# Selected aspects of rotation in MESA

Jakub Ostrowski

Astronomical Institute of the University of Wrocław
December 2, 2013

# Basic effects of rotation

- Centrifugal forces reduce the effective gravity at any point not on the axis of rotation
- Centrifugal force is not, in general, parallel to the gravity so equipotential surfaces are no longer spheres
- Radiative flux varies with the local effective gravity (the von Zeipel effect) => the radiative flux is not constant on an equipotential surface
- Rotation may inhibit certain of convective motions and, thus, directly affect the criterion for convective instability

# Structure equations for rotating stars

$$\frac{dr_P}{dM_P} = \frac{1}{4\pi r_P^2 \overline{\varrho}}$$

$$\frac{dP}{dM_P} = -\frac{G M_P}{4\pi r_P^4} f_P$$

$$\frac{dL_P}{dM_P} = \varepsilon_{\rm nucl} - \varepsilon_\nu + \varepsilon_{\rm grav}$$

$$\frac{d\ln T}{dM_P} = -\frac{G M_P}{4\pi r_P^4} f_P \min\left[\nabla_{\rm ad}, \nabla_{\rm rad}\frac{f_T}{f_P}\right]$$

$$f_P = \frac{4\pi r_P^4}{G M_P S_P}\frac{1}{< g_{\rm eff}^{-1} >}$$

$$f_T = \left(\frac{4\pi r_P^2}{S_P}\right)^2 \frac{1}{< g_{\rm eff} >< g_{\rm eff}^{-1} >}$$

Maeder 2009

# Mixing diffusion equation

$$\left(\frac{\partial X_n}{\partial t}\right)_m = \left(\frac{\partial}{\partial m}\right)_t \left[(4\pi r^2 \rho)^2 D \left(\frac{\partial X_n}{\partial m}\right)_t\right] + \left(\frac{dX_n}{dt}\right)_{\mathrm{nuc}}$$

$$\left.\left(\frac{\partial X_n}{\partial m}\right)_t\right|_{m=0} = 0 = \left.\left(\frac{\partial X_n}{\partial m}\right)_t\right|_{m=M(t)}$$

Heger et al. 2000

# Angular momentum transport

- Diffusion equation:

$$\left(\frac{\partial \omega}{\partial t}\right)_m = \frac{1}{i}\left(\frac{\partial}{\partial m}\right)_t \left[(4\pi r^2 \rho)^2 i \nu \left(\frac{\partial \omega}{\partial m}\right)_t\right] - \frac{2\omega}{r}\left(\frac{\partial r}{\partial t}\right)_m \left(\frac{1}{2}\frac{d\ln i}{d\ln r}\right)$$

$$\nu = D_{\mathrm{conv}} + D_{\mathrm{sem}} + D_{\mathrm{DSI}} + D_{\mathrm{SHI}} + D_{\mathrm{SSI}} + D_{\mathrm{ES}} + D_{\mathrm{GSF}}$$

# Rotationally induced mixing

- Dynamical and secular instabilities

- Endal & Sofia 1978
- Pinsonneault et al. 1989
- Heger et al. 2000 (equations on the next slides)
- Maeder & Meynet 2000
- Maeder 2009

# Dynamical shear instability

- Stability criterion:

$$R_i \equiv \frac{\rho\delta}{P}\left(\nabla_{\rm ad} - \nabla + \frac{\varphi}{\delta}\nabla_\mu\right)\left(g\,\frac{d\ln r}{d\omega}\right)^2 > R_{i,c} \approx \frac{1}{4}$$

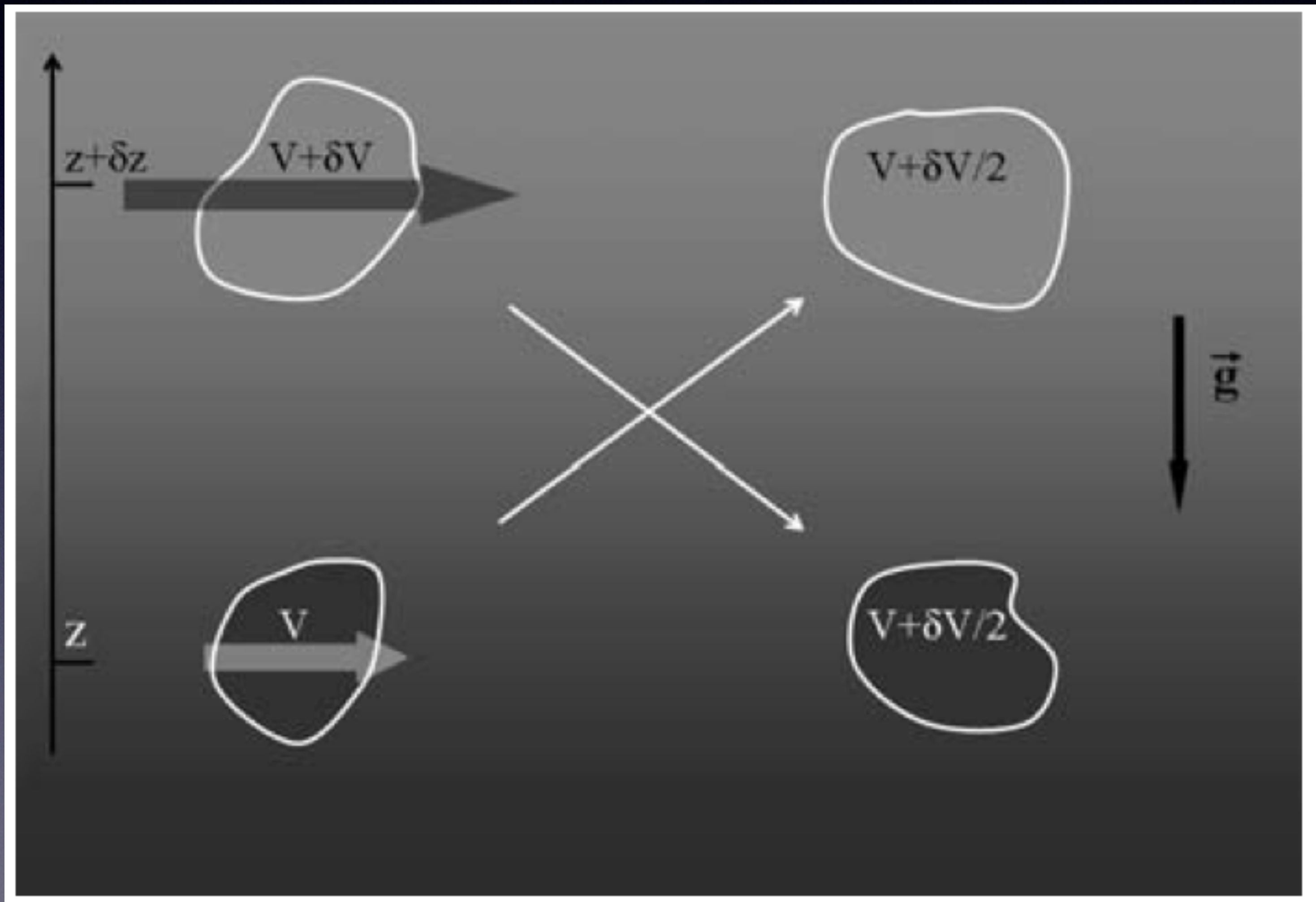
- Diffusion coefficient:

$$D_{\rm DSI} = \left[\min\left\{d_{\rm inst}, H_P\right\}\left(1 - \max\left\{\frac{R_i}{R_{i,c}}, 0\right\}\right)\right]^2 \Big/ \tau_{\rm dyn}$$

$$\tau_{\rm dyn} \equiv \sqrt{r^3/(Gm)}$$

- Density gradients have a stabilising effect

# Dynamical shear instability




Maeder 2009

# Solberg-Høiland instability

- Stability criterion:

$$R_{\mathrm{SH}} \equiv \frac{g\delta}{H_P}\left(\nabla_{\mathrm{ad}} - \nabla + \frac{\varphi}{\delta}\nabla_\mu\right) + \frac{1}{r^3}\frac{d}{dr}(r^2\omega)^2 \geq 0$$

- Diffusion coefficient:

$$D_{\mathrm{SHI}} = \left[\min\{d_{\mathrm{inst}}, H_P\}\left(\frac{rR_{\mathrm{SH}}}{g}\right)\right]^2 \Big/ \tau_{\mathrm{dyn}}$$

# Secular shear instability

- Stability criteria:

$$R_{\mathrm{is},1} \equiv \frac{\mathscr{P}_r R_{e,c}}{8} \frac{\rho\delta}{P} (\nabla_{\mathrm{ad}} - \nabla) \left( g \frac{d\ln r}{d\omega} \right)^2 > R_{i,c}$$

$$R_{\mathrm{is},2} \equiv \frac{\rho\varphi\nabla_\mu}{P} \left( g \frac{d\ln r}{d\omega} \right)^2 > R_{i,c}$$

- where

$$\mathscr{P}_r = \frac{c_V(\mu_p + \mu_r)}{\chi}$$

$$\mu_p \approx 0.406 \frac{\sqrt{m_i (k_{\mathrm{B}} T)^5}}{(Z_i e)^4 \ln \Lambda}, \quad \mu_r = \frac{4aT^4}{15c\kappa\rho}$$

$$\Lambda = \frac{2}{3e^3} \sqrt{\frac{m_i (k_{\mathrm{B}} T)^3}{\pi\rho Z_i^5}}$$

$$v_{\mathrm{SSI}} = \sqrt{\frac{\nu}{R_{e,c}} \frac{d\omega}{d\ln r}}$$

$$\nu = \frac{\mu_p + \mu_r}{\rho}$$

$$H_{v,\mathrm{SSI}} \equiv \left| \frac{dr}{d\ln v_{\mathrm{SSI}}} \right|$$

- Diffusion coefficient:

$$D_{\mathrm{SSI}} = \min\{v_{\mathrm{SSI}}, c_s\} \min\{H_{v,\mathrm{SSI}}, H_P\} \times \left(1 - \frac{\max\{R_{\mathrm{is},1}, R_{\mathrm{is},2}\}}{R_{i,c}}\right)^2 .$$

# Eddington - Sweet circulation

- Circulation velocity:

$$v_e \equiv \frac{\nabla_{\rm ad}}{\delta(\nabla_{\rm ad} - \nabla)} \frac{\omega^2 r^3 l}{(Gm)^2} \left[ \frac{2(\varepsilon_n + \varepsilon_\nu) r^2}{l} - \frac{2r^2}{m} - \frac{3}{4\pi\rho r} \right].$$

- "Stabilising" circulation velocity:

$$v_\mu \equiv \frac{H_P}{\tau^*_{\rm KH}} \frac{\varphi \nabla_\mu}{\delta(\nabla - \nabla_{\rm ad})}$$

- Diffusion coefficient:

$$D_{\rm ES} \equiv \min\{d_{\rm inst}, H_{v,\rm ES}\} v_{\rm ES}$$

$$v_{\rm ES} \equiv \max\{|v_e| - |v_\mu|, 0\}$$

$$H_{v,\rm ES} \equiv \left| \frac{dr}{d\ln v_{\rm ES}} \right|$$

# Goldreich-Schubert-Fricke instability

- Stability conditions:

$$\frac{\partial j}{\partial r} \geq 0 \quad \text{and} \quad \frac{\partial \omega}{\partial z} = 0$$

- Velocities:

$$v_g = \frac{2H_T r}{H_j^2}\left(1 + 2\frac{d\ln r}{d\ln \omega}\right)^{-1} v_e = \frac{2H_T}{H_j}\frac{d\ln \omega}{d\ln r} v_e$$

$$v_{\rm GSF} \equiv \max\{|v_g| - |v_\mu|, 0\}$$

- Diffusion coefficient:

$$D_{\rm GSF} \equiv \min\{d_{\rm inst}, H_{v,{\rm GSF}}\} v_{\rm GSF}$$

$$H_{v,{\rm GSF}} \equiv \left| \frac{dr}{d\ln v_{\rm GSF}} \right|$$

# Enhanced mass loss due to rotation

$$\dot{M}(\omega) \equiv \dot{M}(\omega = 0) \times \left(\frac{1}{1-\Omega}\right)^{\xi},$$

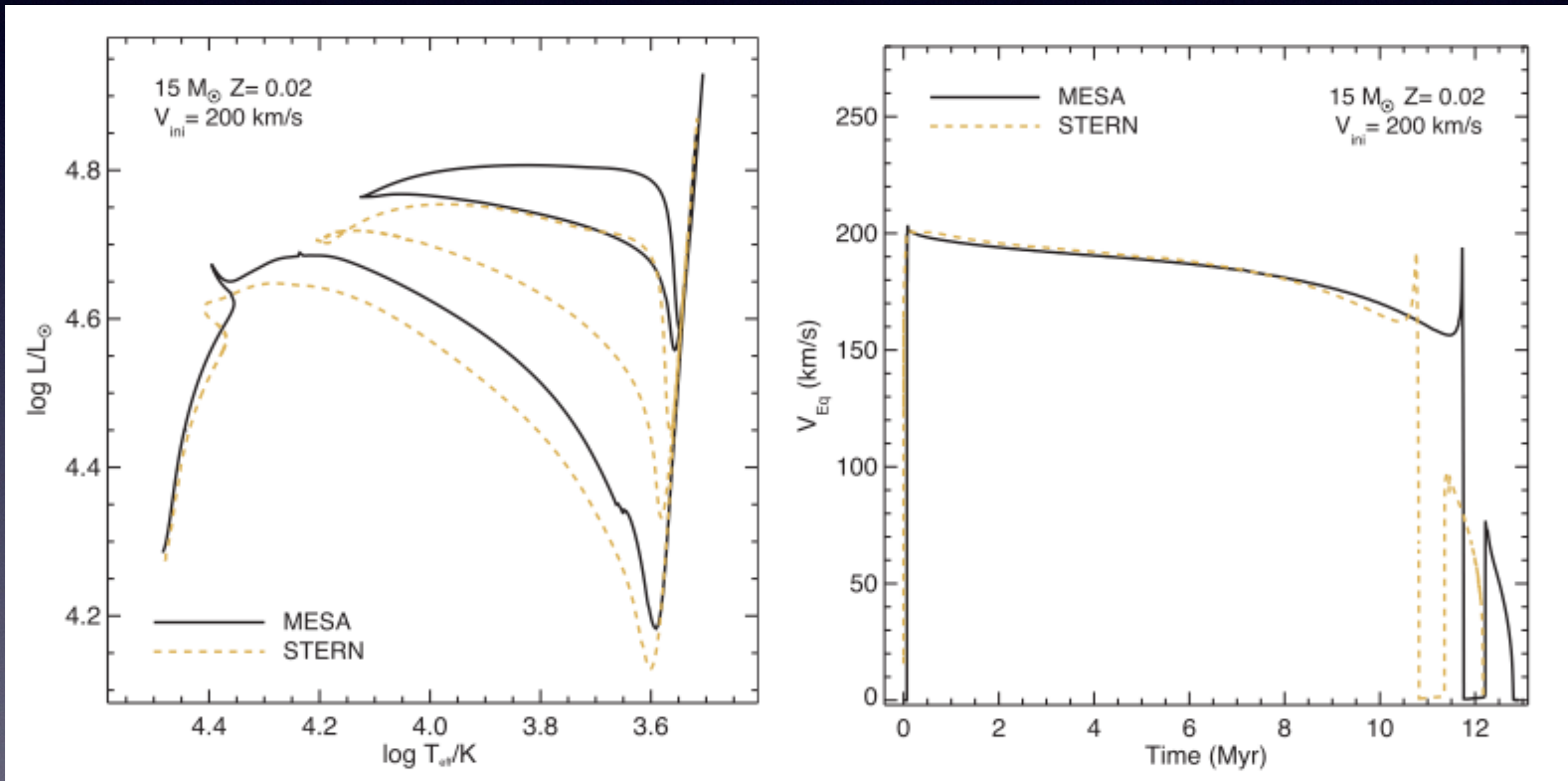
$$\xi \approx 0.43$$

$$\Omega \equiv \frac{v}{v_{\rm crit}}$$

$$v_{\rm crit}^2 \equiv \frac{Gm}{r}(1-\Gamma)$$

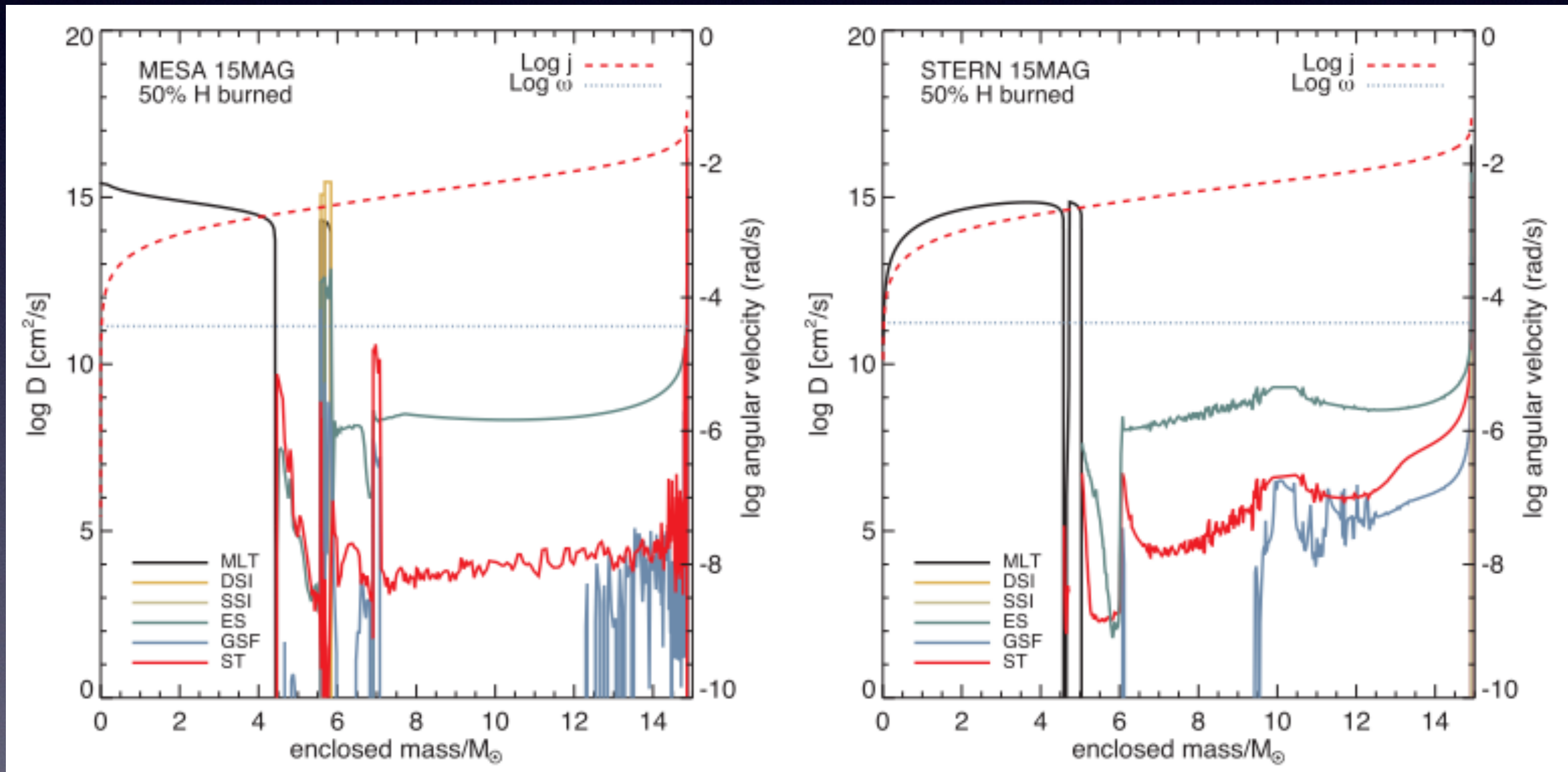
$$\Gamma \equiv \frac{\kappa L}{4\pi c G m}$$

# 15 Ms - MESA vs. STERN



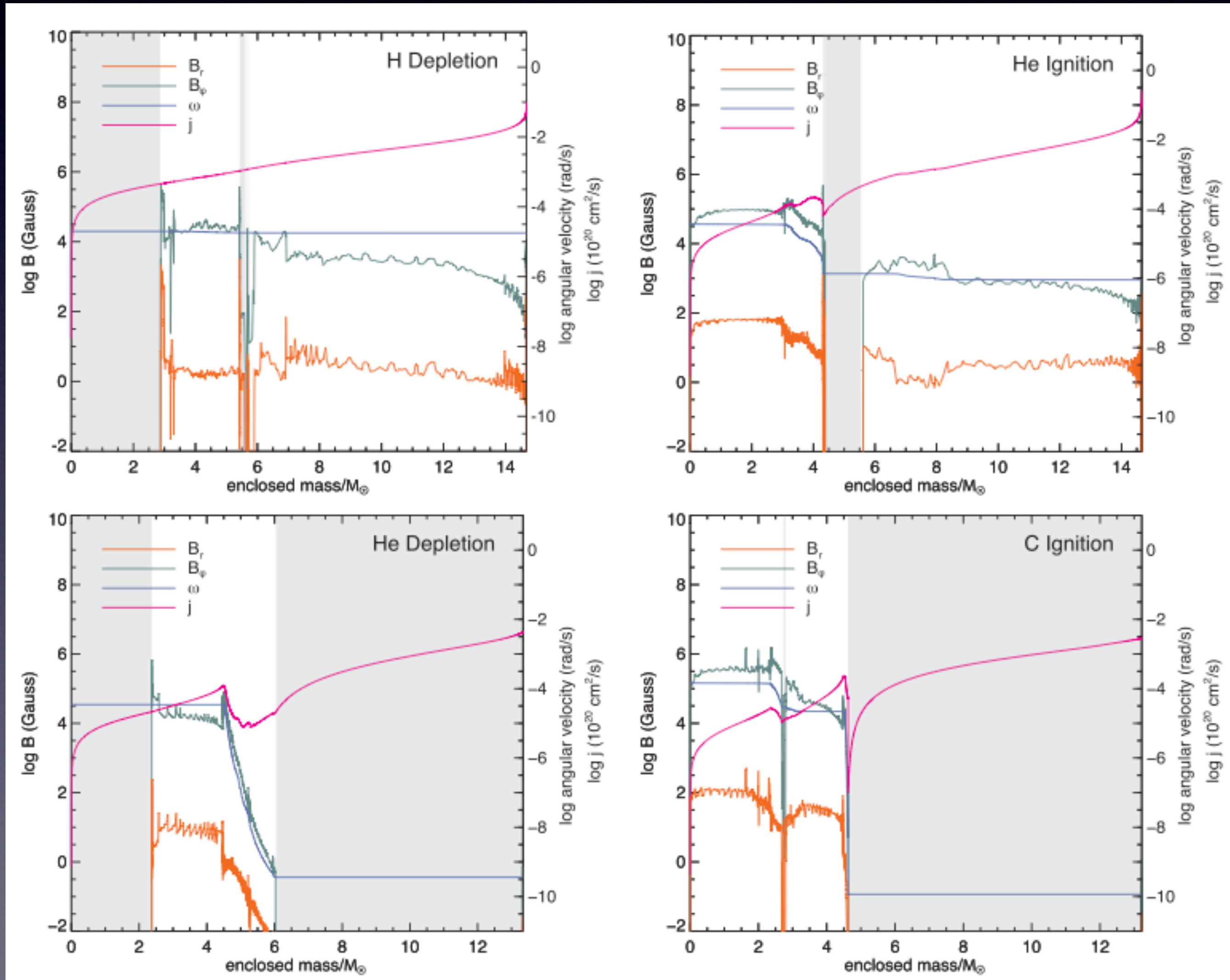

Paxton et al. 2013

# 15 M$_S$ - diffusion coefficients



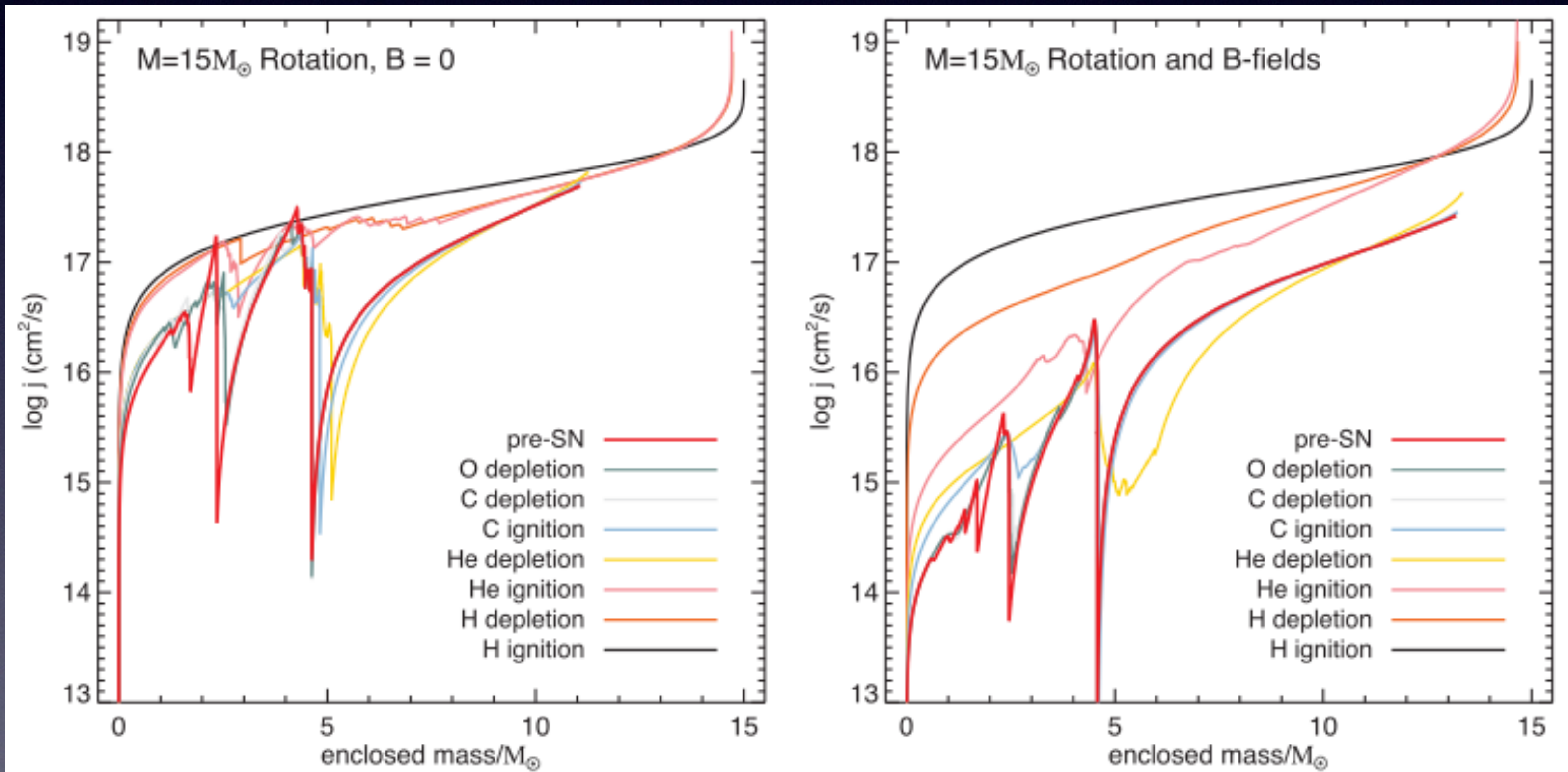

Paxton et al. 2013

# 15 Ms - *j* and Ω



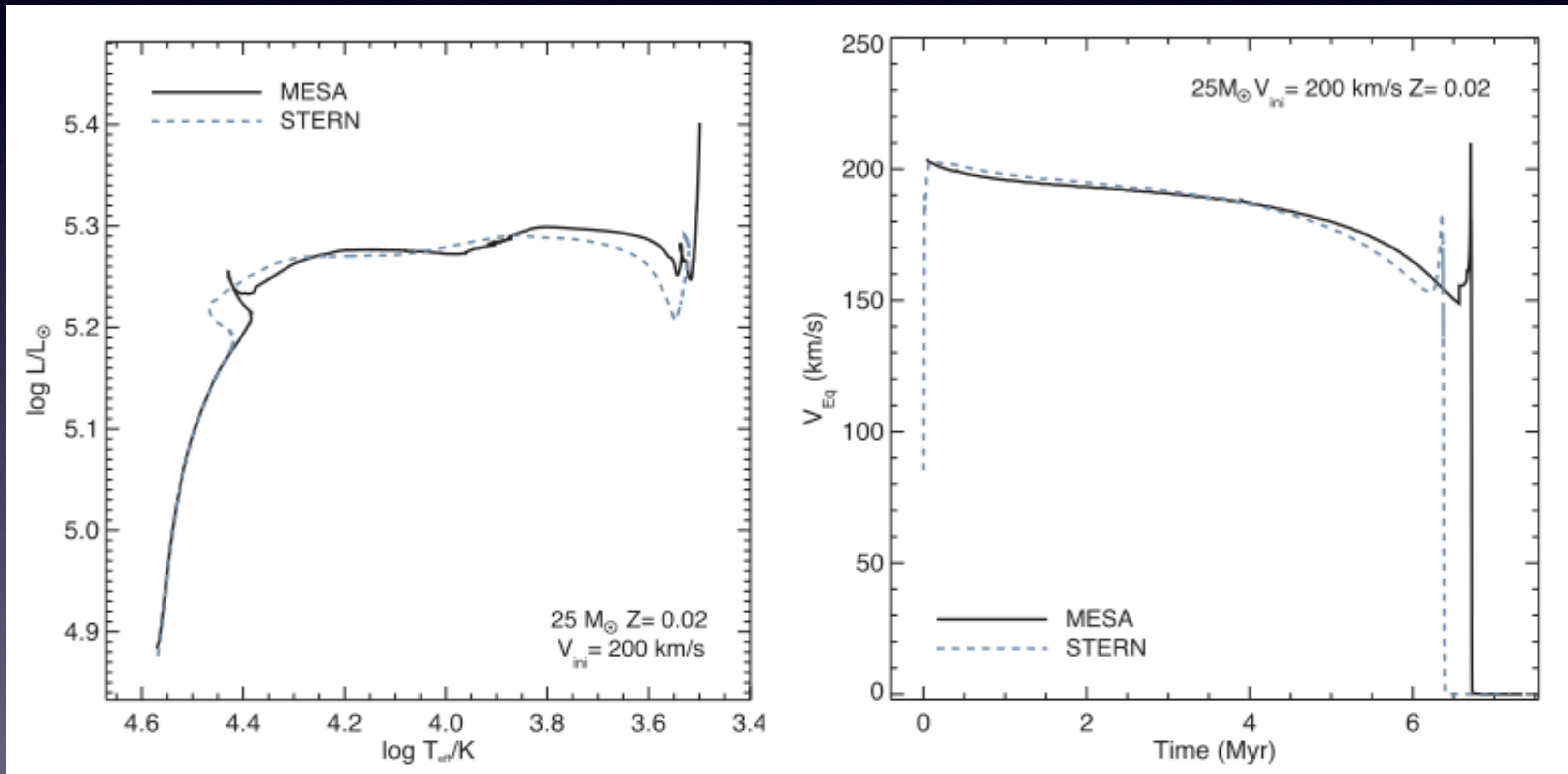

Paxton et al. 2013

# 15 Ms - *j*



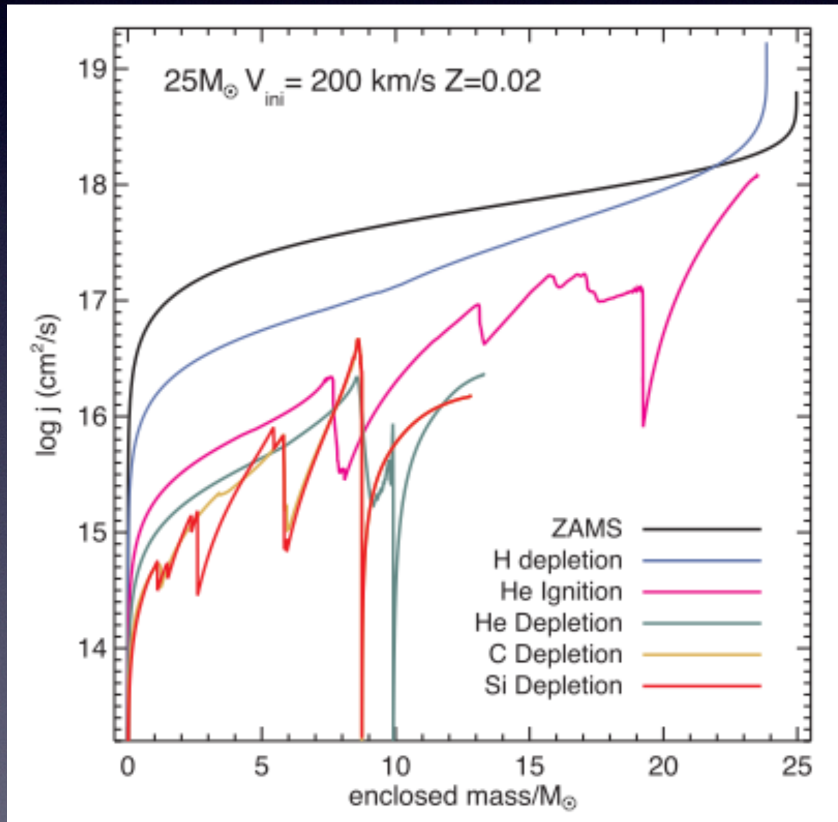

Paxton et al. 2013

# 25 Ms - MESA vs. STERN



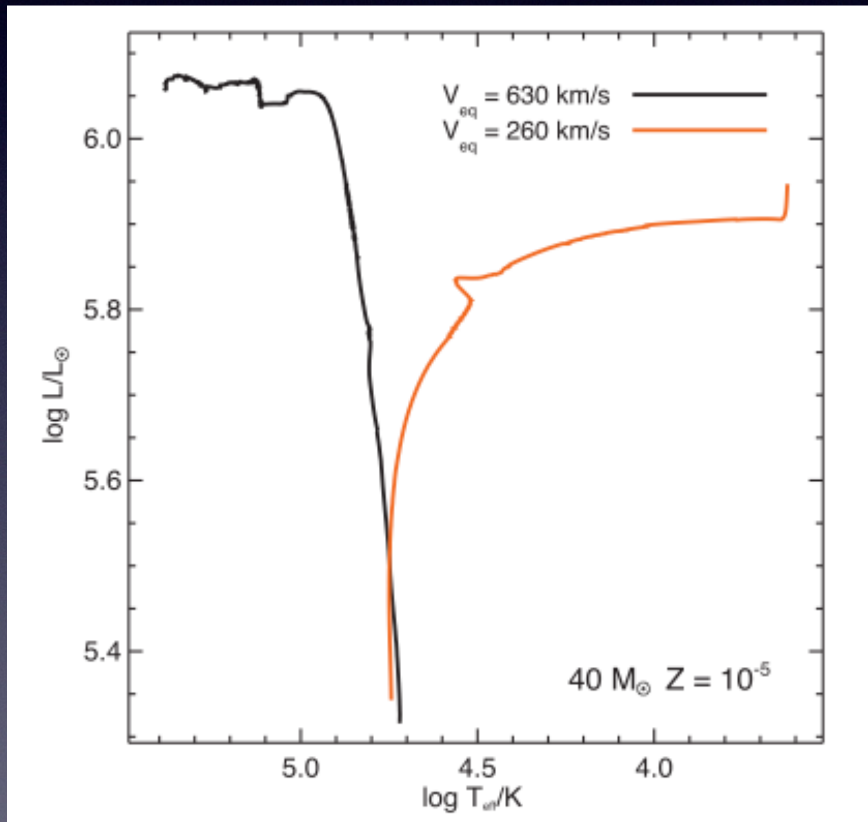

Paxton et al. 2013

# 25 Ms - *j*



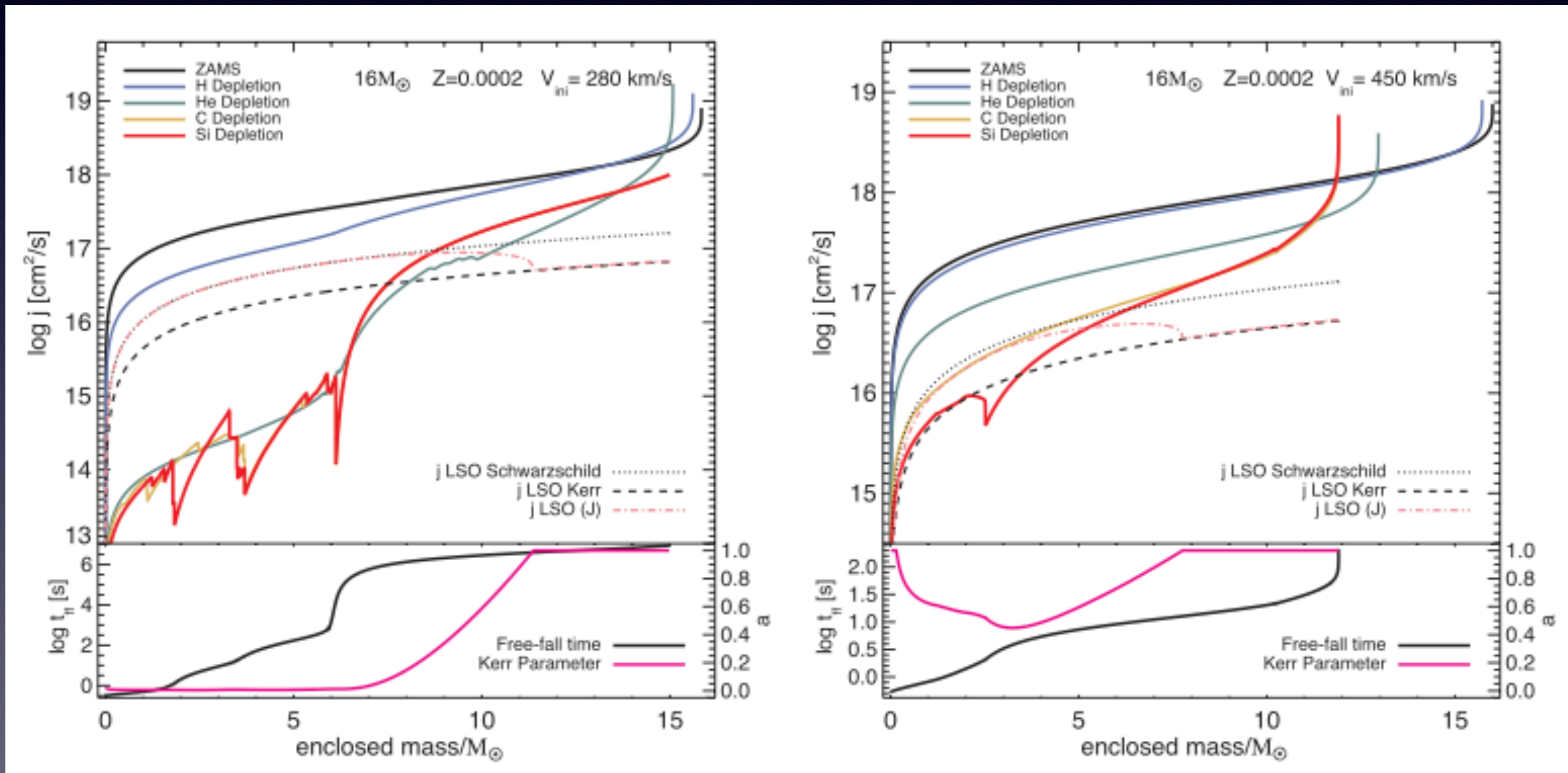

Paxton et al. 2013

# 40 Ms - H-R



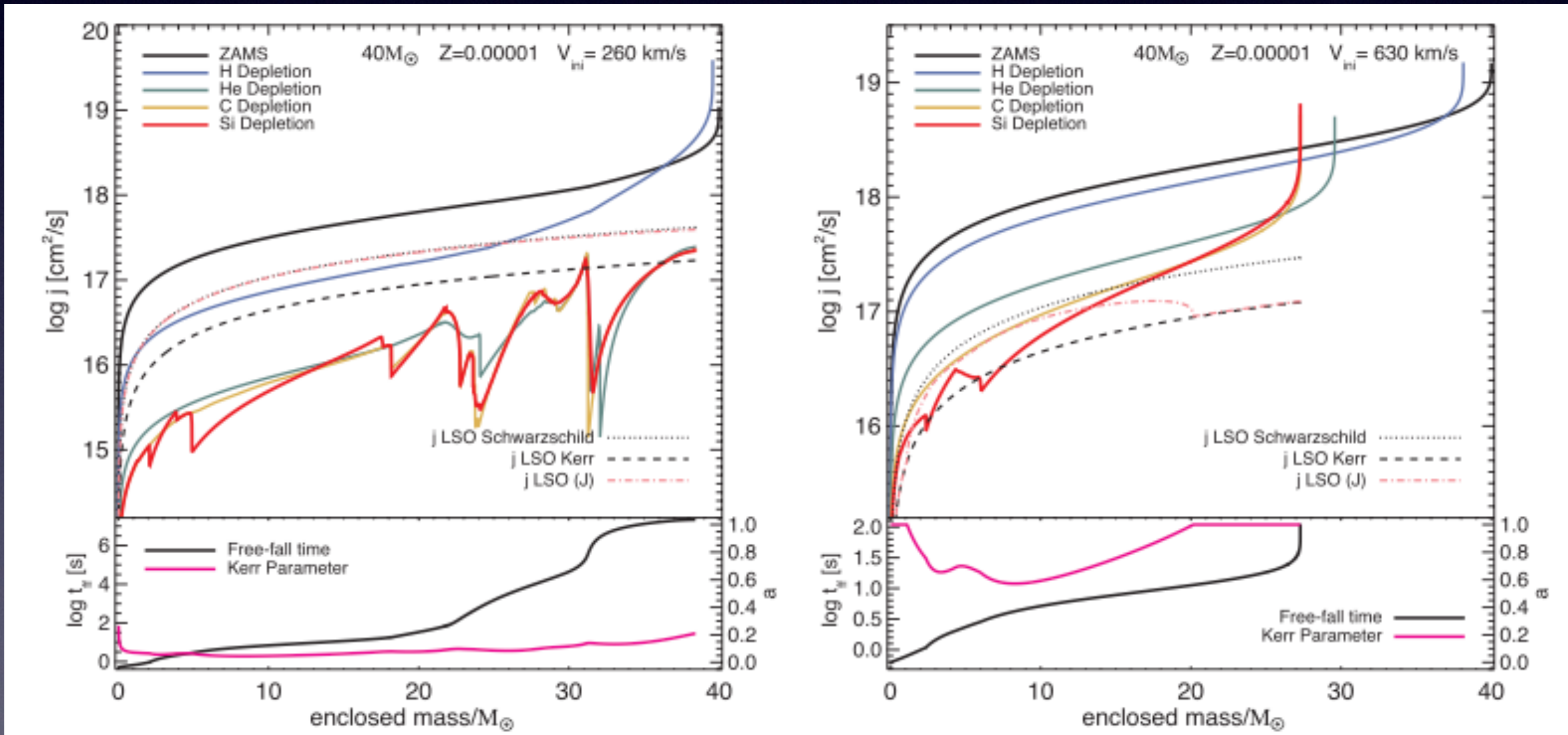

Paxton et al. 2013

# 16 Ms - *j*




Paxton et al. 2013

# 40 Ms - *j*




Paxton et al. 2013

# Conclusions

- Simple modifications to structure equations with presence of rotation
- Significant influence of rotationally induced instabilities on later evolutionary stages of massive stars
- Significant influence of rotation on a fate of massive stars

# Bibliography


- Endal & Sofia 1976, ApJ, 210, 184
- Endal & Sofia 1978, ApJ, 220, 279
- Heger, Langer, Woosley 2000, ApJ, 528, 368
- Maeder 2009; *Physics, Formation and Evolution of Rotating Stars*; Springer
- Maeder & Meynet 2000, ARA&A, 38, 143
- Paxton et al. 2013, ApJSS, 208, 4
- Pinsonneault et al. 1989, ApJ, 338, 424