The Ultimatum Game in Complex Networks

Network Science

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1. Abstract

In this report, it is explored the evolution of the altruistic behaviour networks by making use of repetitive play of the Ultimatum Game around them. It was simulated different Scale-Free (SF) and Erdós–Rény (ER) networks of players and verified how the game is played over time when there is different types of players: Empathetic (offers as much as it is willing to receive), Pragmatic (the value that it offers and the value that it is willing to receive has an inverse relation), and Independent (the values when making the offer and deciding to either accept it or not are independent from each other). (Sinatra et al., 2009)[1]

2. KeyWords

Network Dynamics, Game Theory, Mathematical Economics, Socio-Economic Networks

3. Introduction

This project has been developed in the Complex Networks course at Instituto Superior Técnico (IST) in Lisbon, Portugal by following the work done by Sinatra et al. in "The Ultimatum Game in complex networks" in 2009. The aim of this project, based on Sinatra's work, is to simulate different networks with different types of player and see how its' behaviour evolves by making use of different update rules.

[1] See http://complex.unizar.es/~jesus/pub_files/Jstat09.pdf

The baseline for this project is a network where each node represents a player. "The networks considered in this study individuals can have a homogeneous number of neighbors (Erdós–Rényi graphs) or, in contrast, present a high degree of heterogeneity in the number of contacts (scale-free networks)" (Sinatra et al., 2009). The population will, initially, have an uniform distribution of the different strategies. The players will play the Ultimatum Game multiple rounds against their own neighbours and the strategies on the network will be changed according to predefined updating rules.
The Ultimatum Game, as an experiment, reflects how humans react when faced with unfair situations and their willingness to accept those situations.

4. Playing the Game

Ultimatum game is an experimental economics game played by 2 individuals. On one hand, the first individual (Proposer) has a certain amount of money and has to propose an offer to the second player (Responder) containing a p value. If the second player accepts it, receives p. The Proposer receives the difference between the initial money and the p that it gave away. On the other hand, if the Responder rejects it, both players receive nothing.
Knowing this, we can formulate the payoffs that each player receives during each game. Assuming that the initial amount of money is 1, we have:

-If the Responder accepts the offer:

$$ProposerPayoff = 1 - p$$

$$ResponderPayoff = p$$

-If the Responder rejects the offer:

$$ProposerPayoff = ResponderPayoff = 0$$

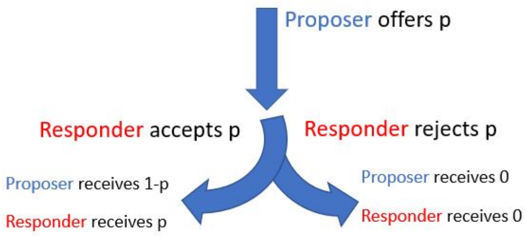

Figure 1: Ultimatum Game Diagram

4.1. Player Behaviours

In each network, every player will have a certain individual strategy of how to play the game. Every player has access to 2 values during its game. It has a p value which is the value that it, as the Proposer, offers to the second player. Also, when playing as the Responder, it has access to a q value which represents the minimum threshold from where it accepts a p offer from the Proposer. This means that if the offer is smaller than the q value, it will reject the offer.
This project supports 3 different types of players:

Empathetic

The Empathetic player i is a player that in its strategy is willing to give to others as much as it is willing to receive.

$$p_i = q_i$$

Pragmatic

The Pragmatic player i does not make a distinction between roles. This way, the Pragmatic player is a player that wants to receive the same in every situation.

$$p_i = 1 - q_i$$

Independent

The Independent player does not have a relation between the value that it offers and the least amount that is willing to receive. This way p and q are independent from one another.

4.2. Update Rules

Since this project aims to analyze the evolution of the strategies and behaviours over time, there was implemented 2 update rules which can be applied individually or together.

Natural Selection

Natural Selection is based on the premises that if someone sees an individual that is performing better than him, it will copy that individual's behaviour to try perform as well or, at least, survive in the long run. This update rule is based on the same formula as in Sinatra's work. At the beginning of each round, before playing against its neighbours, each players selects one random neighbour. After selection, it verifies if the neighbour performed better or worse than it in the previous round. If it did not perform as well, it will remain playing with the previous strategy. Otherwise, the player will adopt the neighbour's strategy with a proportional probability by how much the neighbour strategy is better than its own. Assume that the players payoff is $Payoff_i$ and the neighbours payoff is $Payoff_n$. With this in mind, the probability of using the neighbour's strategy is the follow:

$$Probability = \frac{Payoff_n - Payoff_i}{2max\{k_n, k_i\}}$$

where k_n and k_i are the amount of neighbours of the random neighbour and the player i, respectively.

Social Penalty

Social Penalty is a slower update rule which the premises is that the individuals do not evolve, but the strongest persists. This way, this update rule dictates that, at the end of each round, the player that had the worst performance is eliminated as well as its neighbours. The deletion process in this scenario consists in deleting the strategy of that player and switch for one that is random.
In this experiment, the performance of the players is evaluated round by round, meaning that they do not have a memory of how they performed previously and it has no impact in the more distant generations.

5. Simulating and Results

Due to computational power and time constraints, the simulations were done through 1000 iterations and being composed by 500 players. This relative small sample opens a door to some failures or simulations that might not represent the expected theory results. In a nutshell, the model created supports both ER and SF networks, 3 different types of players behaviours and 2 different update rules that can be used at the same time. The strategies values (p and q) were discretized in 20 intervals of 0.05.

5.1. Natural Selection with Empathetic and Pragmatic Behaviour Players

It was done multiple simulations where SF and ER networks were analyzed with players with Empathetic and Pragmatic behaviours, making use of the natural selection update rule.

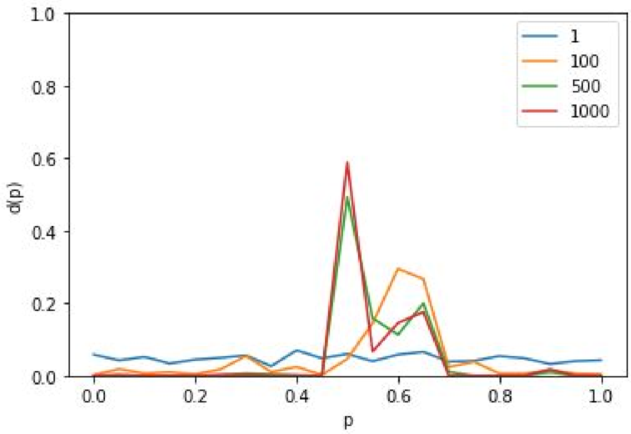

Figure 2: ER network with Empathetic behaviours

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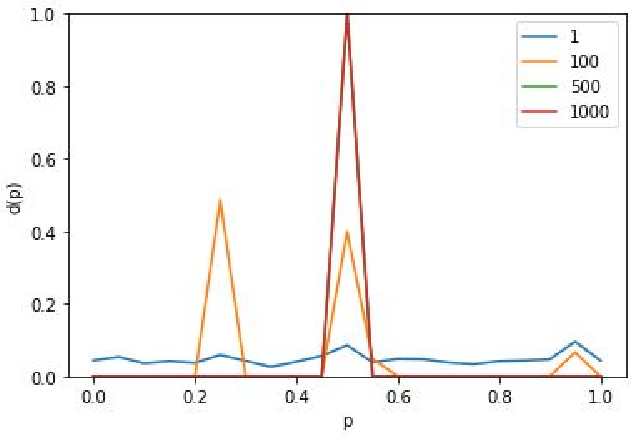

Figure 3: SF network with Empathetic behaviours

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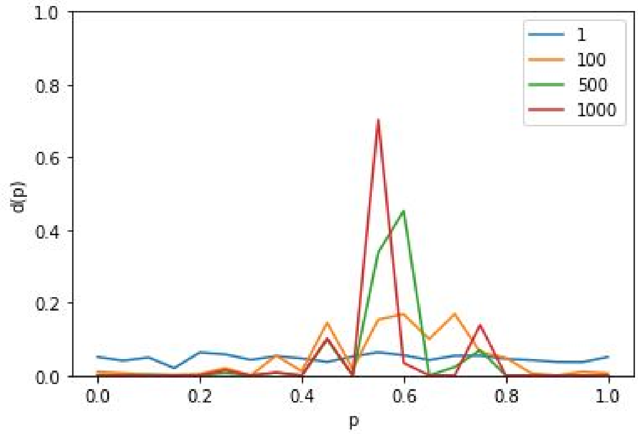

Figure 4: ER network with Pragmatic behaviours

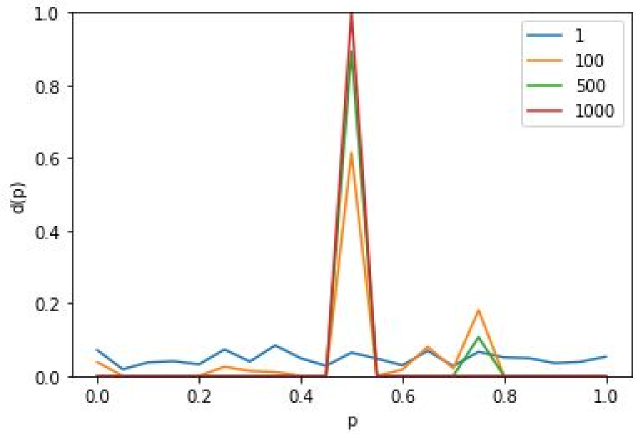

Figure 5: SF network with Pragmatic behaviours

From the analysis of the results, we can assume two main points that will be confirmed during other experiments:

-The convergence of values is different depending on the type of network;

The Scale Free networks provide a more heterogeneous distribution of neighbours which allows for a quicker convergence to the closest ideal strategy in the network or, at least, the strategy that ends up being widely selected is adopted quicker. This can be supported by Figure 1 through 4 which shows, on the Scale Free networks, an almost 100% acceptance of a strategy around the network after 1000 iterations. To be noted that, with a wider network, a higher number of iterations would be needed, naturally.

-The distribution of the offer p, *d(p)*, converges to approximately 0.5 .

The distribution of p, over the updates, is around half of the initial value (0.5). In this case, this value represents what is called a fair value.

5.2. Natural Selection with Independent Behaviour Players

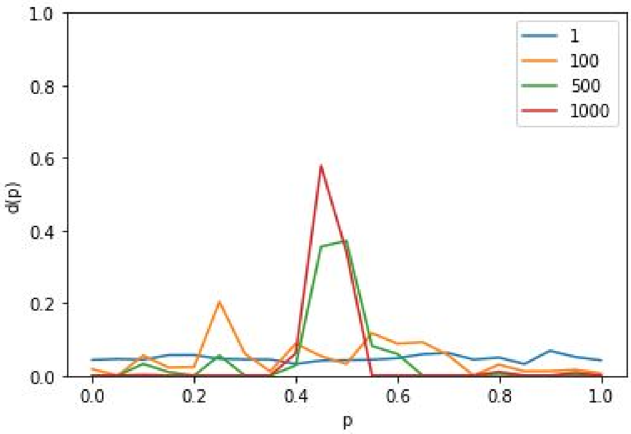

Figure 6: ER network with Independent behaviours

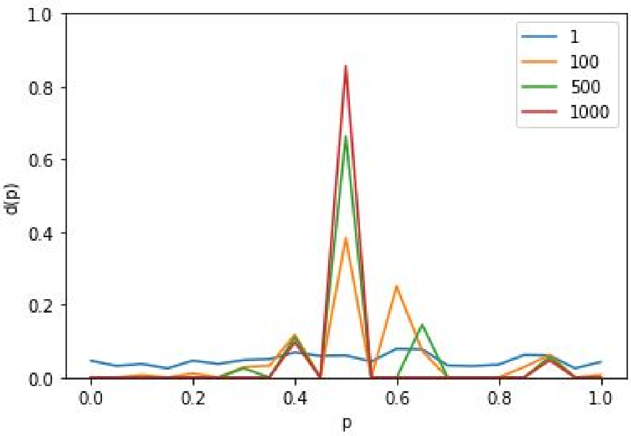

Figure 7: SF network with Independent behaviours

When dealing with Players with Independent behaviour, meaning that the p and q Values in their own strategies have no correlation between each other, the distribution of p Values tend to converge more slowly. The speed of convergence on the Scale Free network is still higher than Erdós–Rényi's network, but when comparing with networks containing players with Independent behaviours and networks containing players with Pragmatic and Empathetic behaviours, the networks with Independent behaviour players provides a slower strategy convergence around the network.

5.3. Social Penalty

It was simulated different networks making use of the Social Penalty update rule. This rule presented in the section 4.2 is considerably slower than the Natural Selection one. The main difference between Social Penalty and Natural Selection is that Natural Selection aims to reshape the strategies by making the players adapt by copying neighbours strategies whereas the Social Penalty update rule focus on the survival of a specific strategy. If that strategy did not perform well enough is removed from that player and so are the strategies from the neighbours around. The strategies removed are replaced with random ones, keeping the layout of network the same. This simulations, in order to reach higher iterations, the number of players were reduced to 100/200 and the game was played throughout 1 000 000 rounds.

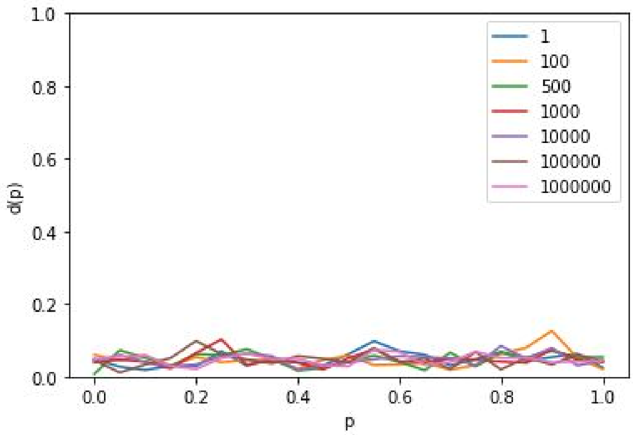

Figure 8: ER network with Empathetic behaviours

As shown in the Figure 8, this update rule did not perform as well in these conditions. The strategies values did not converge to any value. There are two main reasons for why this happen:

-The update rule is not adequate;

-The parameters of the network do not perform well all together (number of players might be too small)

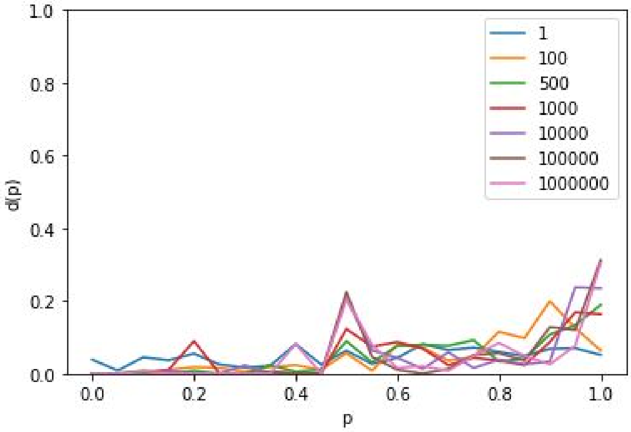

Figure 9: SF network with Empathetic behaviours

The Scale Free networks provided less diversity of strategies in the network after the same number of iterations. Even though it did performed better than ER networks, the values reflect a certain adoption of p=0.5 strategy and maximizing the p=1.0. This might happen due to the heterogeneity of neighbours which allows for a more aggressive removal of players. The nodes with a lower degree are more penalized due to their relative position on the network than their own strategy. This way, the nodes with a lower degree need to adopt a good strategy earlier in the iterations before the more central nodes start being penalized.

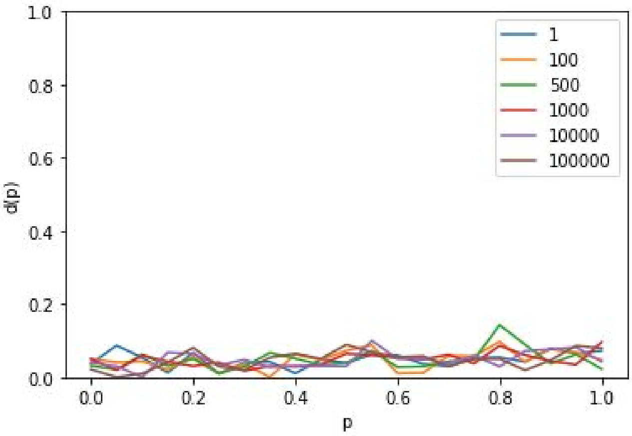

Figure 10: ER network with Pragmatic behaviours

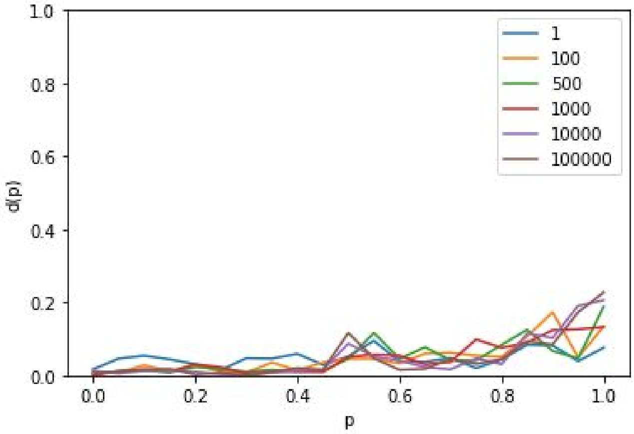

Figure 11: SF network with Pragmatic behaviours

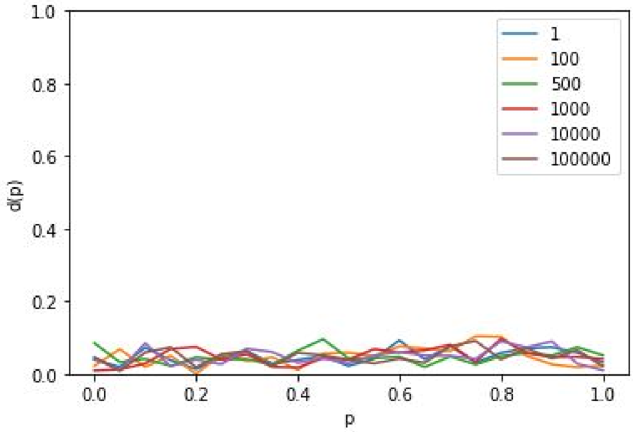

Figure 12: ER network with Independent behaviours

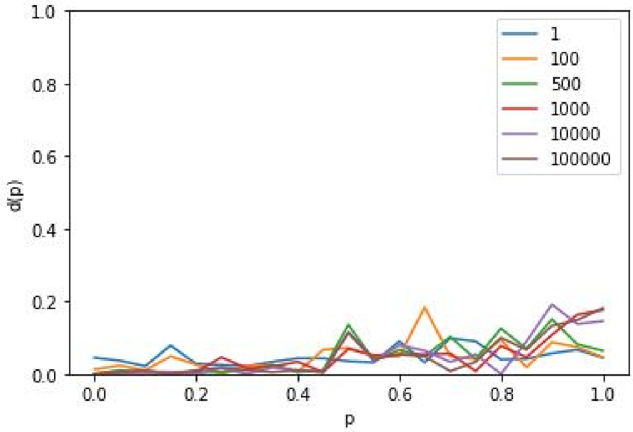

Figure 13: SF network with Independent behaviours

When changing only the types of players in the network from Empathetic to Pragmatic or Independent it is noticeable that the same behaviour occurs over the course of multiple iterations. Further experiments shown that when using both update rules in the model created, there is no convergence. This fact might indicate that Social Penalty or at least the way it was designed/implemented is not working as intended and does not allow the network to converge.

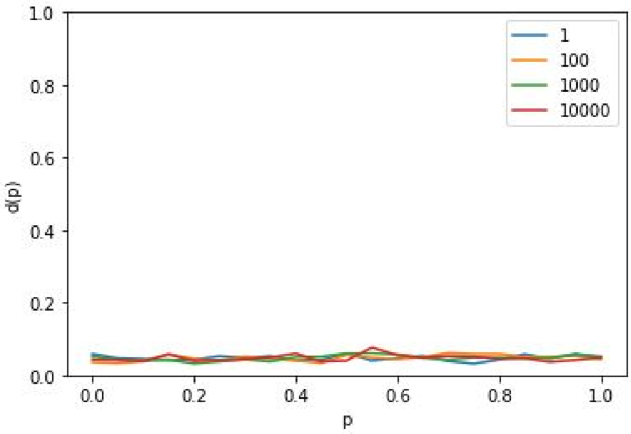

Figure 14: ER network with Empathetic behaviours with both Updating Rules

6. Networks with Multiple Player Behaviours

Sinatra wrote in his article that it would be interesting "incorporating the competition between different kinds of individuals". This section provides a explanation of the approach done. The previous model stayed almost the same, but now there is an equal change of a player in the network having a Empathetic, Pragmatic or Independent behaviour. When using the Natural Selection the players will verify if the random neighbour had a more profitable round than itself. If it did, it will adopt the neighbour's strategy using the same formula as the original model. If nothing more was changed, the behaviour around the network would converge to either one of the three discussed previously. To prevent that, if the player would use the neighbour's strategy, it will copy the p Value. Then, if the player is Empathetic, its q Value would be equal to p. If it is Pragmatic, it would have q being $1\text{-}p$ and, finally, if it is independent, it would copy both p and 1 since the relation between p and q would not matter for that player.

On other experiment it was assumed that the player would be able to change its beliefs. This means that it could, by comparing itself with its neighbours, start believing that it should earn different amounts depending if it is the Proposer or not, against believing that it should always receive the same amount. For that experiment, the Player would simply copy the values and player type from its neighbour. On this section, it is only demonstrated the results using Natural Selection since it was the more promising updating rule from the results with only one player behaviour in the network.

6.1. Natural Selection

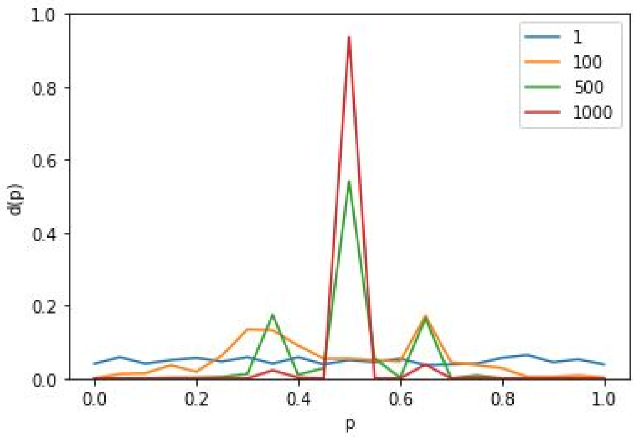

Figure 15: ER network with Natural Selection and no beliefs change

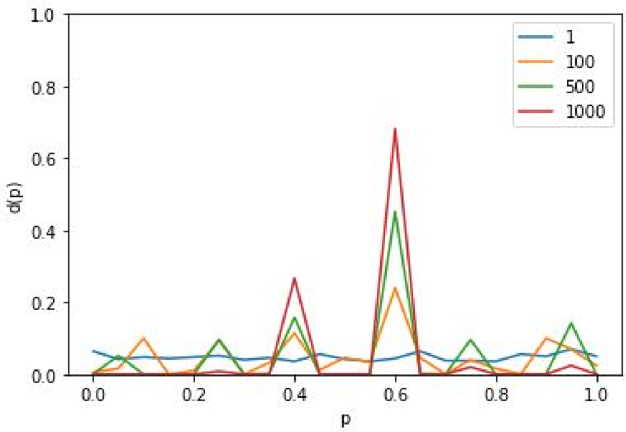

Figure 16: SF network with Natural Selection and no beliefs change

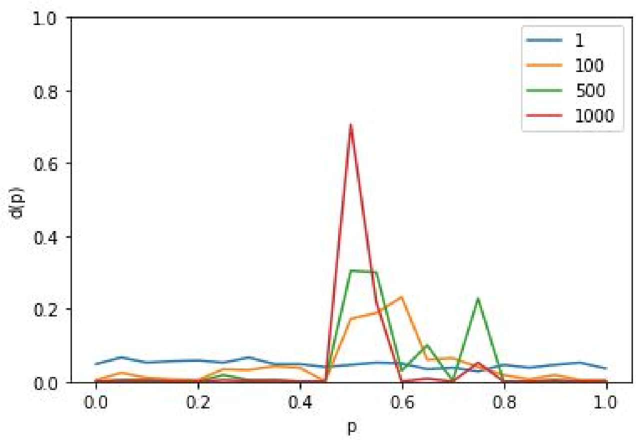

Figure 17: ER network with Natural Selection and beliefs change

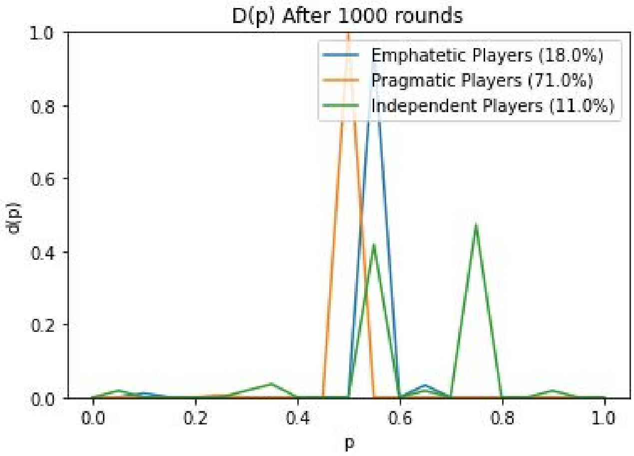

Figure 18: ER network with Natural Selection and beliefs change for each player type

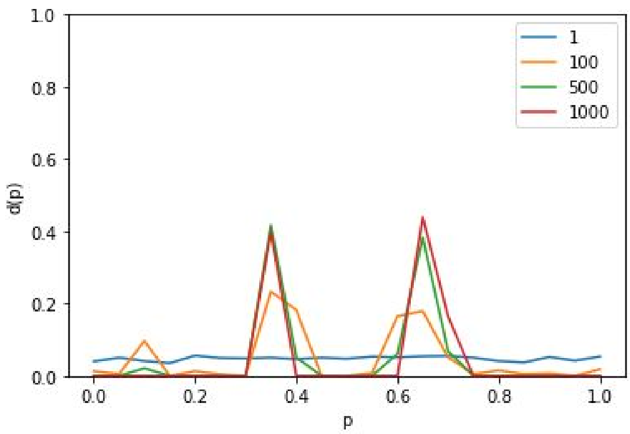

Figure 19: SF network with Natural Selection and beliefs change

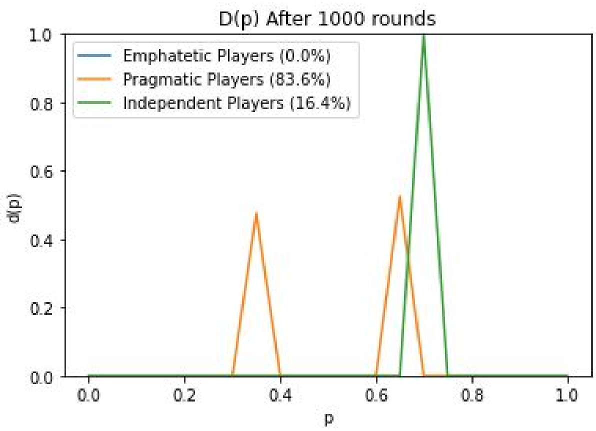

Figure 20: SF network with Natural Selection and beliefs change for each player type

From the simulations we can verify that the values still tend to, more or less, converge to the 50% mark. When allowing players to change their beliefs, it is noticeable that Independent Behaviours do not tend to persist over the course of iterations and Pragmatic Behaviours tend to perform better than the Empathetic ones. This might be explained due to the fact that earlier in the simulations, some Pragmatic Player might have a better payoff than Empathetic ones since they believe they should receive always the same.

7. Conclusion

The experiment of trying to analyze how a network with Players shift is very interesting. It was clear that, in order to survive or, simply, just to have an overall better performance, the players started to use values around 0.5. This is, what we called in a society, a fair trade. The players did not have any memory of previous rounds. They did not know against who they played and if the offers were accepted or not. This way they could not adapt the strategy based on specific outcomes of games, but the values still managed to converge. The reason behind the fact is that if the player was not satisfied with the value offered to it (p offered did not pass the q threshold), it would reject and, that way, both players lose. This means that in the long run, a Proposer should try to also satisfy the Responder. This conclusion goes in touch with some aspects of game-theory. The players payoff need to take in consideration the actions from the other players. In the long run, the values converge to what is called Nash Equilibrium which is the action that provides the best outcome for both players.

7.1. Future Work and Improvements

The model was created successfully and it does work following the research from Sinatra. The values and graphs shown were simulated multiple times and it should be noticed that the amount of players in the network are relatively small which might cause some inconsistency. The Social Penalty update rule could be switched by an update rule that instead of eliminating the player with the worst performance in the network and its neighbours, identify communities and apply some punishment to the player with the worst performance in each community. Even though the Girvan Newman algorithm was not very promising in these scenarios, on more heterogeneous networks it might provide some interesting results.

8. References

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9. Links

For accessing the notebook, see:
`https://colab.research.google.com/drive/1nb04skiiqE1B84_6vQ7CXF9oulbtli6q?usp=sharing`
or:
`https://github.com/rufimelo99/UltimatumGame/blob/main/UltimatumGame.ipynb`