Sam Schreiner

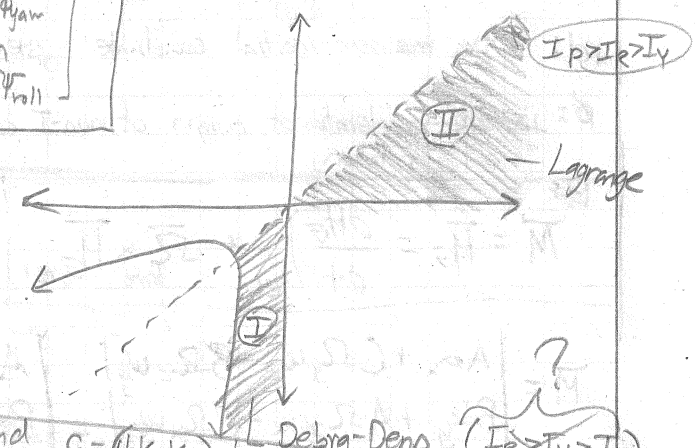
Gravity Gradient Stabalization:

$$B\ddot{\theta} + 3n^2(A-C)\theta = 0$$
$$A\ddot{\psi} + (B-A-C)n\dot{\phi} + 4n^2(B-C)\psi = 0$$
$$C\ddot{\phi} + (A+C-B)n\dot{\psi} + n^2(B-A)\phi = 0$$

Frame:
$$\bar{\Omega} = \begin{bmatrix} n\phi_{yaw} \\ n \\ n\psi_{roll} \end{bmatrix}$$

$$K_Y = \frac{I_P - I_R}{I_Y} \quad \Big\| \quad K_R = \frac{I_P - I_Y}{I_R}$$

Stability: $I_R > I_Y$ $(K_R > K_Y)$ $\quad (1+3K_R+K_RK_Y) > 4\sqrt{K_RK_Y}$ $\quad K_RK_Y > 0$



* For nadir-pointing satellite, region II is good.

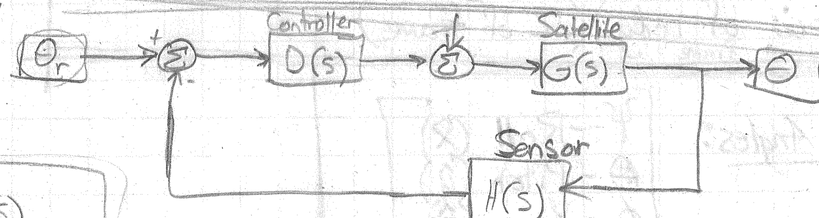
$$\omega_{pitch} = n\sqrt{3\frac{(I_R-I_Y)}{I_P}}$$

$b = (1+3K_R+K_RK_Y)$ and $c = (4K_RK_Y)$

$$\omega_{yaw/roll} = n\sqrt{\frac{1}{2}\left(b \pm \sqrt{b^2-4c}\right)}$$

$\cancel{\omega = \frac{I}{2\pi}}$? $\quad \omega = \frac{2\pi}{T}$

Control Systems:



$$\frac{\theta}{\theta_r} = \frac{D(s)G(s)}{1 + H(s)D(s)G(s)}$$

Root Locus Plots the zeros of $\left[1 + \underbrace{H(s)D(s)G(s)}_{=K_s \cdot L(s)} = 0\right]$ as K changes.

PID Controllers:

$$D(s) = \left\{K_I\frac{1}{s} + K_P + K_D s\right\} = K_S\left\{\frac{(s-z_1)(s-z_2)}{s}\right\}$$

$K_D = K_S$; $K_I/K_D = z_1z_2$; $K_P/K_D = -(z_1+z_2)$

$\rightarrow L(s) = K_S\frac{(s-z_1)(s-z_2)}{s^3}$ (w/ $H(s)=1$) $+\ G(s) = \frac{1}{s^2}$

Laplace Transforms

$f(t)$	$F(s)$
1	$1/s$
$\sin(at)$	$a/(s^2+a^2)$
$\cos(at)$	$s/(s^2+a^2)$
Unit Step @ $t=0$.	$1/s$
$t^n e^{-\alpha t}$	$\frac{n!}{(s+\alpha)^{n+1}}$
$e^{-\alpha t}$	$\frac{1}{s+\alpha}$

duh.

Basics

$$\bar{a}_{I} = \bar{a}_0 + \bar{a}_{rel} + 2\bar{\omega}_0 \times \bar{V}_{rel} + \bar{\alpha}_0 \times \bar{r}_{rel} + \bar{\omega}_0 \times (\bar{\omega}_0 \times \bar{r}_{rel})$$

rel: in the ~~ne~~ non-inertial coordinate system

0: ~~in the~~ acceleration of origin of non-I coord in inertial coord rotation

$$I = \int_M \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} dm$$

$$\bar{M} = \dot{\bar{H}}_G = \left.\frac{d\bar{H}_G}{dt}\right)_{rel} + \bar{\Omega}_{frame} \times \bar{H}_{G,rel} \qquad \bar{H}_G = [I]\bar{\omega}$$

$$T_{rot} = \frac{1}{2}\bar{\omega} \cdot \bar{H}_G$$

$$\bar{M} = \begin{bmatrix} A\alpha_x + C\Omega_y\omega_z - B\Omega_z\omega_y \\ B\alpha_y + A\Omega_z\omega_x - C\Omega_x\omega_z \\ C\alpha_z + B\Omega_x\omega_y - A\Omega_y\omega_x \end{bmatrix} = \begin{bmatrix} A\alpha_x + (C-B)\omega_y\omega_z \\ B\alpha_y + (A-C)\omega_x\omega_z \\ C\alpha_z + (B-A)\omega_x\omega_y \end{bmatrix}$$

$(\alpha = \dot{\omega}_{in\ frame}$ of body$)$ $(\Omega$ of frame$)$

$$\left.\begin{bmatrix} mk_x^2 = A \\ mk_y^2 = B \\ mk_z^2 = C \end{bmatrix}\right\} \text{Radius of Gyration}$$

Euler Angles:

$$\begin{bmatrix} \psi - \text{Roll } (x) \\ \theta - \text{Pitch } (y) \\ \phi - \text{Yaw } (z) \end{bmatrix}$$

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c\psi & s\psi \\ 0 & -s\psi & c\psi \end{bmatrix} \quad R_2 = \begin{bmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{bmatrix} \quad R_3 = \begin{bmatrix} c\phi & s\phi & 0 \\ -s\phi & c\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Direction-Cosine Matrix:

$$[b] = [Q_{Lb}][L] \qquad \phi = \tan^{-1}\left(\frac{Q_{12}}{Q_{11}}\right) \qquad \theta = -\sin^{-1}(Q_{13}) \qquad \psi = \tan^{-1}\left(\frac{Q_{23}}{Q_{33}}\right)$$

For 3-2-1 Euler angle sequence:

$$[Q_{Lb}] = [R_1(\psi)][R_2(\theta)][R_3(\phi)] = \begin{bmatrix} c\phi c\theta & s\phi c\theta & -s\theta \\ c\phi s\theta s\psi - s\phi c\psi & s\phi s\theta s\psi & c\theta s\psi \\ c\phi s\theta c\psi + s\phi s\psi & s\phi s\theta c\psi - c\phi s\psi & c\theta c\psi \end{bmatrix}$$

3-2-1

For 3-2-1:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sin\psi}{\cos\theta} & \frac{\cos\psi}{\cos\theta} \\ 0 & \cos\psi & -\sin\psi \\ 1 & \sin\psi\tan\theta & \cos\psi\tan\theta \end{bmatrix} \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$3\text{-}1\text{-}3 = \begin{bmatrix} -s\phi c\theta s\psi + c\phi c\psi & c\phi c\theta s\psi + s\phi c\psi & s\theta s\psi \\ -s\phi c\theta c\psi - c\phi s\psi & c\phi c\theta c\psi - s\phi s\psi & s\theta c\psi \\ \sin\phi s\theta & -c\phi s\theta & c\theta \end{bmatrix}$$

$$\bar{q} = \begin{bmatrix} \sin\theta/2 \cdot u_1 \\ \sin\theta/2\ u_2 \\ \sin\theta/2\ u_3 \\ \cos\theta/2 \end{bmatrix} \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix}$$

$$[Q_{Lb}] = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

$$\frac{d\bar{q}}{dt} = \frac{1}{2}\begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}_{rel} \cdot \bar{q}$$

LLH Coord: $\begin{bmatrix} x: \text{along velocity} \\ y: \perp \text{ to orbital plane} \\ z: \text{nadir} \end{bmatrix}$

$$q_1 = \frac{Q_{23} - Q_{32}}{4q_4} \qquad q_2 = \frac{Q_{31} - Q_{13}}{4q_4} \qquad q_3 = \frac{Q_{12} - Q_{21}}{4q_4} \qquad q_4 = \frac{1}{2}\sqrt{1 + Q_{11} + Q_{22} + Q_{33}}$$

Spacecraft Attitude Dynamics & Control

$$\boxed{\bar{a}_p = \bar{a}_o + \bar{a}_{p,rel} + 2\bar{\omega}\times\bar{v}_{p,rel} + \bar{\alpha}\times\bar{r}_{p/rel} + \bar{\omega}\times(\bar{\omega}\times\bar{r}_{p,rel})}$$

Transport Theorem:

$$\bar{M} = \dot{\bar{H}}_G = \left.\frac{\partial \bar{H}_G}{\partial t}\right)_{rel} + \bar{\Omega}_{frame}\times\bar{H}_{G_{rel}}$$

$$\bar{M}_G = \dot{\bar{H}}_G$$

$$\bar{H}_G = [I]\omega$$

$$\bar{I} = \int_M \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -yz \\ -xz & -yz & x^2+y^2 \end{bmatrix} dm$$

M

$$\bar{M} = \begin{bmatrix} A\alpha_x + (C-B)\omega_z\omega_y & \hat{i} \\ B\alpha_y + (A-C)\omega_x\omega_z & \hat{j} \\ C\alpha_z + (B-A)\omega_x\omega_y & \hat{k} \end{bmatrix}$$

LLH Coordinate Frame:

x: along velocity
y: $\perp$ to orbital plane
z: Nadir

$$\bar{M} = \begin{bmatrix} A\alpha_x + C\Omega_y\omega_z - B\Omega_z\omega_y \\ B\alpha_y + A\Omega_z\omega_x - C\Omega_x\omega_z \\ C\alpha_z + B\Omega_y\omega_x - A\Omega_x\omega_y \end{bmatrix}$$

$$T_{rot} = \frac{1}{2}\bar{\omega}\cdot\bar{H}_G$$

$mk_x^2 = A$
$mk_y^2 = B$
$mk_z^2 = C$

"Radius of gyration"

Euler Angles:

ψ: Roll ($\hat{x}$)
θ: Pitch ($\hat{y}$)
ϕ: Yaw ($\hat{z}$)

$$R_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\psi & \sin\psi \\ 0 & -\sin\psi & \cos\psi \end{bmatrix}$$

$$R_2 = \begin{bmatrix} \cos & 0 & -\sin\theta \\ 0 & 1 & 0 \\ \sin\theta & 0 & \cos\theta \end{bmatrix}$$

vectors in b, vectors in L

$$[b] = [Q_{Lb}][L]$$

$$[\dot{Q}_{Lb}] = \begin{bmatrix} 0 & \omega_3 & -\omega_2 \\ -\omega_3 & 0 & \omega_1 \\ \omega_2 & -\omega_1 & 0 \end{bmatrix}[Q_{Lb}]$$

$$R_3(\phi) = \begin{bmatrix} \cos\phi & \sin\phi & 0 \\ -\sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\phi = \tan^{-1}\left(\frac{Q_{12}}{Q_{11}}\right) \qquad \theta = -\sin^{-1}(Q_{13}) \qquad \psi = \tan^{-1}\left(\frac{Q_{23}}{Q_{33}}\right) \quad 3\text{-}2\text{-}1$$

For 3-2-1

$$[Q_{Lb}] = [R_1(\psi)][R_2(\theta)][R_3(\phi)] = \begin{bmatrix} c\phi c\theta & s\phi c\theta & -s\theta \\ c\phi s\theta s\psi - s\phi c\psi & s\phi s\theta s\psi + c\phi c\psi & c\theta s\psi \\ c\phi s\theta c\psi + s\phi s\psi & s\phi s\theta c\psi - c\phi s\psi & c\theta c\psi \end{bmatrix} \quad 3\text{-}2\text{-}1$$

For 3-2-1

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 0 & \frac{\sin\psi}{\cos\theta} & \frac{\cos\psi}{\cos\theta} \\ 0 & \cos\psi & -\sin\psi \\ 1 & \sin\psi\tan\theta & \cos\psi\tan\theta \end{bmatrix}\begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}$$

$$\begin{bmatrix} -s\phi c\theta s\psi + c\phi c\psi & c\phi c\theta s\psi + s\phi c\psi & s\theta s\psi \\ -s\phi c\theta c\psi - c\phi s\psi & c\phi c\theta c\psi - s\phi s\psi & s\theta c\psi \\ s\phi s\theta & -c\phi s\theta & c\theta \end{bmatrix} \quad 3\text{-}1\text{-}3$$

$$\bar{q} = \begin{bmatrix} \sin\frac{\theta}{2} \cdot u_1 \\ \sin\frac{\theta}{2} \cdot u_2 \\ \sin\frac{\theta}{2} \cdot u_3 \\ \cos\frac{\theta}{2} \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \end{bmatrix} \qquad [Q_b] = \begin{bmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2(q_1q_2 + q_3q_4) & 2(q_1q_3 - q_2q_4) \\ 2(q_1q_2 - q_3q_4) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2(q_2q_3 + q_1q_4) \\ 2(q_1q_3 + q_2q_4) & 2(q_2q_3 - q_1q_4) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{bmatrix}$$

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$$q_1 = \frac{Q_{23} - Q_{32}}{4q_4} \quad \Big| \quad q_2 = \frac{Q_{31} - Q_{13}}{4q_4} \quad \Big| \quad q_3 = \frac{Q_{12} - Q_{21}}{4q_4} \quad \Big| \quad q_4 = \frac{1}{2}\sqrt{1 + Q_{11} + Q_{22} + Q_{33}}$$

$$\frac{d\bar{q}}{dt} = \frac{1}{2}[\Omega]\bar{q} \implies [\Omega] = \begin{bmatrix} 0 & \omega_z & -\omega_y & \omega_x \\ -\omega_z & 0 & \omega_x & \omega_y \\ \omega_y & -\omega_x & 0 & \omega_z \\ -\omega_x & -\omega_y & -\omega_z & 0 \end{bmatrix}_{rel}$$

ω: in body frame

Star Trackers: We have $\bar{r}_1^L, \bar{r}_2^L, \bar{r}_1^b, \bar{r}_2^b$

$$[Q_{bL}] = [L][b]^{-1}$$

$[L] = [\bar{r}_1^L \ \ \bar{r}_2^L \ \ \bar{r}_1 \times \bar{r}_2^L]$; $[b] = [\text{same}]$; $[L] = [Q_{bL}][b]$

Dual Spinners: $\frac{d}{dt}(\omega_\perp^2) = \frac{2}{A_\perp}\left[\bar{T}_p + \frac{C_r}{C_r - A_\perp}\bar{T}_r\right] \implies C_r > A_\perp$: unconditionally stable

Momentum Wheels:

$$M_1 = A\dot{\omega}_1 + (C-B)\omega_2\omega_3 + [(I+2J)\dot{\omega}_1] + I[\dot{\omega}_{rel}^{(1)} - \omega_{rel}^{(2)}\omega_3 + \omega_{rel}^{(3)}\omega_2]$$

$$M_2 = B\dot{\omega}_2 + (A-C)\omega_1\omega_3 + [(I+2J)\dot{\omega}_2] + I[\dot{\omega}_{rel}^{(2)} - \omega_{rel}^{(3)}\omega_1 + \omega_{rel}^{(1)}\omega_3]$$

$$M_3 = C\dot{\omega}_3 + (B-A)\omega_1\omega_2 + [(I+2J)\dot{\omega}_3] + I[\dot{\omega}_{rel}^{(3)} - \omega_{rel}^{(1)}\omega_2 + \omega_{rel}^{(2)}\omega_1]$$

$\dot{\omega}_{rel}^{(i)} =$ relative spin of ith wheel in its own local frame

Yo-Yo Despin:

$$\left[K = 1 + \frac{C}{2mR^2}\right]$$

mass of single yo-yo

tangential release: $\ell_f = R\sqrt{K\frac{(\omega_0 - \omega_f)}{(\omega_0 + \omega_f)}}$ $\quad \Delta t = \frac{\Delta\ell}{R\omega_0}$

radial release: $\ell_f = R(\sqrt{K} - 1)$ $\quad (\omega_f = 0)$