NPTEL MOOC,JAN-FEB 2015
Week 3, Module 1

DESIGN AND ANALYSIS OF ALGORITHMS

Introduction to graphs

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Map Colouring

- Assign each state or country a colour
- States that share a border should be coloured differently
- How many colours do we need?

Map Colouring

* Mark each state

Map Colouring

- Mark each state
- Connect states that share a border

Map Colouring

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- Assign colours to dots so that no two connected dots have the same colour

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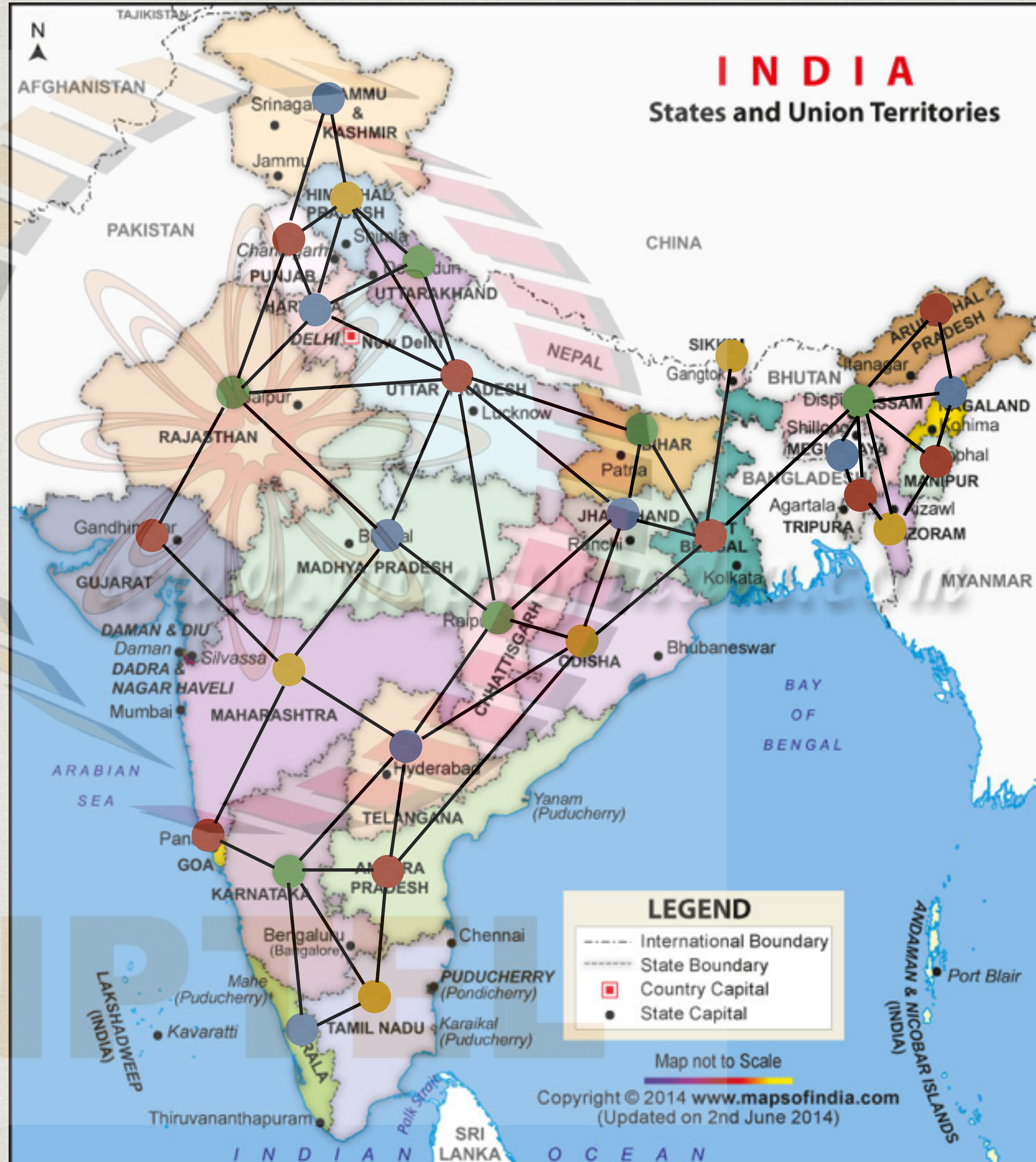

Map Colouring

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- In fact, the actual map is irrelevant!

Map Colouring

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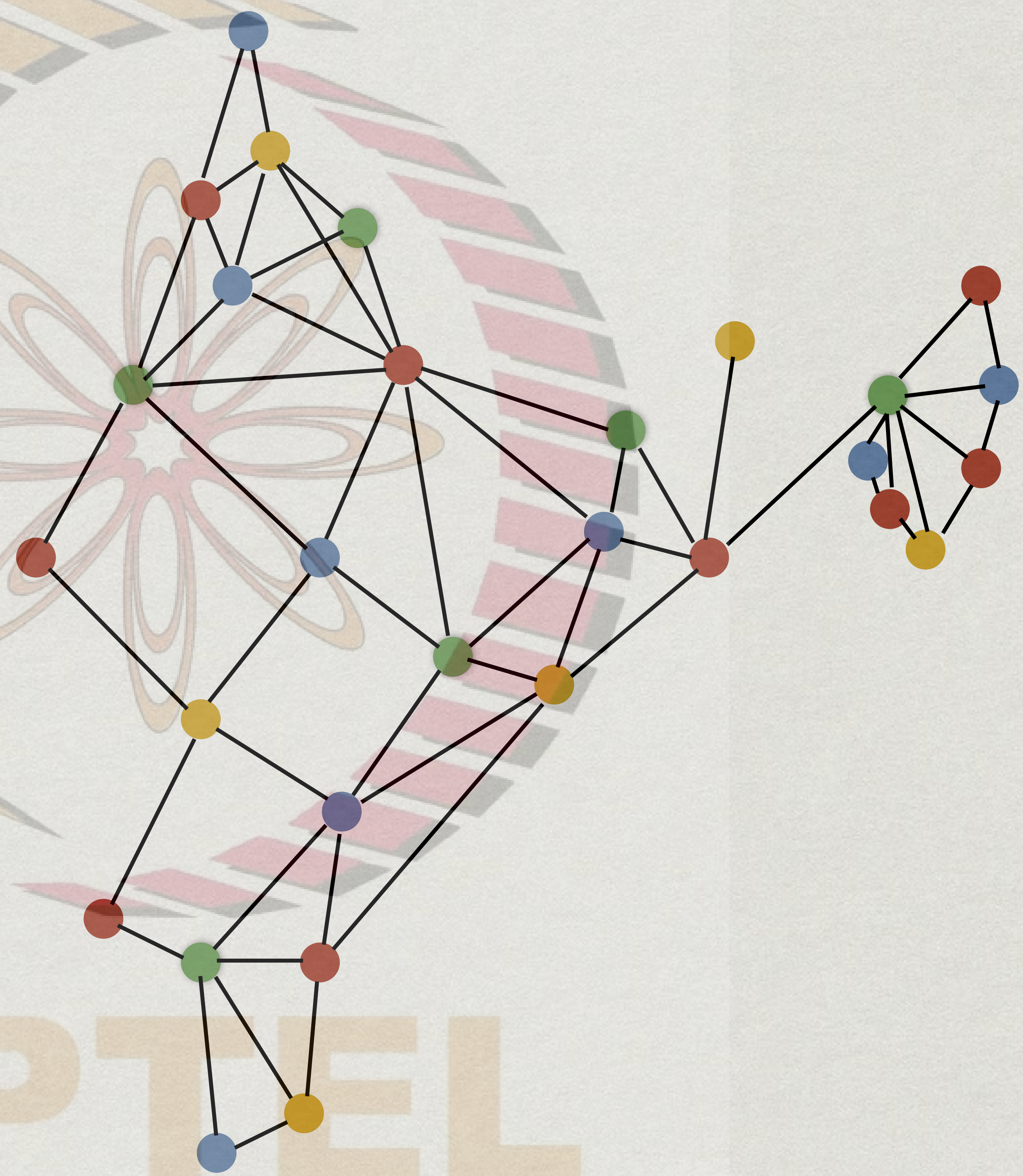
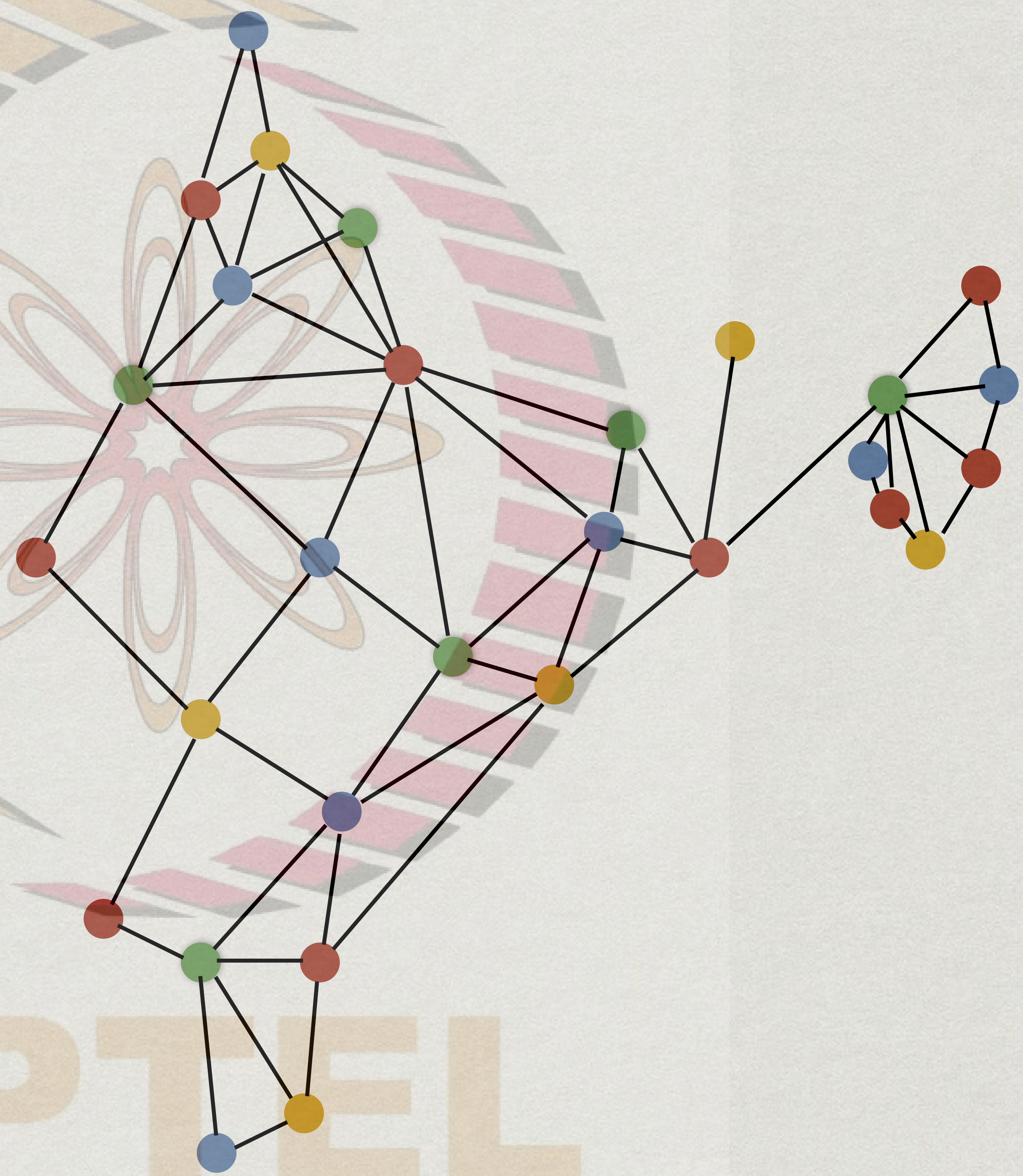
- All we need is the underlying pattern of dots and connections



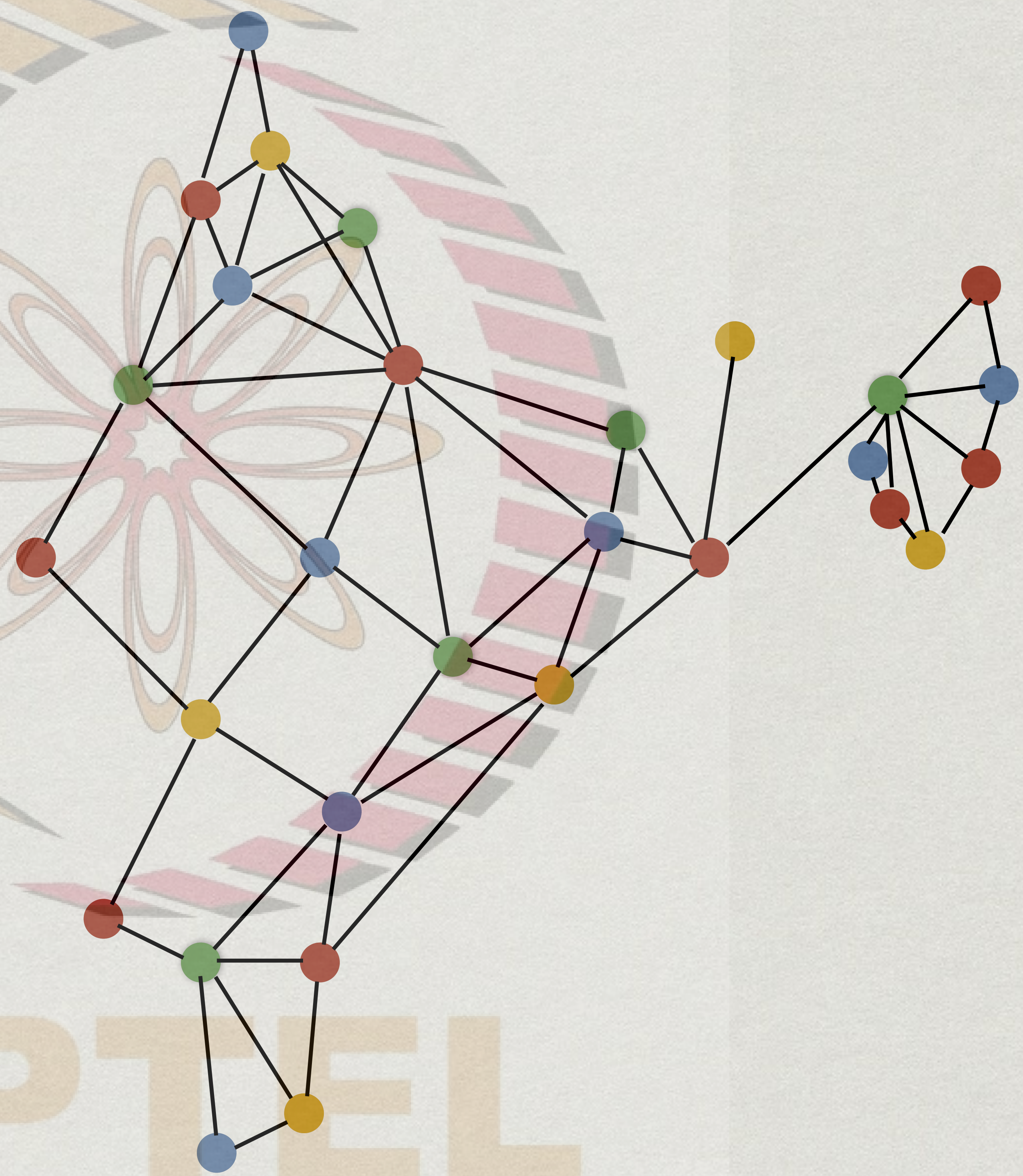
Map Colouring

- This kind of diagram is called a graph
- Dots are **nodes** or **vertices**
    - One vertex, many vertices
- Connections are **edges**



Graph Colouring

- The problem we have solved is called **graph colouring**
- We used 4 colours
- In fact, 4 colours are always enough for such maps
- This is a **theorem** that is surprisingly hard to prove!



Graph Colouring

- Observe that the original map used more than 4 colours

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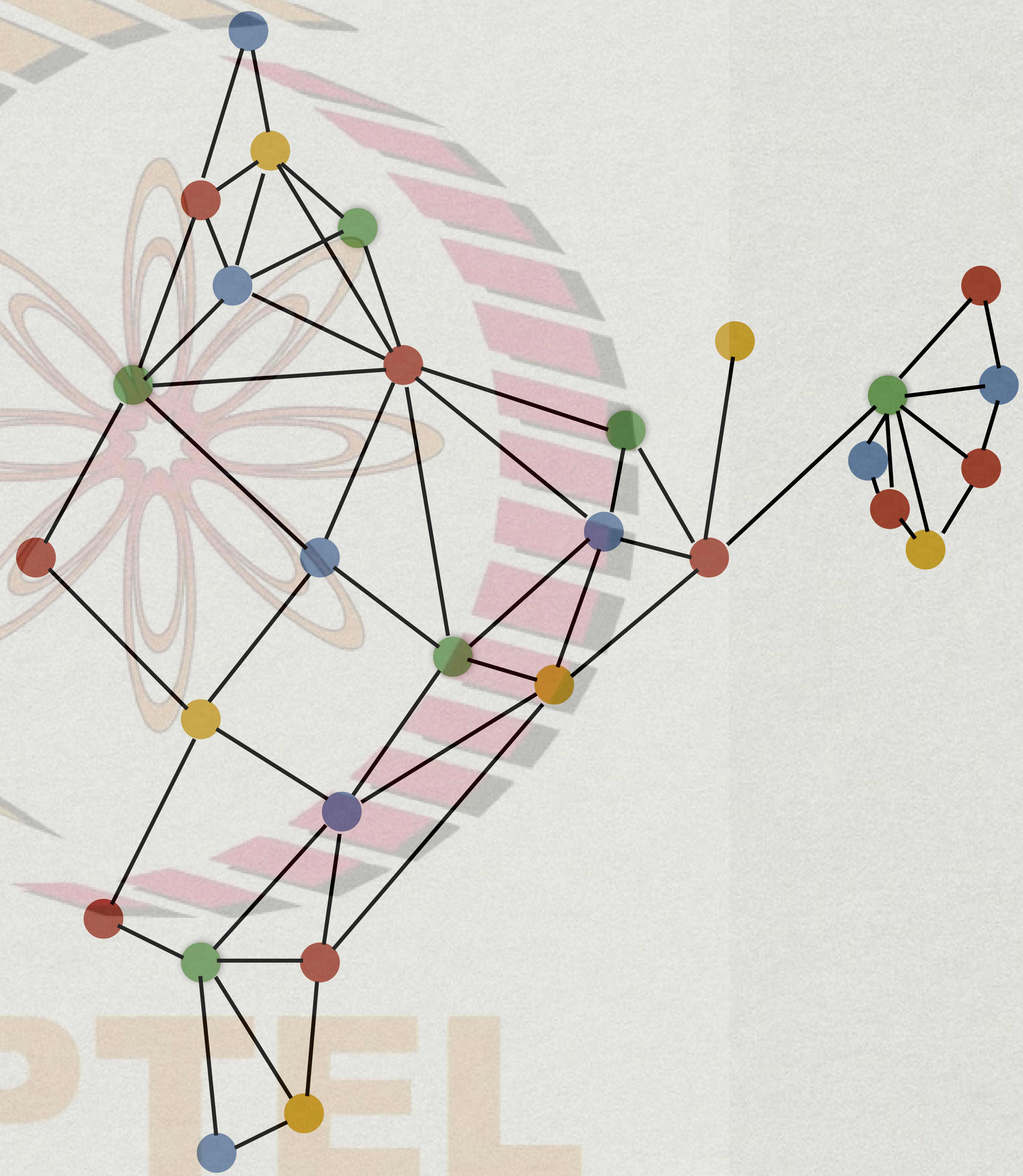
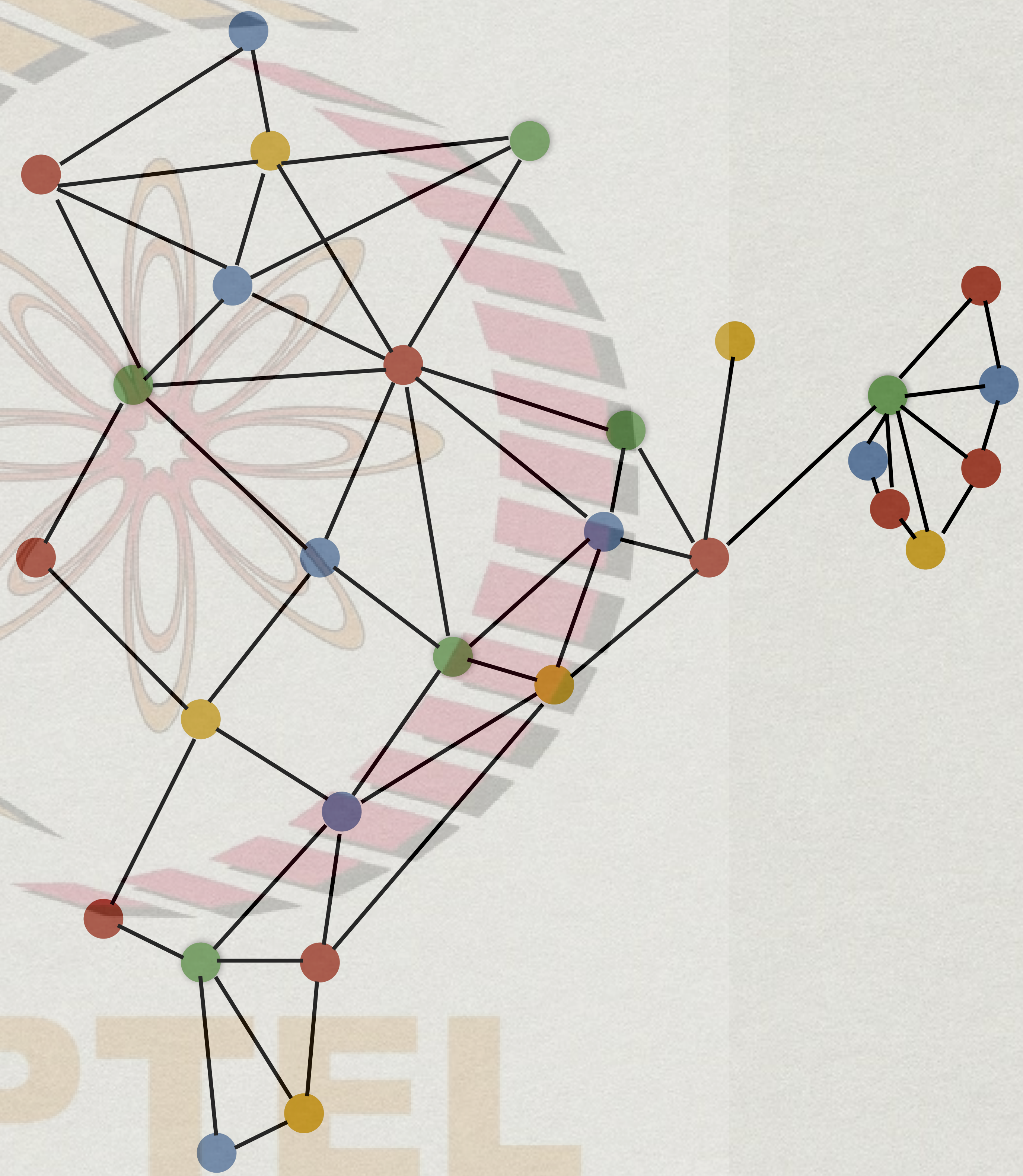
Graph Colouring

- The graph emphasizes the essential features of the problem
  - What is connected to what?



Graph Colouring

- The graph emphasizes the essential features of the problem
    - What is connected to what?
- We can distort this figure and the problem remains the same



More graph problems

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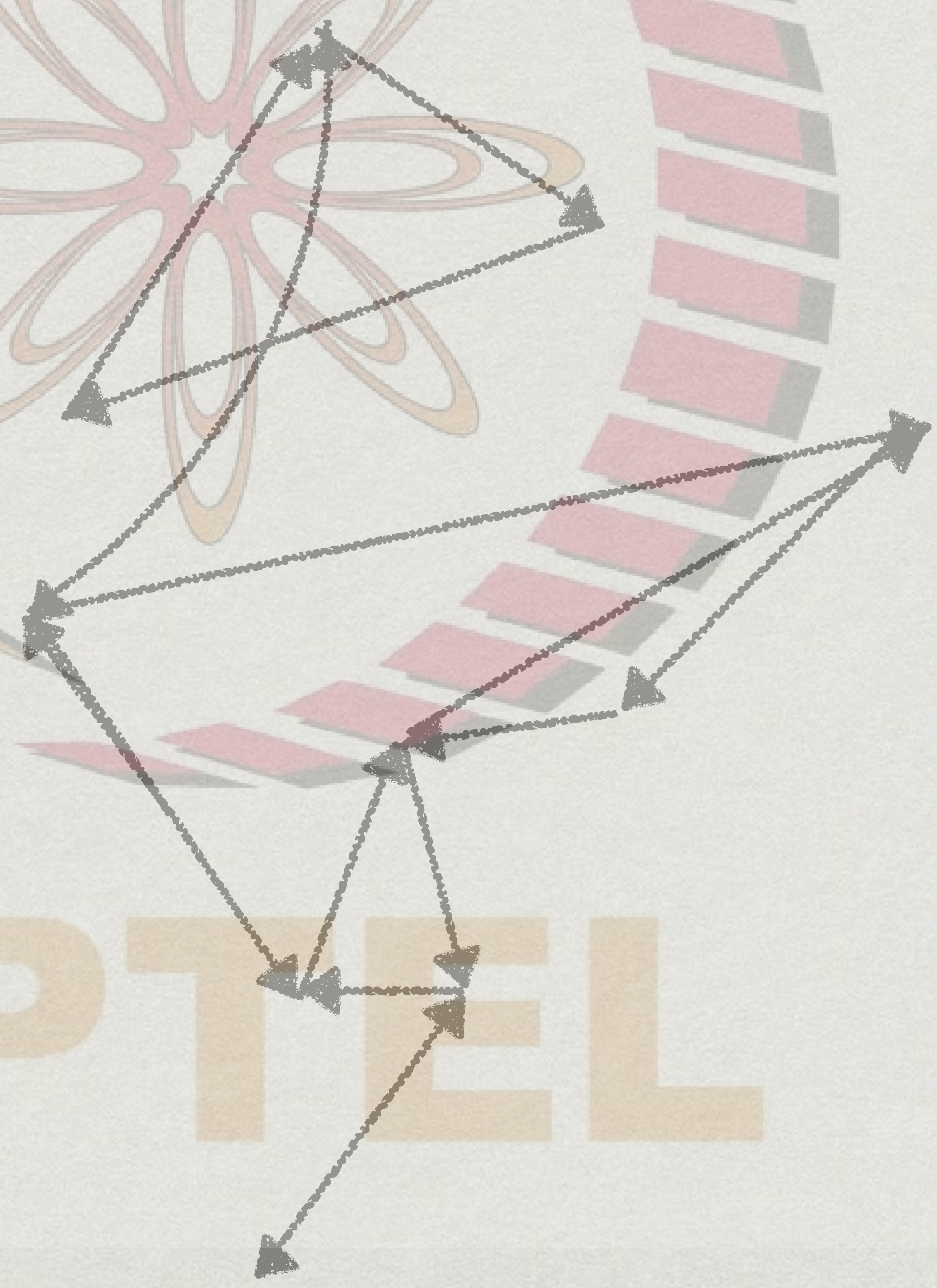
* Airline routing

More graph problems

- Airline routing
- Can I travel from New Delhi to Trivandrum without changing airlines?

More graph problems

- Airline routing
- Can I travel from New Delhi to Trivandrum without changing airlines?
- Again, all that is important is the underlying graph



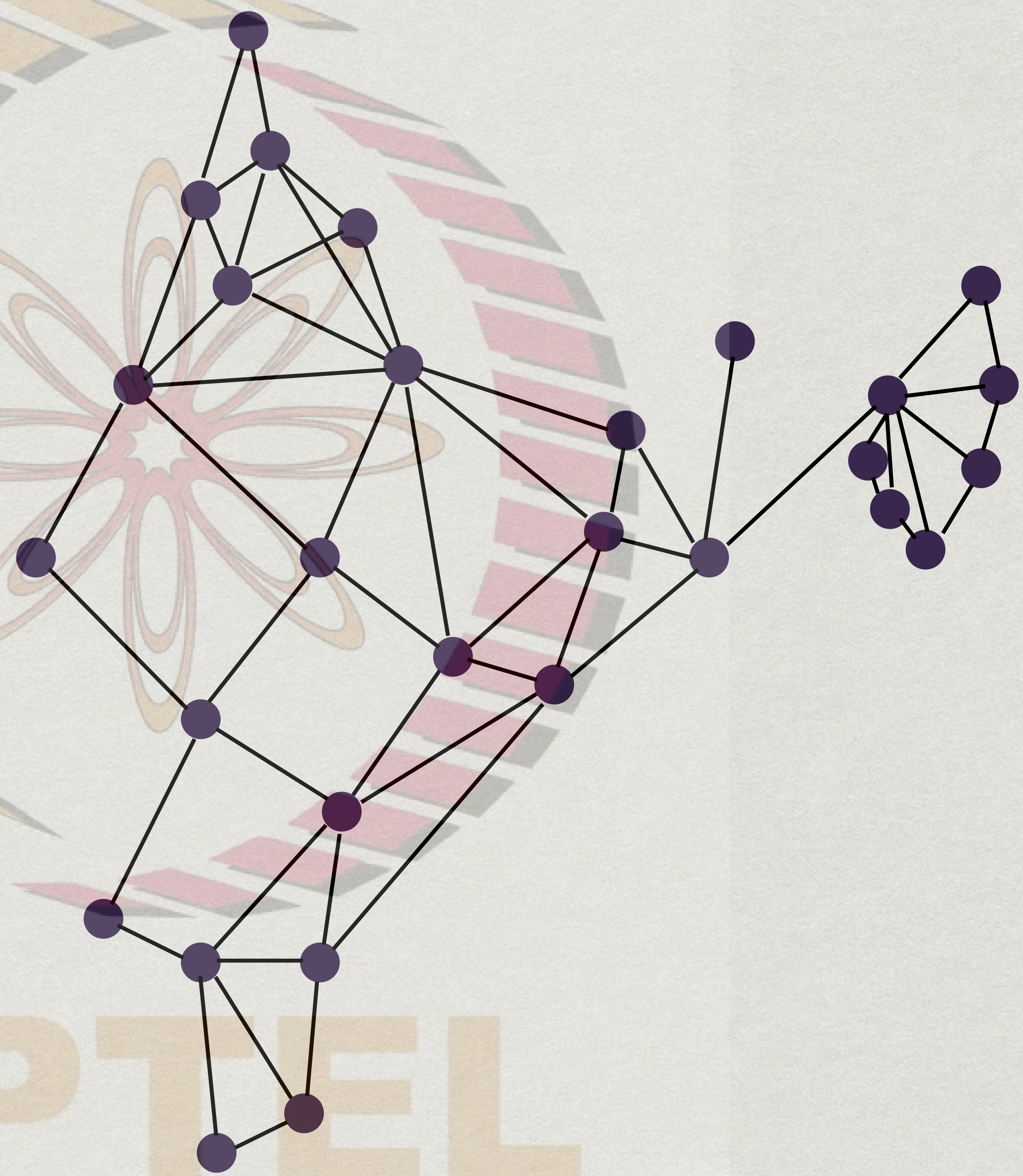
Graphs, formally

$G = (V,E)$

- Set of vertices V
- Set of edges E
  - E is a subset of pairs (v,v'): $E \subseteq V \times V$
  - Undirected graph: (v,v') and (v',v) are the same edge
  - Directed graph:
    - (v,v') is an edge from v to v'
    - Does not guarantee that (v',v) is also an edge

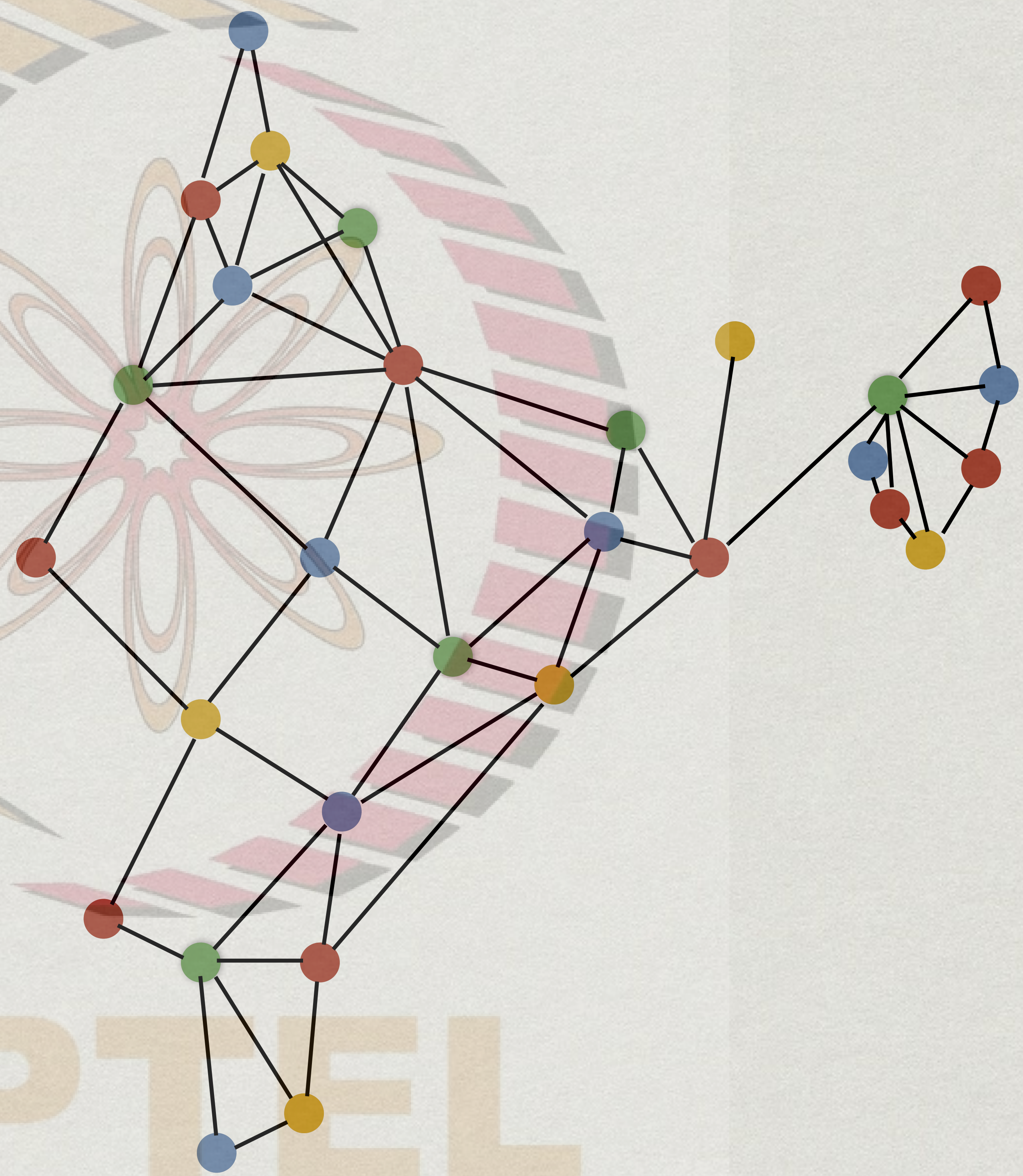
Graph Colouring

* Undirected graph



Graph Colouring

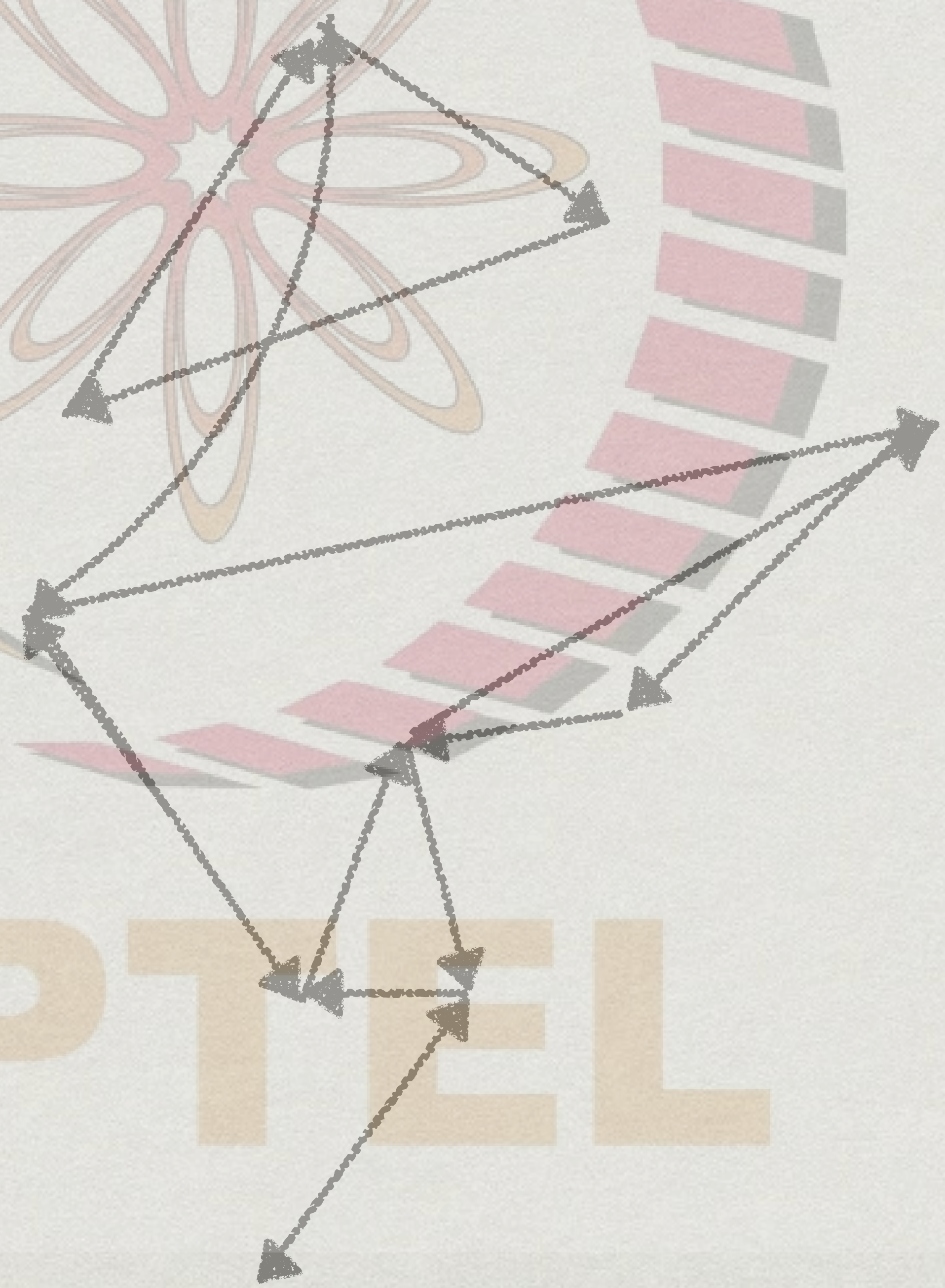
- Undirected graph
- Colouring C assigns each vertex v a colour C(v)
- Legal colouring: if (v,v') is in E, then C(v) ≠ C(v')



Finding a route

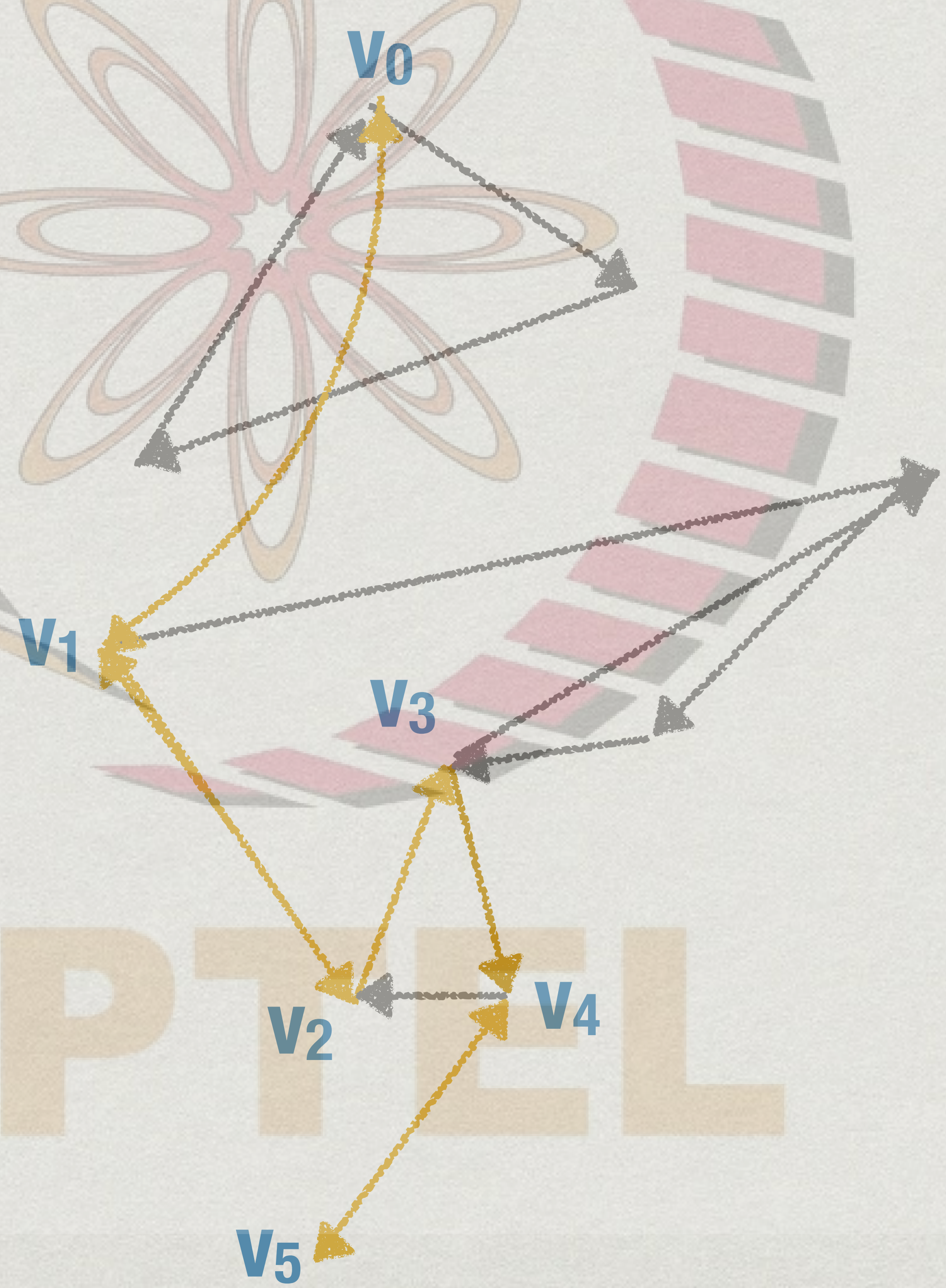
Finding a route

* Directed graph



Finding a route

- Directed graph
- Find a sequence of vertices v_0, v_1, …, v_k such that
  - v_0 is New Delhi
  - Each (v_i, v_{i+1}) is an edge in E
  - v_k is Trivandrum



Finding a route

- Also makes sense for undirected graphs
- Find a sequence of vertices v_0, v_1, …, v_k such that
  - v_0 is New Delhi
  - Each (v_i, v_{i+1}) is an edge in E
  - v_k is Trivandrum