NPTEL MOOC,JAN-FEB 2015
Week 5, Module 2

DESIGN AND ANALYSIS OF ALGORITHMS

Union-Find data structure using pointers

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Union-Find data structure

- A set of elements S partitioned into subsets, or components, $\{C_1, C_2, \ldots, C_k\}$
  - Each s in S belongs to exactly one C_j
- Support the following operations
  - MakeUnionFind(S) — set up initial components, each s in S is a separate singleton component {s}
  - Find(s) — returns the component containing s
  - Union(C,C') — merges the components C and C'

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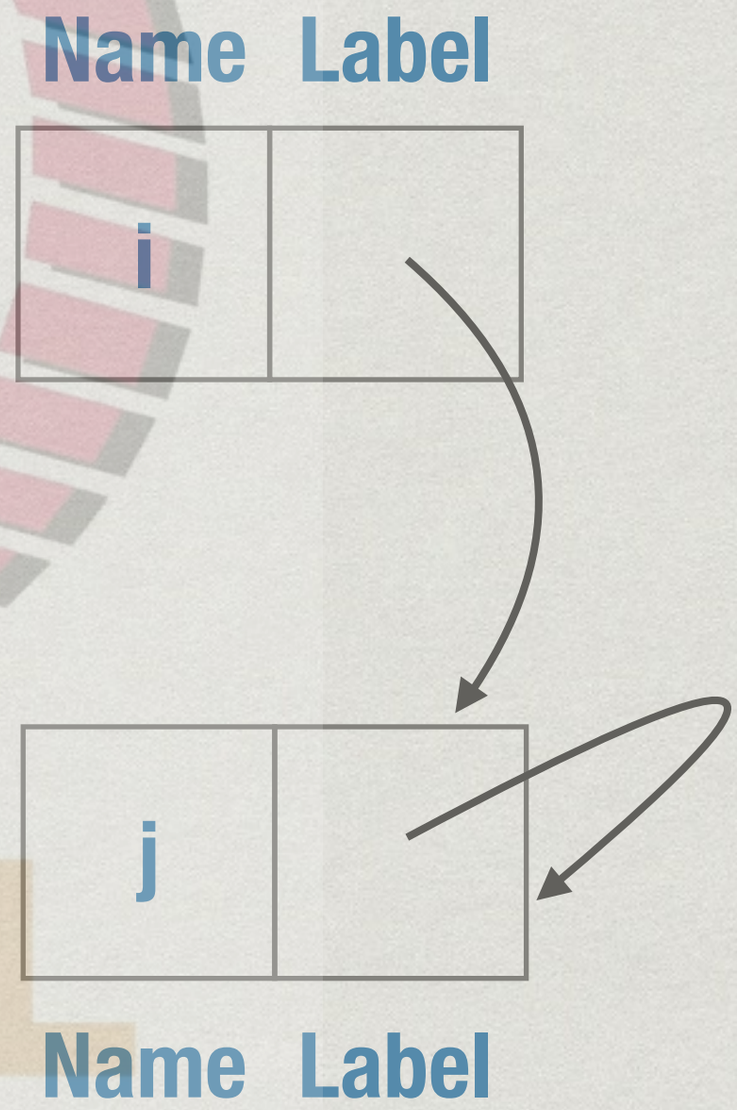
Implement with arrays/lists

- Implement Union-Find using array Component[1..n], lists Member[1..n] and array Size[1..n]
- MakeUnionFind(S) is O(n)
- Find(s) is O(1)
- Amortized complexity of each Union(k,k') is O(log m) over a sequence of m operations

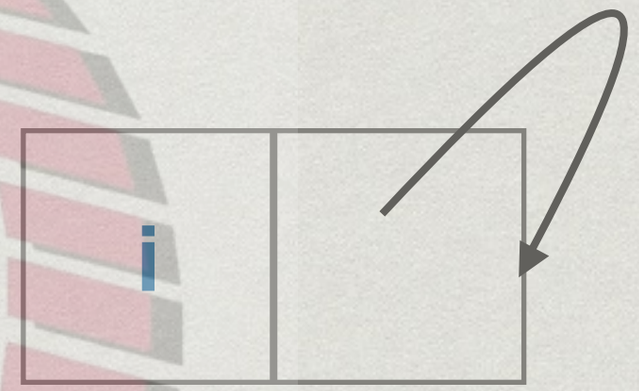
Implementing with pointers

- Each element of the set is a node with two fields
  - Name: the name of the element
  - Label: pointer to the set containing the element
  - Recall that we use same set S for component labels



Pointers …

- Initially, each node points to itself



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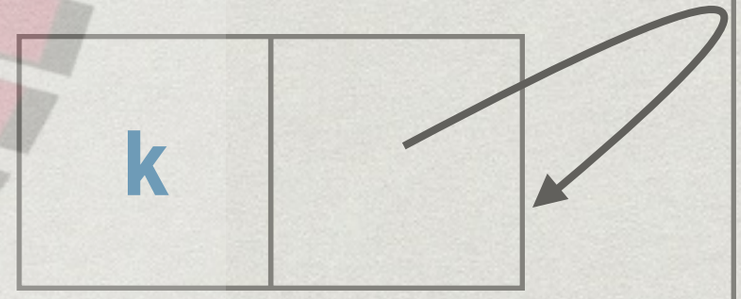
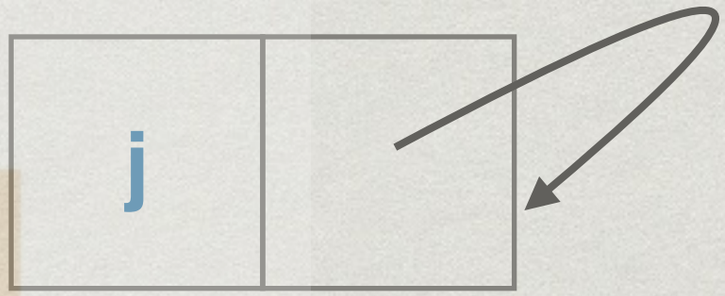


- Initially, each element s is in component s



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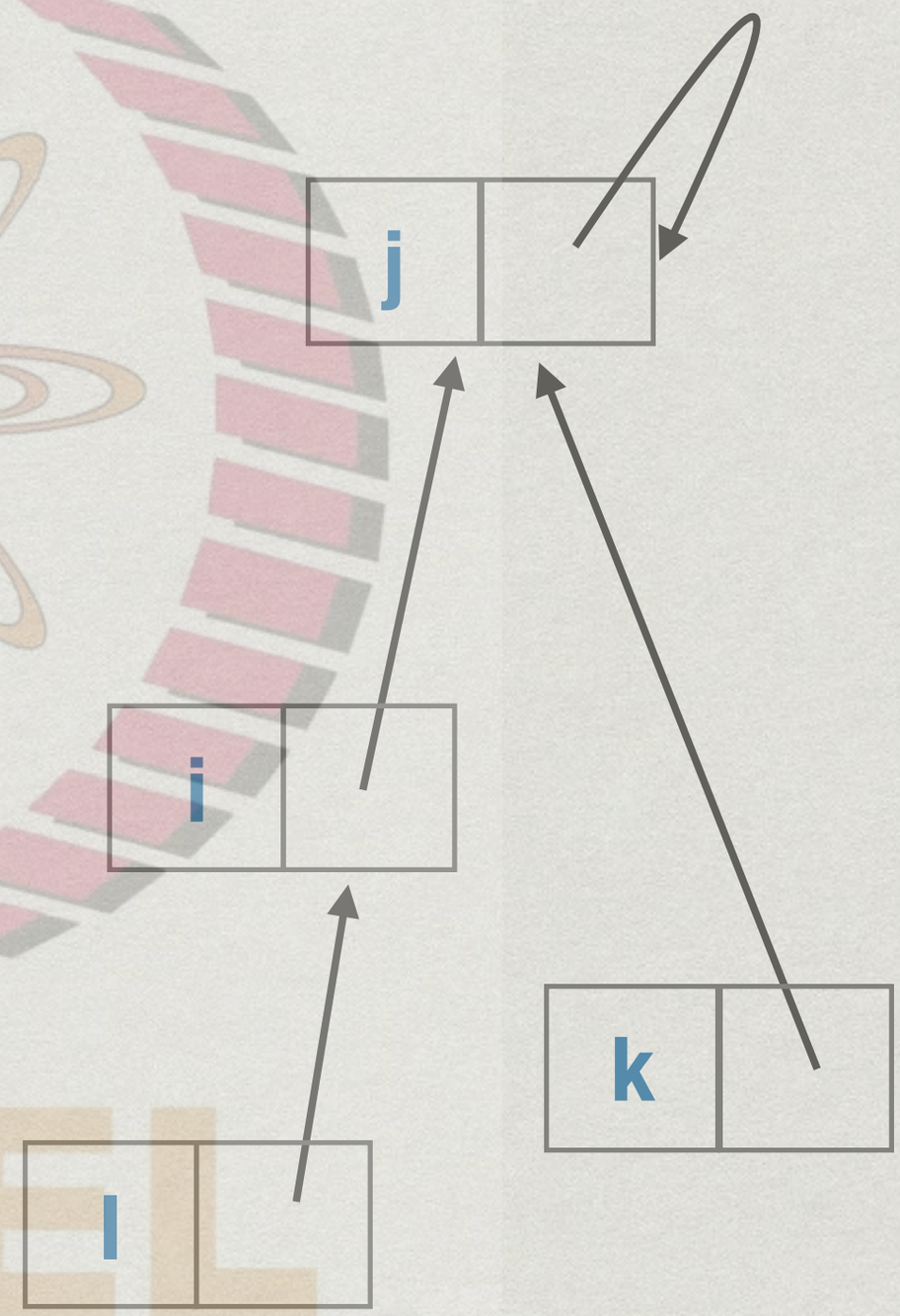
Pointers …

- A component is a tree
  - Root element names the component, points to itself
  - For other elements, follow path to root to find the name of the component



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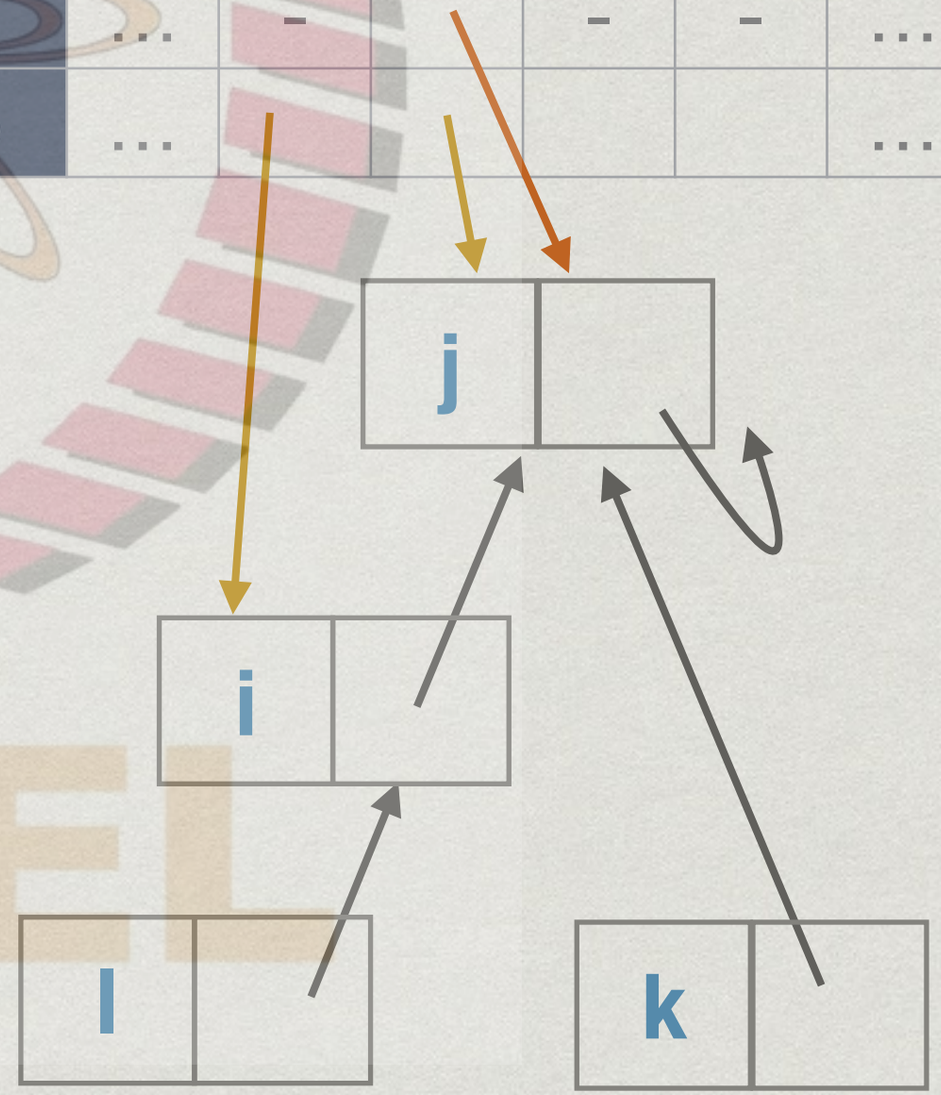
Auxiliary structures

- Node[1..n]
  - Node[k] points to node containing element k
- Root[1..n]
  - Root[k] points to root node of tree for component k
- Size[k]
  - Size of component k

	…	i	j	k	l	…
Size	…	-	4	-	-	…
Root	…	-		-	-	…
Node	…					…



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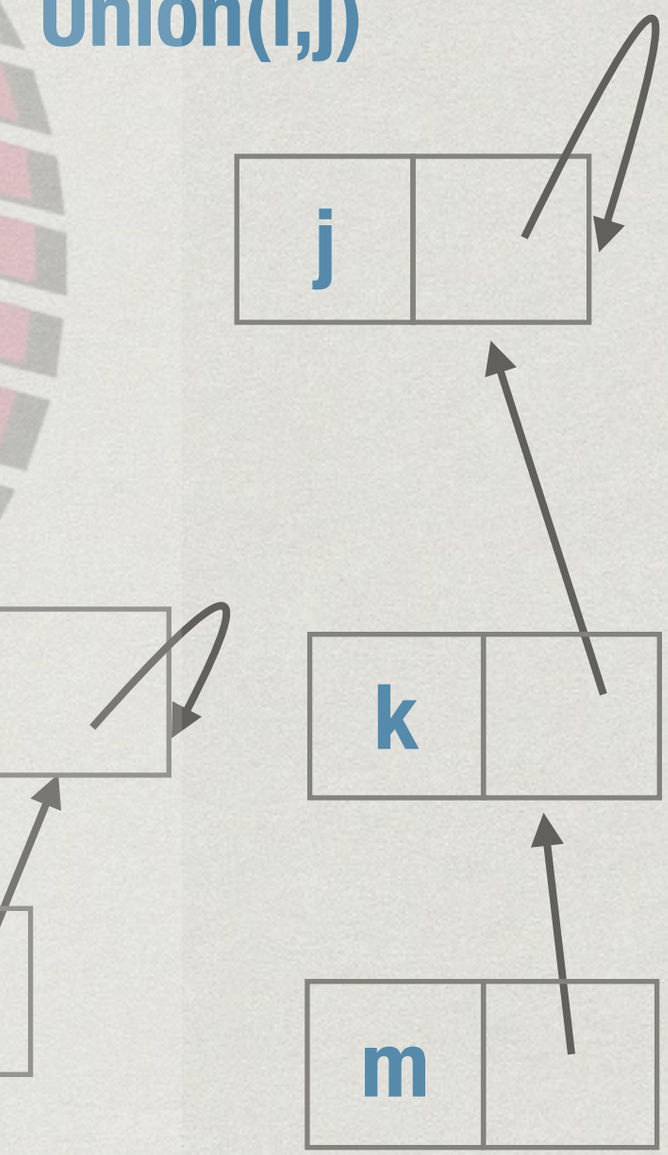
Union(k,k’)

- Root of one component points to root of the other
  - Becomes a direct child of the other root node
  - As usual, merge smaller component into larger one
- O(1) operation
- Size information in Size[k]
- Root[k] points to root of k



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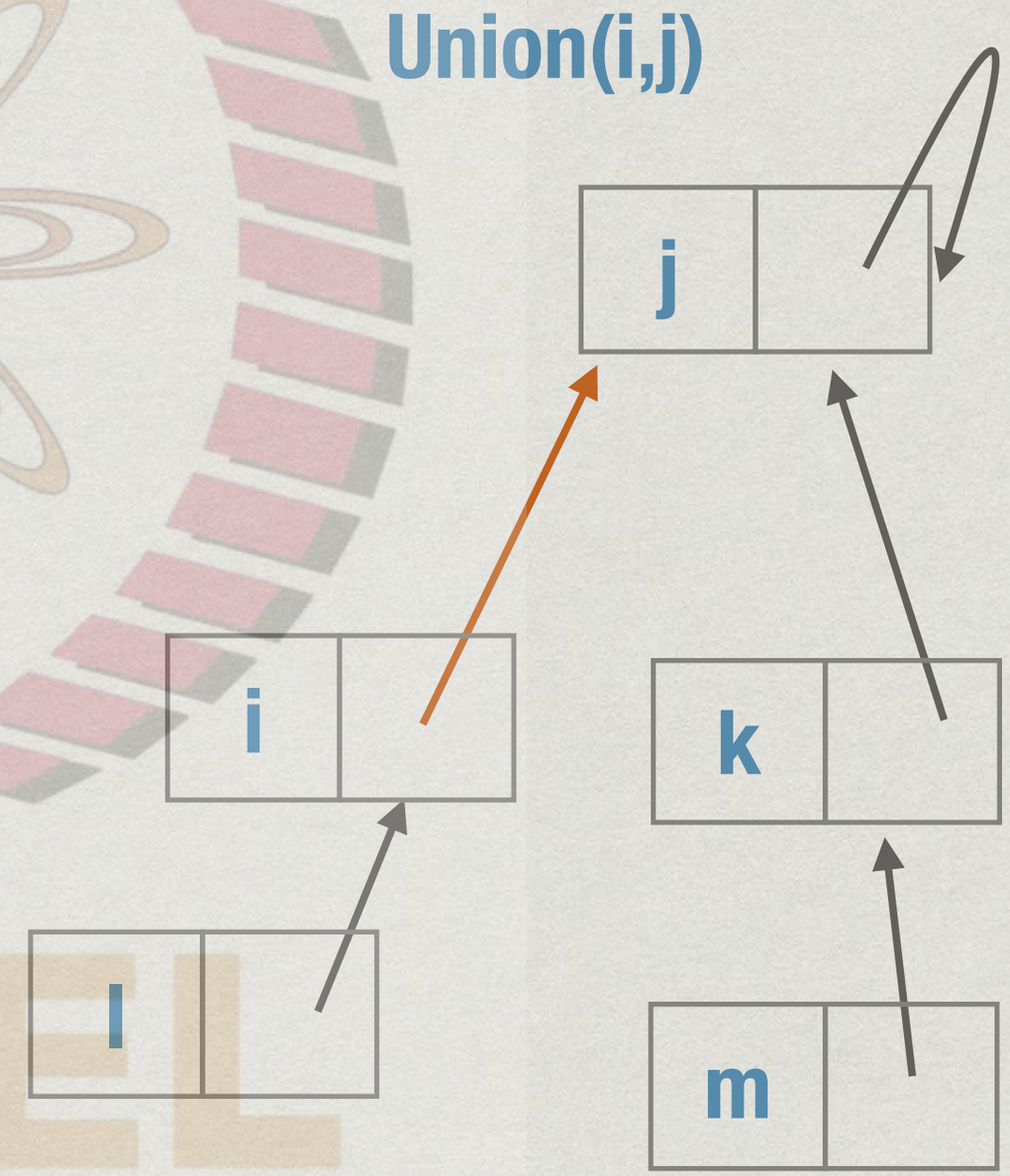

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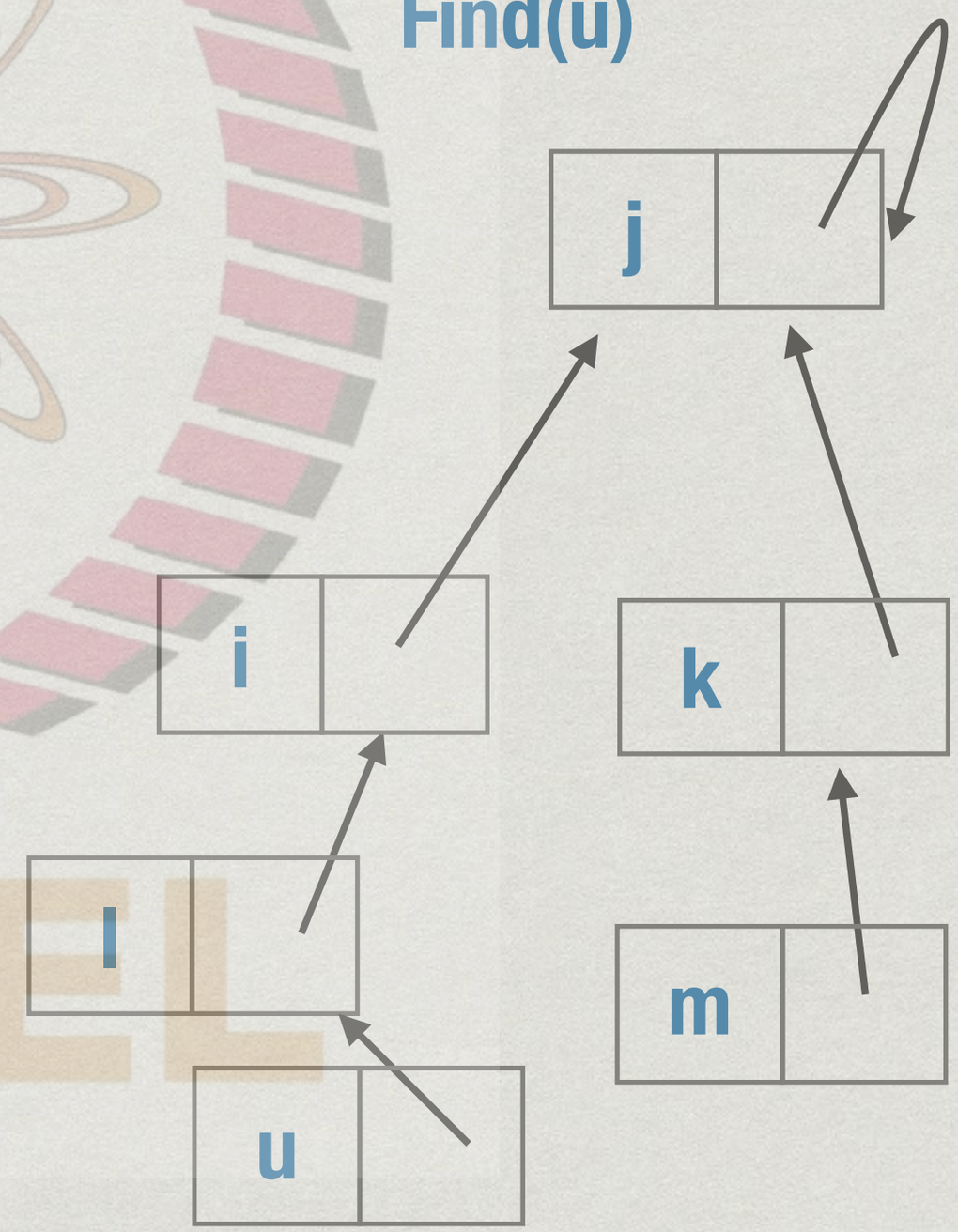
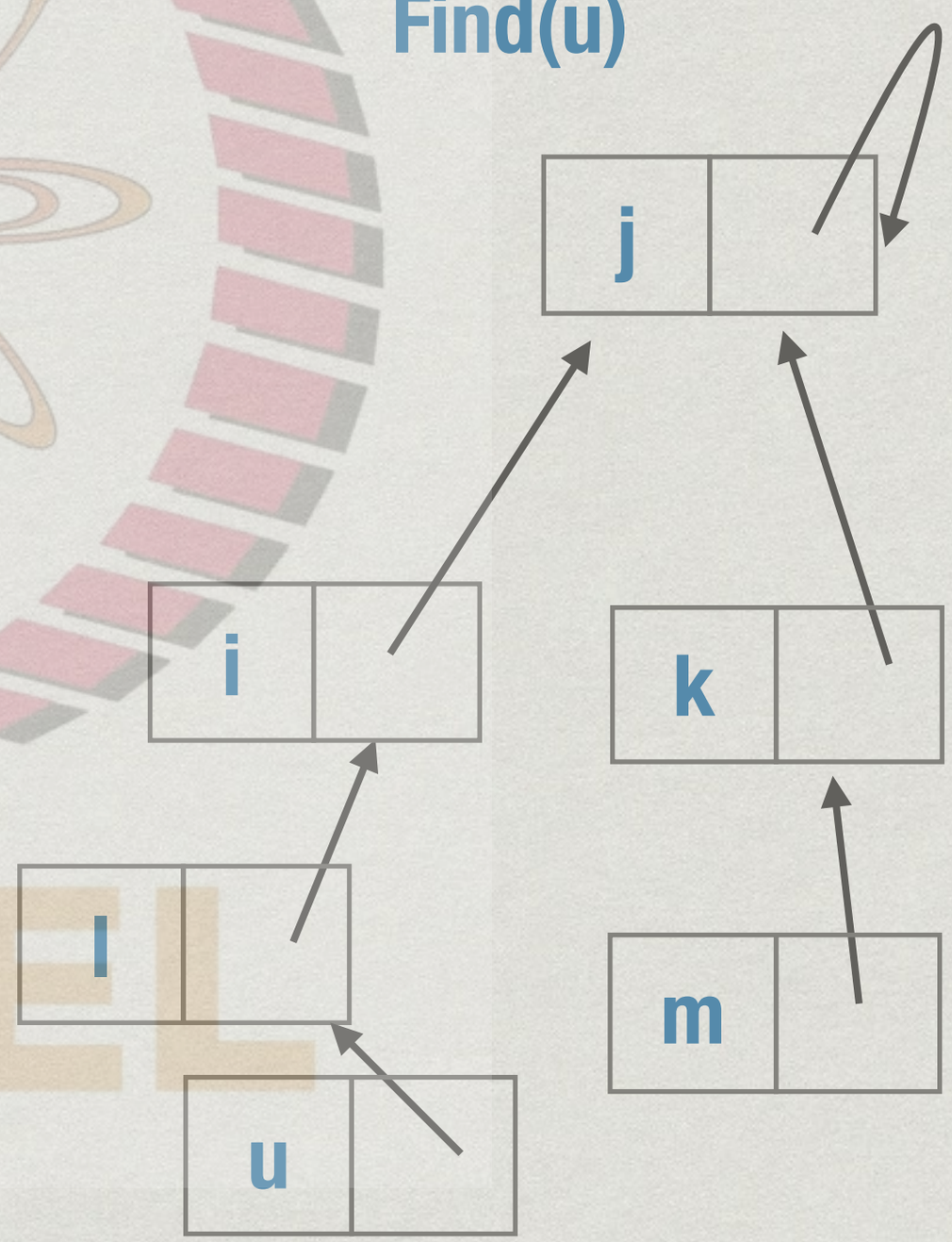
Find(j)

- Need to traverse the path from j to root
- Path increases by 1 each time label of j changes
- Component size doubles with each merge
- Max component size is n
- Path is at most log n



Path compression

- Each Find(j) traverses path from j to root
  - Remember the answer: make j point directly point to root
- After each Find(j), retrace path and reset all pointers to point directly to root
  - "Flatten" the tree



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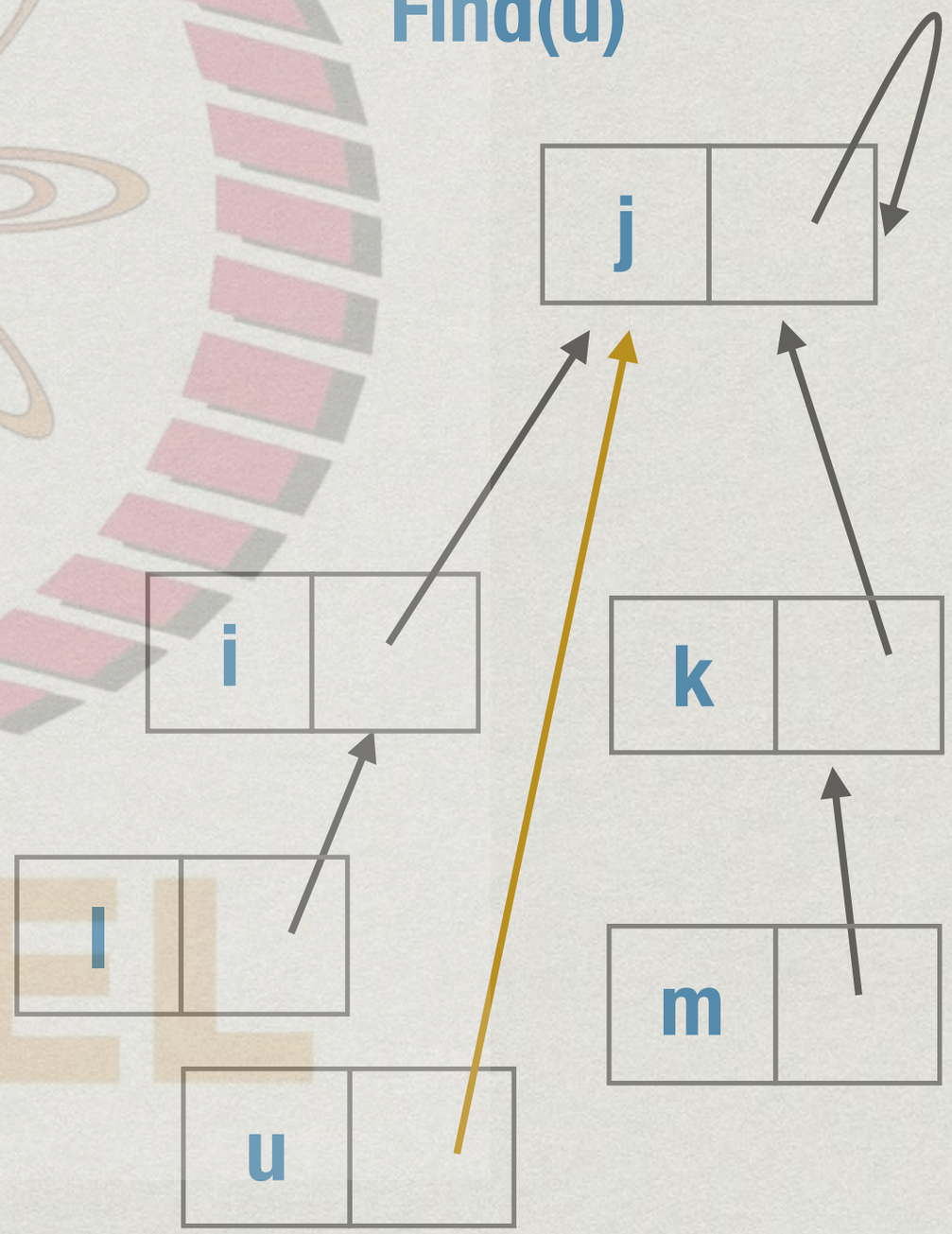

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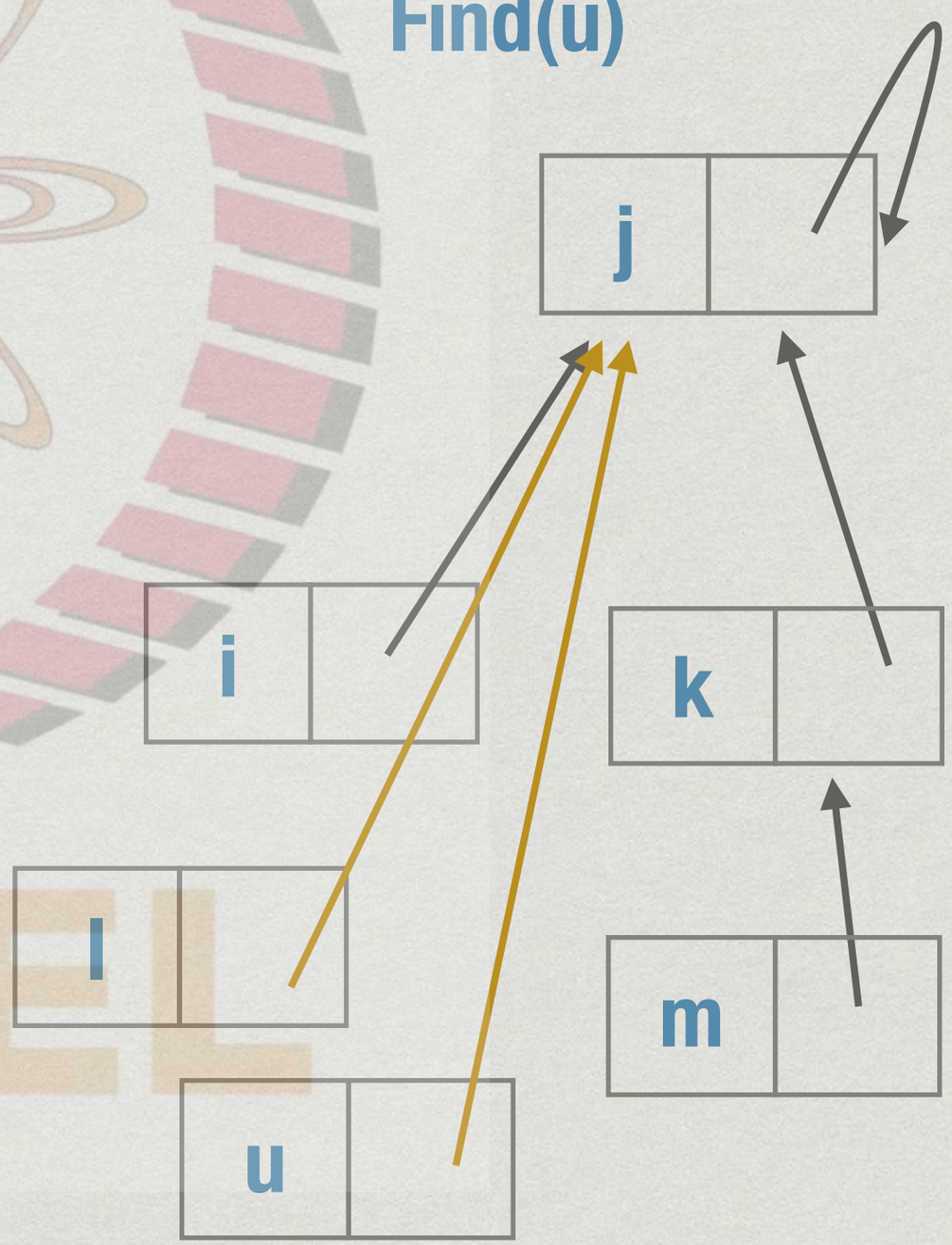
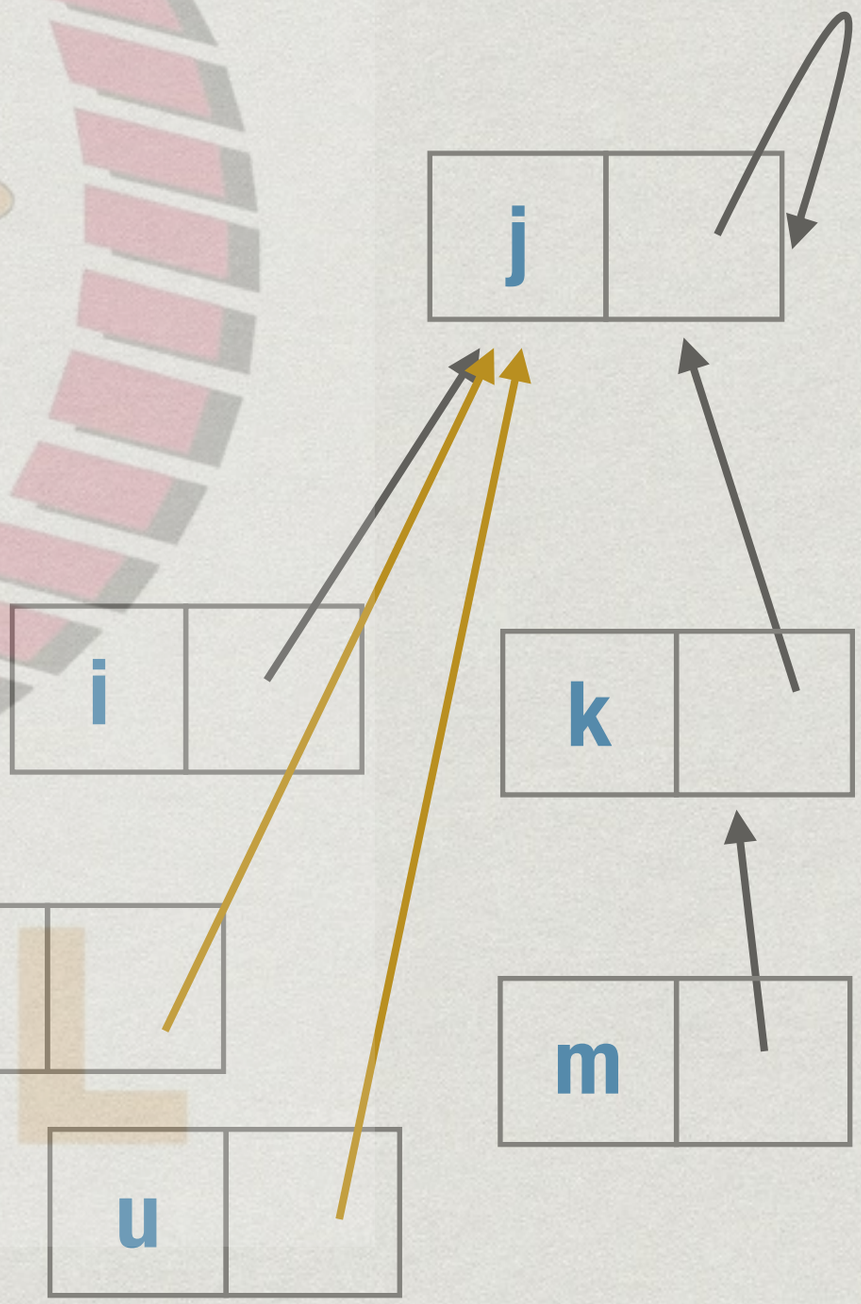


Path compression

- First Find(j) takes log n
- Then O(1)
- Overall, n finds using path compression take time almost linear in n
  - O(n alpha(n))
  - alpha(n) : inverse Ackermann function, grows very slowly



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Summary

- Implement Union-Find using nodes with pointers
- MakeUnionFind(S) is O(n)
- Union(k,k') is O(1)
- Find(s) is O(n alpha(n))
  - Use path compression to speed up repeated Find(s) operations

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