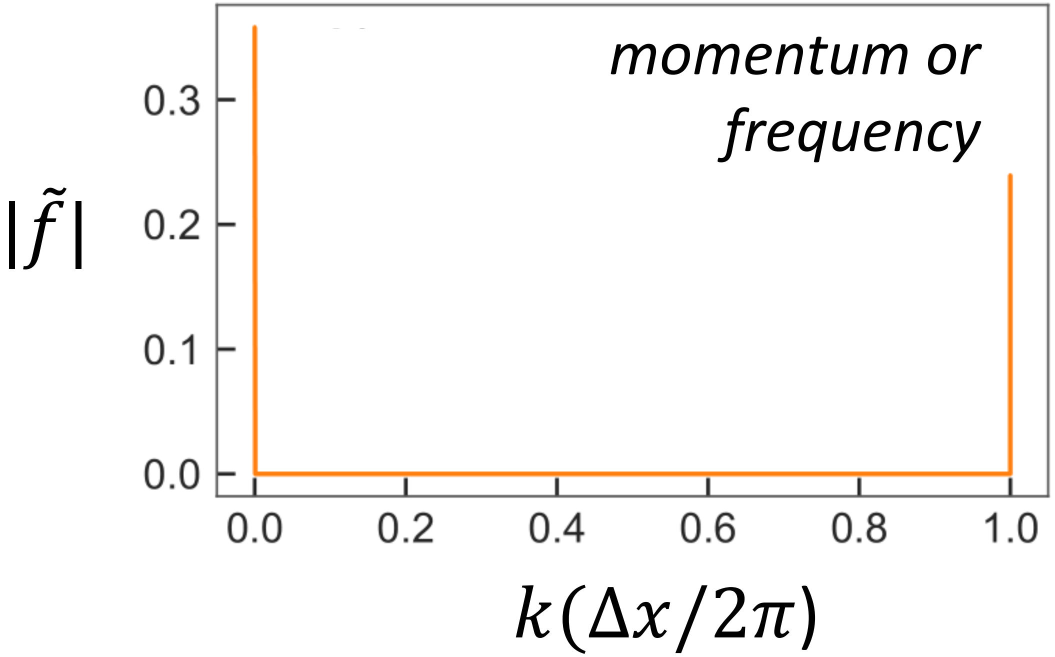
$$f(x_n) = \sum_{k_m} \tilde{f}(k_m) \frac{e^{-ik_m x_n}}{\sqrt{N}}$$

Direct and inverse transform

$$\tilde{\boldsymbol{f}}_m = (F\boldsymbol{f})_m$$
$$\boldsymbol{f}_n = \left(F^{-1}\tilde{\boldsymbol{f}}\right)_n$$

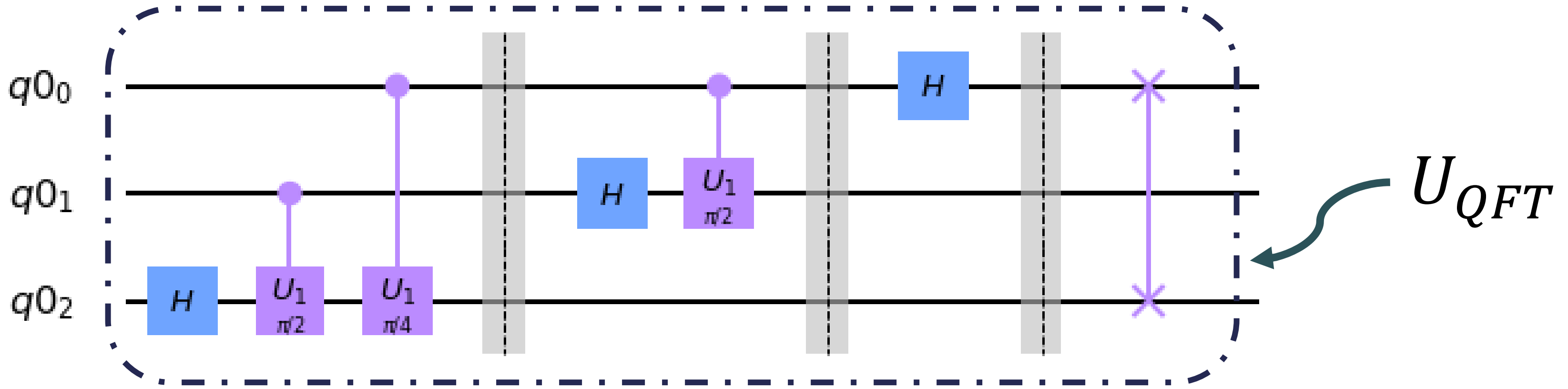
"F" is a unitary matrix!



# Quantum Fourier Transform

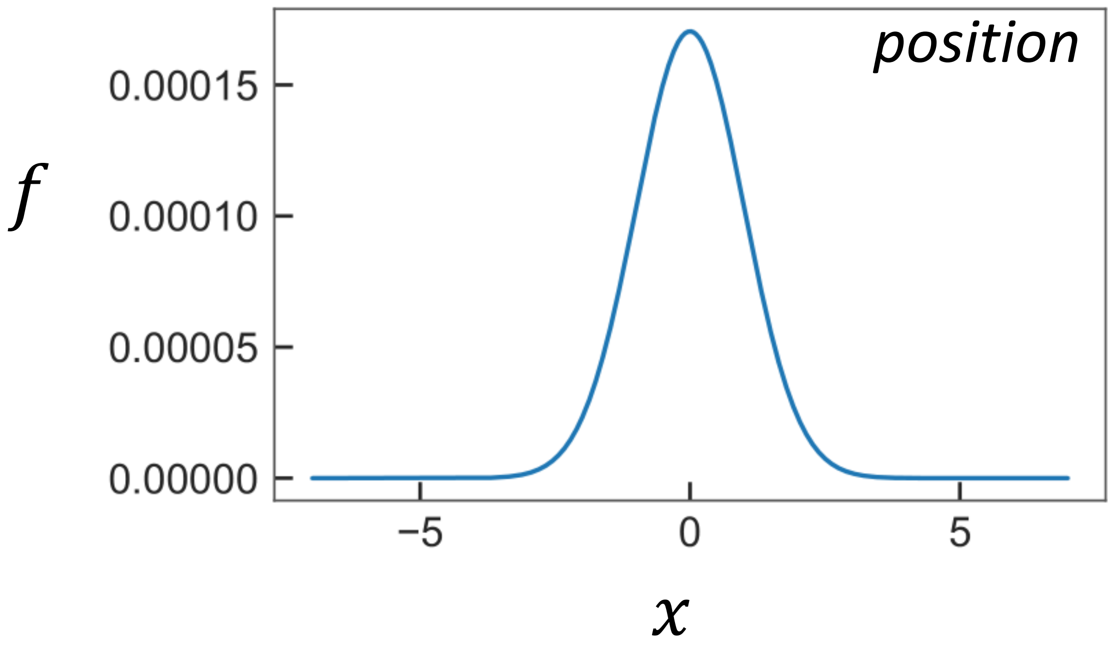
The Fourier transform is a unitary matrix. When dimension is a power of 2, it can be implemented using quantum gates

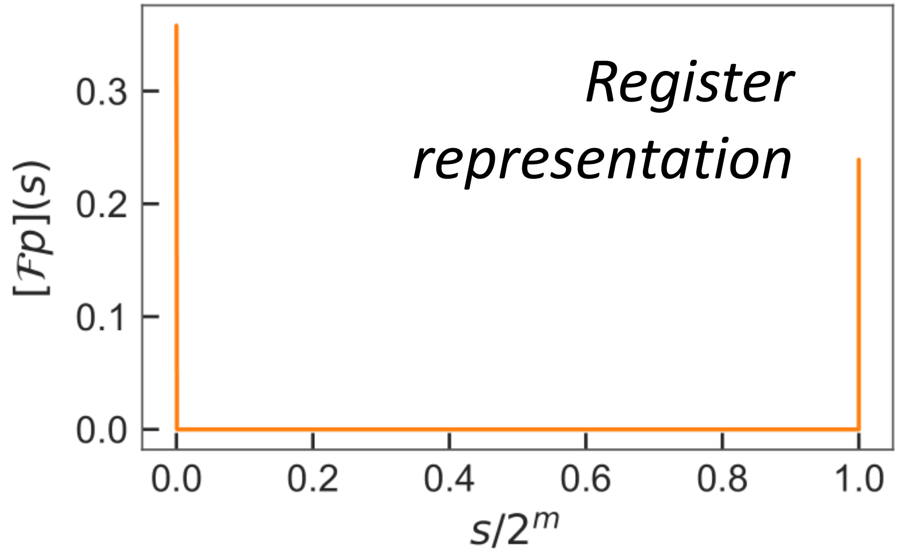


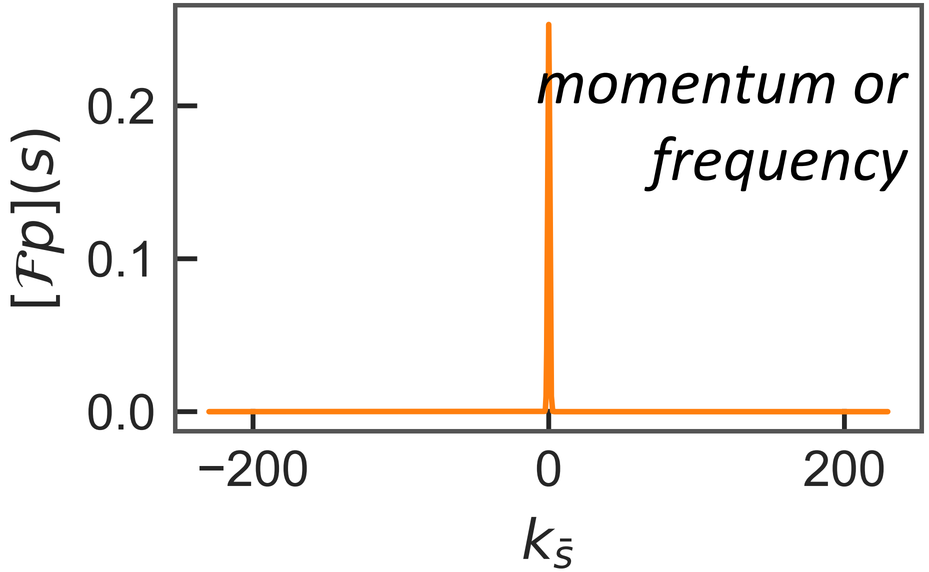

$$|f\rangle = \sum_s f(x_s)|s_1 s_2 \dots s_N\rangle \rightarrow |\tilde{f}\rangle = \sum_s \tilde{f}(k_s)|s_1 s_2 \dots s_N\rangle$$

$$|\tilde{f}\rangle = U_{QFT}|f\rangle$$

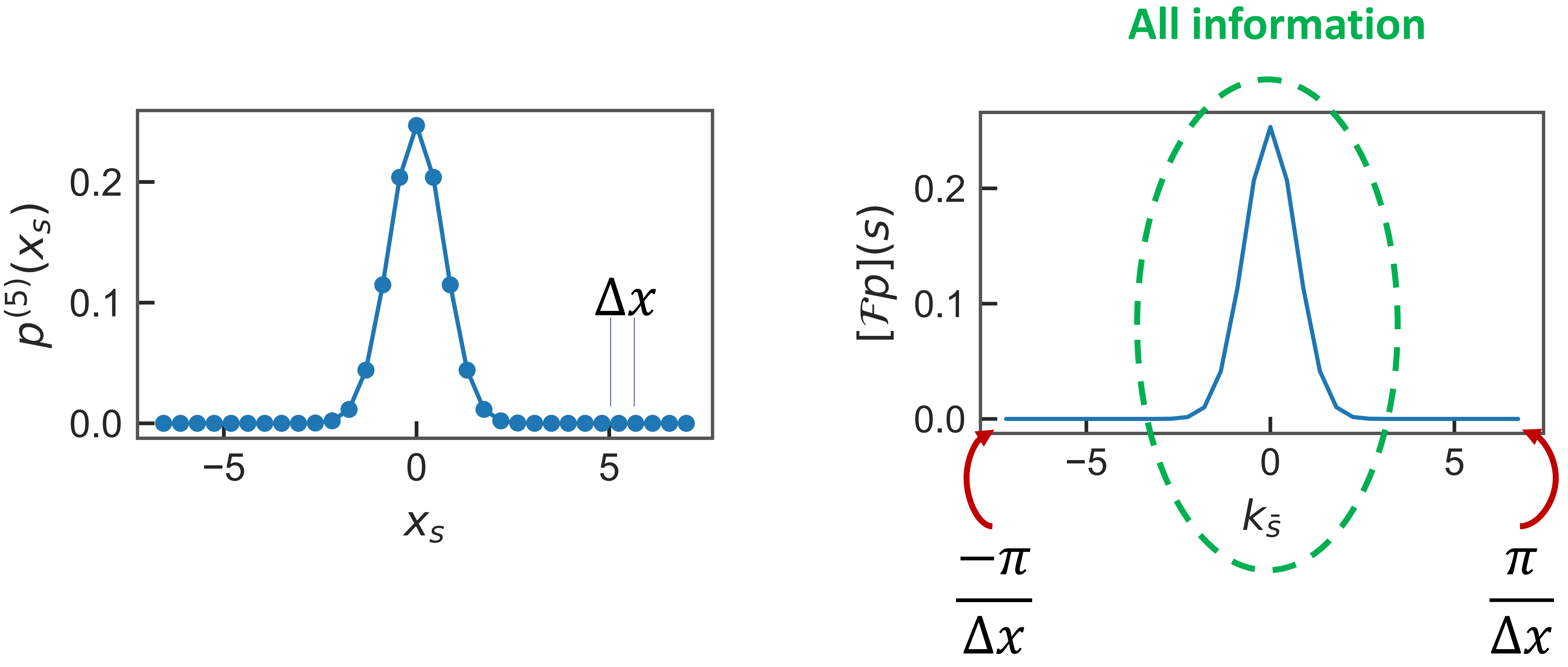
# A quirk of DFT encoding



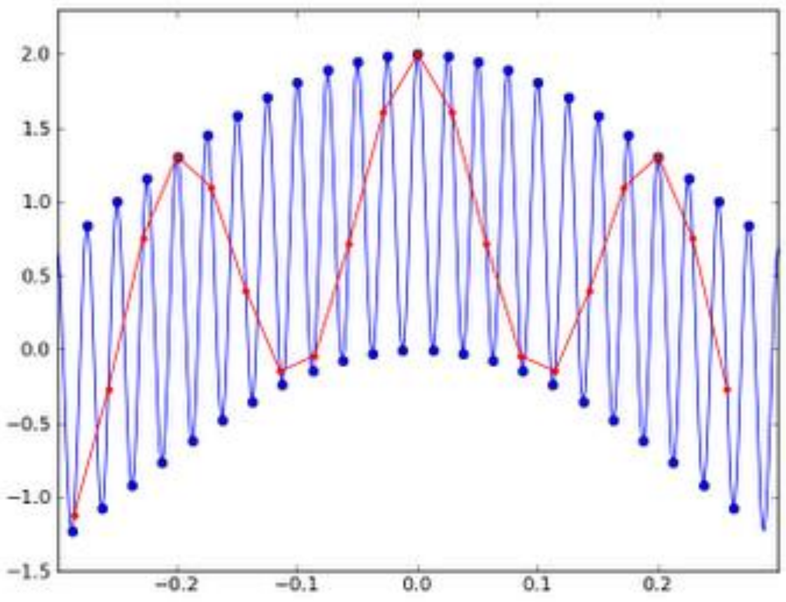



# Finite-bandwidth functions

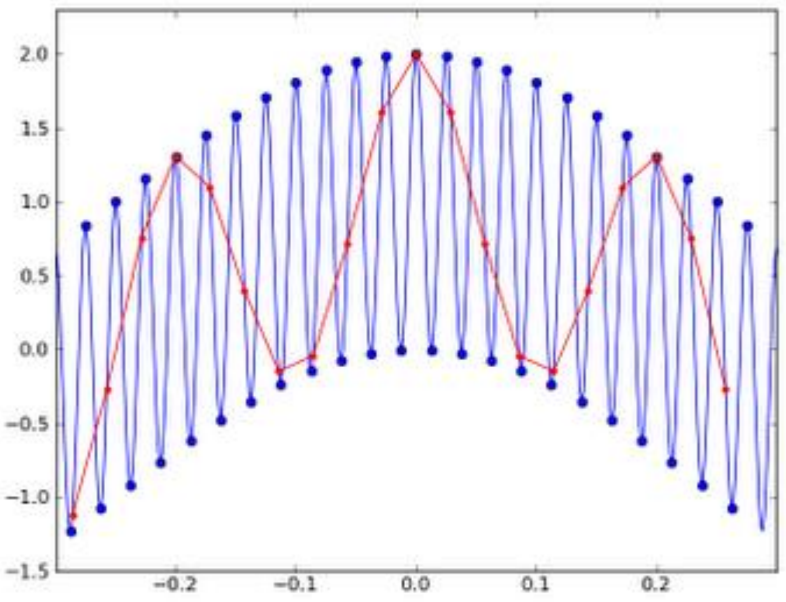


# Nyquist-Shannon sampling theorem

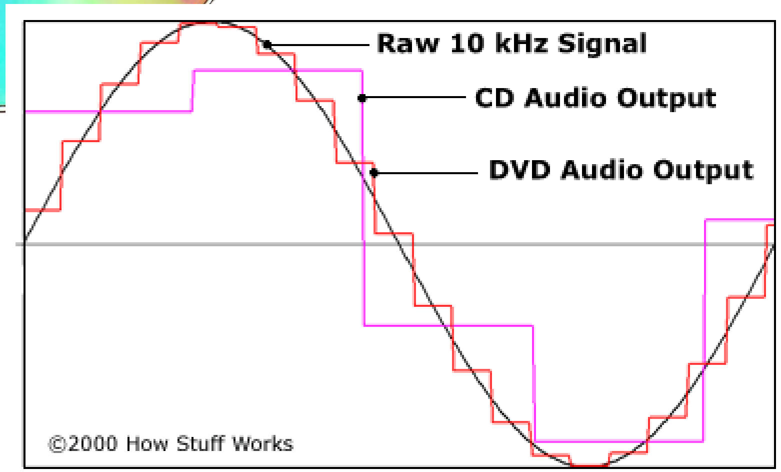



$$\nu_{sampling} \geq 2\nu_{signal}$$

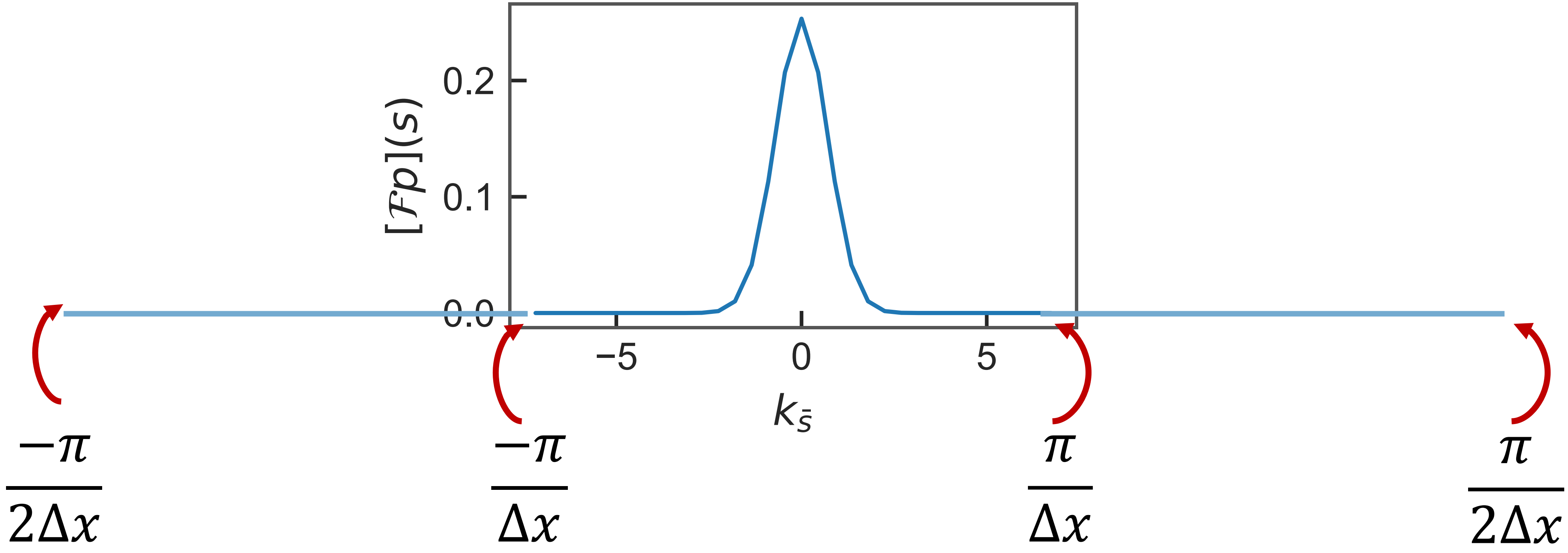
# Nyquist-Shannon sampling theorem




$$\nu_{sampling} \geq 2\nu_{signal}$$

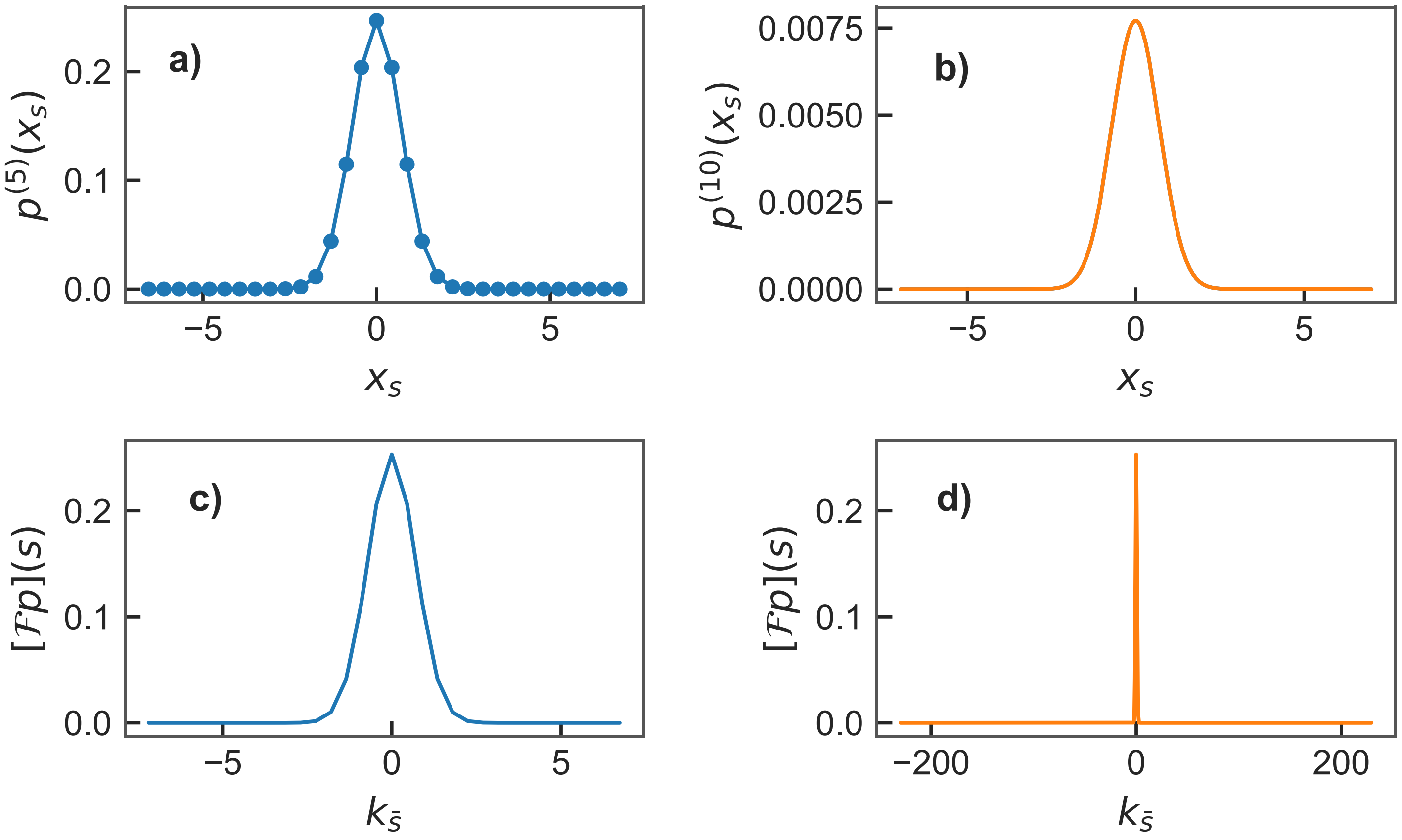


# Interpolation in frequency space




By extrapolating (with zeros) we get the Fourier transform of a function defined with finer resolution

# Fourier interpolation




$$|p^{(m+k)}\rangle = \hat{\mathcal{F}}^{(m+k)} U_{2c}^{m+k} \left[ \left( U_{2c} \hat{\mathcal{F}}^{(m)} \, |p^{(m)}\rangle \right) \otimes |0_2, 0_3, \ldots, 0_{k+1}\rangle \right]$$

*JJGR, arXiv:1909.06619*

# Solving differential equations

Partial differential equation (e.g. Schrödinger!)

$$-\frac{d^2}{dx^2} f(x) + V(x) f(x) = E f(x)$$

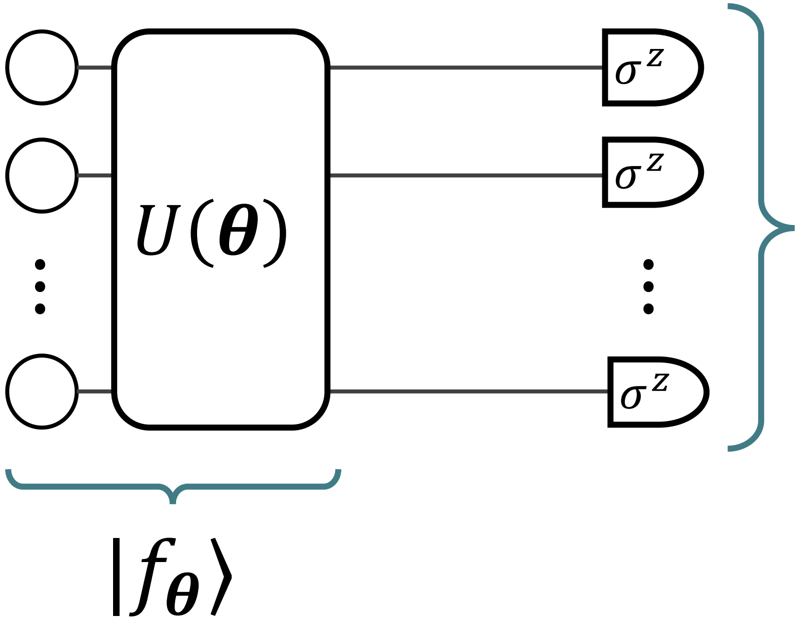
Equivalent variational formulation for min. E

$$\min_{f(x)} \int f(x)^* \left[ -\frac{d^2}{dx^2} + V(x) \right] f(x) dx$$

We need to compute

$$\langle V \rangle_f = \int f(x)^* V(x) f(x) dx \,, \qquad \langle \partial_x^2 \rangle_f = \int f(x)^* \frac{d^2}{dx^2} f(x) dx$$

# Variational integration




Measurement outcomes (random variables)

$$\{s_1^m, s_2^m, \dots, s_N^m\}, m = 1 \dots M$$

Position “value” (random variable too)

$$x^m = \sum_i \frac{\Delta x}{2^i} s_i^m, m = 1 \dots M$$

Average potential (unbiased estimator)

$$\overline{V(x)} = \frac{1}{M} \sum_m V(x^m) = \langle V(x) \rangle + O\left(\frac{1}{\sqrt{M}}\right)$$

# Differentiation

Fourier interpolation associates a continuous function to discrete values

$$f^{interp}(x) = \sum_{k_m} \tilde{f}(k_m) \frac{e^{-ik_m x}}{\sqrt{N}} \simeq f(x)$$

We can use this to estimate derivatives

$$\frac{\mathrm{d}}{dx} f(x) \simeq \frac{d}{dx} f^{interp}(x) = \sum_{k_m} (-ik_m) \tilde{f}(k_m) \frac{e^{-ik_m x}}{\sqrt{N}}$$

# “Measure derivatives”

Remember that our “momentum” operator is a combination of bits

$$k_s = \frac{2\pi}{\Delta x}\left(\frac{1}{2}s_0 + \frac{1}{4}s_1 + \cdots\right)$$

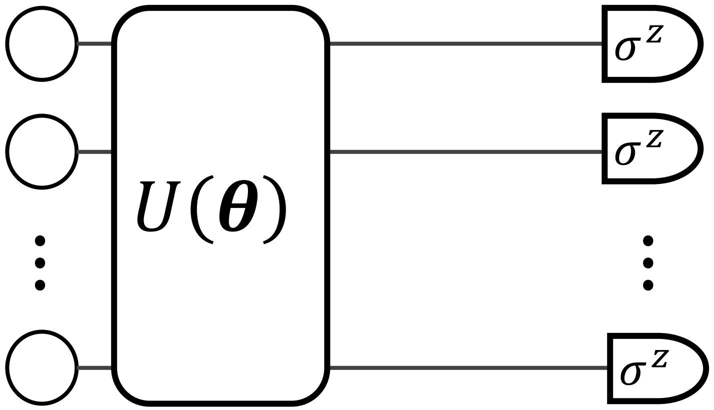
We can use this to compute some interesting integrals

$$\int f(x)\frac{\mathrm{d}}{dx}f(x) \simeq \sum_{k_m}(-ik_m)\left|\tilde{f}(k_m)\right|^2$$

On the quantum computer we can estimate these integrals with the QFT state

$$\int f(x)\frac{\mathrm{d}}{dx}f(x) \simeq -i\frac{2\pi}{\Delta x}\left(\frac{1}{2}\langle\tilde{f}|\sigma_0^z|\tilde{f}\rangle + \frac{1}{4}\langle\tilde{f}|\sigma_1^z|\tilde{f}\rangle + \cdots\right)$$

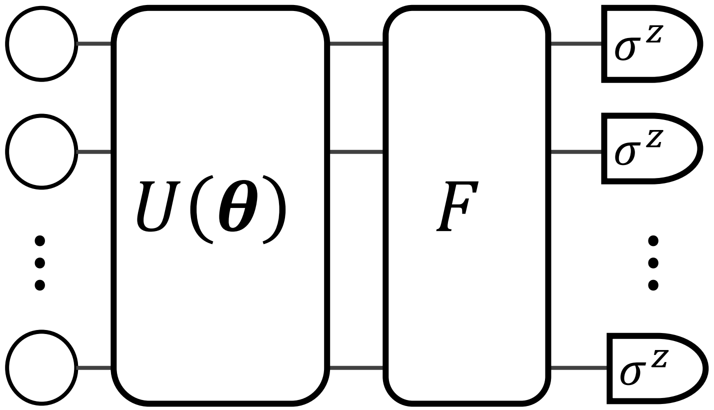
# Variational formulation




$$|f_{\boldsymbol{\theta}}\rangle$$

$$\langle \tilde{f}_{\boldsymbol{\theta}}|\sigma_i^z|\tilde{f}_{\boldsymbol{\theta}}\rangle, \langle \sigma_i^z \sigma_j^z\rangle, \ldots$$

$$\langle V(x)\rangle = \int f_{\boldsymbol{\theta}}(x) V(x) f_{\boldsymbol{\theta}}(x)$$

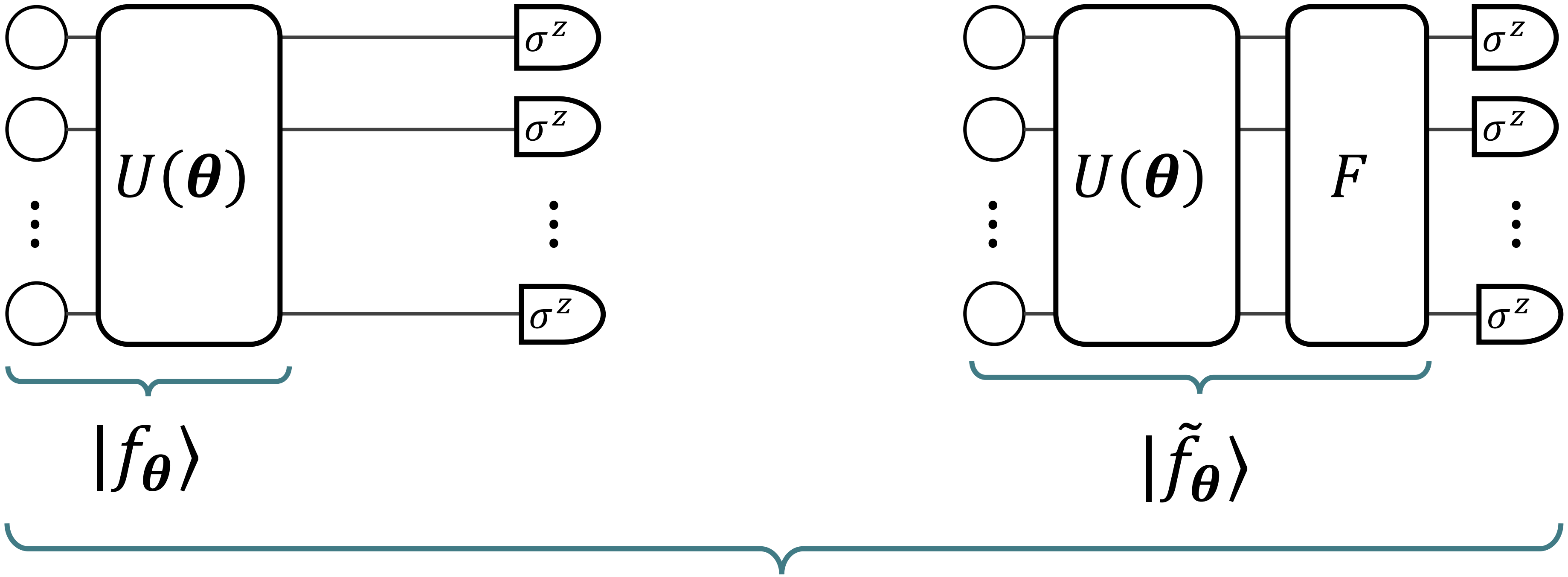



$$|\tilde{f}_{\boldsymbol{\theta}}\rangle$$

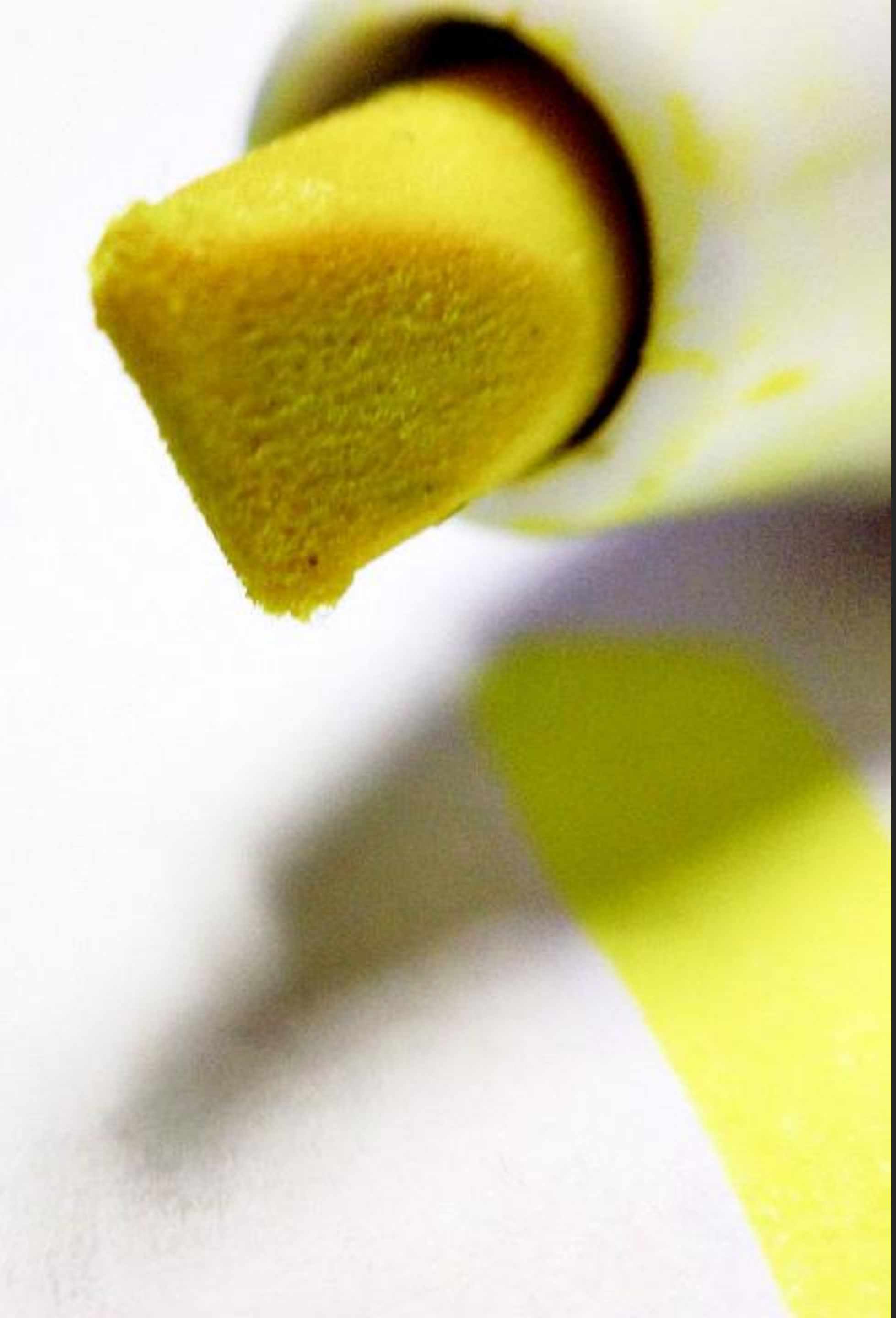
$$\langle \tilde{f}_{\boldsymbol{\theta}}|\sigma_i^z|\tilde{f}_{\boldsymbol{\theta}}\rangle, \langle \sigma_i^z \sigma_j^z\rangle, \ldots$$

$$\langle -i\partial_x\rangle = \int f_{\boldsymbol{\theta}}(x)\left(-i\,\frac{\mathrm{d}}{dx}\right) f_{\boldsymbol{\theta}}(x)\,, \langle \partial_x^2\rangle, \ldots$$

# Variational formulation




$$\operatorname{argmin}_{\boldsymbol{\theta}} \left\langle f_{\boldsymbol{\theta}}(x) \left| -\frac{d^2}{dx^2} + V(x) \right| f_{\boldsymbol{\theta}}(x) \right\rangle$$



# Main ideas

- Quantum registers can encode efficiently multivariate functions.
- Quantum computers can construct, manipulate and interrogate functions.
- There are many algorithms with a potential gain, even if <u>not exponential</u>.
- Applications to other domains: numerical analysis, finances, Physics, engineering…

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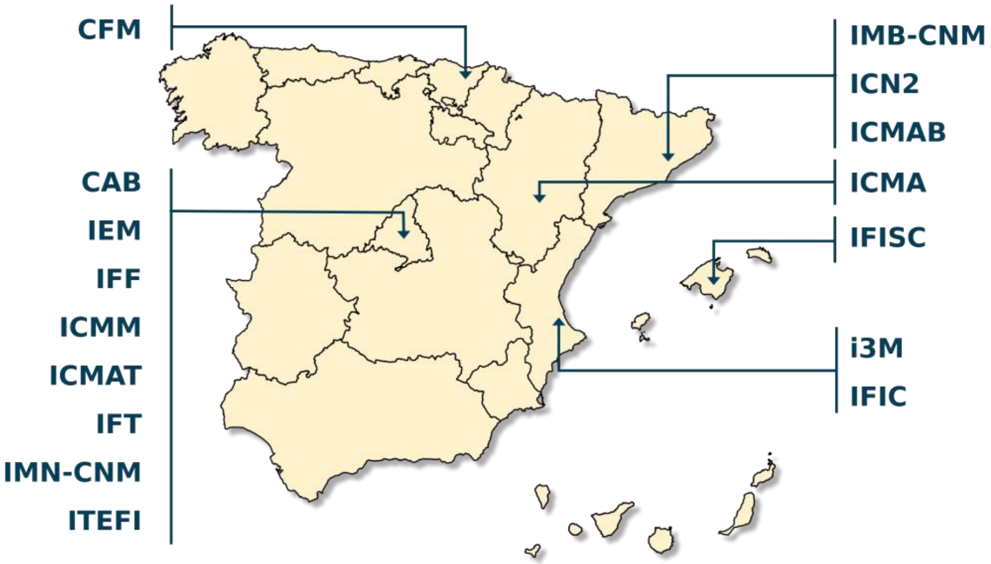
CSIC

**QTEP** ABOUT RESEARCH NEWS IBM-Q


## CSIC enters the IBM-Q network

CSIC and IBM have signed a contract to allow researchers at CSIC and Universidad Autónoma de Madrid to work with IBM's superconducting quantum computers, collaborate in training a new generation of quantum software developers and translating quantum computing technologies to Spain's industrial ecosystem. This agreement was celebrated in an event that took place at CSIC's...

**Continue Reading**

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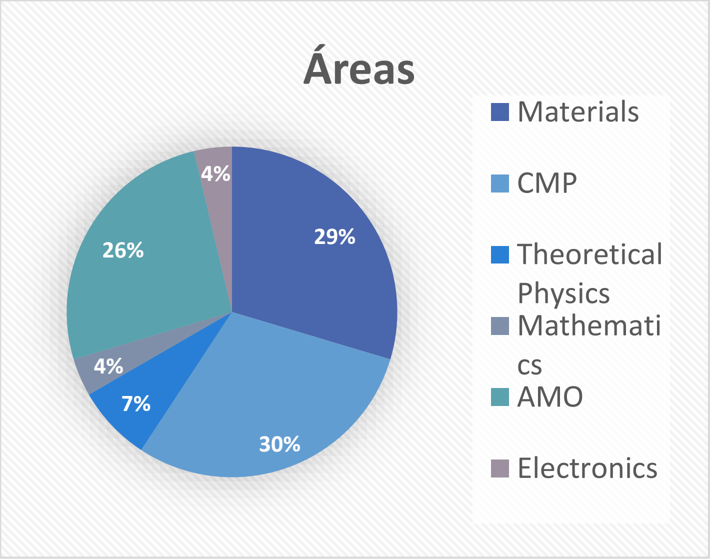

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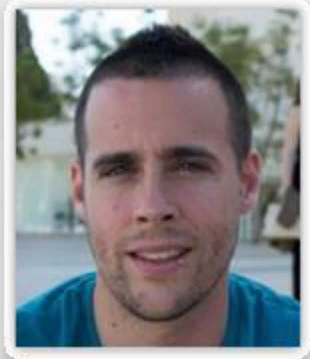
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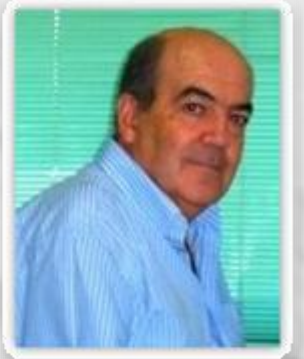
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Diego G. Olivares

Thanks!