Circuito 1 → LAB 1

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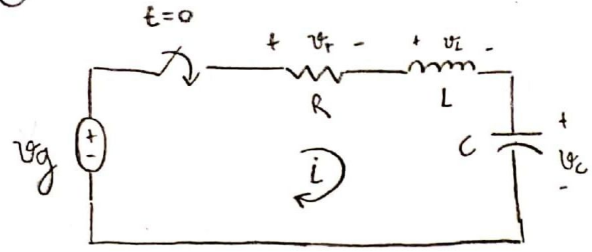

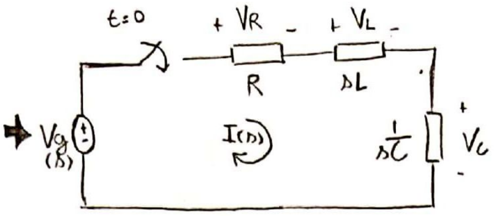

domínio do tempo

domínio da frequência

$$-V_g(s) + I(s)\left(R + sL + \frac{1}{sC}\right) = 0$$

$$I(s)\left(\frac{RsC + s^2LC + 1}{sC}\right) = V_g(s)$$

$$\boxed{H_1(s) = \frac{I(s)}{V_g(s)} = \left(\frac{sC}{s^2LC + sRC + 1}\right)}$$

quando $v_g = A \cdot u(t)$ temos:

$$V_g(s) = \frac{A}{s} \text{ ; logo, } I(s) = H_1(s) \cdot \frac{A}{s}$$

portanto,

$$\boxed{I(s) = \left(\frac{AC}{s^2LC + sRC + 1}\right)}$$

②

$$I(s) = \frac{V_C(s)}{\frac{1}{sC}} = V_C(s) \cdot sC$$

logo,

$$H_2(s) = \frac{V_C(s)}{V_g(s)} = \frac{I_g(s)}{V_g(s)\, sC} = \frac{H_1}{sC}$$

$$\therefore \quad \boxed{H_2(s) = \frac{1}{s^2 LC + sRC + 1}}$$

quando $v_g(t) = A \cdot u(t)$, temos $V_g(s) = \frac{A}{s}$

logo, $V_C(s) = H_2(s) \cdot \frac{A}{s} = \boxed{\frac{A}{s(s^2LC + sRC + 1)}}$

③

Atribuindo valores para R, L e C:

$$\boxed{\alpha = \frac{R}{2L}, \quad \omega_0 = \frac{1}{\sqrt{LC}}}$$

ⓐ <u>SUPERAMORTECIDA</u>

$$I(s) = \frac{A/L}{\left(s^2 + s\frac{R}{L} + \frac{1}{LC}\right)} = \frac{A/L}{\left(s + \alpha + \sqrt{\alpha^2 - \omega_0^2}\right)\left(s + \alpha - \sqrt{\alpha^2 - \omega_0^2}\right)}$$

, tomando $\beta = \sqrt{\alpha^2 - \omega_0^2}$

$$I(s) = \frac{A/L}{[s + (\alpha + \beta)] \cdot [s + (\alpha - \beta)]} = \frac{K_1}{[s + (\alpha + \beta)]} + \frac{K_2}{[s + (\alpha - \beta)]}$$

$$K_1 = \left.\frac{A/L}{[s+\alpha-\beta]}\right|_{s=-\alpha-\beta} = \frac{A/L}{-\alpha-\beta+\alpha-\beta} = \boxed{\frac{A/L}{-2\beta}}$$

$$K_2 = \left.\frac{A/L}{[s+\alpha+\beta]}\right|_{s=-\alpha+\beta} = \boxed{\frac{A/L}{2\beta}}$$

logo,

$$\boxed{i(t) = \left[\left(\frac{-A}{2L\beta}\right) e^{-(\alpha-\beta)t} + \left(\frac{A}{2L\beta}\right) e^{-(\alpha+\beta)t} \right] u(t)}$$

Ⓑ SUB-AMORTECIDA

$$I(s) = \frac{A/L}{(s+\alpha+j\beta)(s+\alpha-j\beta)} = \frac{K_1^*}{(s+\alpha+j\beta)} + \frac{K_1}{(s+\alpha-j\beta)}$$

$$K_1 = \left.\frac{A/L}{(s+\alpha+j\beta)}\right|_{s=-\alpha+j\beta} = \frac{A/L}{-\alpha+j\beta+\alpha+j\beta} = \boxed{\frac{A/L}{j2\beta}}$$

portanto,

$$\boxed{i(t) = \left\{ \left(\frac{A}{jL\beta}\right) e^{-\alpha t} \cos\left[\beta t + tg^{-1}\left(\frac{\beta}{\alpha}\right)\right] \right\} u(t)}$$

Ⓒ CRITICAMENTE AMORTECIDA:

$$I(s) = \frac{A/L}{(s+\alpha)^2} = \frac{A_0}{(s+\alpha)^2} + \frac{A_1}{(s+\alpha)}$$

$$G(s) = \frac{A/L}{1},$$

$$\boxed{A_0 = A/L}$$

$$A_1 = \frac{1}{1!}\frac{d}{ds}[G(s)] = 0$$

$$\boxed{i(t) = \left[(A/L)\, t\, e^{-(\alpha t)} \right] u(t)}$$

Agora, para $V_C(s)$, temos:

Ⓐ SUPERAMORTECIDA

$$V_C(s) = \frac{A}{s(s^2LC + sRC + 1)} = \frac{A/LC}{s[s+(\alpha+\beta)][s+(\alpha-\beta)]}$$

$$V_C(s) = \frac{K_1}{s} + \frac{K_2}{s+(\alpha+\beta)} + \frac{K_3}{s+(\alpha-\beta)}$$

$$K_1 = \frac{A/LC}{(\alpha+\beta)(\alpha-\beta)} = \boxed{\frac{A/LC}{\alpha^2-\beta^2}}$$

$$K_2 = \frac{A/LC}{s[s+(\alpha-\beta)]}\Bigg|_{s=-\alpha-\beta} = \frac{A/LC}{(-\alpha-\beta)[-\alpha-\beta+\alpha-\beta]}$$

$$K_2 = \frac{A/LC}{-(\alpha+\beta)(-2\beta)} = \boxed{\frac{A/LC}{2\alpha\beta+2\beta^2}}$$

$$K_3 = \frac{A/LC}{s[s+(\alpha+\beta)]}\Bigg|_{s=-\alpha+\beta} = \frac{A/LC}{(-\alpha+\beta)[(-\alpha+\beta+\alpha+\beta)]}$$

$$K_3 = \frac{A/LC}{(-\alpha+\beta)(2\beta)} = \boxed{\frac{A/LC}{2\beta^2-2\beta\alpha}}$$

logo,

$$\boxed{v_c(t) = \left[\left(\frac{A/LC}{\alpha^2-\beta^2}\right) + \left(\frac{A/LC}{2\beta^2+2\beta\alpha}\right)e^{-(\alpha+\beta)t} + \left(\frac{A/LC}{2\beta^2-2\beta\alpha}\right)e^{-(\alpha-\beta)t}\right]u(t)}$$

Ⓑ SUB-AMORTECIDA

$$V_C(s) = \frac{K_1}{s} + \frac{K_2}{s+\alpha-j\beta} + \frac{K_2^*}{s+\alpha+j\beta}$$

$$K_1 = \frac{A/LC}{(\alpha-j\beta)(\alpha+j\beta)} = \boxed{\frac{A/LC}{\alpha^2+\beta^2}}$$

$$K_2 = \frac{A/LC}{(-\alpha+j\beta)(-\alpha+j\beta+\alpha+j\beta)} = \frac{A/LC}{(-\alpha+j\beta)(2j\beta)}$$

$$= \frac{A/LC}{-2j\alpha\beta - 2\beta^2} = \boxed{\frac{-A/2LC}{\beta^2 + j\alpha\beta}}$$

$$\boxed{v_c(t) = \left\{\left(\frac{A/LC}{\alpha^2+\beta^2}\right) - \left(\frac{A/LC}{\beta^2+j\alpha\beta}\right)e^{-\alpha t}\cos\left[\beta t + tg^{-1}\left(\frac{\beta}{\alpha}\right)\right]\right\}u(t)}$$

Ⓒ CRITICAMENTE AMORTECIDA

$$V_C(s) = \frac{K_1}{s} + \frac{A_0}{(s+\alpha)^2} + \frac{A_1}{(s+\alpha)}$$

$$K_1 = \frac{A/LC}{\alpha^2}$$

$$A_0 = \frac{-A/LC}{\alpha}$$

$$A_1 = (G'(s))\Big|_{s=-\alpha} = \frac{-A/LC}{\alpha^2}$$

$$\boxed{v_c(t) = \left(\frac{A}{LC\alpha^2} - \frac{A}{LC\alpha}\,t\,e^{-\alpha t} - \frac{A}{LC\alpha^2}\,e^{-\alpha t}\right)u(t)}$$