NPTEL MOOC,JAN-FEB 2015
Week 4, Module 5

DESIGN AND ANALYSIS OF ALGORITHMS

Minimum cost spanning trees

MADHAVAN MUKUND, CHENNAI MATHEMATICAL INSTITUTE
http://www.cmi.ac.in/~madhavan

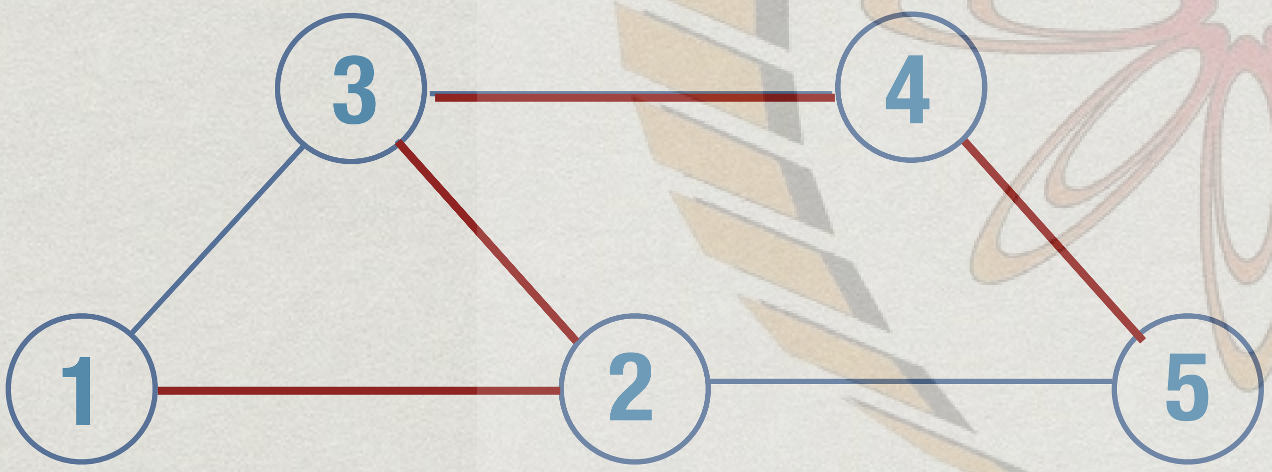
Example: Road network

- District hit by a cyclone, damaging the roads
- Government sets to work to restore the roads
- Priority is to ensure that all parts of the district can be reached
- What set of roads should be restored first?

Spanning tree

- Minimum connectivity: no loops
  - Removing an edge from a loop cannot disconnect graph
- Connected acyclic graph — tree
- Spanning tree

Spanning tree



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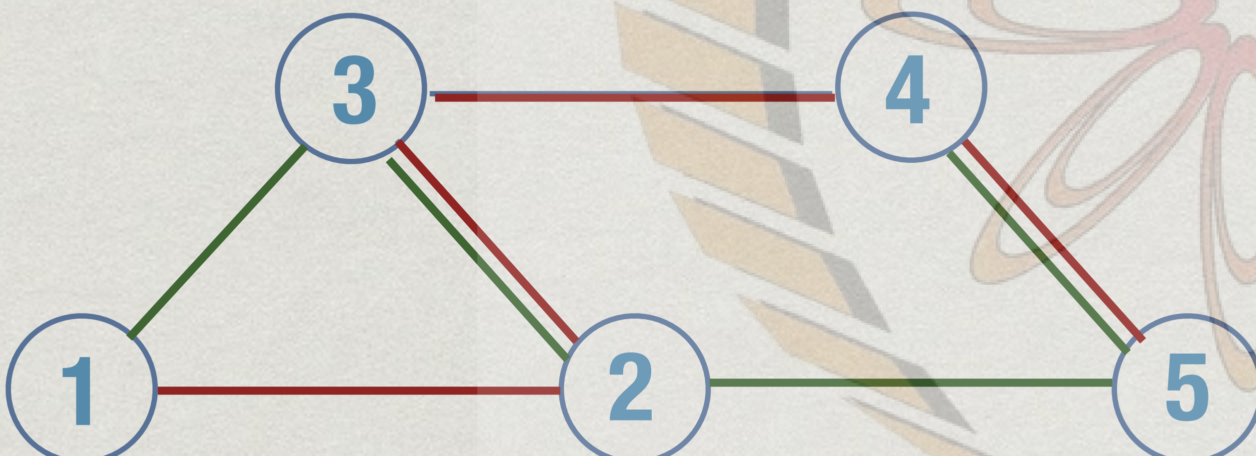

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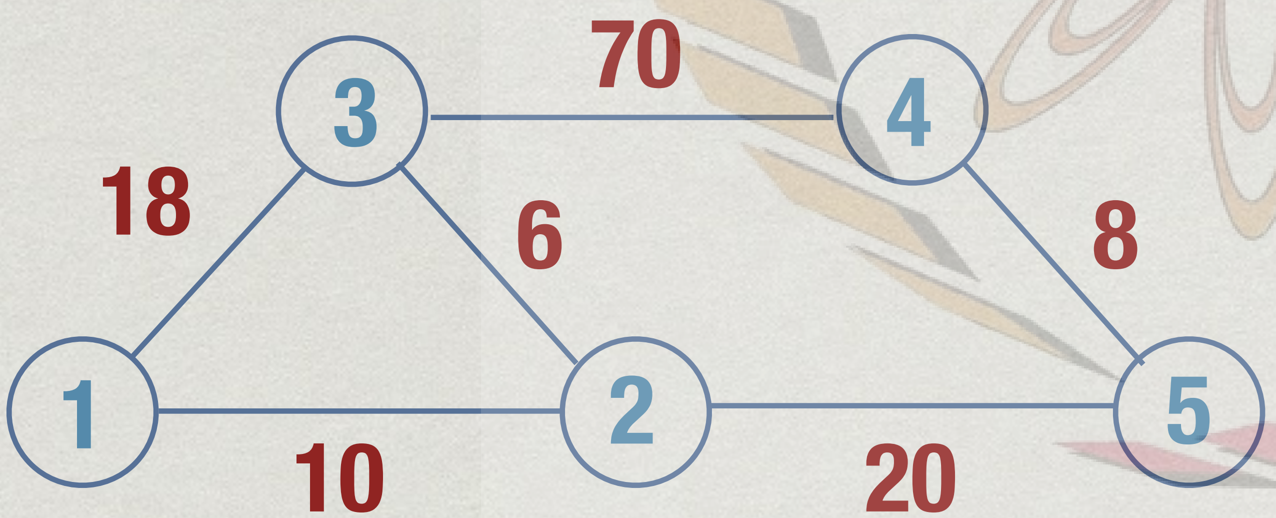
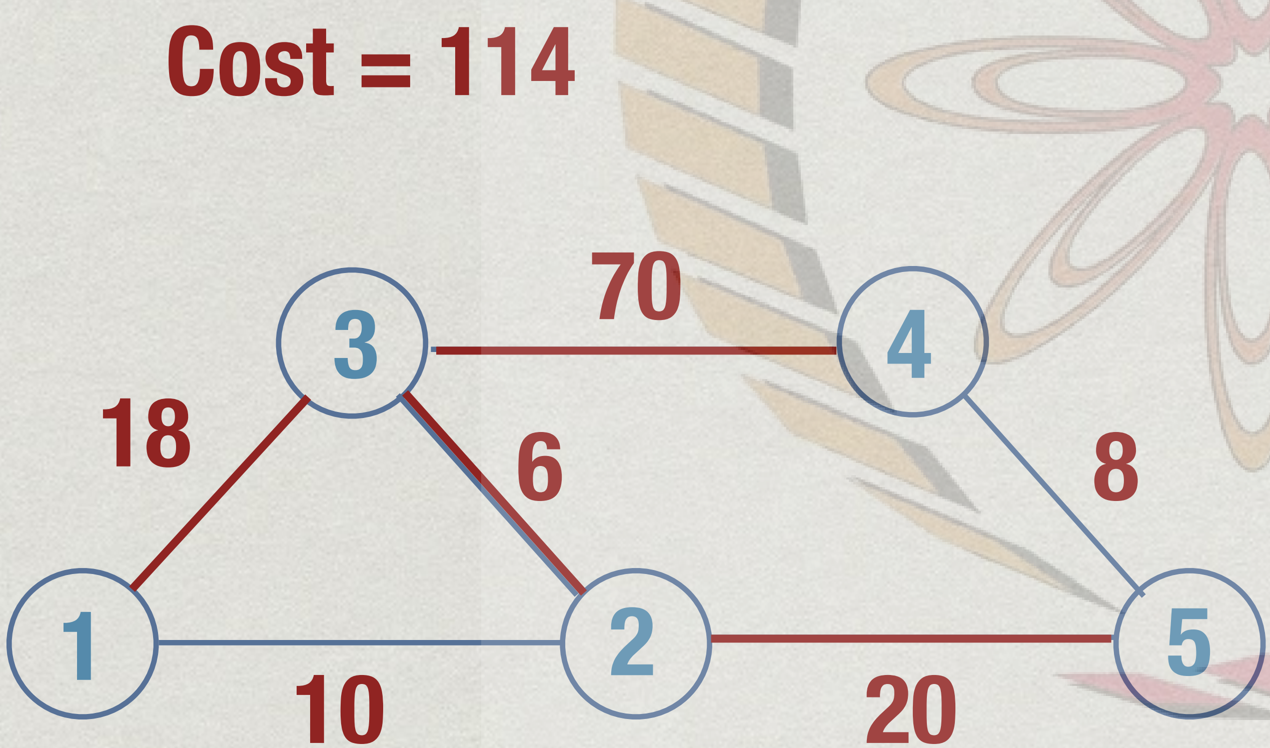
Spanning tree with costs



- Restoration of each road has a cost
- Among the different spanning trees, choose the one with minimum cost
- Minimum cost spanning tree

Spanning tree with costs



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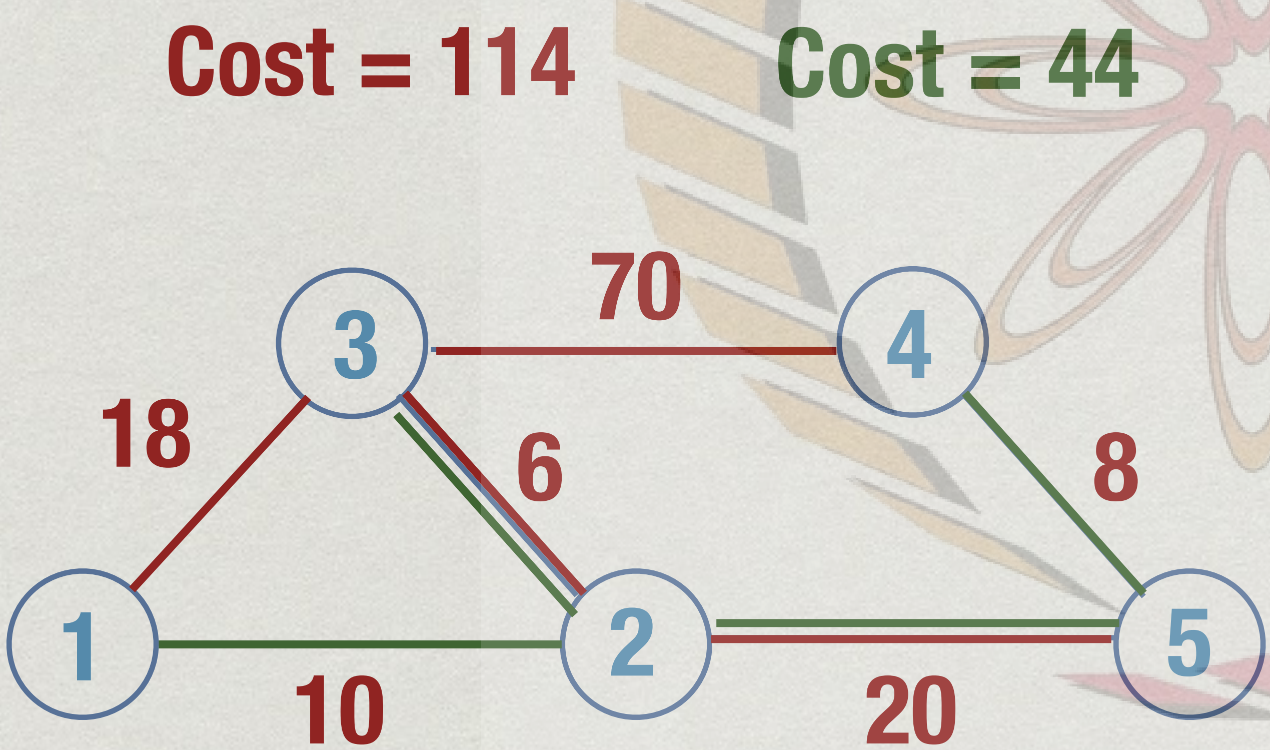

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Facts about trees

Definition: A tree is a connected acyclic graph

Fact 1: A tree on n vertices has exactly n-1 edges

- Start with a tree and delete edges
- Initially one single component
- Deleting an edge must split a component into two
- After n-1 edge deletions, n components, each an isolated vertex

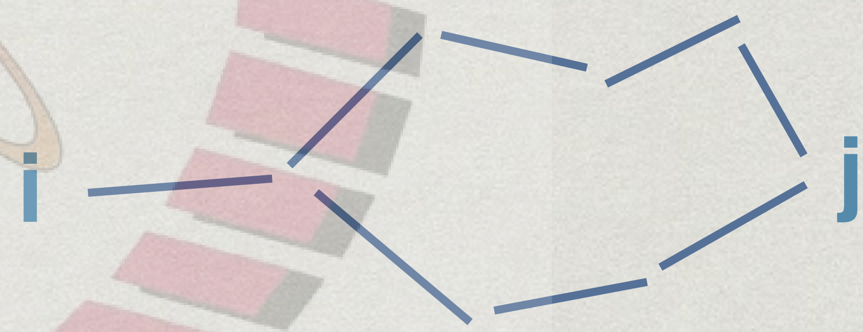
Facts about trees

Fact 2: Adding an edge to a tree must create a cycle

- Suppose we add an edge (i,j)
- Tree is connected, so there is already a path p from i to j
- New edge (i,j) plus path p creates a cycle

Facts about trees

Fact 3: In a tree, every pair of nodes is connected by a unique path



- If there are two paths from i to j, there must be a cycle

Facts about trees

Any two of the following facts about a graph G implies the third

- G is connected
- G is acyclic
- G has n-1 edges

Building a minimum cost spanning trees

Two natural strategies

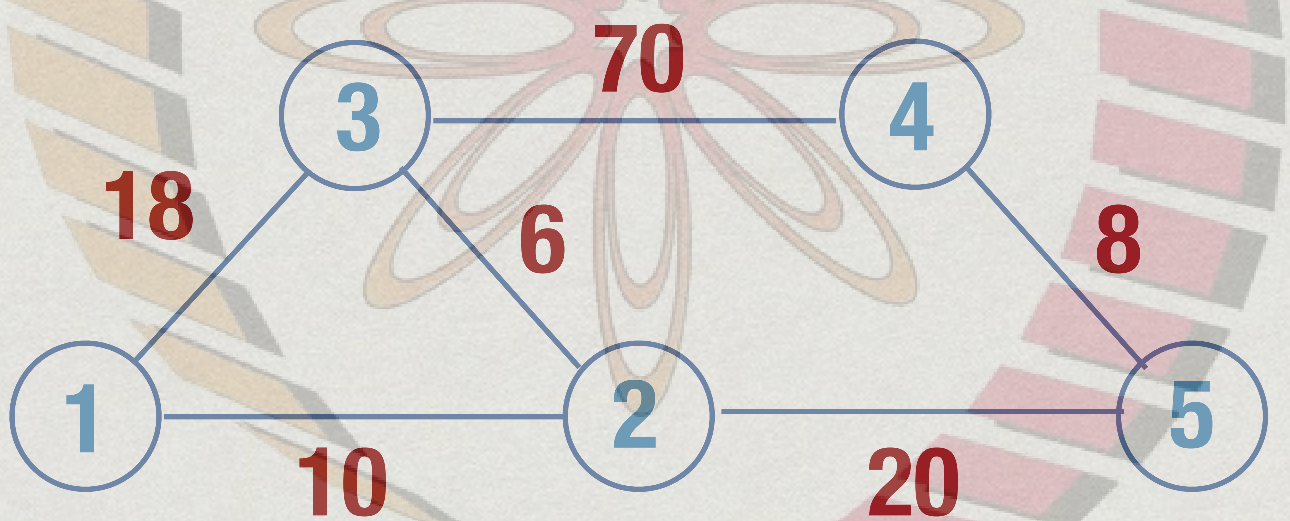
- Start with smallest edge and grow it into a tree

  Prim's Algorithm

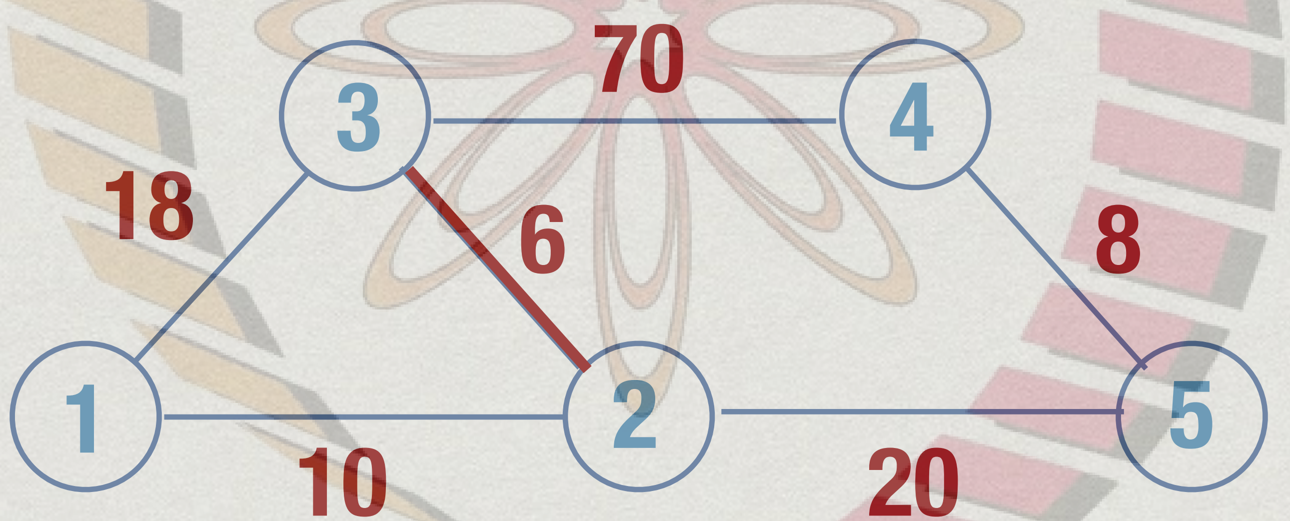
- Scan edges in ascending order of cost and connect components to form a tree

  Kruskal's Algorithm

Prim's algorithm



Prim's algorithm



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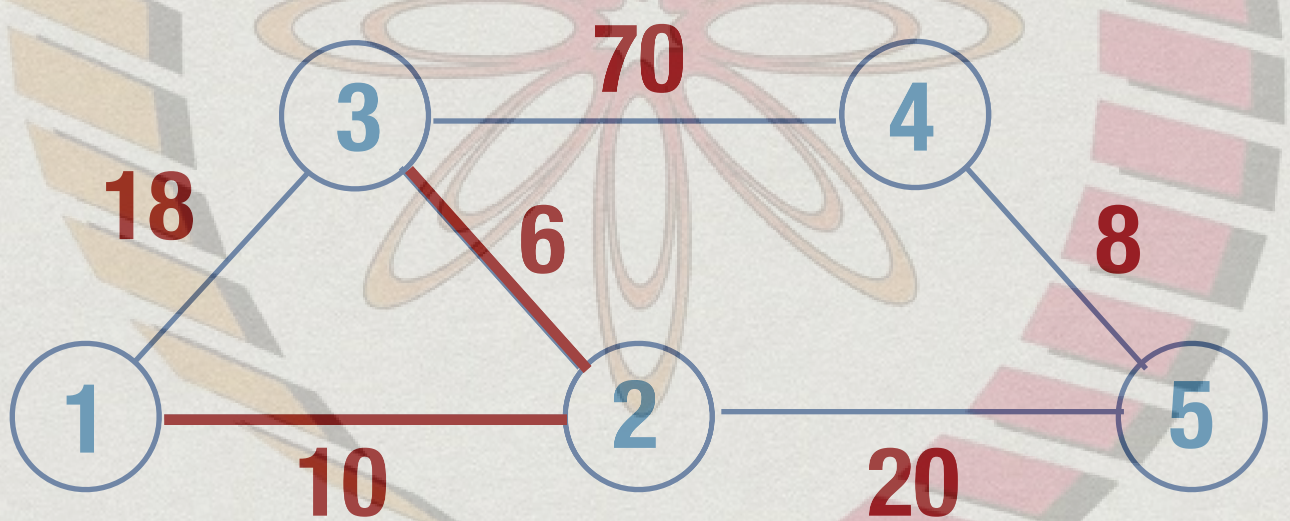

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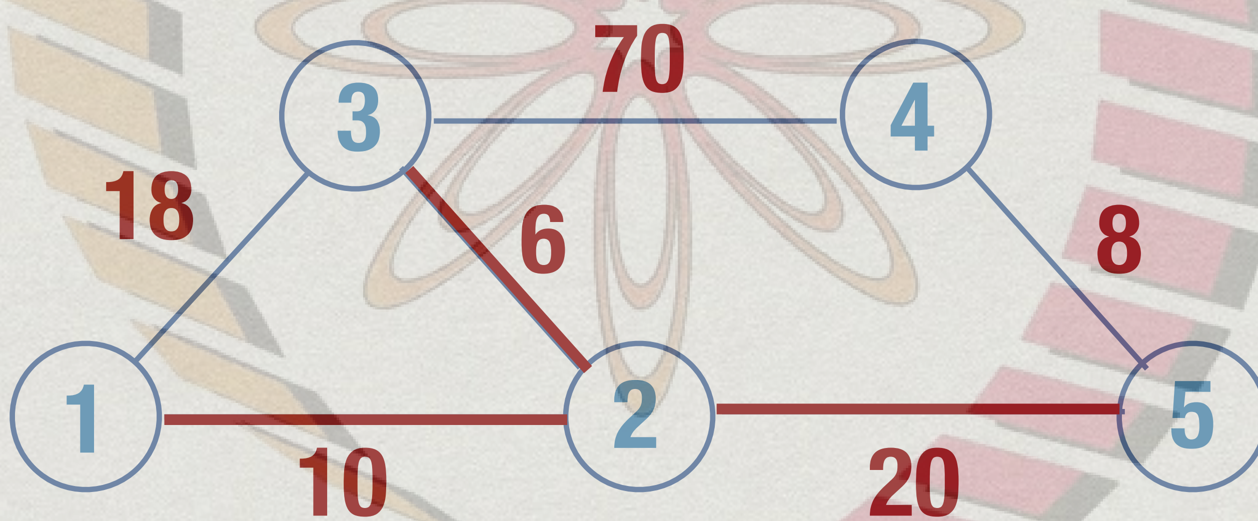

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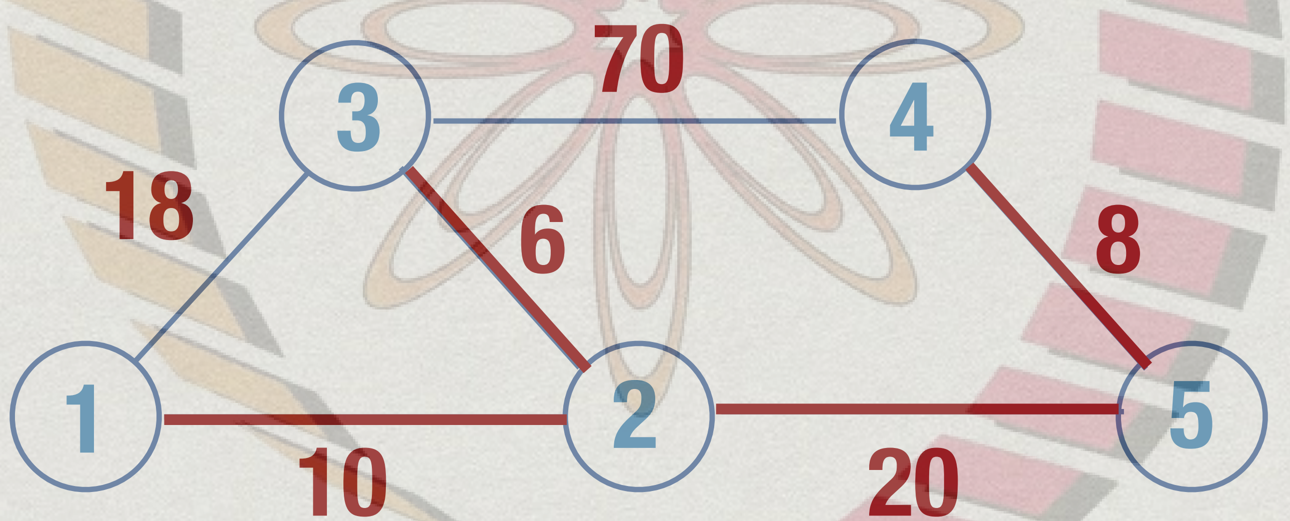
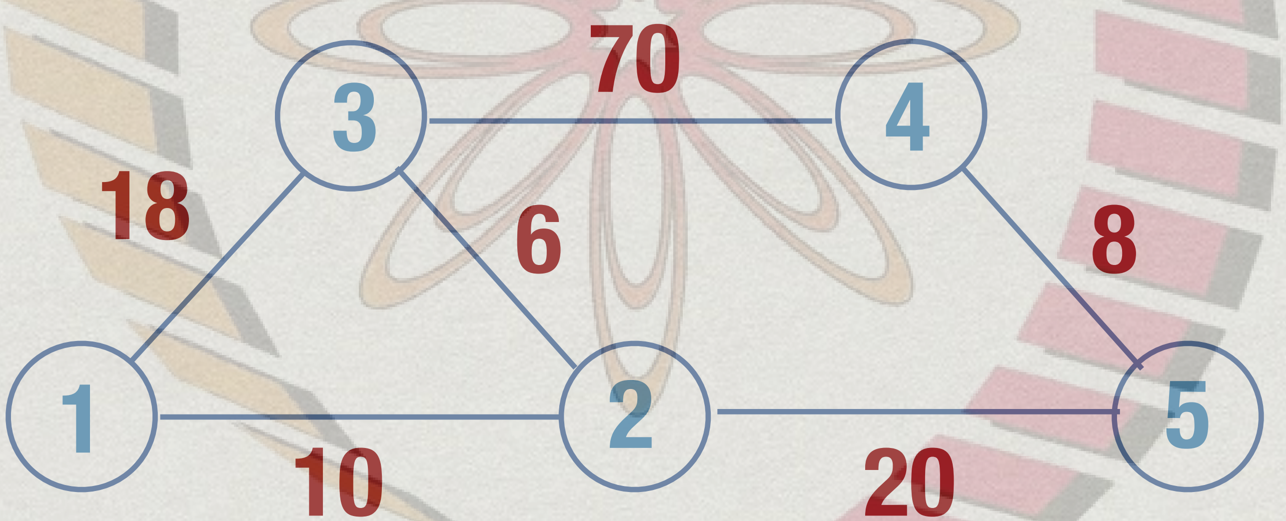


Kruskal's algorithm



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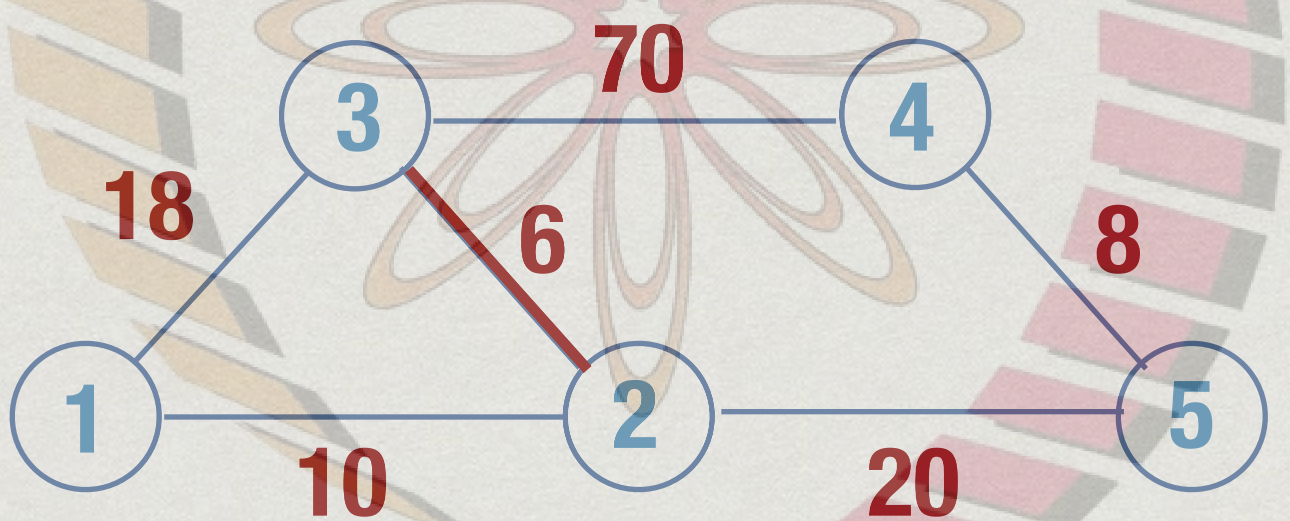

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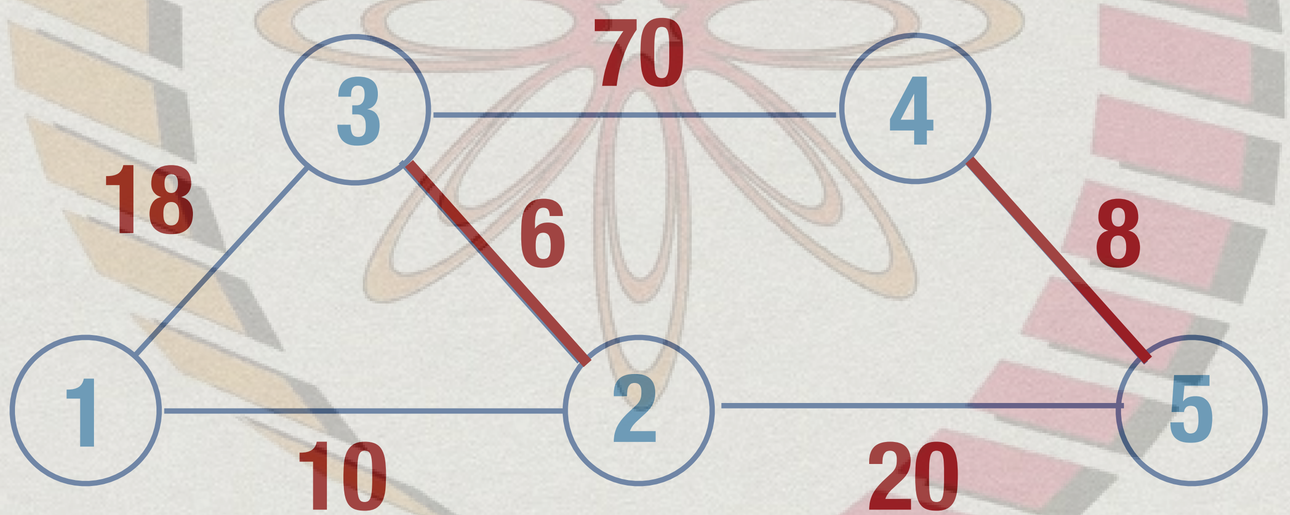

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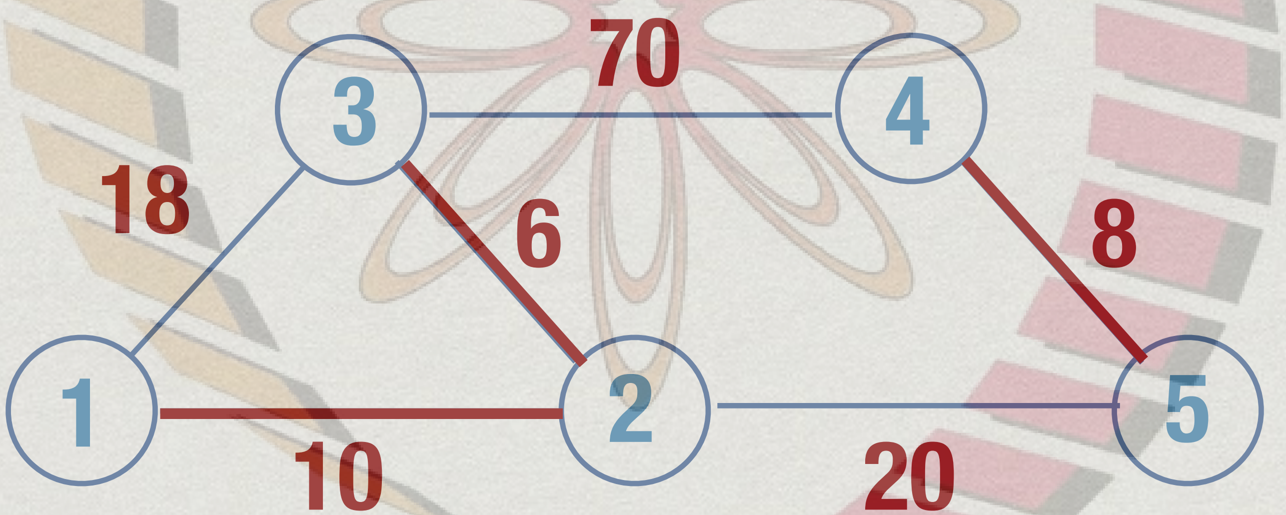

Kruskal's algorithm